Applications of Renormalization Group Methods in Nuclear Physics – 6

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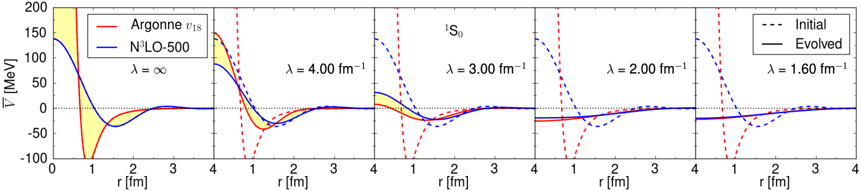
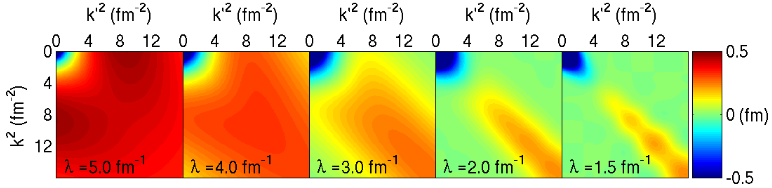
HUGS 2014





Outline: Lecture 6

Lecture 6: High-res. probes of low-res. nuclei

- Recap: Running Hamiltonians
- Parton distributions as paradigm
- Summary and challenges
- Extra: High-res. probes of low-res. nuclei

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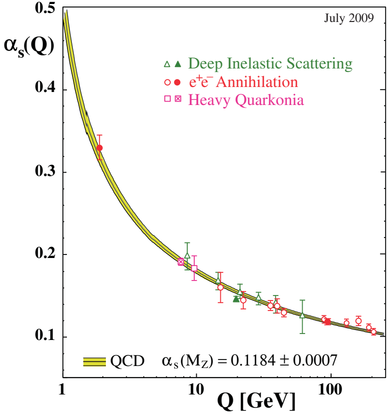
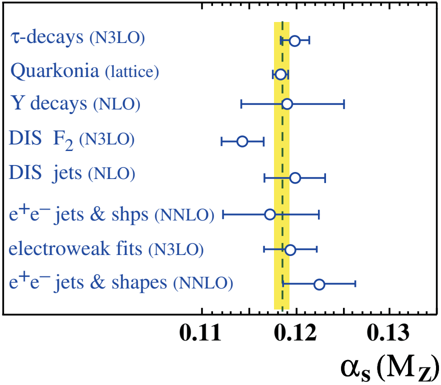
"Measuring" the QCD Hamiltonian: Running $\alpha_s(Q^2)$



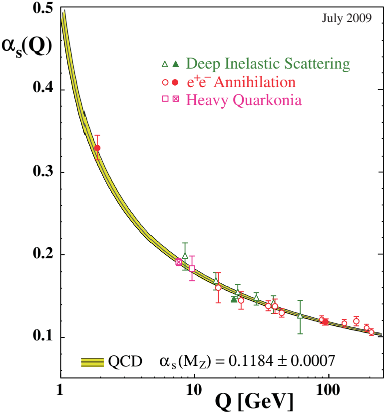
- The QCD coupling is *scale* dependent ("running"): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\mathrm{QCD}})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\mathrm{MS}}$)

- Extractions from experiment can be compared (here at M_Z):



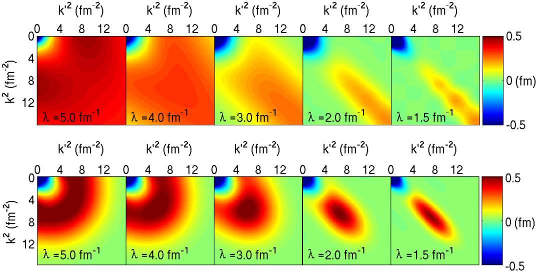
- cf. QED, where $\alpha_{em}(Q^2)$ is effectively constant for soft Q^2:
  $\alpha_{em}(Q^2 = 0) \approx 1/137$
  $\therefore$ fixed H for quantum chemistry

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



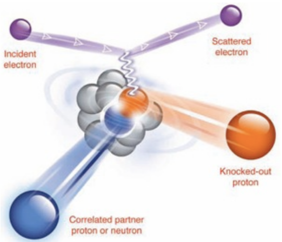
- The QCD coupling is *scale* dependent (cf. low-E QED): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\rm MS}$)

- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\text{low }k}$) running of initial potential with λ (decoupling or separation scale)

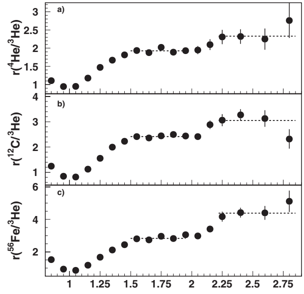


- Scheme dependence: AV18 vs. N^{3}LO (plus associated 3NFs)
- But all are (NN) phase equivalent!
- Shift contributions between interaction and sums over intermediate states

JLab: Understanding “short-range correlations” in nuclei



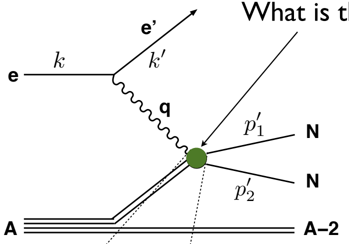
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



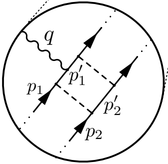
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N\nu}$$

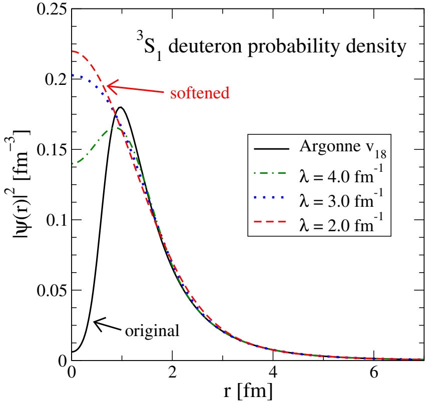
Higinbotham, arXiv:1010.4433

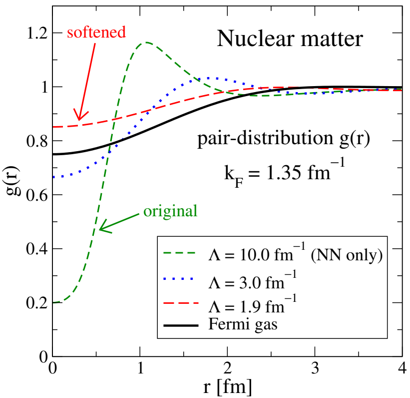


SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

Nuclear structure natural with *low* momentum scale

But soft potentials don't lead to short-range correlations (SRC)!





