Fundamental of Accelerators

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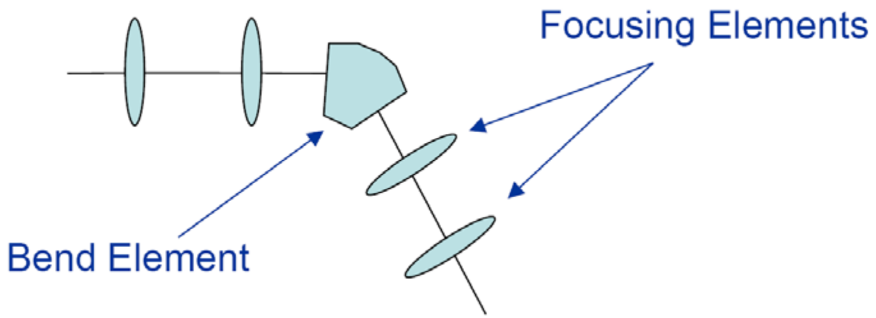
Jefferson Lab
Thomas Jefferson National Accelerator Facility

Definition of Beam Optics

- Beam Optics: the process of guiding a charged particle beam from A to B using magnets
- An array of magnets which accomplishes this is a transport system, or magnetic lattice



- Recall Lorentz Force on a particle

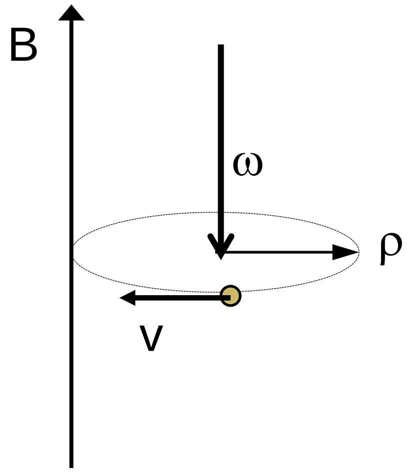
$$\frac{d(\gamma m\vec{\upsilon})}{dt} = q(\vec{E} + \vec{\upsilon} \times \vec{B})$$

  - Energy changes from electric fields
  - Direction changes (energy conservative) from magnetic fields

Force on a Particle in a Magnetic Field

- The simplest type of magnetic field is a constant field. A charged particle in a constant field executes a circular orbit, with radius ρ and frequency ω

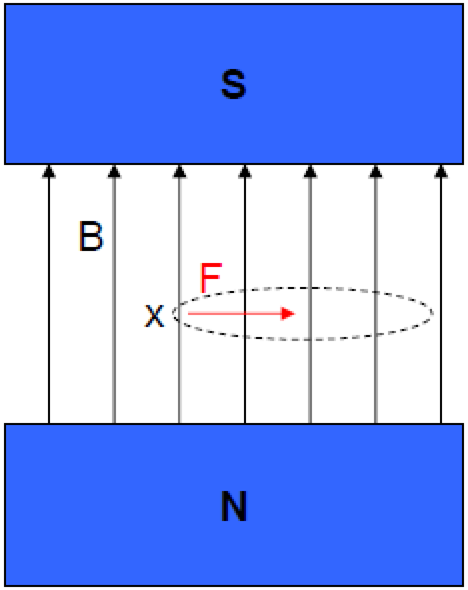
  - To find the direction of the force on the particle, use the right-hand-rule.



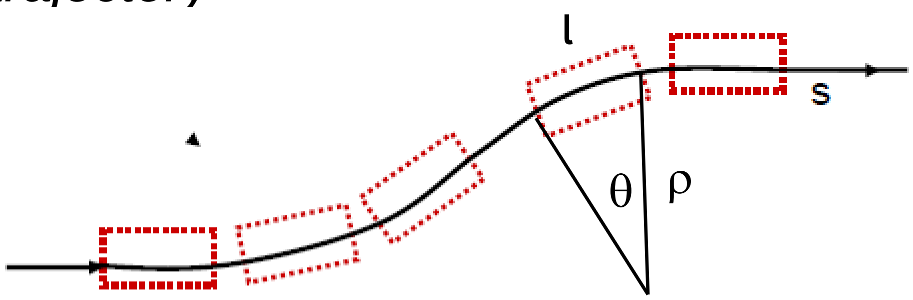
- *What would happen if the initial velocity had a component in the direction of the field ?*

Dipole Magnets

- A dipole magnet gives a constant field B
  - Field lines in a magnet run from the North to South
- Symbol convention:
  - ✕ : traveling into the page
  - ● : traveling out of the page
- In the field shown, for a positively charged particle traveling into the page, the force is to the right.



- In an accelerator lattice, dipoles are used to bend the beam trajectory. The set of dipoles in a lattice defines the *reference trajectory*

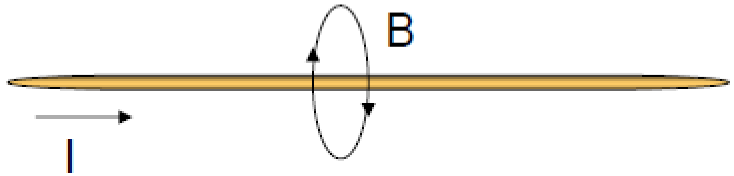


$$\frac{p[GeV/c]}{q[e]} \approx 0.3B[T]\rho[m]$$

$$\theta = \frac{l}{\rho}$$

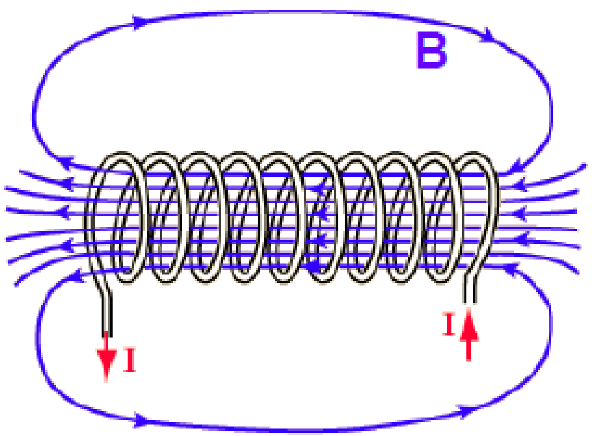
Generating B Field from a Current

- A current in a wire generates a magnetic field B which curls around the wire:



- Winding many turns on a coil generates a strong uniform magnetic field

- Field strength is given by one of Maxwell's equations: $$\frac{\nabla \times \vec{B}}{\mu} = \vec{J}$$

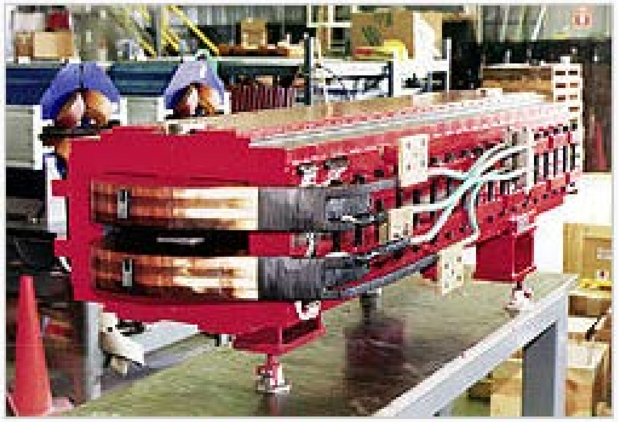


or Ampere circuital law

$$\oint_C \frac{\vec{B}}{\mu} \cdot d\vec{l} = \iint_s \vec{J} . \cdot d\vec{S} = I_{enc}$$

