The Transverse Spin Structure of the Nucleon

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Lessons Scheme

LECTURES 1 & 2

- Introduction
- Deep Inelastic Scattering and 1D parton distribution functions
- From 1D to 3D nucleon structure: Transverse Momentum Dependent (TMD) parton distribution functions
- TMD Measurements @ Jlab in Hall B

LECTURES 3 & 4

- Data analysis
- Monte Carlo simulations
- Asymmetries extraction
- TMDs extraction

LECTURE 5

- Where are we? What's next

TMDs with a Transversely Polarized Target

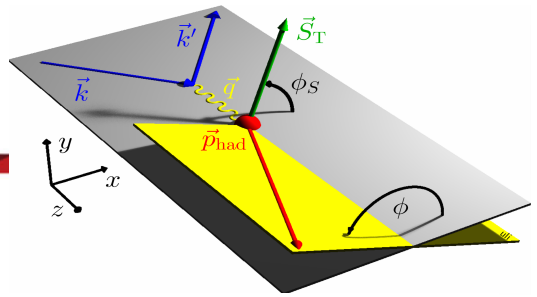
N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

Sivers and Collins Asymmetries

Sivers $\mathbf{f_{1T}^\perp} \otimes \mathbf{D_1}$

Collins $\mathbf{h_1} \otimes \mathbf{H_1^\perp}$

SIDIS Cross Section



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)}$$

$$\times \Big\{ F_{UU,T} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + S_L\,\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}$$

$$+ S_L\lambda_e\sqrt{1-\varepsilon^2}\,F_{LL} + |\boldsymbol{S}_T| \Big[\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} \quad \mathbf{f_{1T}^{\perp}}$$

$$+ \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \Big]$$

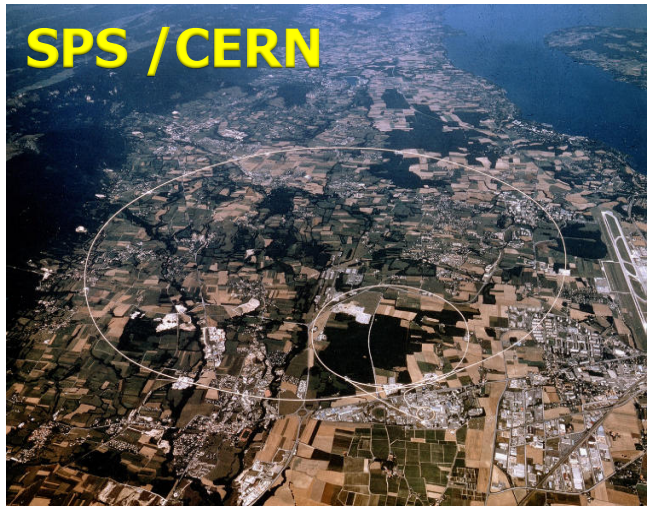
$$\mathbf{h_1}$$

$$+ |\boldsymbol{S}_T|\,\lambda_e \Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \Big] \Big\},$$

TMDs with a Transversely Polarized Target

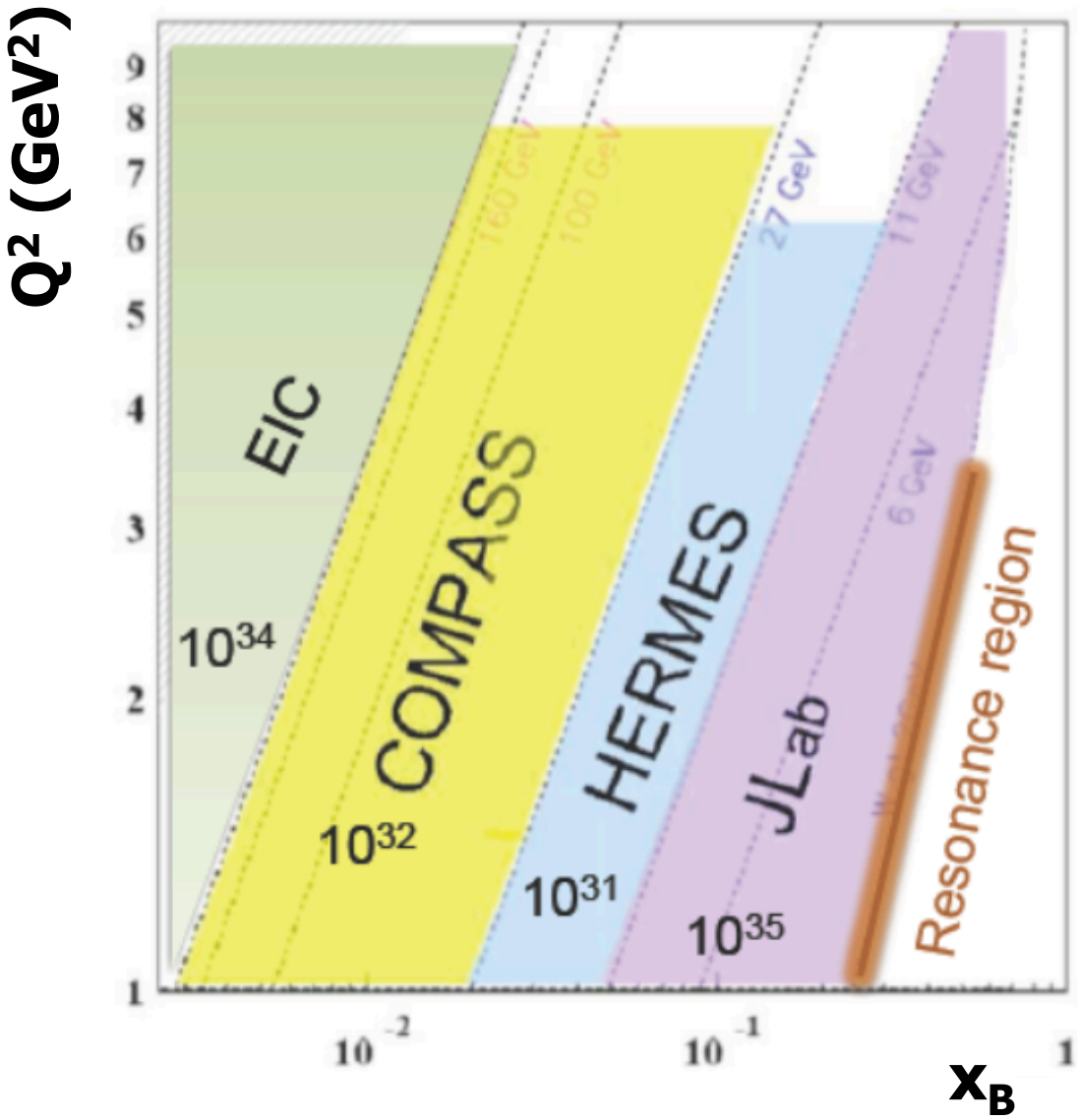
Polarized 27 GeV e+/e-
Polarized pure gaseous H&D targets
Excellent Particle ID
COLLINS & SIVERS π/k - H target

HALL-A
Polarized 6 GeV e^-
Polarized 3He target
High- Luminosity
COLLINS & SIVERS π



Polarized 160 GeV μ
Polarized ^{6}LiD & NH_3 targets
High-Energy
COLLINS & SIVERS π/k – H&D targets

Kinematical Coverage



FIXED TARGET EXPERIMENTS

COMPASS, HERMES
→ 0.006/0.02<xB<0.3 :
gluons/valence and sea quarks

JLab/JLab@12GeV
→ $0.1<x_B<0.7$
valence quarks

FUTURE COLLIDER EXP.: EIC?

$10^{-4}<x_B<0.02$:
Gluons and sea quarks

Sivers

Sivers PDF describes the number density of unpolarised quarks inside a transversely polarised proton:
correlation between the intrinsic transverse momentum of quarks and the nucleon spin.

$$\mathbf{A_{Siv}} = \frac{\sigma_{\mathbf{UT}}}{\sigma_{\mathbf{UU}}} \propto \mathbf{A_{UT}^{\sin(\phi_h - \phi_S)}} \mathbf{\sin(\phi_h - \phi_S)}$$

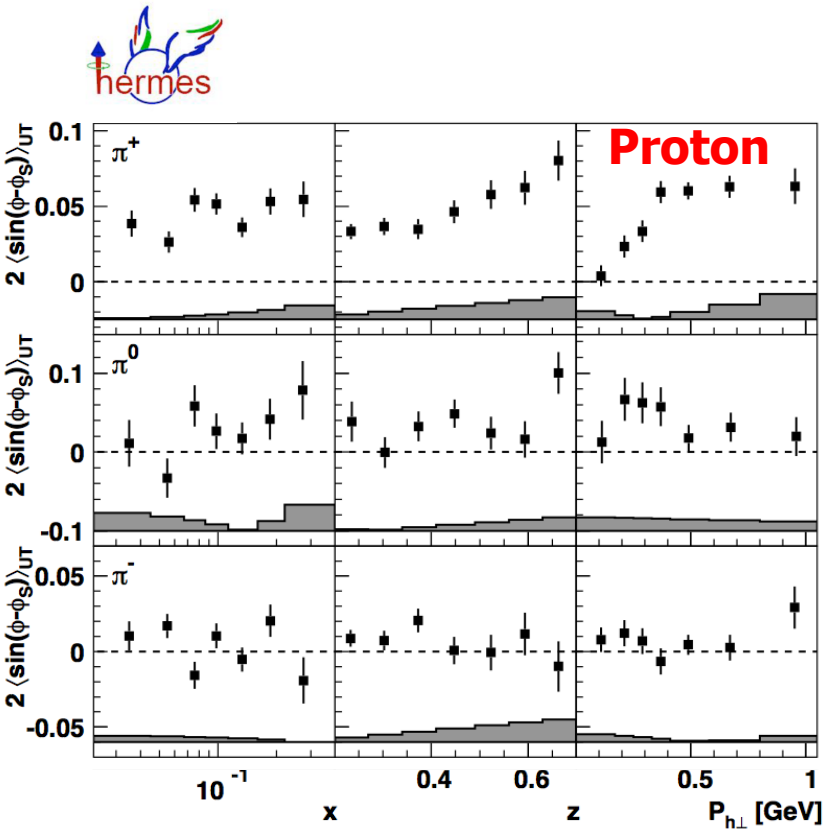
How to extract it from asymmetries measurements?