- Continuously transformed potential $\Longrightarrow$ variable SRC's in wf!
- Therefore, it seems that SRC's are *very scale/scheme dependent*
- Analog in high energy QCD: parton distributions

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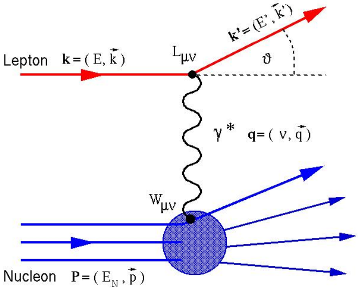
Parton distributions as paradigm [C. Keppel]

DIS Kinematics



$L_{\mu\nu}$: lepton tensor

$W_{\mu\nu}$: hadron tensor

*a virtual photon of four-momentum **q** is able to resolve structures of the order ħ/√q²*

- Four-momentum transfer:

$$q^2 = (E-E')^2 - (\vec{k}-\vec{k}')\cdot(\vec{k}-\vec{k}') =$$
$$= m_e^2 + m_{e'}^2 - 2(EE' - |\vec{k}||\vec{k}'|\cos\theta) =$$
$$\boxed{\approx \; -4EE'\sin^2\tfrac{\theta}{2} \equiv -Q^2}$$

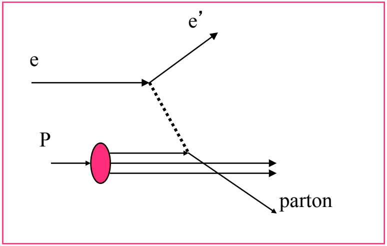
- Mott Cross Section ($\hbar c = 1$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\frac{\theta}{2}\cdot\frac{E'}{E}$$
$$= \frac{4\alpha^2 E'^2}{16E^2E'^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\cdot\frac{1}{1+\frac{E}{M}(1-\cos\theta)}$$
$$= \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\cdot\frac{1}{1+\frac{E}{M}(2\sin^2\frac{\theta}{2})}$$

Electron scattering of a spinless point particle

Parton distributions as paradigm [C. Keppel]

Simple parton model



$$p_{\text{quark}} = xP_{\text{proton}} \qquad x = Q^2/2P \cdot q$$

- Bjorken scaling $\Longrightarrow$ structure function F_2 independent of Q^2
- Measured $F_2(x)$ gives quark momentum distribution

$$F_2(x, Q^2) \approx F_2(x) = \sum_q e_q^2 \, x \, q(x)$$

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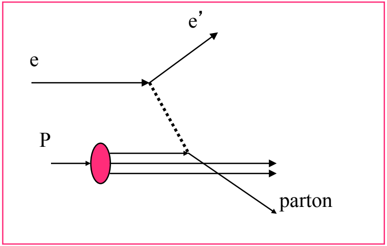

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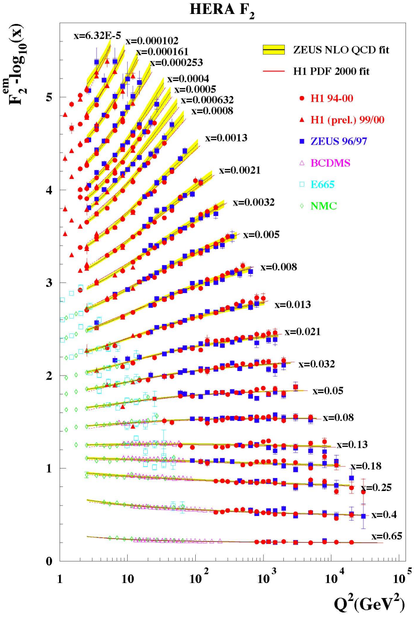

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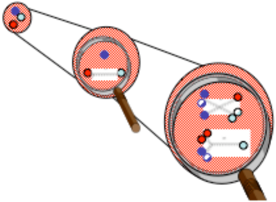
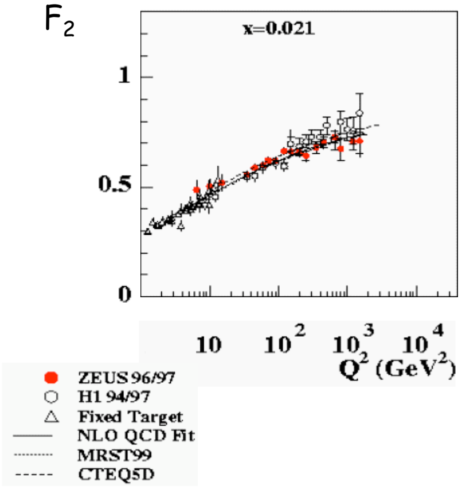
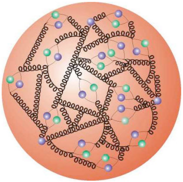


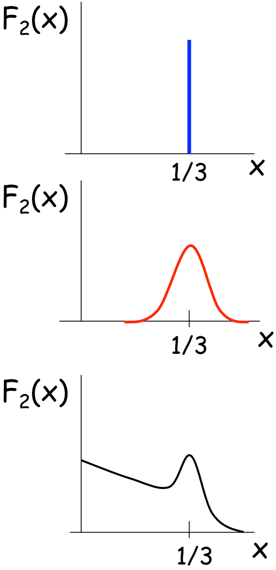


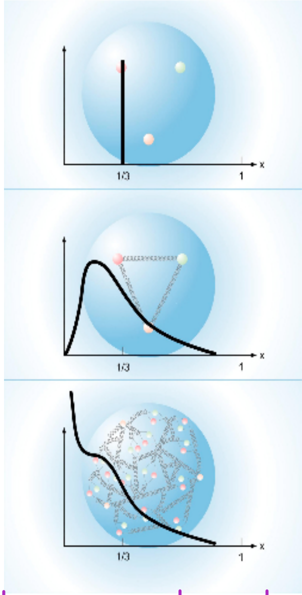
Higher the resolution (i.e. higher the Q^2) more low x partons we "see".



So what do we expect F_2 as a function of x at a fixed Q^2 to look like?

Parton distributions as paradigm [C. Keppel]





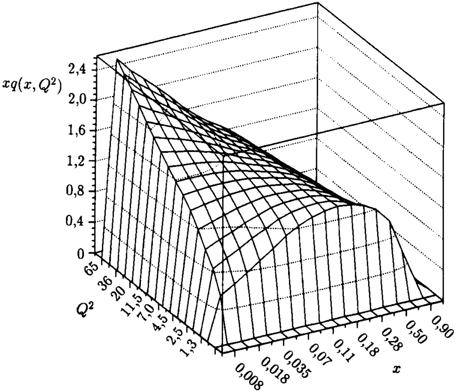
Three quarks with 1/3 of total proton momentum each.