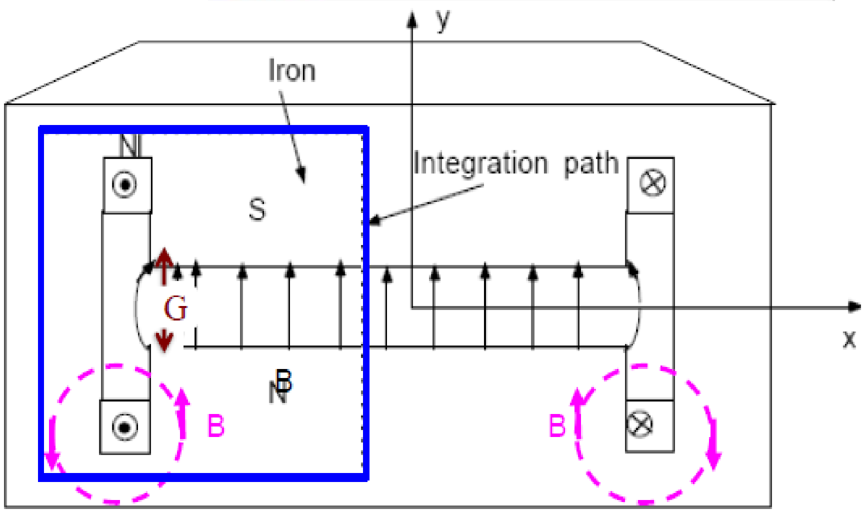
Dipole Current to Field

- In an accelerator, we use current-carrying wires and metal cores of high μ (magnetic permeability) to set up a strong dipole field
- N turns of current generate a field perpendicular to the pole tip surface
- Relationship between B in the gap and I in the wire:



Dipole magnet from the Advanced Photon Source

$$\oint_C \frac{\vec{B}}{\mu}\cdot d\vec{l} = \int_0^G \frac{B}{\mu_\circ}\cdot d.l + \underbrace{\int_{iron} \frac{B}{\mu_{iron}}.dl}_{\text{negligible, since } \mu_{iron} >> \mu_o} = I_{enc} = 2I_{coil}$$



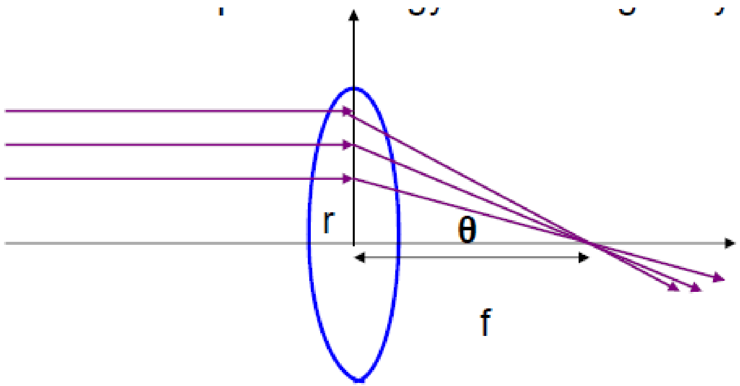
Pure dipole: NI turns/pole

$$I_{coil} = NI_{wire} = \frac{GB}{2\mu_\circ}$$

Focusing Particles w/ Magnets

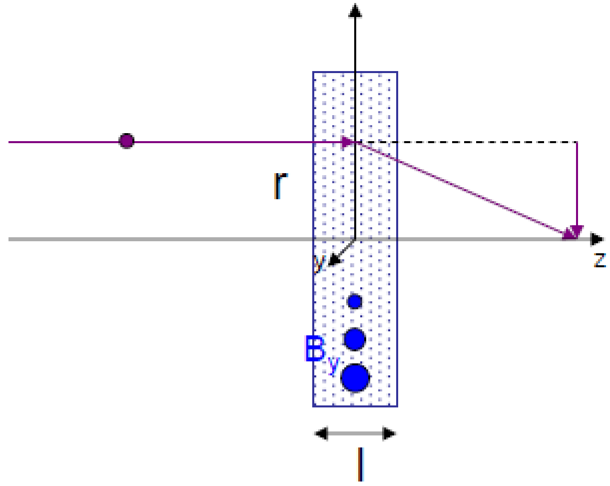
- Consider the optical analogy of focusing a ray light through a convex lens:
  - The focusing angle depends on the the distance from the center. The farther off axis, the stronger the focusing effect.

$$\theta = -\frac{r}{f}$$



- Now consider a magnetic lens

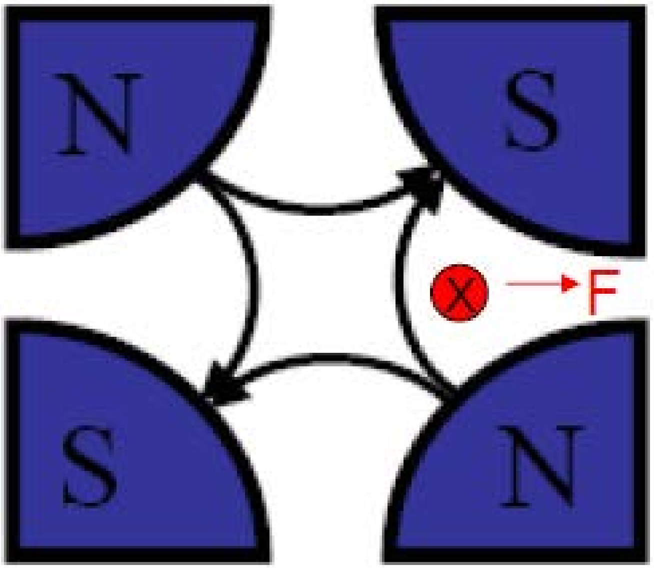
$$\theta = -\frac{l}{\rho_y} = \frac{l}{(p/e)/B_y} = \frac{B_y l}{p/e} = \frac{grl}{B\rho}$$



  - Here
    - Focusing strength $g = \frac{\partial B_y}{\partial r}$
    - Beam rigidity $\frac{p}{e} = (B\rho)_{dipole}$

Quadrupole Magnets

- A quadrupole magnet imparts a force proportional to distance from the center. This magnet has 4 poles.
- Consider a positive particle traveling into the the page in the quadrupole magnetic field.