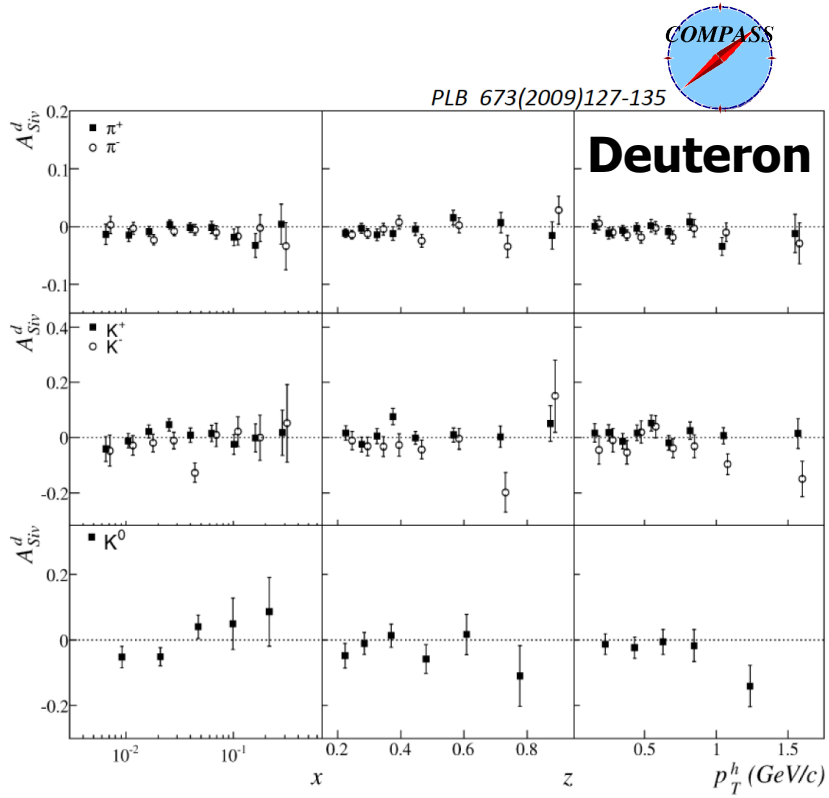
$$\mathbf{A_{Siv}} = \frac{\sum_q \mathbf{e_q^2} \cdot \mathbf{f_{1T}^{\perp q}} \otimes \mathbf{D_{1q}^h}}{\sum_q \mathbf{e_q^2} \cdot \mathbf{f_1^q} \cdot \mathbf{D_{1q}^h}}$$

Sivers Function Extraction: step 1

1_Take the measured asymmetries

(Proton data from HERMES & COMPASS, deuteron data from COMPASS)





- Asymmetries on deuteron compatible with zero
  → cancellation between $f_{1T}^{\perp u}(x)$ $f_{1T}^{\perp d}(x)$?

Sivers Function Extraction : step 2

2_Write the explicit Sivers expressions for pions on a proton and deuteron target

proton

$$A_{Sivers}^{p,\pi^+} = -\frac{4f_{1T}^{\perp u}D_1 + f_{1T}^{\perp d}D_2 + 4f_{1T}^{\perp \bar{u}}D_2 + f_{1T}^{\perp \bar{d}}D_1 + (f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{4uD_1 + dD_2 + 4\bar{u}D_2 + \bar{d}D_1 + (s+\bar{s})D_2}$$

$$A_{Sivers}^{p,\pi^-} = -\frac{4f_{1T}^{\perp u}D_2 + f_{1T}^{\perp d}D_1 + 4f_{1T}^{\perp \bar{u}}D_1 + f_{1T}^{\perp \bar{d}}D_2 + (f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{4uD_2 + dD_1 + 4\bar{u}D_1 + \bar{d}D_2 + (s+\bar{s})D_2}$$

deuteron

$$A_{Sivers}^{d,\pi^+} = -\frac{(f_{1T}^{\perp u} + f_{1T}^{\perp d})(4D_1 + D_2) + (f_{1T}^{\perp \bar{u}} + f_{1T}^{\perp \bar{d}})(4D_2 + D_1) + 2(f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{(u+d)(4D_1 + D_2) + (\bar{u}+\bar{d})(4D_2 + D_1) + 2(s+\bar{s})D_2}$$

$$A_{Sivers}^{d,\pi^-} = -\frac{(f_{1T}^{\perp u} + f_{1T}^{\perp d})(4D_2 + D_1) + (f_{1T}^{\perp \bar{u}} + f_{1T}^{\perp \bar{d}})(4D_1 + D_2) + 2(f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{(u+d)(4D_2 + D_1) + (\bar{u}+\bar{d})(4D_1 + D_2) + 2(s+\bar{s})D_2}$$

$D_{1,2}$ = Fragmentation Function (FFs)

$u, d, s, \bar{u}, \bar{d}, \bar{s}$ = f(x) Upolarized Parton Distribution Functions (PDFs)

Sivers Function Extraction : step 2

2_Write the explicit Sivers expressions for pions on a proton and deuteron target

proton

$$A^{p,\pi^+}_{Sivers} = -\frac{4f_{1T}^{\perp u}D_1 + f_{1T}^{\perp d}D_2 + 4f_{1T}^{\perp \bar{u}}D_2 + f_{1T}^{\perp \bar{d}}D_1 \; \cdots \; + f_{1T}^{\perp \bar{s}})D_2}{4uD_1 + dD_2 + 4\bar{u}D_2 + \bar{d}D \; \cdots \; \bar{s})D_2}$$

$$A^{p,\pi^-}_{Sivers} = -\frac{4f_{1T}^{\perp u}D_2 + f_{1T}^{\perp d}D_1 + 4f_{1T}^{\perp \bar{u}}D_1 \; \cdots \; D_2 + (f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{4uD_2 + dD_1 + 4\bar{u} \; \cdots \; D_2 + (s + \bar{s})D_2}$$

deuteron

$$A^{d,\pi^+}_{Sivers} = -\frac{(f_{1T}^{\perp u} + f_{1T}^{\perp d})(4D_1 + D_2 \; \cdots \; + f_{1T}^{\perp \bar{d}})(4D_2 + D_1) + 2(f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{(u+d)(4D_1 + D \; \cdots \; + (\bar{u} + \bar{d})(4D_2 + D_1) + 2(s + \bar{s})D_2}$$

$$A^{d,\pi^-}_{Sivers} = -\frac{(f_{1T}^{\perp u} + f_{1T}^{\perp d})(4D_2 + D_1) + (f_{1T}^{\perp \bar{u}} + f_{1T}^{\perp \bar{d}})(4D_1 + D_2) + 2(f_{1T}^{\perp s} + f_{1T}^{\perp \bar{s}})D_2}{(u+d)(4D_2 + D_1) + (\bar{u} + \bar{d})(4D_1 + D_2) + 2(s + \bar{s})D_2}$$

Homework: derive them!

$D_{1,2}$ = Fragmentation Function (FFs)

$u, d, s, \bar{u}, \bar{d}, \bar{s}$ = f(x) Upolarized Parton Distribution Functions (PDFs)

Sivers Function Extraction : step 2 (con't)

$$\pi^+ = u\bar{d}$$
$$\pi^- = d\bar{u}$$

Note 1.

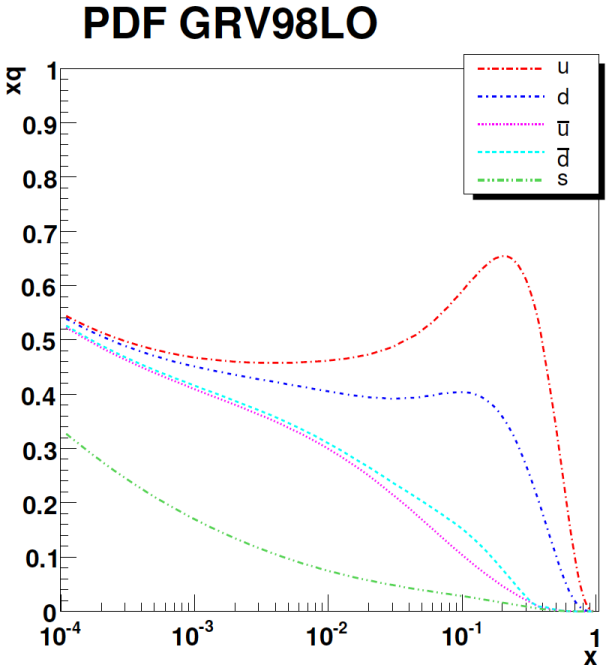
$$D_d^{\pi^+} = D_{s\bar{s}}^{\pi^+} < D_u^{\pi^+} = D_{\bar{d}}^{\pi^+}$$ Derived from the valence structure of pions

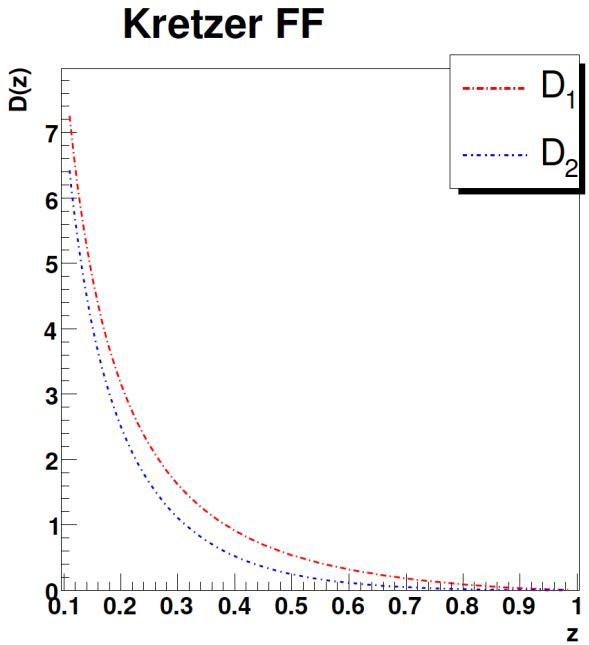
$$D_1 = D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} = D_{\bar{u}}^{\pi^-} = D_d^{\pi^-}$$

$$D_2 = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} = D_d^{\pi^+} = D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}$$

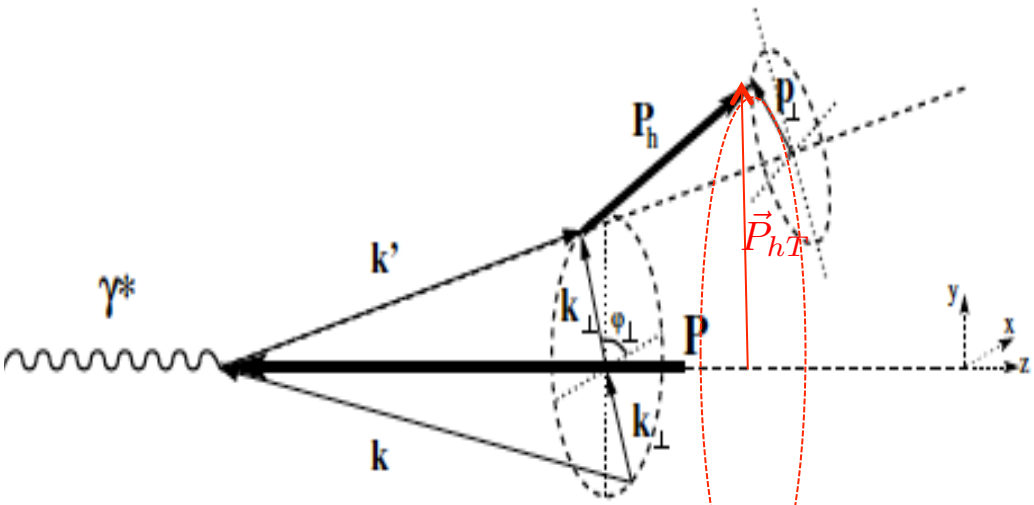
Note 2.