Three quarks with some momentum smearing.

The three quarks radiate partons at low x.

....The answer depends on the Q^2!

Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

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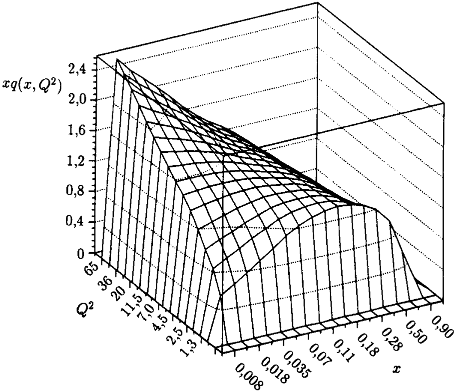

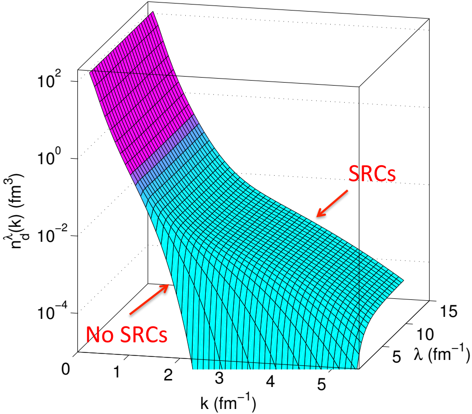

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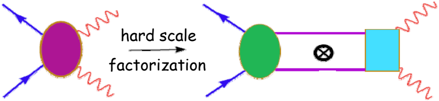
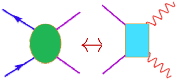
- Deuteron momentum distribution is scale *and* scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\mathrm{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$

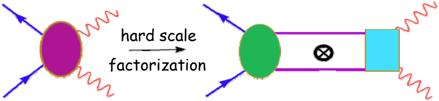
long-distance parton density



short-distance Wilson coefficient

- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Q^2 running of $f_a(x, Q^2)$ comes from choosing μ_f to optimize extraction from experiment

Factorization: high-E QCD vs. low-E nuclear



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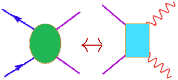

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_f - J_i| \le j \le J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What does it mean to be *consistent* between structure and reaction models? Treat separately? (No!)
- How does scale/scheme dependence come in?
- What are the trade-offs? (Does simpler structure always mean much more complicated reaction?)

Scheming for parton distributions

Need schemes for both renormalization and factorization

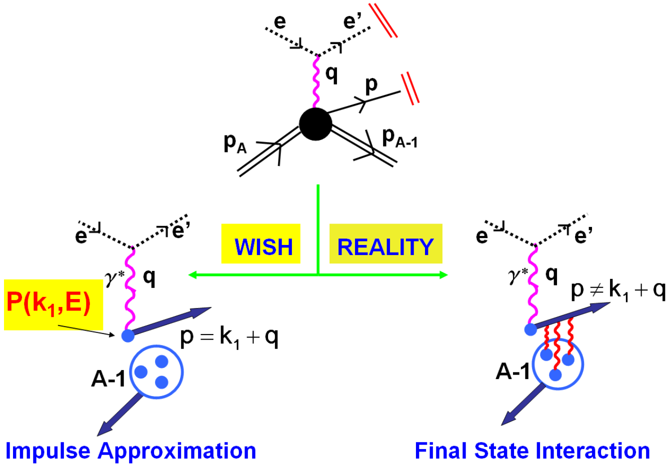
From the "Handbook of perturbative QCD" by G. Sterman et al.

> *"Short-distance finite parts at higher orders may be apportioned arbitrarily between the C's and ϕ's. A prescription that eliminates this ambiguity is what we mean by a factorization scheme. . . . The two most commonly used schemes, called DIS and $\overline{\text{MS}}$, reflect two different uses to which the freedom in factorization may be put."*

> *"The choice of scheme is a matter of taste and convenience, but it is absolutely crucial to use schemes consistently, and to know in which scheme any given calculation, or comparison to data, is carried out."*

Specifying a scheme in low-energy nuclear physics includes specifying a potential, including regulators, and how a reaction is analyzed.

Standard story for $(e, e'p)$ [from C. Ciofi degli Atti]



- In IA: "missing" momentum $p_m = k_1$ and energy $E_m = E$
- Common assumption: FSI and two-body currents treatable as independent *add-ons* to impulse approximation
- Is this valid?

Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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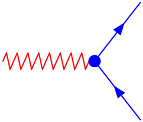
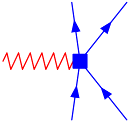
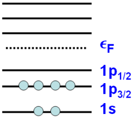
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states. So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

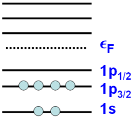
All pieces mix with unitary transformation

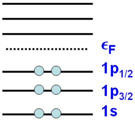
- A one-body current becomes many-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^{\dagger} =$  $+\,\alpha$  $+ \cdots$

- New wf correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+ \cdots \Longrightarrow Z$ 

$+\,\alpha$ 

$+ \cdots$

  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^{\dagger}|\Psi_n^{A-1}\rangle$
  - Thus *spectroscopic factors* are scale dependent

- Final state interactions (FSI) are also modified by $\widehat{U}$
- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, …

How should one choose a scale and/or scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - Low-k potential: improve many-body convergence, or to make microscopic connection to shell model or ...
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology?
- Cleanest extraction from experiment
  - Can one "optimize" validity of impulse approximation?
  - Ideally extract at one scale, evolve to others using RG
- Plan: use range of scales to test calculations and physics
  - Find (match) Hamiltonians and operators with EFT
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguities (e.g., in spectroscopic factors)

Operator flow in practice [e.g., see arXiv:1008.1569]

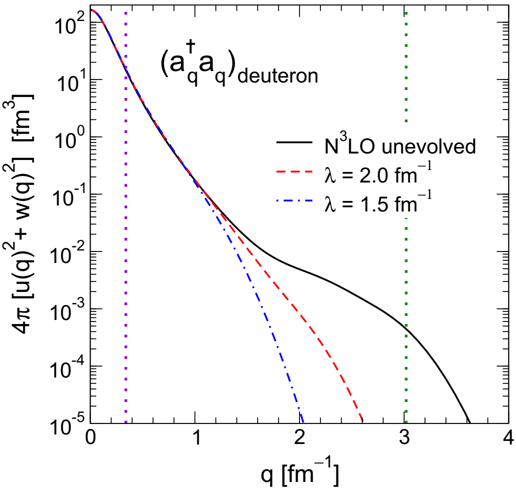
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

so O_s evolves via

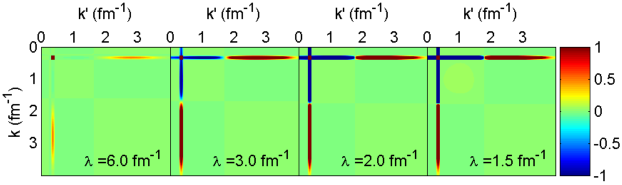
$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$ or solve dU_s/ds flow
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\mathrm{fm}^{-1}$ in deuteron