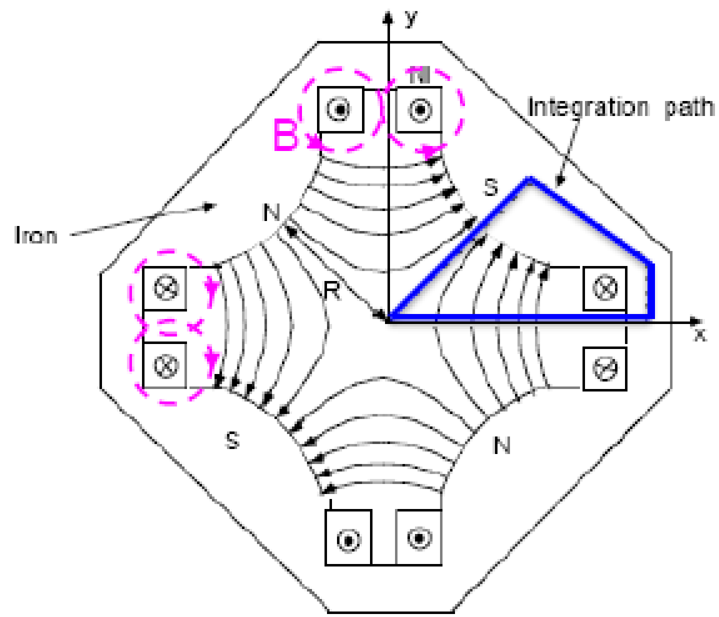
- This magnets is horizontally defocusing.
- What about in the vertical direction?
  - Focusing !
- A quadrupole which defocuses in one plane focuses in the other !

Quadrupole Current to Field

- As with a dipole, we can use current-carrying wires wrapped around metal cores to create a quadrupole magnet
- The field lines are denser near the edges of the of the magnet, meaning the field is stronger there
- The strength of B is function of the distance to the center of magnet
- Relationship between focusing strength g and current I in the wire:

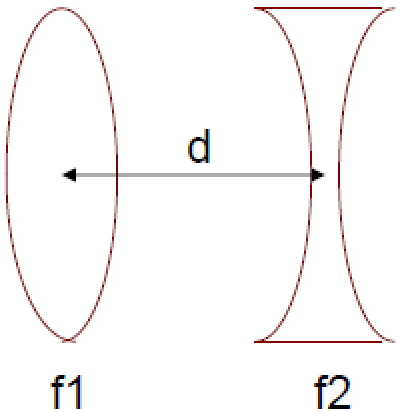
$$\oint_C \frac{\vec{B}}{\mu}\cdot d\vec{l} = \int_0^R \frac{B_r}{\mu_\circ}\cdot d.r + \underbrace{\int_{iron} \frac{B_{iron}}{\mu_{iron}}.dl_{iron}}_{\text{negligible, since } \mu_{iron} >> \mu_o} + \underbrace{\int_0^X \frac{B_y}{\mu_\circ}\cdot d.x}_{=0\text{, since } B_y \perp x} = I_{enc}$$

$$I_{enc} = \frac{gR^2}{2\mu_\circ}$$



Focusing Using Arrays of Quadrupoles

- Quadrupoles focus in one plane while defocusing in the other. So, how can this be used to provide net focusing in an accelerator?
- Consider again the optical analogy of two lenses, with focal length f_1 and f_2, separated by a distance d



The combined $f_{combined}$ is $$\frac{1}{f_{combined}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

if $f_1 = -f_2$, the net effect is focusing(positive)

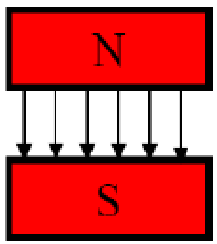
$$\frac{1}{f_{combined}} = \frac{d}{f_1 f_2}$$

- To focus particles in both planes in an accelerator, one need arrays of quadruple magnets !

Other Types of Magnets

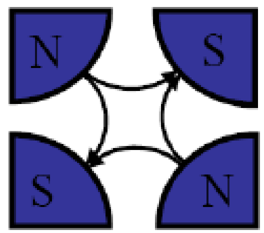
- Examples of magnets:

n=1: Dipole



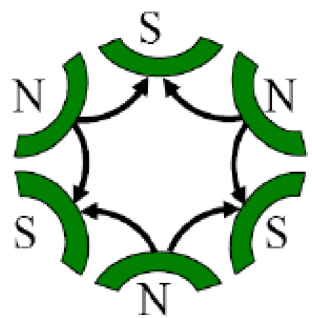
180° between poles

n=2: Quadrupole



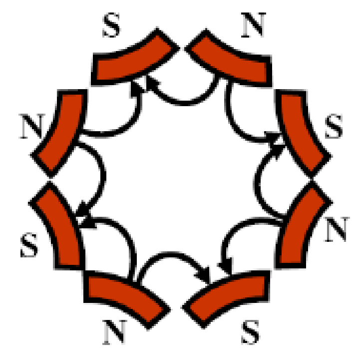
90° between poles

n=3: Sextupole



60° between poles

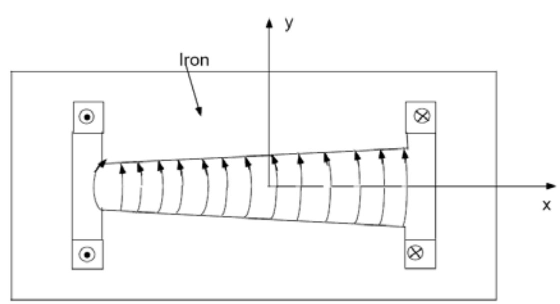
n=4: Octupole



45° between poles

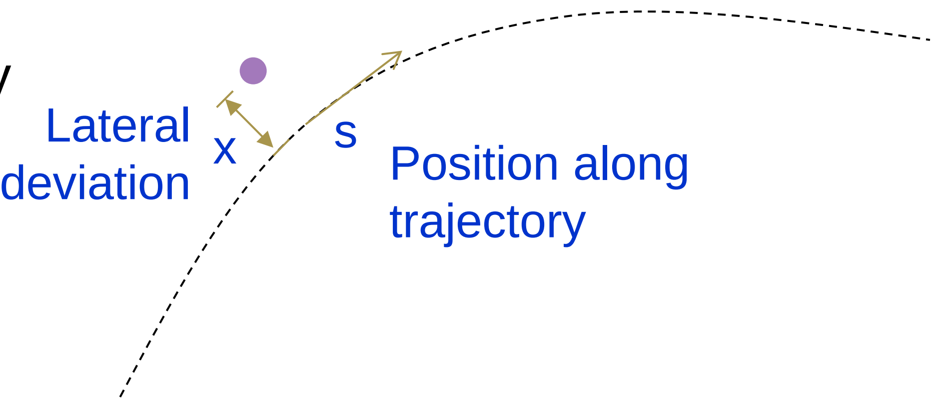
  - In general, poles are 360°/2n apart
  - The skew version of the magnet is obtained by rotating the upright magnet by 180°/2n