PDFs & FFs are taken from existing parametrization





Hadron Transv. Momentum vs Quark Transv. Momentum



$$\vec{P}_{hT} = z\vec{k}_\perp + \vec{p}_\perp$$

This is the observable

These variables are not experimentally accessible. They are convoluted in an integral and can assume all values from 0 to ∞

- γ^* hits quark with transverse momentum $k_\perp$
- Quark propagates in the k' direction then hadronizes in the hadron h
- The detected hadron has:
  - momentum P_h
  - transverse momentum P_{hT}
  - Energy z
  - has been generated by a quark with transverse momentum $p_\perp$

Sivers Function Extraction : step 3

3_Choice of the functional form & transverse momentum dependence (ex.: M. Anselmino et al arX:1107.4446)

Adopt the usual (and convenient) gaussian factorization for the transverse momentum dependent distribution and fragmentation functions

$$f_{q/p}(x,k_\perp) = f_q(x)\frac{1}{\pi < k_\perp^2 >}e^{-\frac{k_\perp^2}{<k_\perp^2>}}$$

$$D_{q/p}(z,p_\perp) = D_q(z)\frac{1}{\pi < p_\perp^2 >}e^{-\frac{p_\perp^2}{<p_\perp^2>}}$$

$$< p_\perp^2 >= 0.20 GeV^2$$
$$< k_\perp^2 >= 0.25 GeV^2$$

Estimate average values of the transverse momenta of quarks inside a proton, **k**⊥ and of final hadrons inside the fragme[illegible] [illegible]ark **p**⊥ from unpolarized cross [illegible]

$$f_{1T}^{\perp}(x,k_\perp) = 2N_q(x)h(\perp)f_{q/p}(x,k_\perp)$$

$$N_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) =$$

Any contribution of the sea Sivers function

N_u α_u β_u N_d α_d β_d M_1 ➜ 7 free parameters

Contraint on for the Sivers functions: positivity bound $|f_{1T}^{\perp}| < f(x)$

Sivers Function Extraction : step 4

4_Minimization of the χ^2

$$\chi^2 = \sum_1 \frac{(A_i^{exp} - A_i^{th}(\alpha_1, ..., \alpha_N))^2}{\sigma_1^2}$$

α_i = parameters of the unknown function

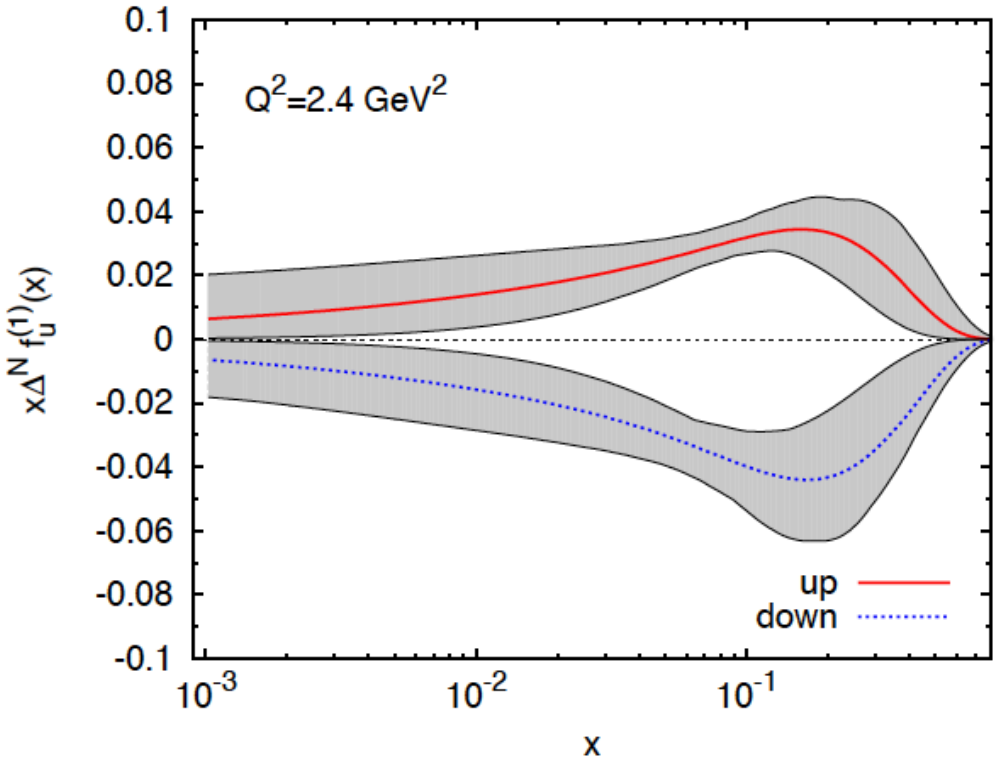
A_i^{exp} = measured asymmetries

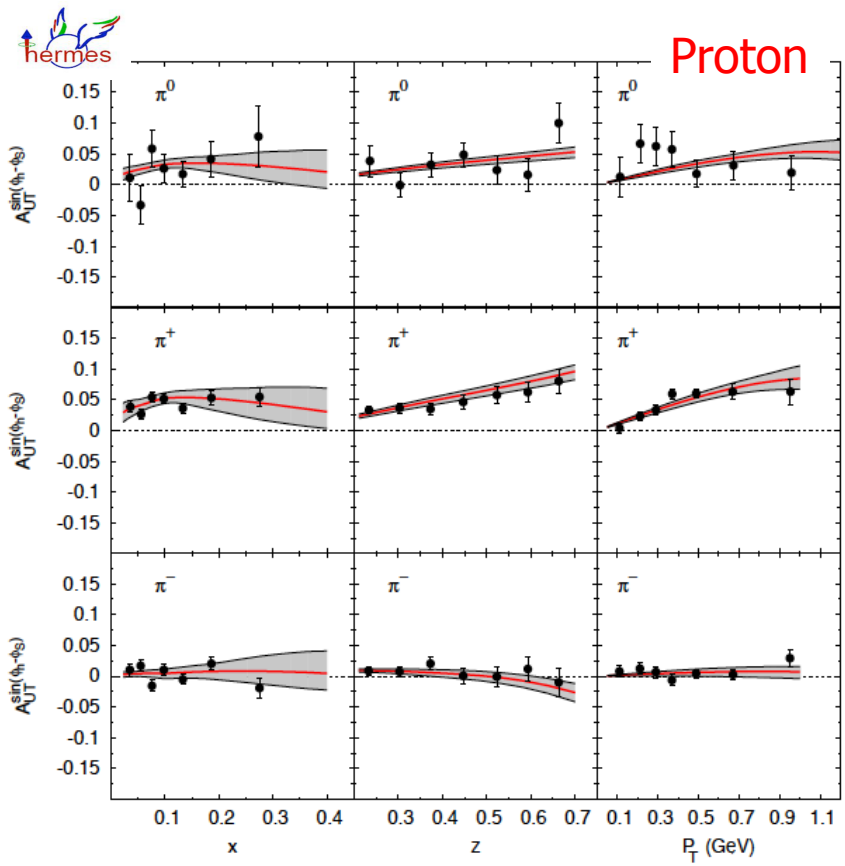
σ_i = errors of the measured asymmetries

A_i^{th} = theoretical expression for the asymmetries

The minimization of the χ^2 gives the set of parameters, together with their errors, that fit the experimental data

Sivers Function





- $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$ In agreement with the large color N_C QCD prediction
- Fit consistent with the QCD prediction of large x behaviour, $(1-x)^5$, for the Sivers function (β parameter ~5)