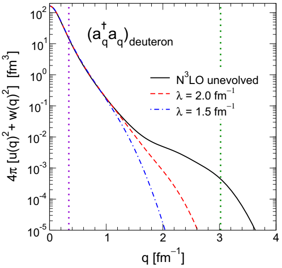


High and low momentum operators in deuteron

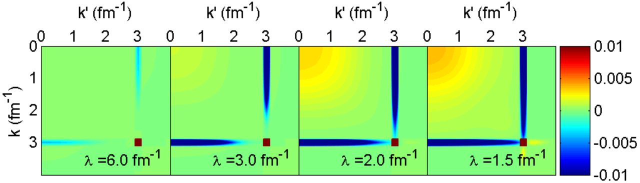
- **Integrand** of $(U a_q^\dagger a_q U^\dagger)$ for $q = 0.34\,\mathrm{fm}^{-1}$



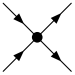
- Momentum distribution



- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$

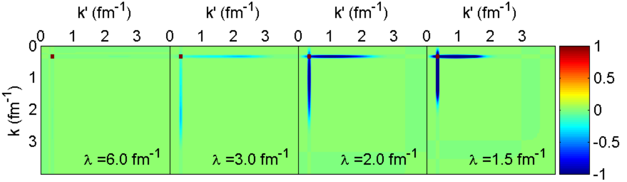


- One-body operator does not evolve (for “standard” SRG)
- Induced two-body operator ≈ regularized delta function:

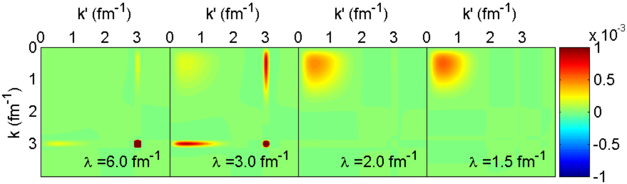


High and low momentum operators in deuteron

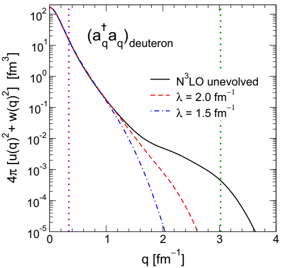
- **Integrand** of $\langle\psi_d|\,(Ua_q^\dagger a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$

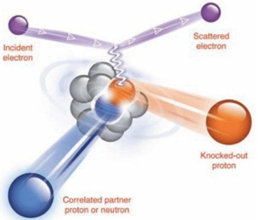


- Momentum distribution

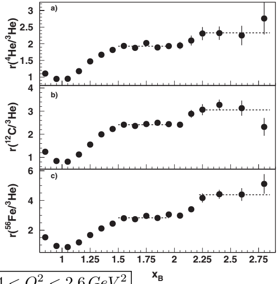


- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$ $\left\langle \frac{1}{r}\right\rangle$, $\langle G_C\rangle$, ...

Looking for missing strength at large Q^2



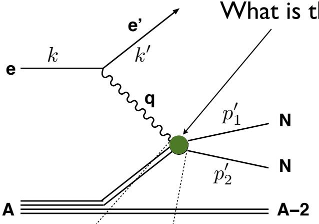
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



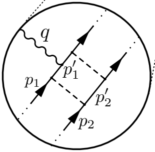
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N\nu}$$

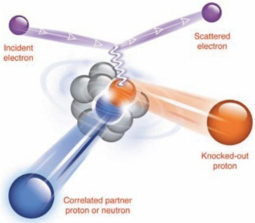
Higinbotham, arXiv:1010.4433



SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

- SRC explanation relies on high-momentum nucleons in structure!

Looking for missing strength at large Q^2



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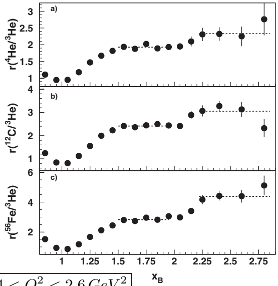

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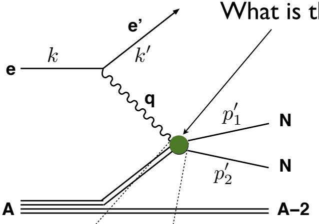

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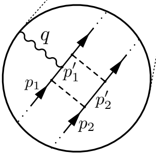

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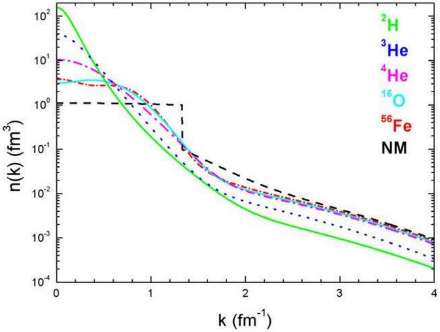
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

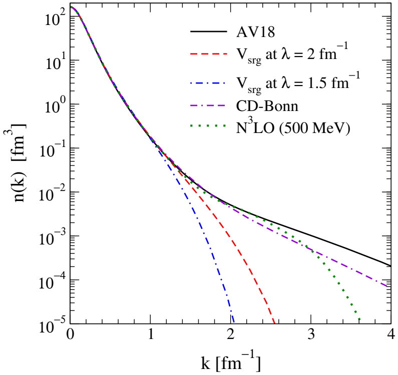
- Changing resolution changes physics interpretation!

Changing the separation scale with RG evolution

- Conventional analysis has (implied) high momentum scale
  - Based on potentials like AV18 and one-body current operator



[From C. Ciofi degli Atti and S. Simula]



- With RG evolution, probability of high momentum decreases, but

$$n(k) \equiv \langle A | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | A \rangle = \left(\langle A | \widehat{U}^\dagger \right) \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger \left(\widehat{U} | \Psi_n \rangle \right) = \langle \widetilde{A} | \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger | \widetilde{A} \rangle$$

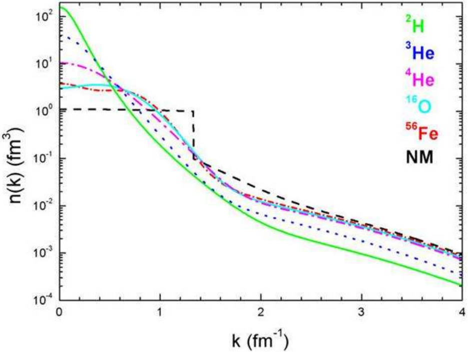
is unchanged! $|\widetilde{A}\rangle$ is easier to calculate, but is operator too hard?