- Combined function magnet: bend and focus simultaneously

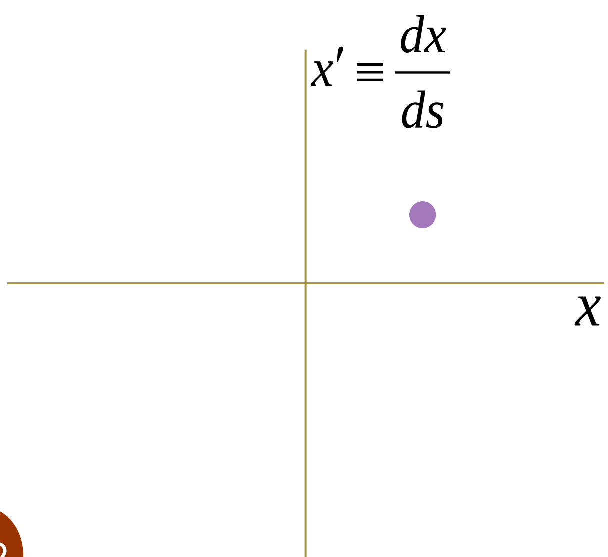


Trajectories and Phase Space

- In general, in an accelerator we assume the dipoles define the nominal particle trajectory, and we solve for lateral deviations from that trajectory



- At any point along the trajectory, each particle can be represented by its position in “phase space”

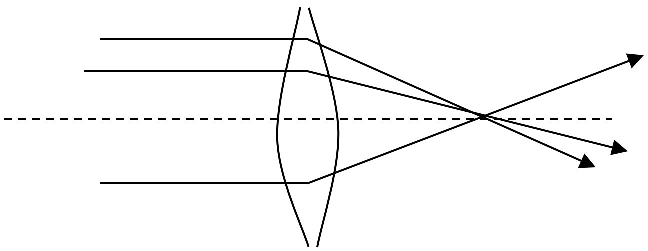


- We would like to solve for x(s)
- We assume:
  - Both transverse planes are independent
    - no coupling
  - All particles independent from each other
    - no space charge effects

Transfer Matrices

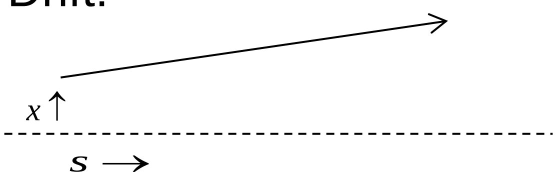
- The simplest magnetic lattice consists of quadrupoles and the spaces in between them (drift). We can express each of these as a linear operation in phase space.

Quadrupole:



$$x = x(0)$$
$$x' = x'(0) - \frac{1}{f}x(0)$$
$$\Rightarrow \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

Drift:



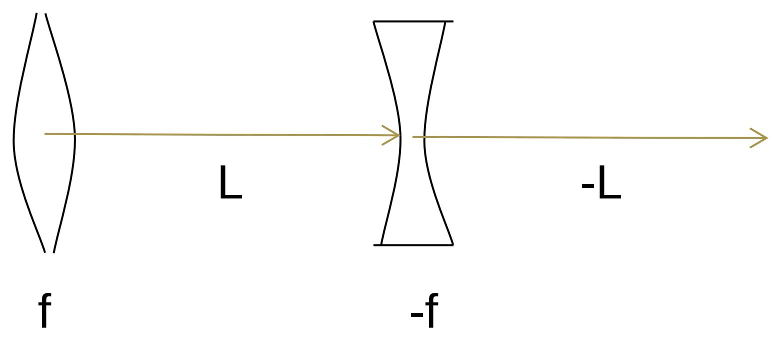
$$x(s) = x(0) + sx'(0)$$
$$x'(s) = x'(0)$$
$$\Rightarrow \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

- By combining these elements, we can represent an arbitrarily complex ring or line as the product of matrices.

$$M = M_N \ldots M_2 M_1$$

Example: FODO Cell

- At the heart of every beam line or ring is the FODO cell, consisting of a focusing and a defocusing element, separated by drifts:



- The transfer matrix is then

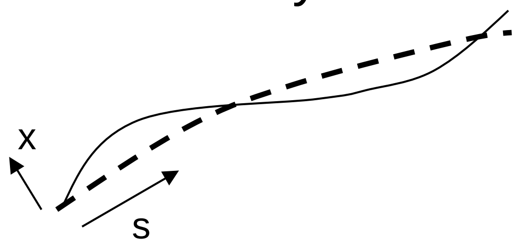
$$\Rightarrow M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ +\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f} - \left(\frac{L}{f}\right)^2 & 2L + \frac{L^2}{f} \\ -\frac{L}{f^2} & 1 + \frac{L}{f} \end{pmatrix}$$

- We can build a ring out of N of these, and the overall transfer matrix will be

$$M = M_{FODO}^{N}$$

Betatron Motion

- Skipping a lot of math, we find that we can describe particle motion in terms of initial conditions and a "beta function" *β(s)*, which is only a function of location in the nominal path.



Lateral deviation in one plane

$$x(s) = A[\beta(s)]^{1/2} \sin(\psi(s) + \delta)$$

Phase advance

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The "betatron function" *β(s)* is effectively the local wavenumber and also defines the beam envelope.

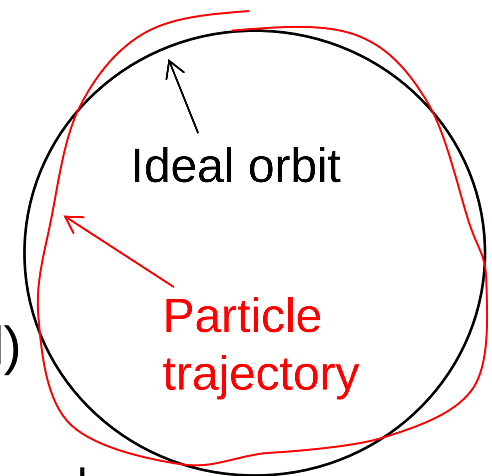
- Closely spaced strong quads -> small β -> small aperture, lots of wiggles
- Sparsely spaced weak quads -> large β -> large aperture, few wiggles

- Minor but important note: we need constraints to define *β(s)*
  - For a ring, we require periodicity (of β, NOT motion): $\beta(s+C)=\beta(s)$
  - For beam line: matched to ring or source

Betatron Tune

- As particles go around a ring, they will undergo a number of betatron oscillations ν given by

$$\nu = \frac{1}{2\pi}\int_{s}^{s+C}\frac{ds}{\beta(s)}$$

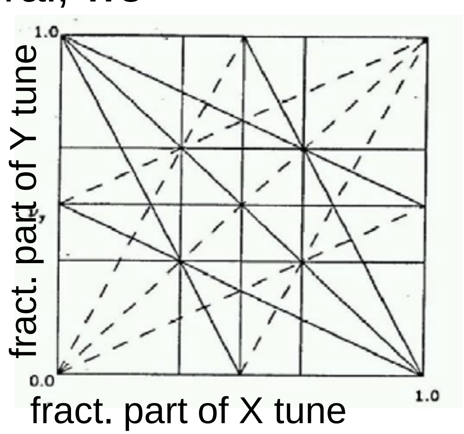


- This is referred to as the "tune" (horizontal and vertical)
  - If the tune is an integer, or lower order rational number, the effect of any imperfection or perturbation will be reinforced on subsequent orbits
  - If we also consider coupling between the planes, in general, we want to avoid

$$k_x\nu_x \pm k_y\nu_y = \text{integer} \Rightarrow (\text{resonant instability})$$