TMDs Evolution

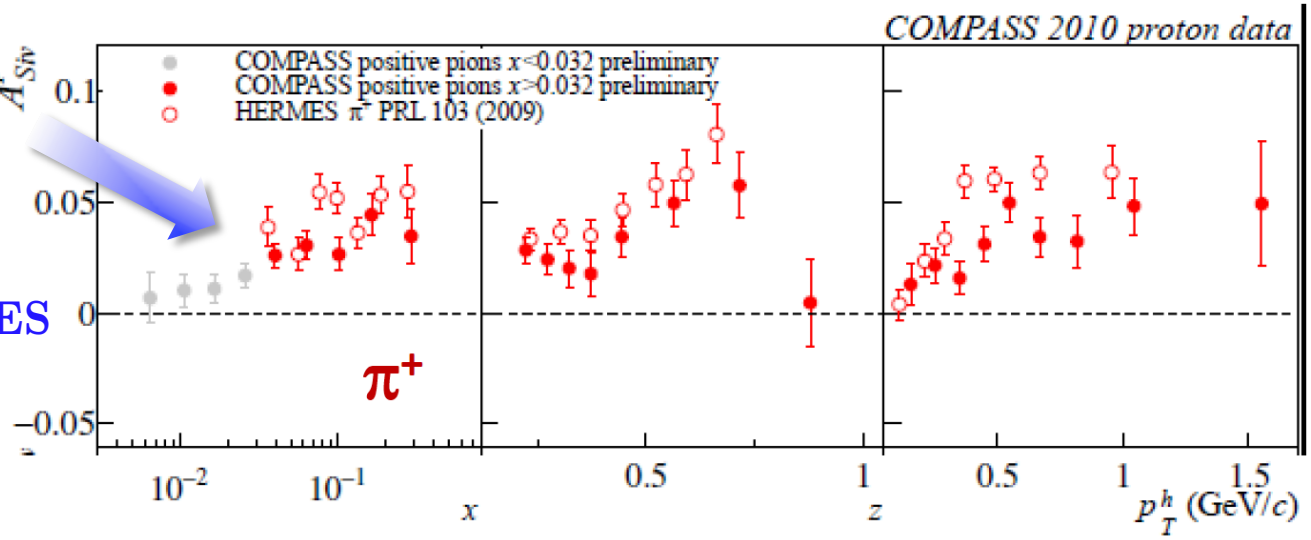
$A_{Siv}^{HERMES} > A_{Siv}^{COMPASS}$

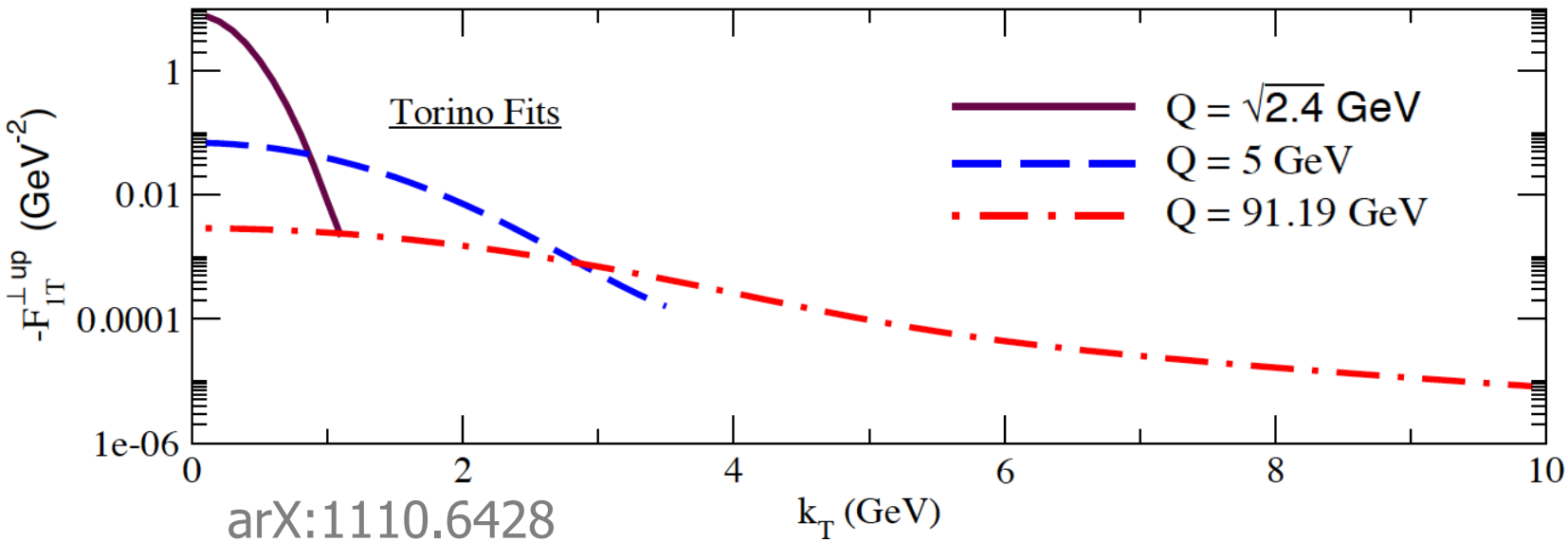
But…

$Q^2_{COMPASS} > Q^2_{HERMES}$

factor 2-3 in the last x bins

➜ Q^2 dependence of the Sivers effect plays a role



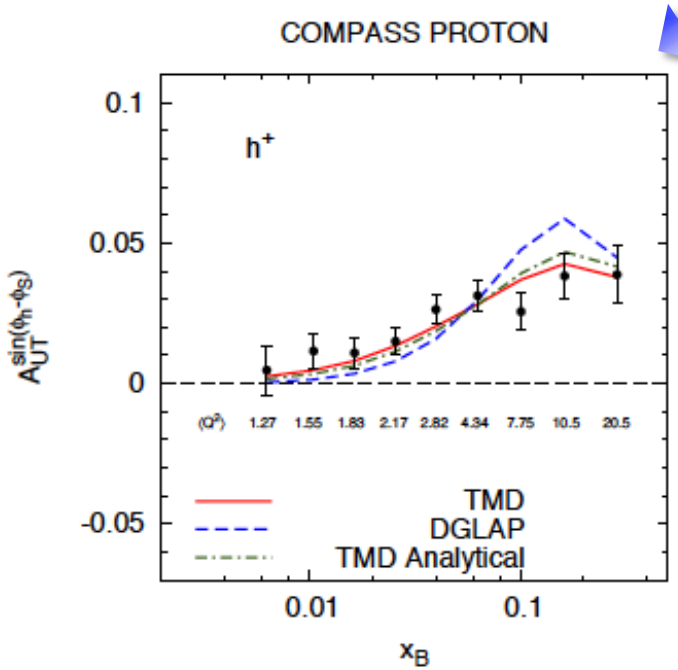


arX:1110.6428

Sivers Function

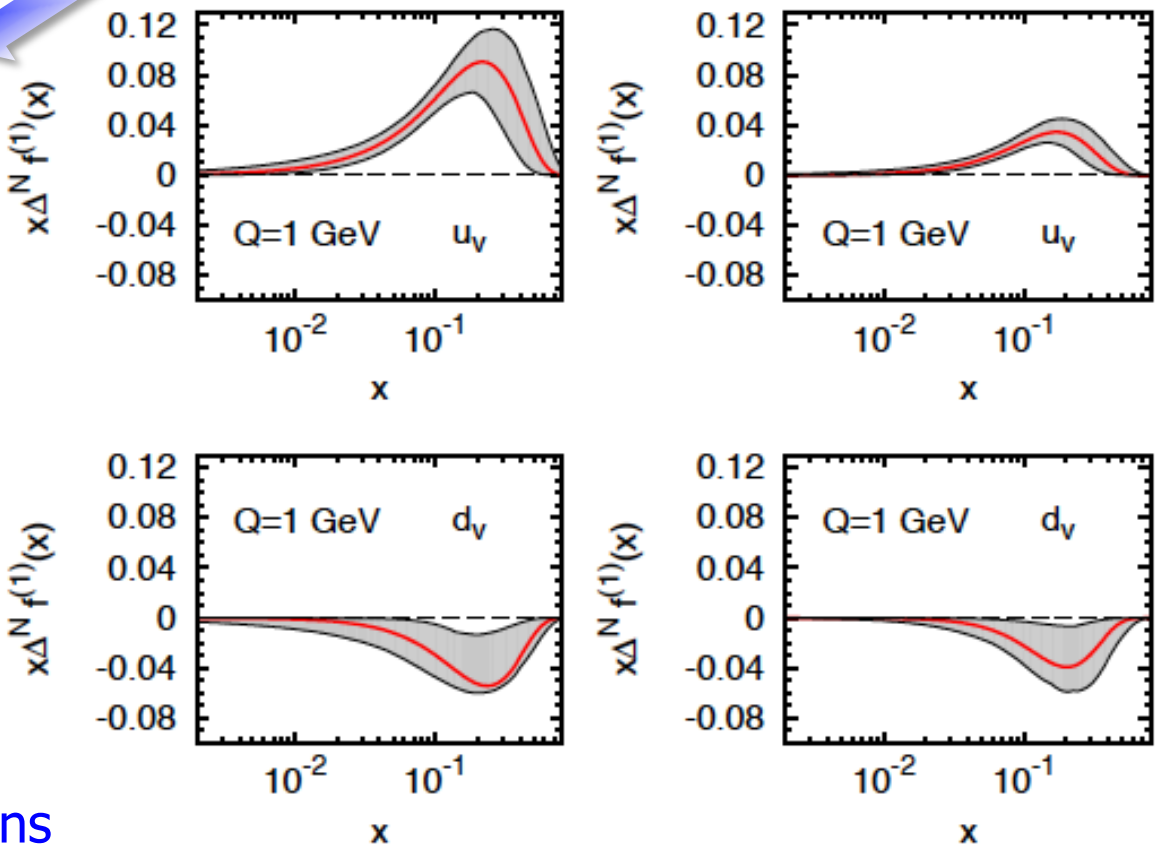
TMD Q^2 evolution has been worked out and added in global fits

(M. Anselmino et al arX:1204.1239)