Nuclear scaling from factorization (schematic!)

- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k,q) \to K_\lambda(k)Q_\lambda(q)$

$$\frac{n_A(q)}{n_d(q)} = \frac{\langle\widetilde{A}|\widehat{U}a^\dagger_{\mathbf{q}}a_{\mathbf{q}}\widehat{U}^\dagger|\widetilde{A}\rangle}{\langle\widetilde{d}|\widehat{U}a^\dagger_{\mathbf{q}}a_{\mathbf{q}}\widehat{U}^\dagger|\widetilde{d}\rangle} = \frac{\langle\widetilde{A}|\int U_\lambda(k',q')\delta_{q'q}U^\dagger_\lambda(q,k)|\widetilde{A}\rangle}{\langle\widetilde{d}|\int U_\lambda(k',q')\delta_{q'q}U^\dagger_\lambda(q,k)|\widetilde{d}\rangle}$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



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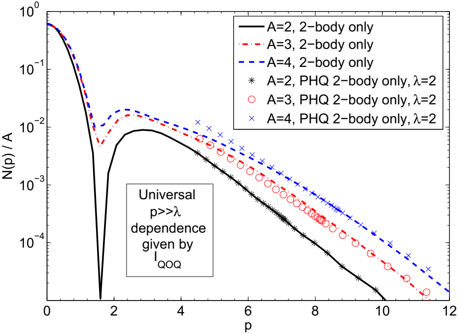
Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]

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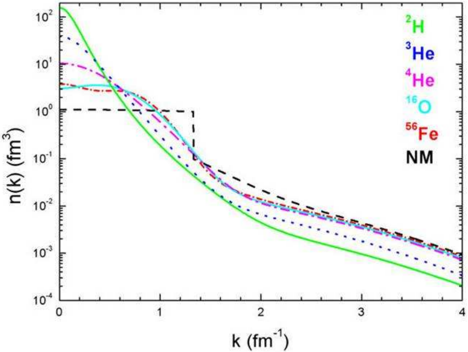

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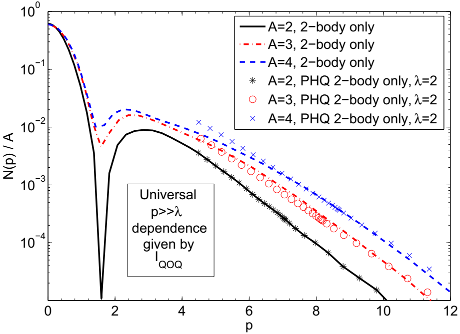

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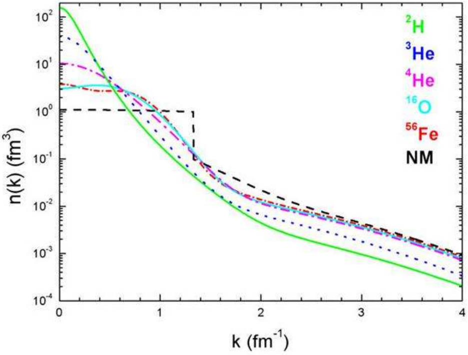

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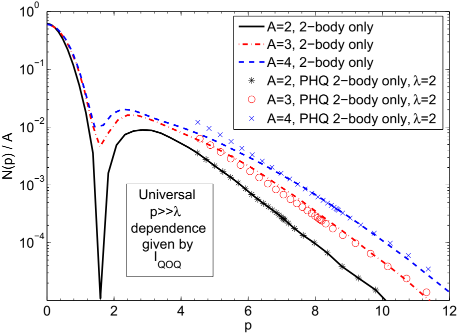

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- Factorization: $U_\lambda(k,q) \to K_\lambda(k)Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q)\int_0^\lambda p^2dp\, Z(\lambda)\Psi_\alpha^\lambda(p) + \eta^\lambda(q)\int_0^\lambda p^2dp\, p^2\, Z(\lambda)\,\Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k)Q_\lambda(q)$

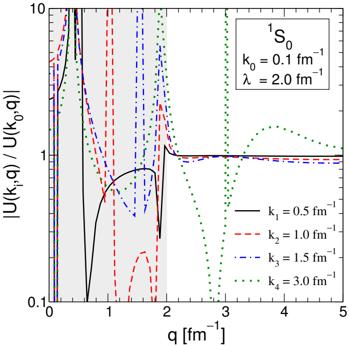
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}}\langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\Big]\gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



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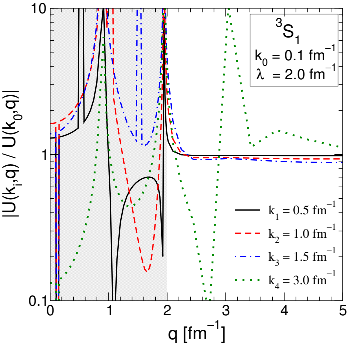

Outline: Lecture 6

Lecture 6: High-res. probes of low-res. nuclei

- Recap: Running Hamiltonians
- Parton distributions as paradigm
- Summary and challenges
- Extra: High-res. probes of low-res. nuclei

Summary: Atomic Nuclei at Low Resolution

- Strategy: Lower the resolution and track dependence
  - High resolution $\Longrightarrow$ high momenta can be painful!
    (“It hurts when I do this.” “Then don’t do that.”)
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  - Nuclear case: evolve until few-body forces start to explode
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- Applications to nuclei and beyond
  - CI, coupled cluster, ... converge faster $\Longrightarrow$ greater reach
  - IM-SRG, microscopic shell model $\Longrightarrow$ role of 3-body forces
  - MBPT works $\Longrightarrow$ improved nuclear density functional theory

Confluence of progress in theory and experiment

- Theory advances, catalyzed by large-scale collaboration
  - Explosion of complementary many-body methods
  - Computational power and advanced algorithms
  - Inputs: effective field theory and renormalization group
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  - $\vdots$
- *Precision* comparisons are increasingly possible *if* we can
  - control (and minimize) model dependence
  - quantify theory error bars from all sources

Challenges: Precision nuclear structure and reactions

- We're in a golden age for low-energy nuclear physics
  - Many complementary methods able to incorporate 3NFs
  - Synergies of theory and experiment
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  - Scale and scheme dependence is inevitable $\Longrightarrow$ deal with it!
- Challenges for which EFT/RG perspective + tools can help
  - Can we have controlled factorization at low energies?
  - How should one choose a scale/scheme in particular cases?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What are the roles of short-range/long-range correlations?
  - How do we consistently match Hamiltonians and operators?
  - ...and many more. Calculations are in progress!