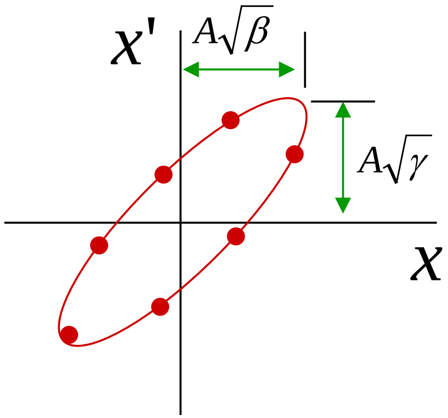
"small" integers

➔Avoid lines in the "tune plane"



Twiss Parameters: α, β, γ

- As a particle returns to the same point s on subsequent



$$\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2 = A^2$$

Twiss Parameters

$$\beta = (\text{betatron function})$$

$$\alpha = -\frac{1}{2}\frac{d\beta}{ds}$$

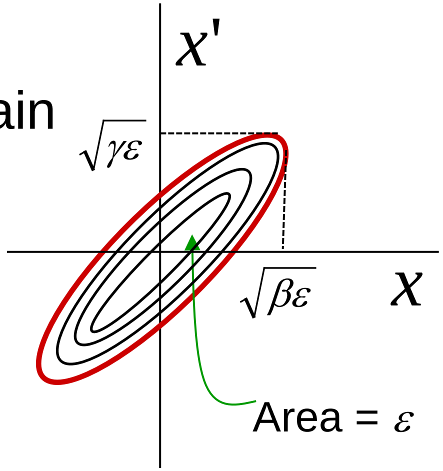
$$\gamma = \frac{1+\alpha^2}{\beta}$$

- As we examine different locations on the ring, the parameters will change, but the area A remains constant

Emittance

- If each particle is described by an ellipse with a particular amplitude, then an ensemble of particles will always remain within a bounding ellipse of a particular area:

$$\gamma(s)x^2 + 2\alpha xx' + \beta x'^2 = \varepsilon$$



- Since these distributions often have long tails, we typically define the "emittance" as an area which contains some specific fraction of the particles.

Typically, $\varepsilon_{rms} = \dfrac{\sigma^2}{\beta}$ contains 39% of Gaussian particles

$\varepsilon_{95} = 6\varepsilon_{rms}$ contains 95% of Gaussian particles

Dispersion and Chromaticity

- Up until now, we have assumed that momentum is constant
- Real beams will have a distribution of momenta
- Two most important parameters describing the behavior of off-momentum particles are:
  - *Dispersion*: describes the position dependence on momentum
    - Most important in the bend plane

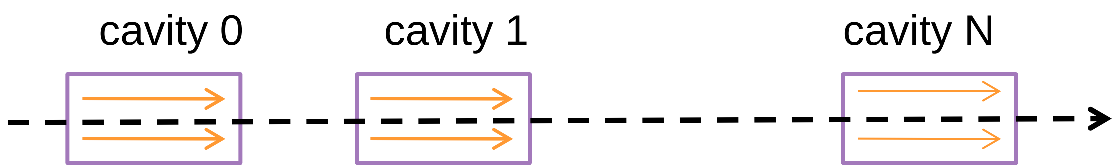
$$D_x \equiv \frac{\Delta x}{(\Delta p / p)}$$

  - *Chromaticity*: describes the tune dependence on momentum

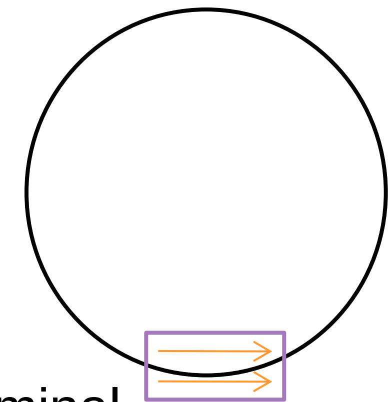
$$\xi_x \equiv \frac{\Delta \nu}{(\Delta p / p)} \quad \text{OR} \quad \xi_x \equiv \frac{\Delta \nu / \nu}{(\Delta p / p)}$$

Longitudinal Motion

- We generally accelerate particles using structures that generate time-varying electric fields (RF cavities), either in a linear arrangement or located within a circulating ring

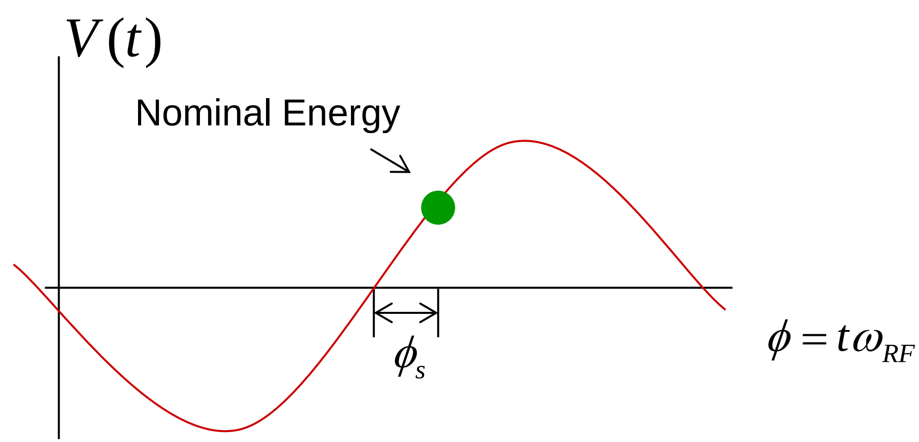


$E = E(t) = E_0 \sin \omega t + \delta_0$ $E = E(t) = E_0 \sin \omega t + \delta_1$ $E = E(t) = E_0 \sin \omega t + \delta_N$



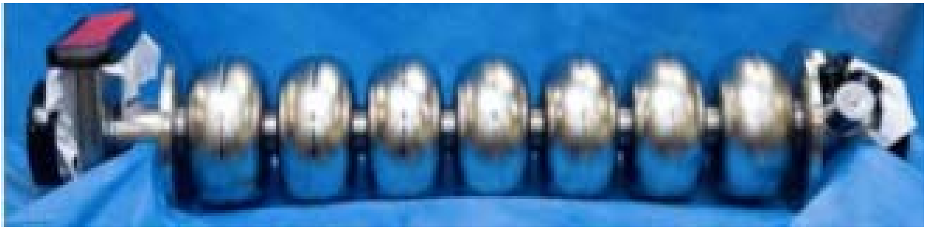
- In both cases, we phase the RF cavities so that a nominal arriving particle will see the same accelerating voltage and therefore get same boost in energy