With TMD Q^2 evolution

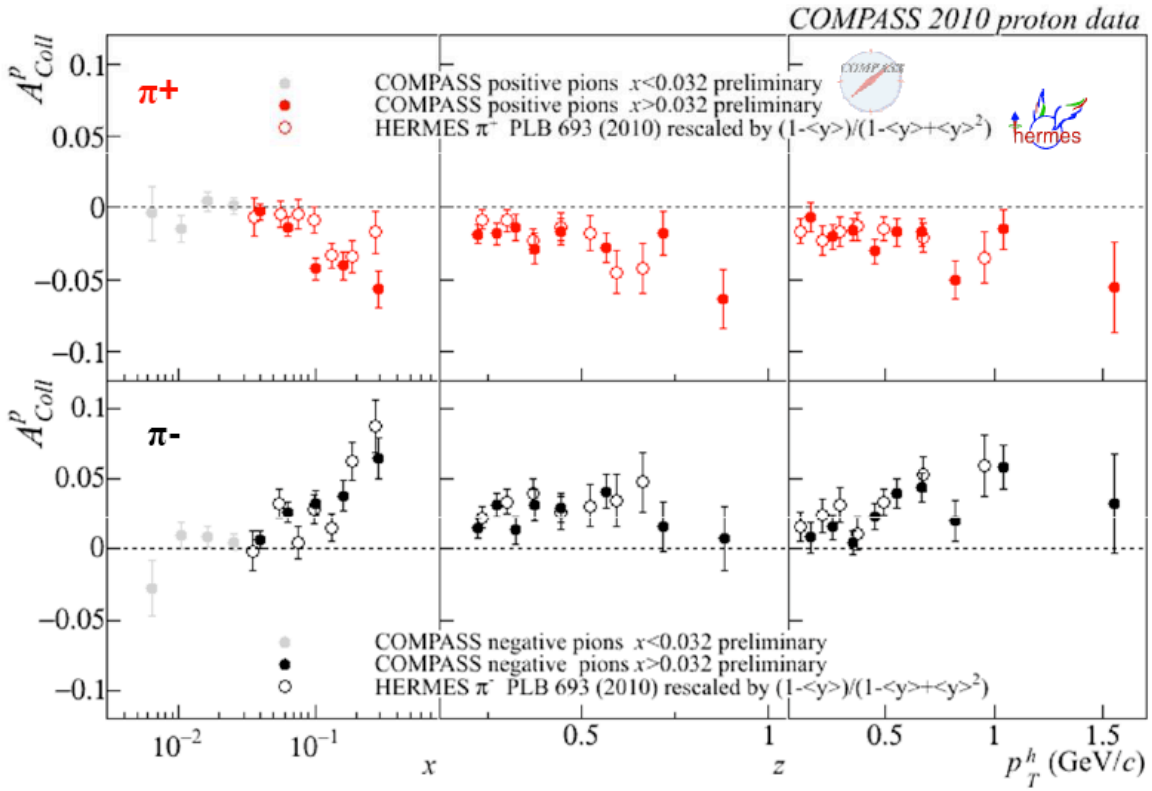
Without

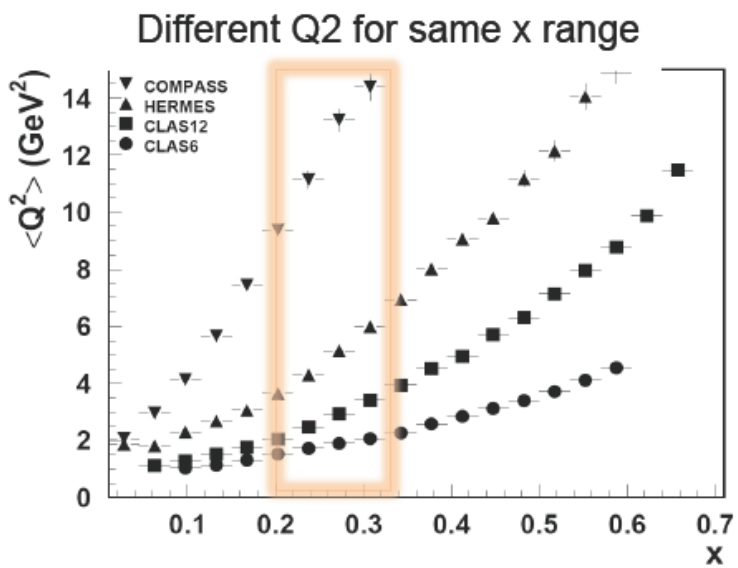


evidence of the phenomenological success of the TMD evolution equations

The Collins Amplitude

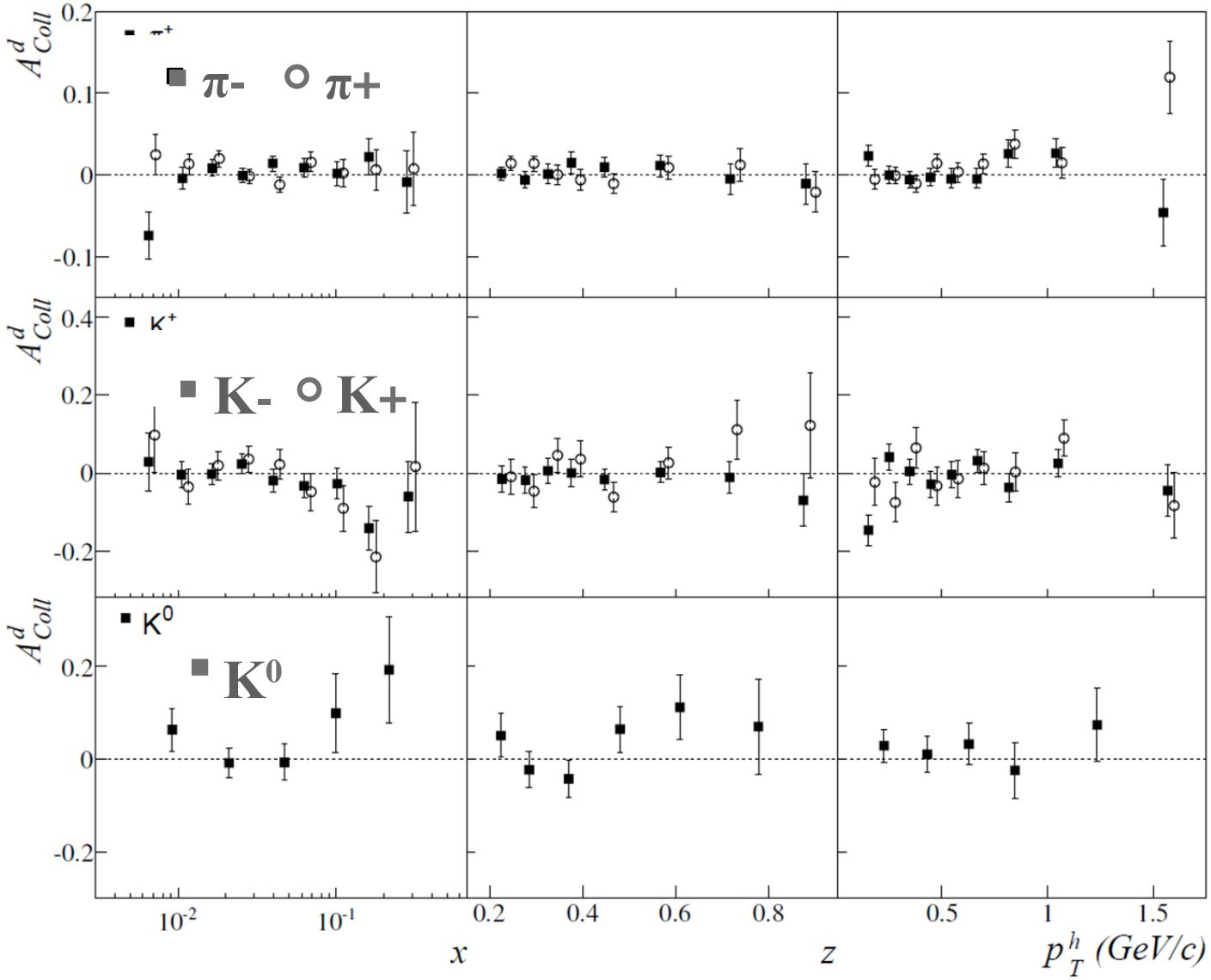
$h_1 \otimes H_1^{\perp}$





- Opposite sign for pions reveals Collins features: $H_{1fav}^{\perp} = -H_{1unf}^{\perp}$
- Consistent results at different Q^2 → No strong evolution

The Collins Asymmetry on Deuteron



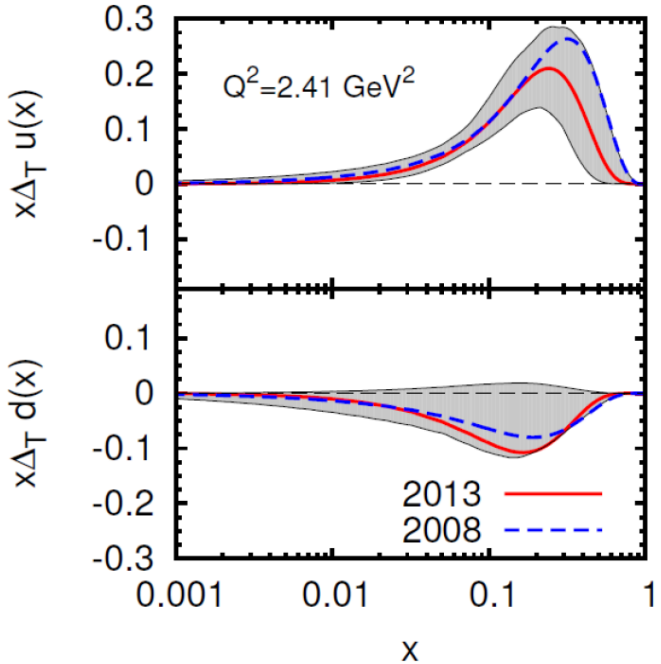
PLB 673 (2009) 127

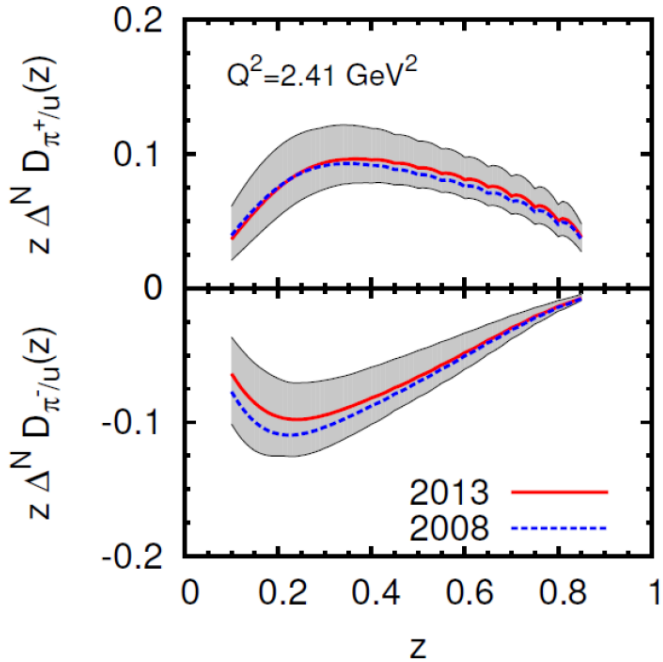
understood as

- u – d cancellation
- favored/ unfavored Collins FF

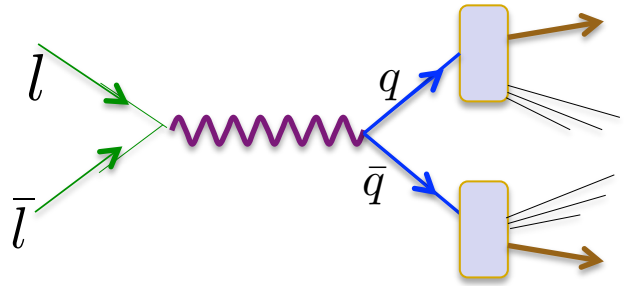
Extraction of the Collins & Transversity Functions

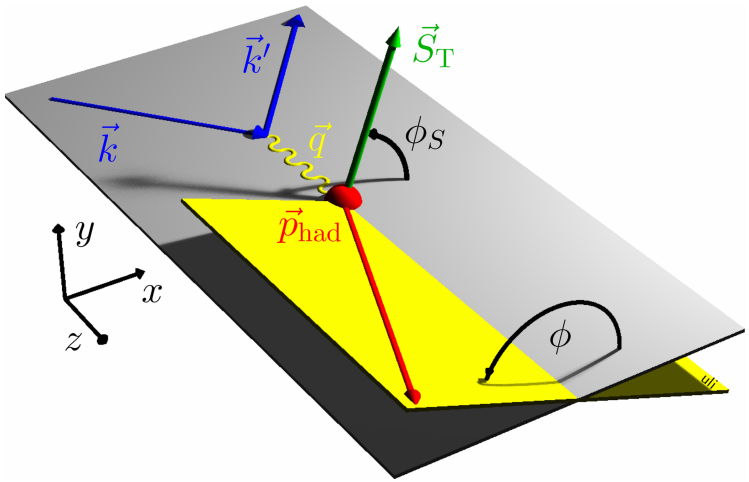
- The Collins case is more complicated with respect to the Sivers one:
  → in the asymmetry formula two set of unknown functions appear!
- Additional information on the Collins function can be achieved from e^+e^- process
- Different approaches in literature.The latest one makes a **simultaneous fit** of HERMES p, COMPASS p and d, Belle e^+e^- data (M. Anselmino et al., arXiv:1303.3822)