Long-term gameplan: Fully connected descriptions

Nuclei

$\Uparrow$

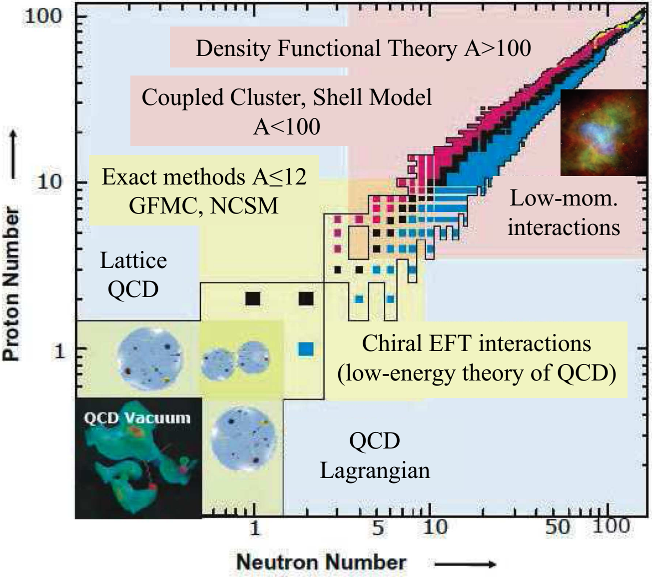
RG evolution
$NN \cdots N$

$\Uparrow$

Chiral EFT
$NN \cdots N$

$\Uparrow$

Lattice QCD
$\Longrightarrow$ LEC's



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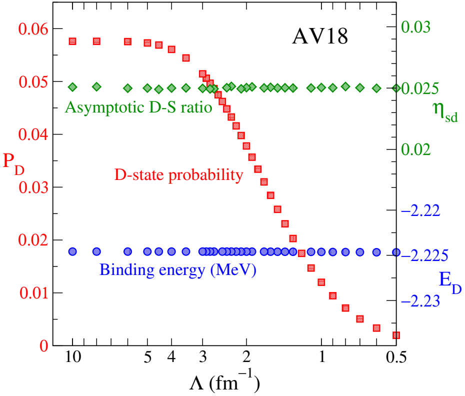
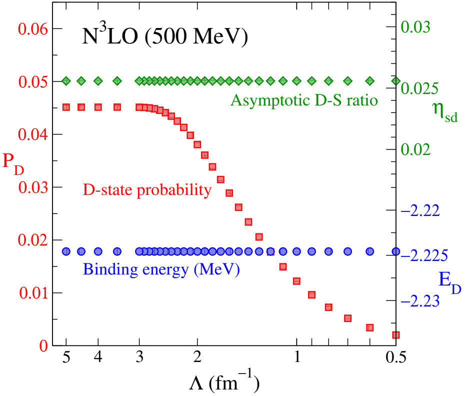
What parts of wf's can be extracted from experiment?

- Measurable: asymptotic (IR) properties like phase shifts, ANC's
- Not "pure" observables, but well-defined theoretically given a Hamiltonian: interior quantities like spectroscopic factors
  - These depend on the scale and the scheme
  - Extraction from experiment requires robust factorization of structure and reaction; only the combination is scale/scheme independent (cross sections) [What if weakly dependent?]
- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)
  - Are there RG-invariant quantities to extract?
- High-momentum tails of momentum distributions?
  - Nuclear tails depend on scale and scheme

Deuteron scale-(in)dependent observables

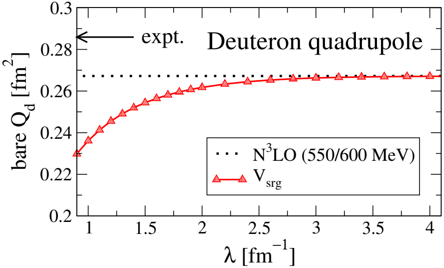


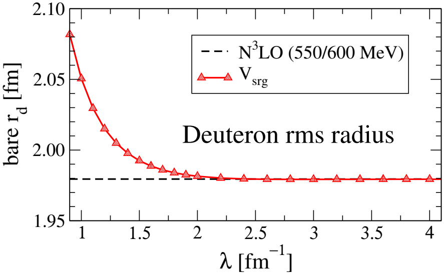


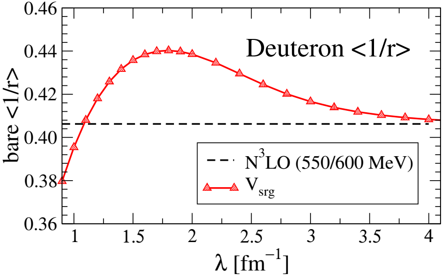
- $V_{\text{low }k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)

Unevolved long-distance operators change slowly with λ

- Matrix elements dominated by long range run slowly for $\lambda \gtrsim 2\,\text{fm}^{-1}$
- Here: examples from the deuteron (compressed scales)
- Which is the correct answer?
- Are we using the complete operator for the experimental quadrupole moment?







EMC effect from the EFT perspective

- Exploit scale separation between short- and long-distance physics
  - *Match* complete set of operator matrix elements (power count!)
  - Cf. needing a *model* of short-distance nucleon dynamics
  - Distinguish long-distance nuclear from nucleon physics
- EMC and effective field theory (examples)
  - "DVCS-dissociation of the deuteron and the EMC effect"
    [S.R. Beane and M.J. Savage, Nucl. Phys. A 761, 259 (2005)]

    "By constructing all the operators required to reproduce the matrix elements of the twist-2 operators in multi-nucleon systems, one sees that operators involving more than one nucleon are not forbidden by the symmetries of the strong interaction, and therefore must be present. While observation of the EMC effect twenty years ago may have been surprising to some, in fact, its absence would have been far more surprising."
  - "Universality of the EMC Effect"
    [J.-W. Chen and W. Detmold, Phys. Lett. B 625, 165 (2005)]

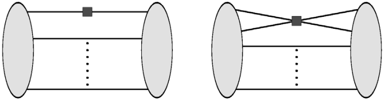
Dependence of EMC effect on A is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2\, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators $\Longrightarrow$ parton dist. moments



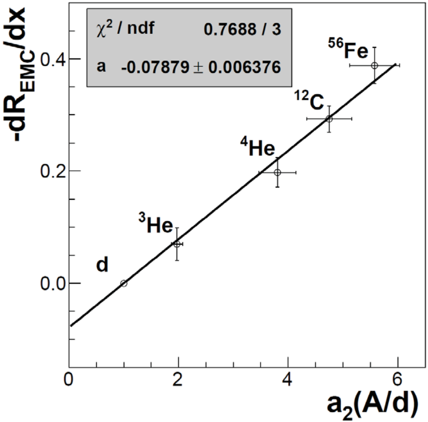
$$\Longrightarrow \langle x^n \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

Scaling and EMC correlation via low resolution

- SRG factorization, e.g., $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
  - Dependence on high-q independent of A $\Longrightarrow$ universal
  - A dependence from low-momentum matrix elements $\Longrightarrow$ calculate!
- EMC from EFT using OPE:
  - Isolate A dependence, which factorizes from x
  - EMC A dependence from long-distance matrix elements



L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

If same leading operators dominate, then linear A dependence of ratios?