$$\frac{\Delta E_s}{\Delta n} = eV_0 \sin \phi_s$$



Examples of Accelerating RF Structues

Fermilab Drift Tube Linac (200MHz): oscillating field uniform along length



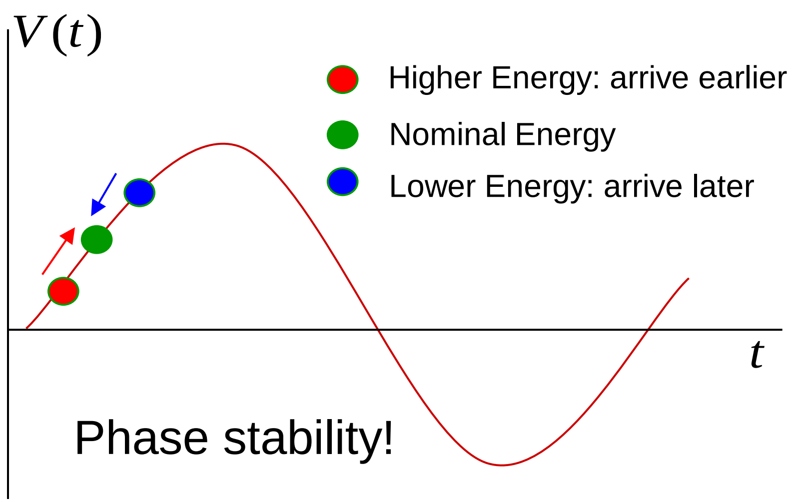
Jlab C100 cavity: string of eight 7-cell cavities, 1497MHz, design gradient 19.2MW/m average.

C-100 Cryomodule assembly

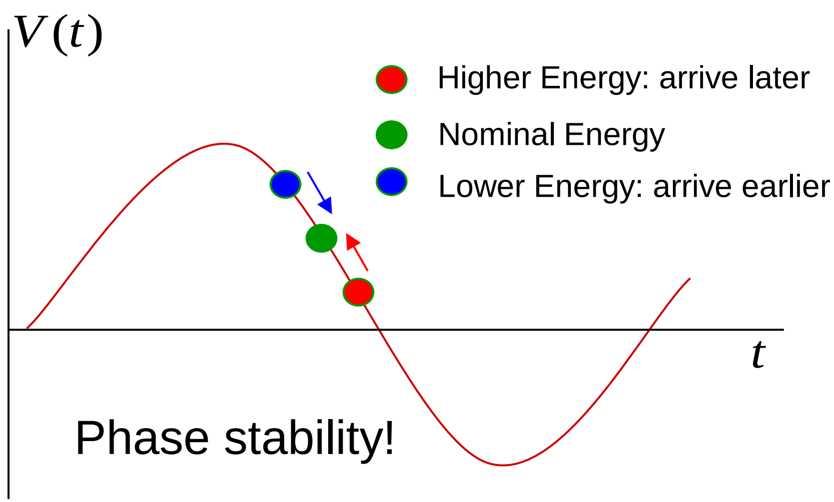
Transition Energy and Phase Stability

- Transition energy γ_t is determined by the lattice design. At transition energy, all particles at different energies travel around the ring with equal revolution frequency and experience the same acceleration at RF cavities.
- While

Below γ_{t} : velocity dominates



Above γ_t : path length dominates



- Rings have been designed (but never built) with $\gamma => \gamma_t$

The Case for Colliding Beams

- For a relativistic beam hitting a fixed target, the center of mass energy is :

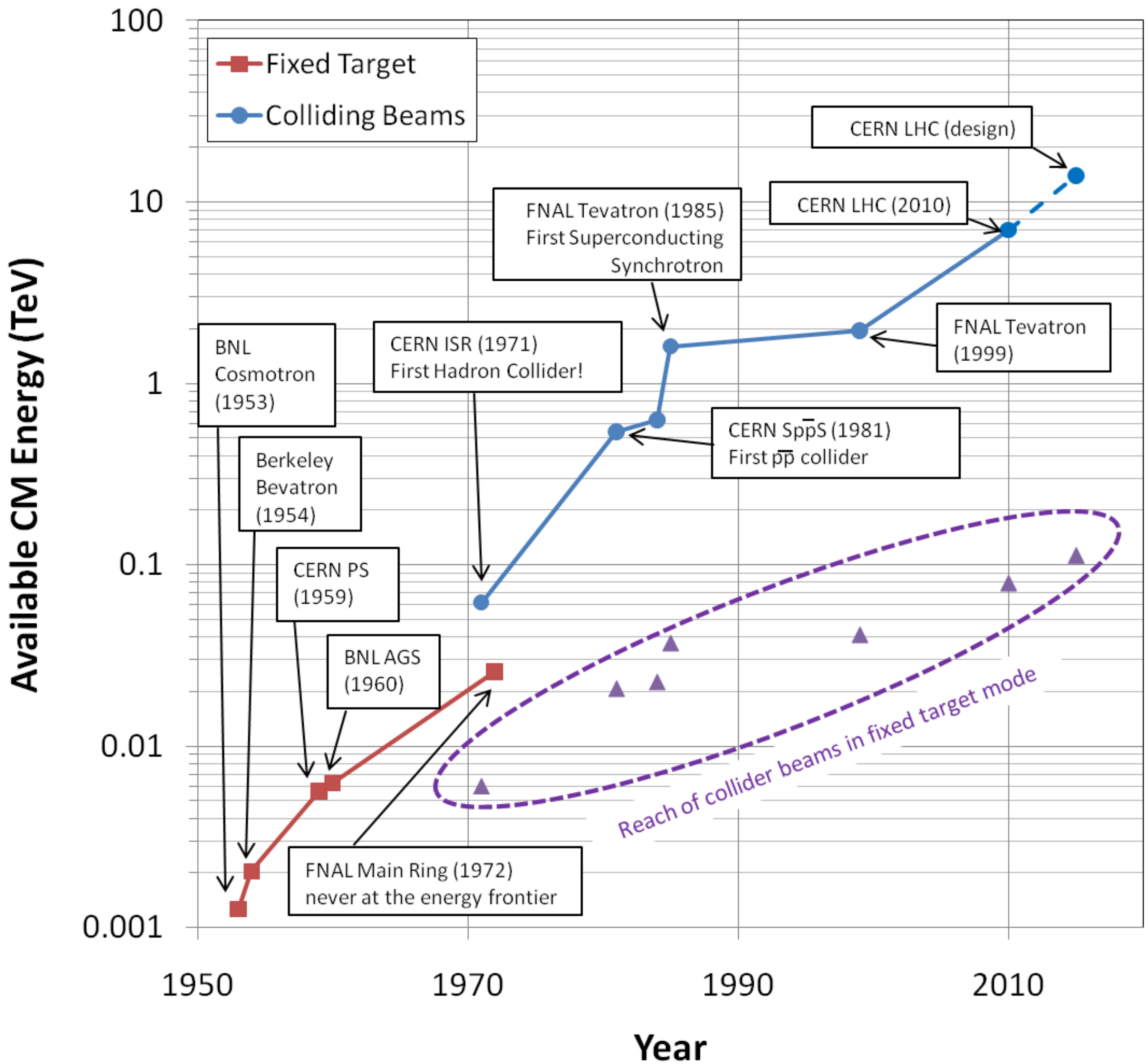
$$E_{\text{CM}} = \sqrt{2E_{\text{beam}} m_{\text{target}} c^2}$$