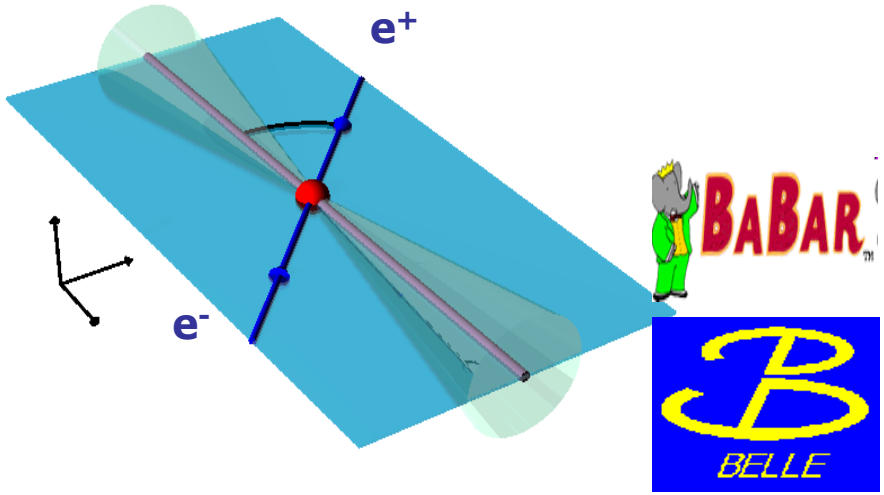




Collins Fragmentation Function in e^+e^-



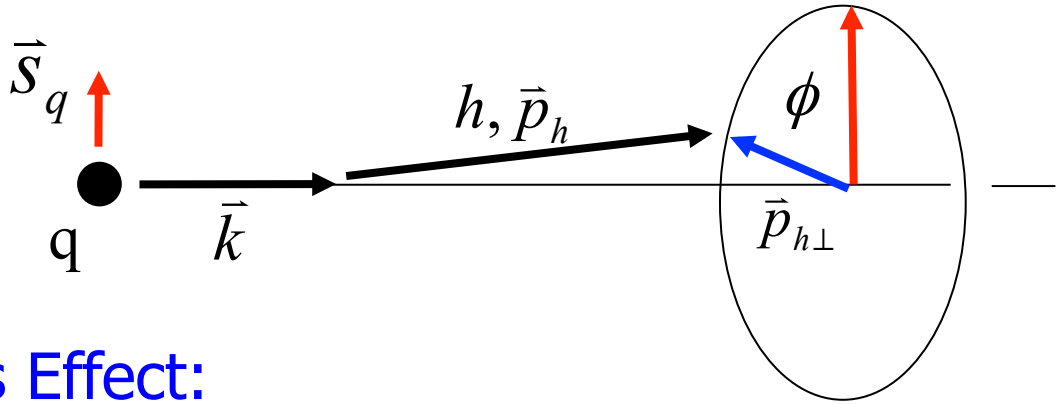




$$\vec{e}\vec{p} \rightarrow e'\pi X$$

$$e^+e^- \rightarrow \pi^+_{jet1}\pi^-_{jet2}X$$

Collins Effect in Quark Fragmentation



J.C. Collins, Nucl. Phys. B396, 161(1993)

Collins Effect:

Fragmentation of a transversely polarized quark *q* into spin-less hadron *h* carries an azimuthal dependence:

$$\propto (\vec{k} \times \vec{p_{h\perp}}) \cdot \vec{s}_q \propto sin\phi \qquad \rightarrow (\vec{k} \times \vec{s}_q) \cdot \vec{p_{h\perp}} \propto sin\phi$$

General Form of Fragmentation Functions:

Number density for finding hadron *h* from a transversely polarized quark, *q*:

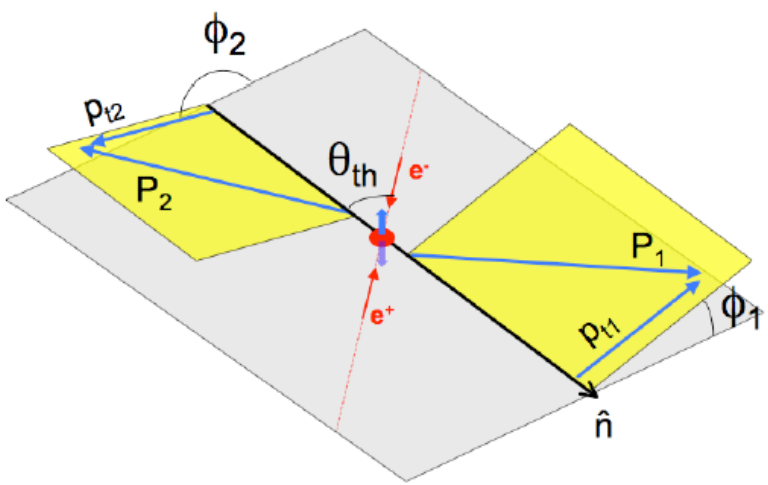
$$D^h_{q^\uparrow}(z, \vec{p}_{h\perp}) = \underbrace{D_1^{q,h}(z)}_{\text{unpolarized FF}} + \underbrace{H_1^{\perp q,h}(z, p^2_{h\perp})}_{\text{Collins FF}} \frac{\left(\hat{k} \times \vec{p}_{h\perp}\right) \cdot \vec{s}_q}{zM_h}$$

Collins FF in e+e- : Need Correlation between Hemispheres

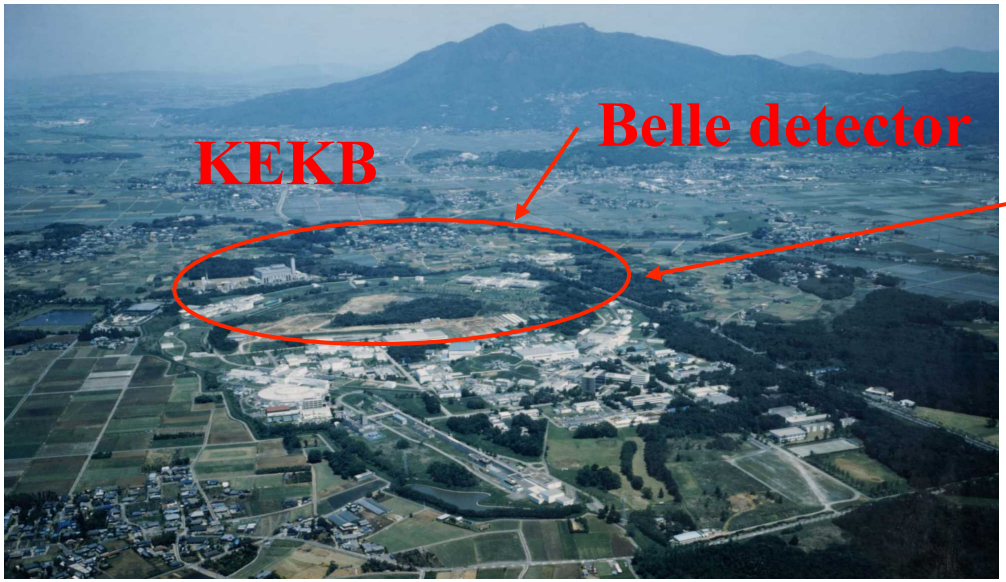
- Quark spin direction unknown: measurement of Collins function in one hemisphere is not possible: sinϕ modulation will average out.

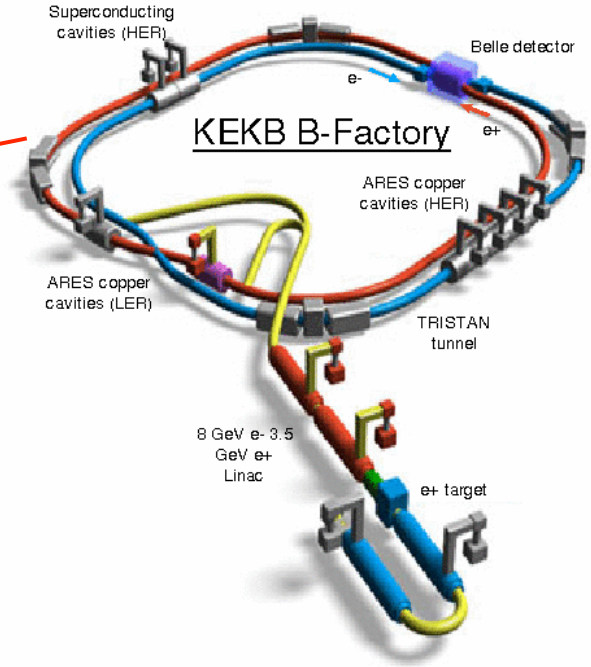
- Correlation between two hemispheres with sinϕ Collins single spin asymmetries results in cos(ϕ_1+ϕ_2) modulation of the observed di-hadron yield.

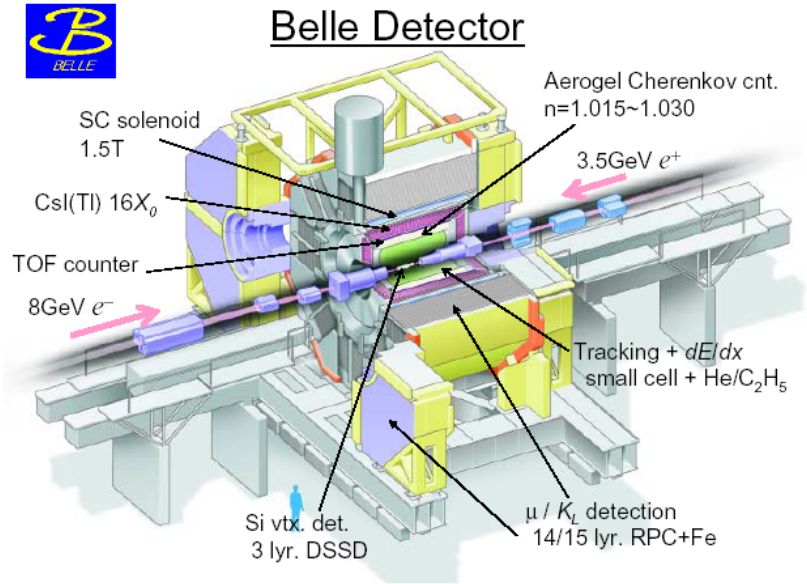
Measurement of azimuthal correlations for pion pairs around the jet axis in two-jet events