‘Non-observables’ vs. Scheme-dependent observables

- Some quantities are *in principle* not observable
  - T.D. Lee: “The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”
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  - E.g., cross sections and energies
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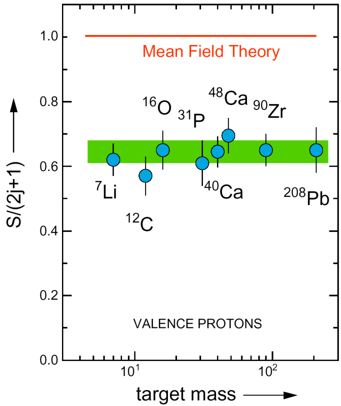
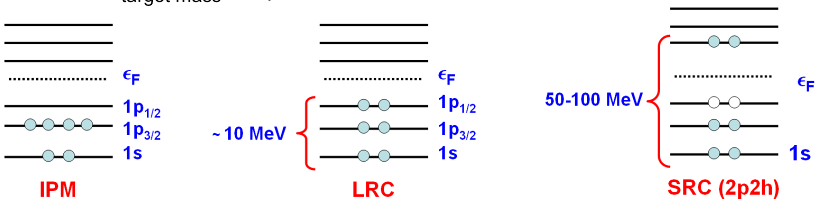
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!
- Scale- and scheme-dependent derived quantities
  - Critical questions to address for each quantity:
    - What is the ambiguity or convention dependence?
    - Can one convert between different prescriptions?
    - Is there a consistent extraction from experiment such that they can be compared with other processes and theory?
  - Physical quantities can be *in-practice* clean observables if scheme dependence is negligible (e.g., $(e, 2e)$ from atoms)
  - How do we deal with dependence on the Hamiltonian?

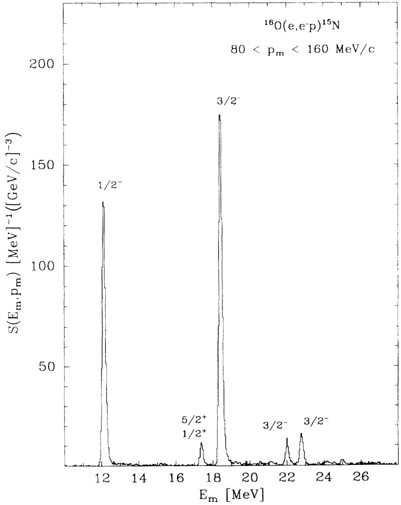
Scale/scheme dependence: spectroscopic factors



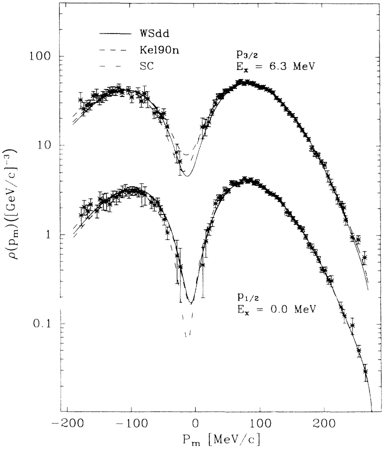
- Spectroscopic factors for valence protons have been extracted from $(e, e'p)$ experimental cross sections (e.g., Nikhef 1990's at left)
- Used as canonical evidence for "correlations", particularly short-range correlations (SRC's)
- But if SFs are scale/scheme dependent, how do we explain the cross section?



(Assumed) factorization of $(e, e'p)$ cross section



Missing energy spectrum for
$^{16}O(e, e'p)^{15}N$ [Leuschner (1994)]



$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \ \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

$\Longrightarrow p_{1/2}$ spectroscopic factor ≈ 0.63

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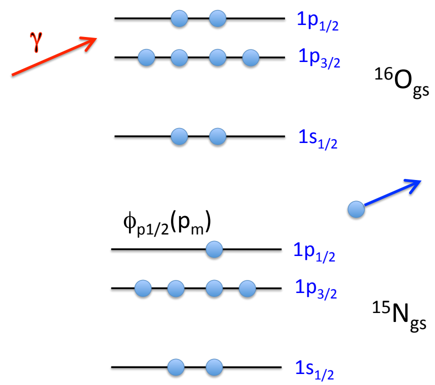

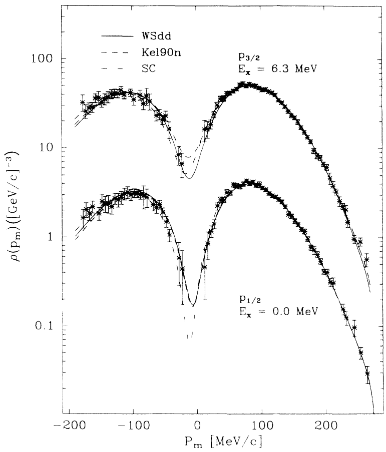

- Knock out $p_{1/2}$ proton from ^{16}O to ^{15}N ground state in IPM
- Adjust s.p. well depth and radius to identify $\phi_\alpha(\mathbf{p}_m)$
- Final state interactions (FSI) added using optical potential(s)

$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

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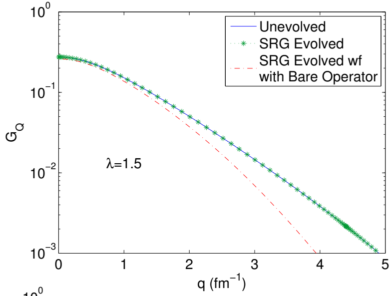
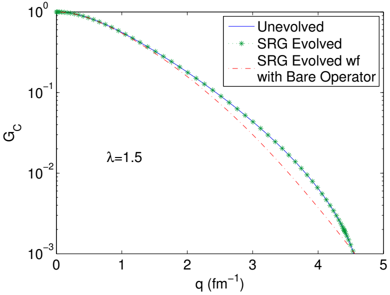
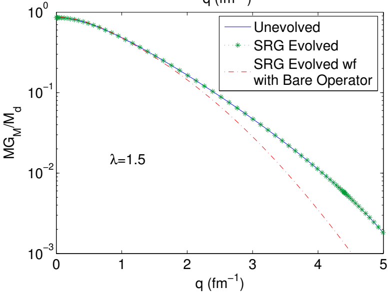
Deuteron electromagnetic form factors