- For colliding beams (of equal mass and energy)

$$E_{\text{CM}} = 2E_{\text{beam}}$$

  - To get 14TeV CM design energy of the LHC with a single beam on a fixed target would require that beam to have an energy of 100,000 TeV
    - *Would required a ring 10 times the diameter of the Earth !!!*

Evolution of Energy Frontier



~a factor of 10 every 15 years

Luminosity

- The relationship of the beams to the rate of observed physics processes is given by the "luminosity"

$$R = L\sigma$$

Rate

Luminosity

Cross section (physics)

- Standard unit for luminosity is $cm^{-2}s^{-1}$
- Standard unit for cross section is "barn"$=10^{-24}$ cm^2
- Integrated luminosity is usually in $barn^{-1}$,where

$$b^{-1} = (1\,\mathrm{sec}) \times (10^{24}\ \mathrm{cm^{-2}s^{-1}})$$

$nb^{-1}= 10^9 b^{-1}$, $fb^{-1}= 10^{15} b^{-1}$,etc

- For (thin) fixed target:

Target thickness

$$R = N\rho_n t\sigma \Rightarrow L = N\rho_n t$$

Incident rate

Target number density

Luminosity (cont'd)

- For equally intense Gaussian beams

Collision frequency

Particles in two bunches

Geometrical reduction factor:
- crossing angle
- hourglass effect

$$L = f\,\frac{N_b^{\,2}}{4\pi\sigma^2}\,G(R)$$

Transverse size (RMS)

- Expressing this in terms of usual beam parameters

$$L = f_{rev}\,\frac{1}{4\pi}\,nN_b^2\,\frac{\gamma}{\beta^*\varepsilon_N}\,G(R)$$

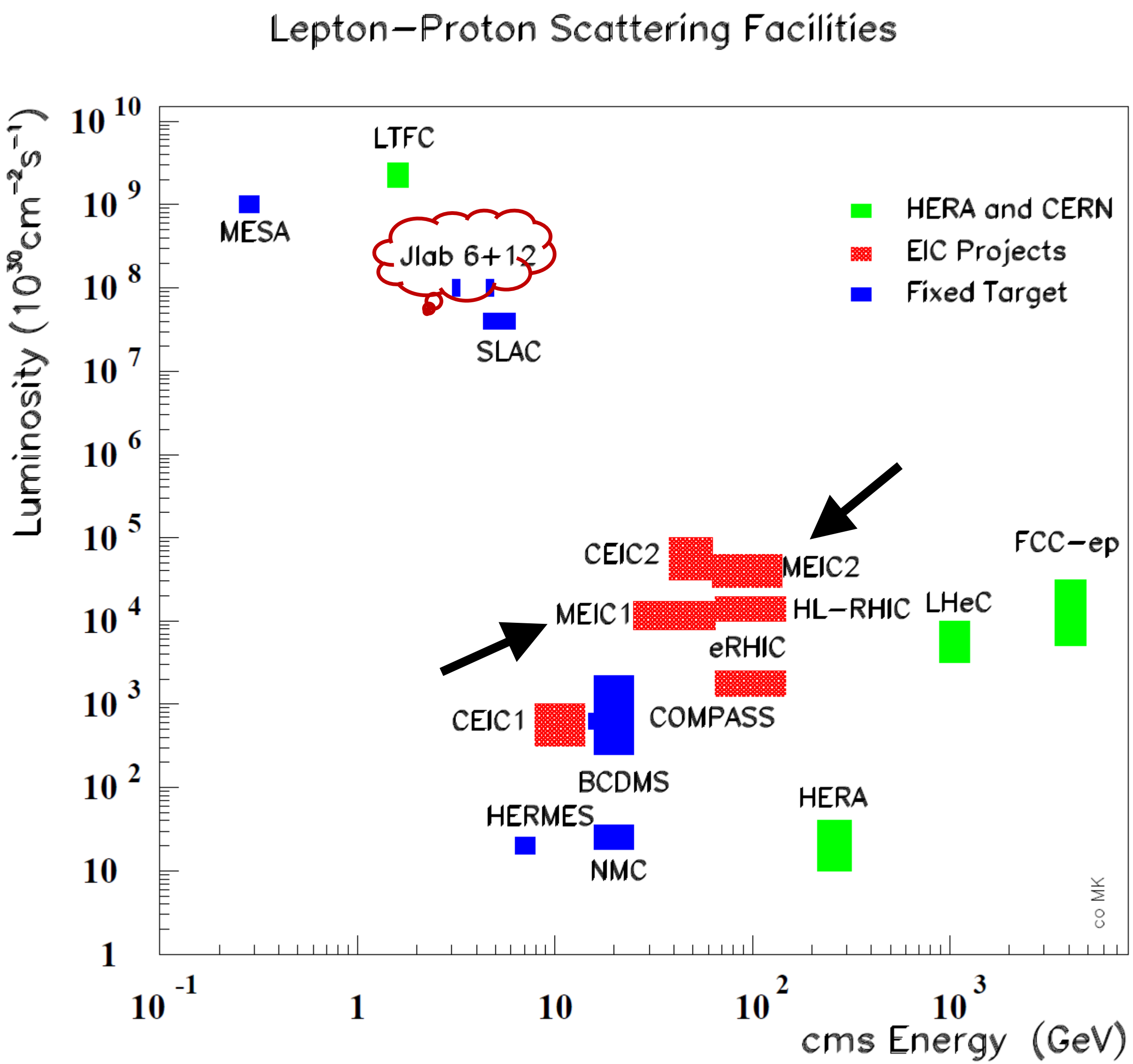
Revolution frequency

Normalized emittance

Number of bunches

Betatron function at collision point

Energy vs Luminosity Landscape



Spin

- Classical definition
  - the body rotation around its own axis
- Particle spin
  - an intrinsic property, like mass and charge
  - a quantum degree freedom associated with the intrinsic magnetic moment