- G_C, G_Q, G_M in deuteron with chiral EFT at leading order (Valderrama et al.)
- NNLO 550/600 MeV potential
- Unchanged at low q with unevolved operators
- Independent of λ with evolved operators



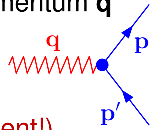




Generic knockout reaction [e.g., Dickhoff/Van Neck text]

- Consider a scalar external probe that just transfers momentum $\mathbf{q}$

$$\rho(\mathbf{q}) = \rho_0 \sum_{j=1}^{A} e^{-i\mathbf{q}\cdot\mathbf{r}} \Longrightarrow \widehat{\rho}(\mathbf{q}) = \rho_0 \sum_{\mathbf{p},\mathbf{p}'} \langle \mathbf{p} | e^{-i\mathbf{q}\cdot\mathbf{r}} | \mathbf{p}' \rangle a^\dagger_{\mathbf{p}} a_{\mathbf{p}'}$$



  - First assumption: one-body operator (scale dependent!)
- Then the cross section from Fermi's golden rule is

$$d\sigma \sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2$$

- Complication: ejected final particle A interacts on way out (FSI)

$$H_A = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V(i,j) = H_{A-1} + \frac{p_A^2}{2m} + \sum_{i=1}^{A-1} V(i,A)$$

  - If we neglect this interaction $\Longrightarrow$ PW (no FSI)

$$|\Psi_i\rangle = |\Psi_0^A\rangle \ , \qquad |\Psi_f\rangle = a^\dagger_{\mathbf{p}} |\Psi_n^{A-1}\rangle \Longrightarrow \langle \Psi_f | = \langle \Psi_n^{A-1} | a_{\mathbf{p}}$$

$\Longrightarrow$ factorized knockout cross section $\propto$ hole spectral fcn:

$$d\sigma \sim \rho_0^2 \sum_n \delta(E_m - E_0^A + E_n^{A-1}) |\langle \Psi_n^{A-1} | a_{\mathbf{p}_m} | \Psi_0^A \rangle|^2 = \rho_0^2 \, S_h(\mathbf{p}_m, E_m)$$

- Does it still factorize when corrected for (scale dependent!) FSI?

Now repeat with a unitary transformation $\widehat{U}$

- The cross section is *guaranteed* to be the same from $\widehat{U}^\dagger \widehat{U} = 1$

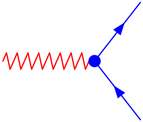
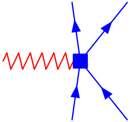
$$
\begin{aligned}
d\sigma &\sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2 \\
&= \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | (\widehat{U}^\dagger \widehat{U}) \widehat{\rho}(\mathbf{q}) (\widehat{U}^\dagger \widehat{U}) | \Psi_i \rangle|^2 \\
&= \sum \delta(\omega + E_i - E_f) |(\langle \Psi_f | \widehat{U}^\dagger)(\widehat{U} \widehat{\rho}(\mathbf{q}) \widehat{U}^\dagger)(\widehat{U} | \Psi_i \rangle)|^2
\end{aligned}
$$

  but the pieces are different now.

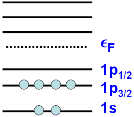
- Schematically, the SRG has $\widehat{U} = 1 + \frac{1}{2}(U - 1) a^\dagger a^\dagger a a + \cdots$
  - U is found by solving for the unitary transformation in the $A = 2$ system (this is the easy part!)
  - The $\cdots$'s represent higher-body operators
  - One-body operators ($\propto a^\dagger a$) gain many-body pieces (EFT: there are always many-body pieces at some level!)
  - Both initial and final states are modified (and therefore FSI)

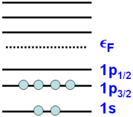
New pieces after the unitary transformation

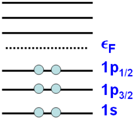
- The current is no longer just one-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger =$  $+\,\alpha$  $+\cdots$

- New correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+\cdots \Longrightarrow$ 

$+\,\alpha$ 

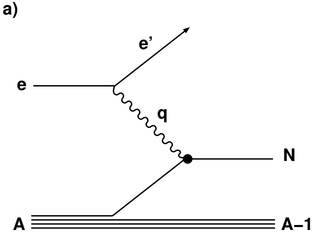
$+\cdots$

  - Similarly with $|\Psi_f\rangle = a^\dagger_{\mathbf{p}}|\Psi_n^{A-1}\rangle$
  - So the spectroscopic factors are modified

- Final state interactions are also modified by $\widehat{U}$
- Bottom line: the cross section is unchanged *only* if all pieces are included, with the same U: $H(\lambda)$, current operator, FSI, …

Changing resolution shifts physics: Not unique!

- From D. Higinbotham, arXiv:1010.4433



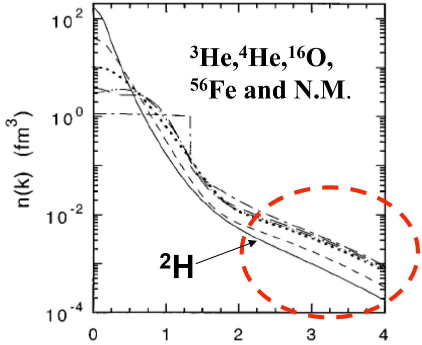
"The simple goal of short-range nucleon-nucleon correlation studies is to cleanly isolate diagram b) from a). Unfortunately, there are many other diagrams, including those with final-state interactions, that can produce the same final state as the diagram scientists would like to isolate. If one could find kinematics that were dominated by diagram b) it would finally allow electron scattering to provide new insights into the short-range part of the nucleon-nucleon potential."

- What is in the blob in b)? A one-body vertex and an SRC, or a two-body vertex? Depends on the resolution! (Also FSI+ will mix.)

Deuteron-like scaling at high momenta from factorization

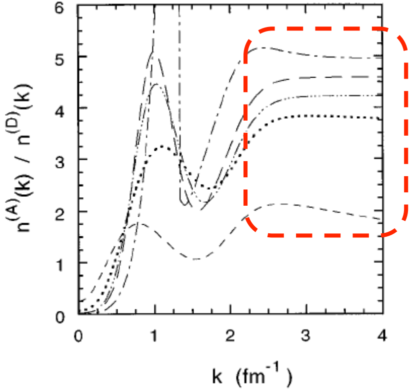
C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



n(k) at high Momentum regions are similar to it of the Deuteron

Ratio to the Deuteron



Almost Flat!

High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

Lower resolution $\Longrightarrow$ lower separation scale $\Longrightarrow$ fall-off depends on V_λ