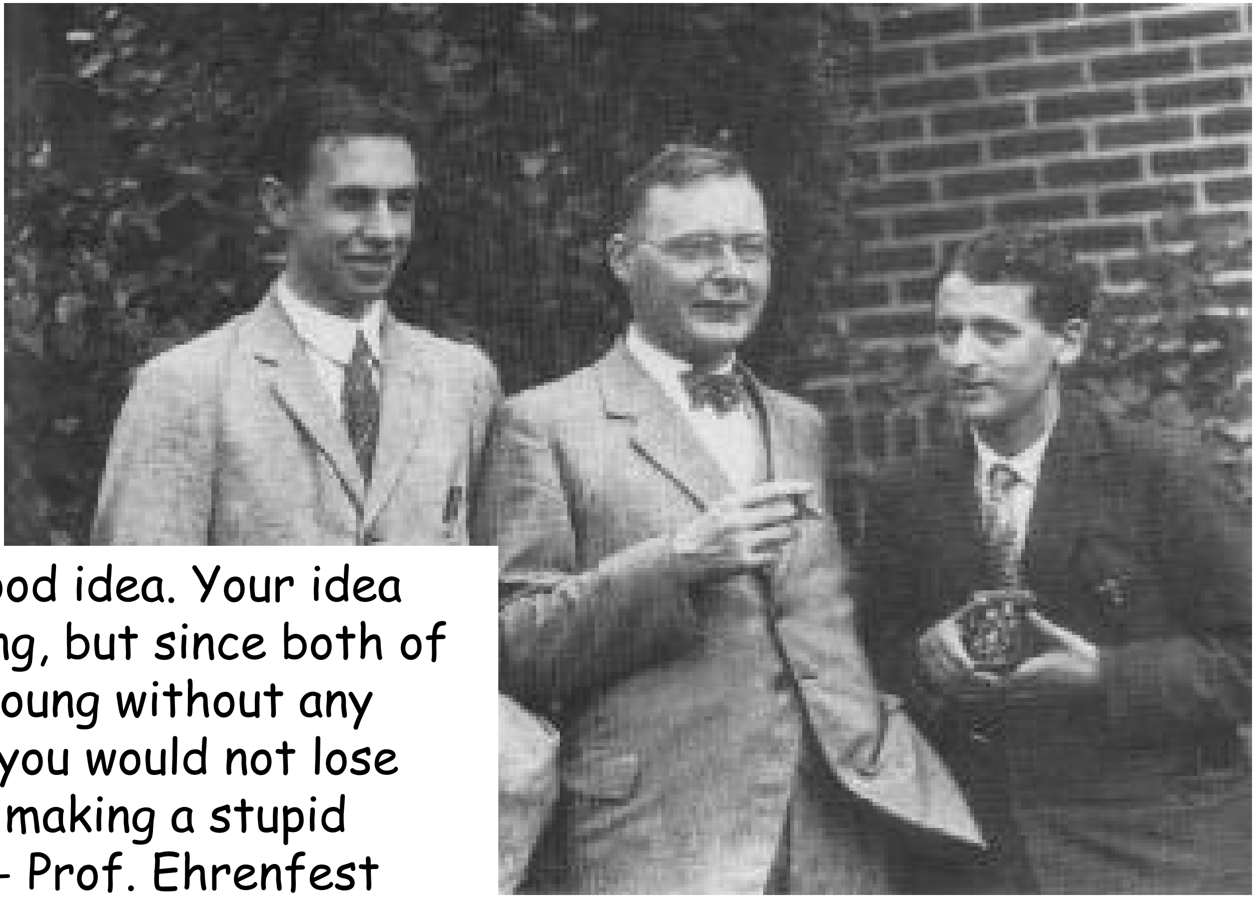
q: electrical charge of particle

$$\mu_s = (1+G)\frac{q}{m}S$$

m: particle mass

G: anomalous gyromagnetic factor, describes the particle internal structure. For particles:
point-like: G=0
electron: G=0.00115965219
muon: G=0.001165923
proton: G=1.7928474

Discovery of Spin: 1925



“This is a good idea. Your idea may be wrong, but since both of you are so young without any reputation, you would not lose anything by making a stupid mistake.” --- Prof. Ehrenfest

G.E. Uhlenbeck and S. Goudsmit, Naturwissenschaften ***47*** *(1925) 953. A subsequent publication by the same authors, Nature* ***117*** *(1926) 264,*

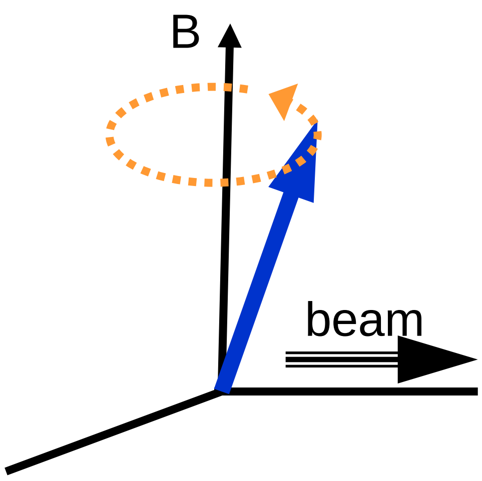
Spin Motion

- Spin motion follows Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} = -\frac{e}{\gamma m}[G\gamma \vec{B}_{\perp} + (1+G)\vec{B}_{//}] \times \vec{S}$$

Spin vector in particle's rest frame

- In a perfect accelerator, spin vector precesses around the bending dipole field direction: vertical

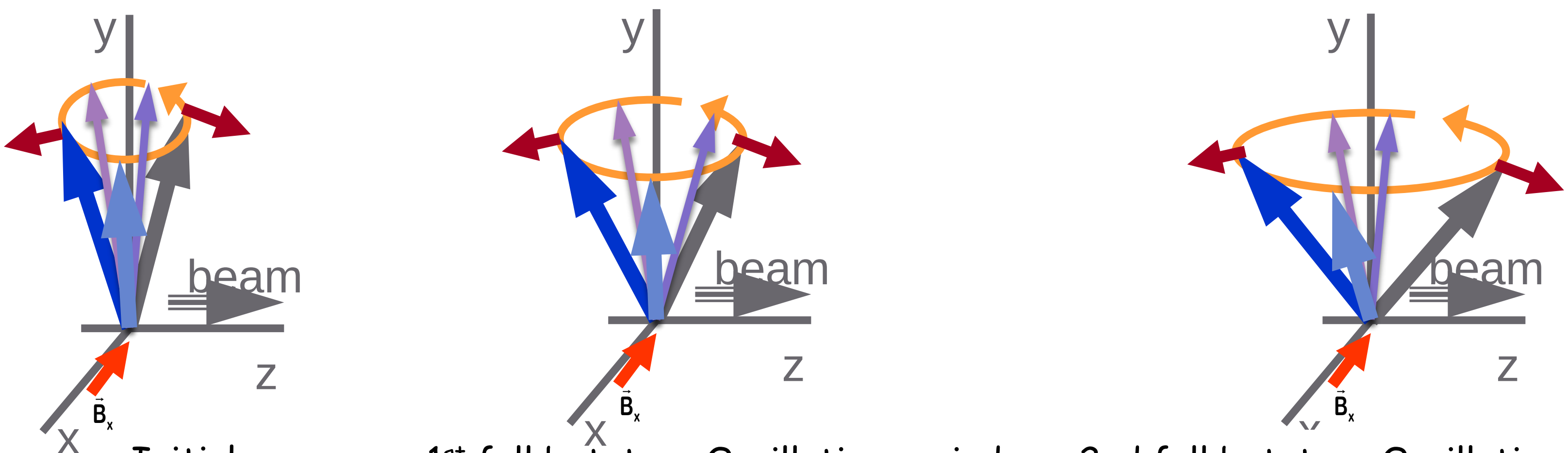


- Spin tune Qs: number of spin precessions in one orbital revolution. In general,

$$\mathbf{Q_s} = \mathbf{G}\gamma$$

Beam Polarization

- Polarization: statistical average of all the spin vectors over number of particles in the beam
  - zero polarization: spin vector point to all directions
  - 100% polarization: beam is fully polarized if all spin vectors point to the same direction
- Depolarization: come from the horizontal magnetic field which kicks the spin vector away from its vertical direction
  - coherent build-up of perturbation on spin vector when the spin vector gets kicked at the same frequency as its precession frequency



Initial   1st full betatron Oscillation period   2nd full betatron Oscillation period

References

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