Belle @ KEKB

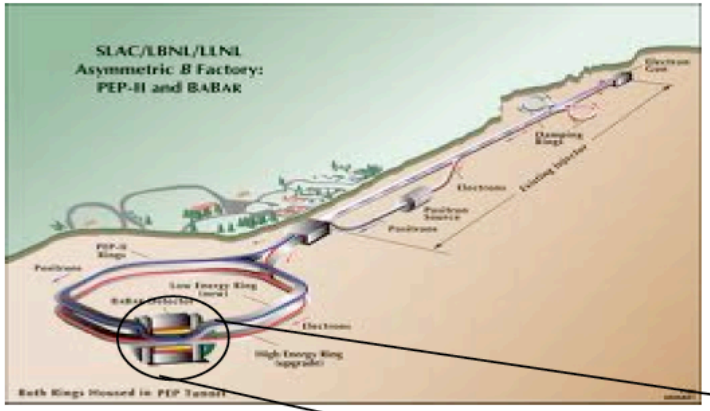






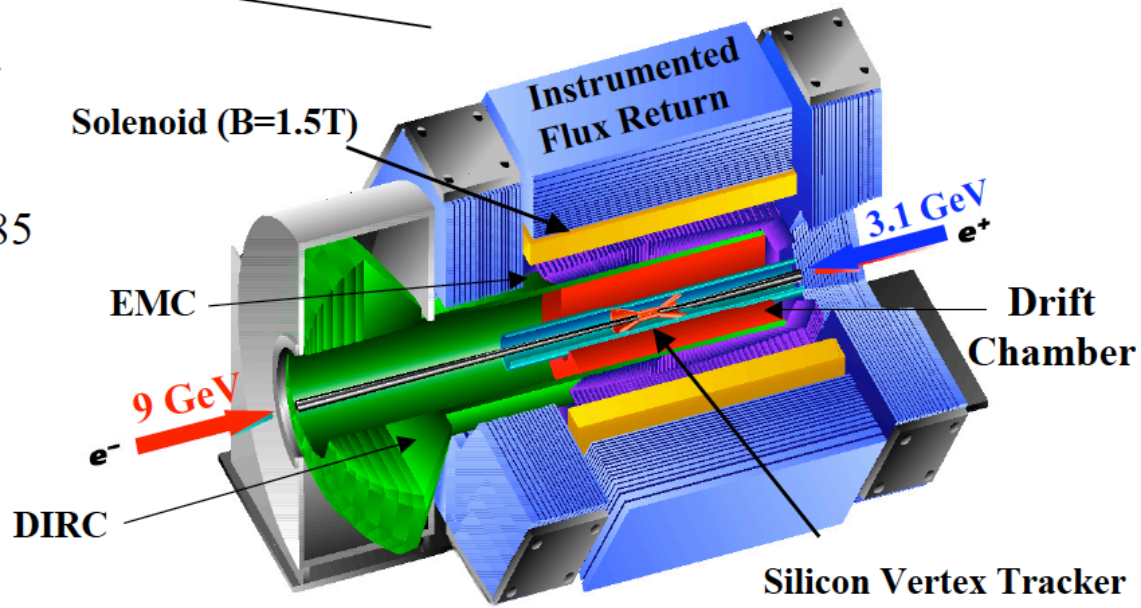
- Asymmetric collider
- 8GeV e^- + 3.5GeV e^+
- $\sqrt{s}$ = 10.58GeV (Y(4S))
- **L>1.5x10^{34}cm^{-2}s^{-1}**
- **Integrated Luminosity: >700 fb^{-1}**

PEP-II and the BaBar detector at SLAC



- Asymmetric-energy e^+e^- collider operating at the $\Upsilon(4S)$ resonance ($\sqrt{s}$=10.58 GeV)
  - High Energy Ring (**HER**): 9.0 GeV e^-
  - Low Energy Ring (**LER**): 3.1 GeV e^+
  - c.m.-lab boost, $\beta\gamma \approx 0.56$
- High luminosity: $\int \mathcal{L} \sim 500\ \mathrm{fb}^{-1}$

- Asymmetric detector
  - c.m. acceptance $-0.9<\cos\theta^*<0.85$ wrt e^- beam
- Excellent performance
  - good tracking, mass resolution
  - good γ, π^0 reconstruction
  - full e, μ, π, K, and p identification

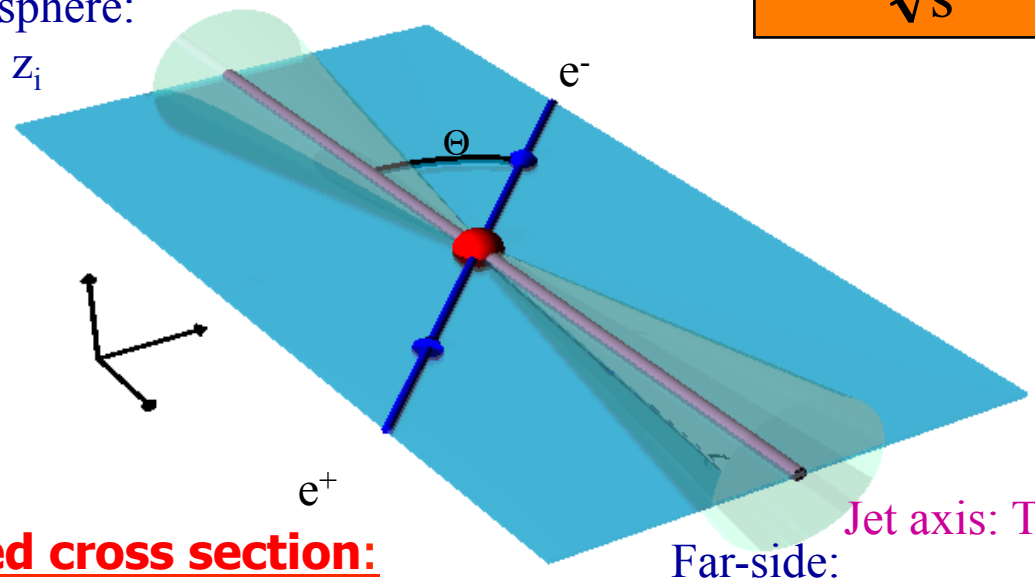


Event Structure @ Belle

e^+e^- CMS frame:

$$z = \frac{2E_h}{\sqrt{s}}, \quad \sqrt{s} = 10.52\ GeV$$

Near-side Hemisphere:
h_i, $i=1,N_n$ with z_i



Jet axis: Thrust

Far-side:
hj , j=1,Nf with zj

Spin averaged cross section:

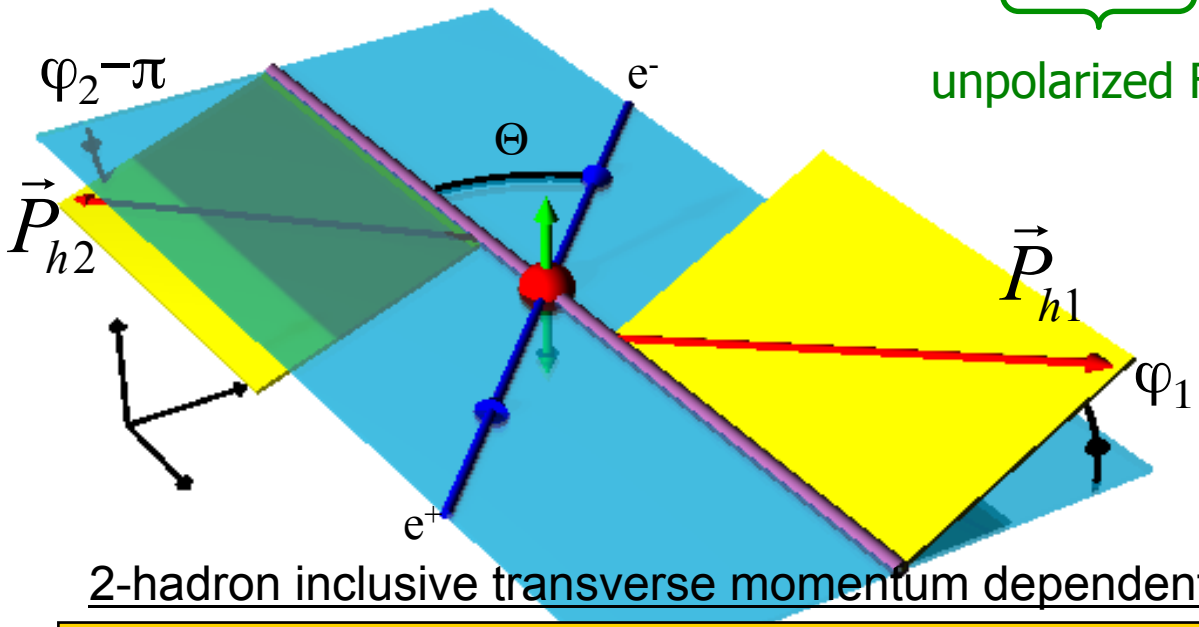
$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2} = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a^2 D_1(z_1) \bar{D}_1(z_2)$$

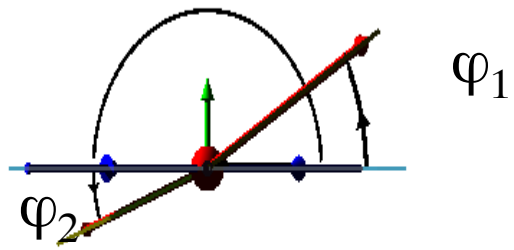
$$A(y) = \left(\frac{1}{2} - y + y^2\right) \overset{(cm)}{=} \frac{1}{4}\left(1 + cos^2\Theta\right)$$

Cross Section cos ($\phi_1+\phi_2$) method

e+e- CMS frame:

$$D^{h}_{q\uparrow}(z,\vec{p}_{h\perp}) = \underbrace{D_1^{q,h}(z)}_{\text{unpolarized FF}} + \underbrace{H_1^{\perp q,h}(z,p_{h\perp}^2)}_{\text{Collins FF}} \frac{\left(\hat{k}\times\vec{p}_{h\perp}\right)\cdot\vec{s}_q}{zM_h}$$





Observable: yield, $N_{12}(\varphi_1+\varphi_2)$ of $\pi^+\pi^-$ pairs

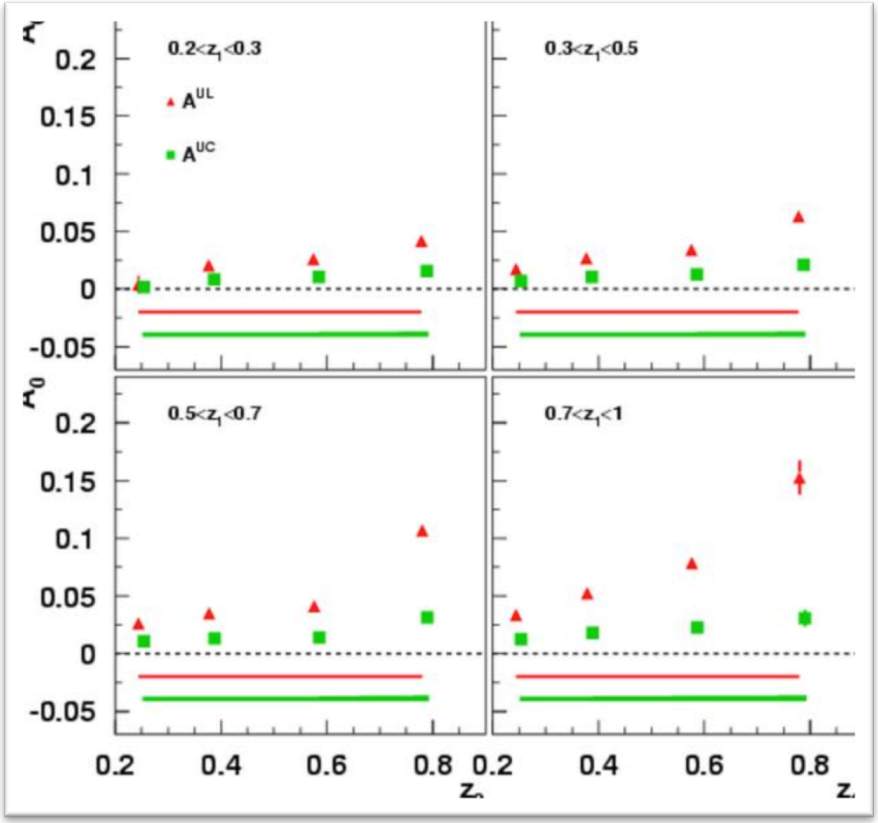
2-hadron inclusive transverse momentum dependent cross section:

$$\frac{d\sigma(e^+e^- \to h_1h_2X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} = \cdots B(y)\cos(\varphi_1+\varphi_2)H_1^{\perp[1]}(z_1)\overline{H}_1^{\perp[1]}(z_2)$$

$$B(y) = y(1-y) \overset{cm}{=} \frac{1}{4}\sin^2\Theta$$

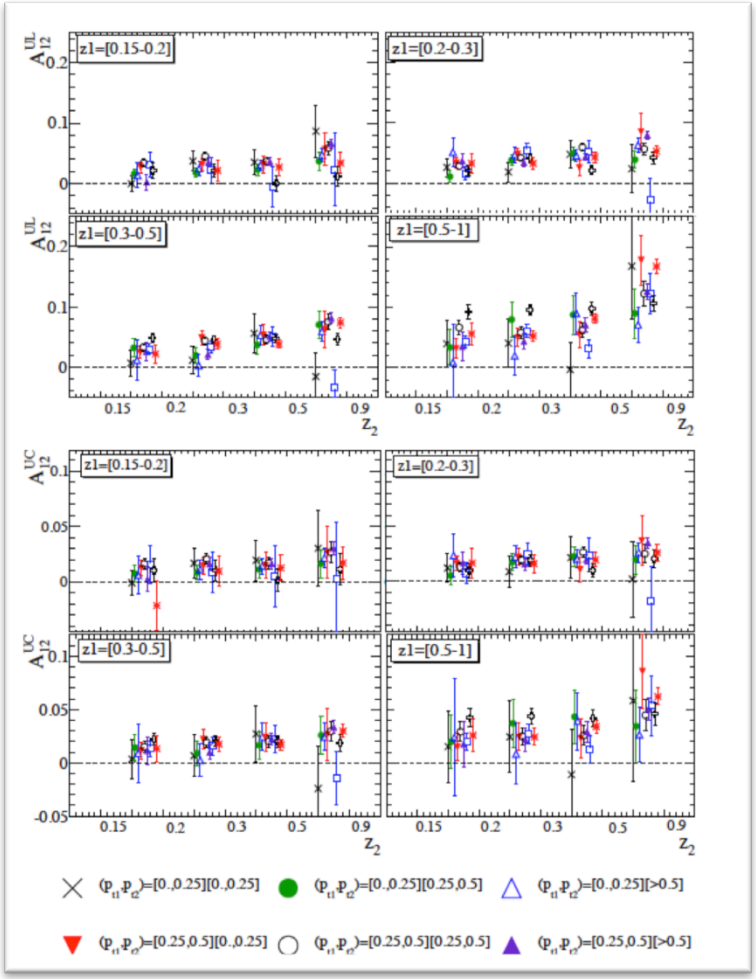
Final Collins Results

Belle



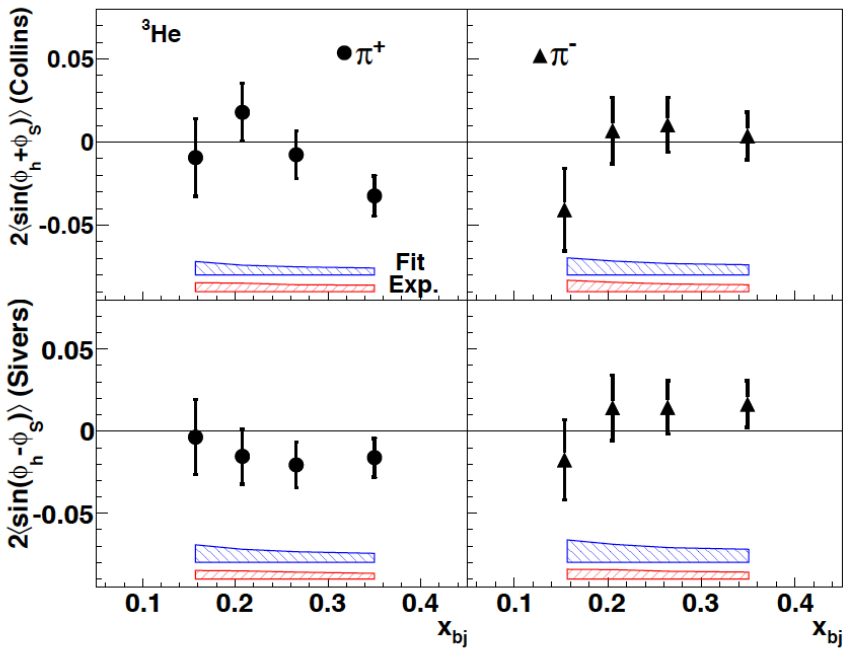
Phys.Rev.D78:032011,2008

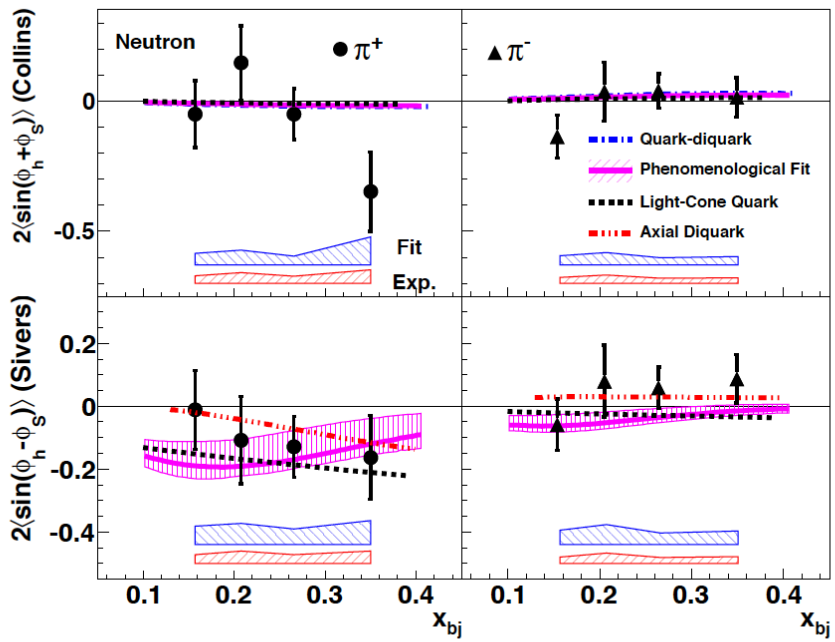
BaBar



arxiv:1309.5278

Neutron data





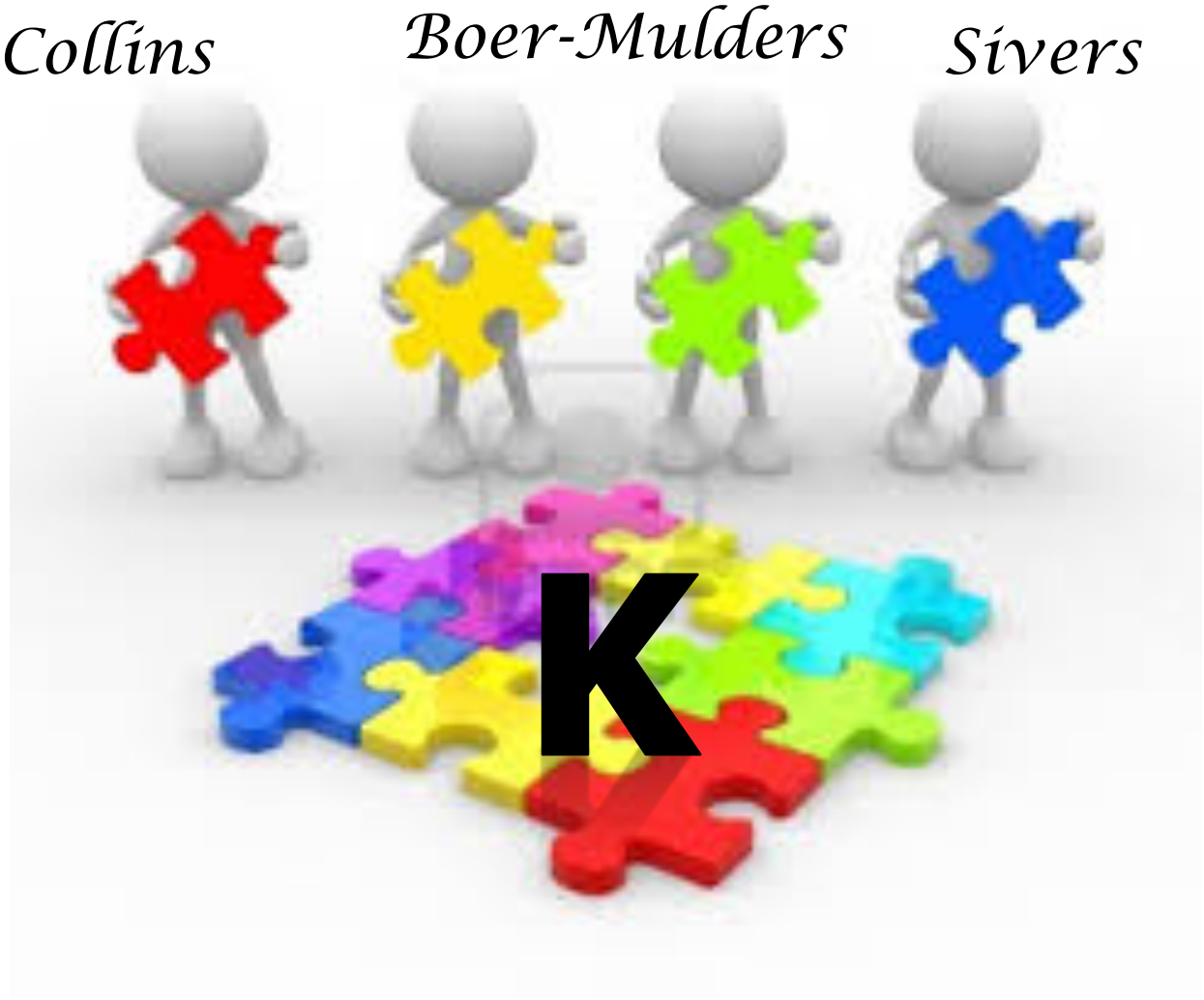
^{3}He:

Collins moments: consistent with zero, except for the π^+ moment at x = 0.35
Sivers moments: consistent with zero for π^- while for π^+ favor negative values.

Neutron:

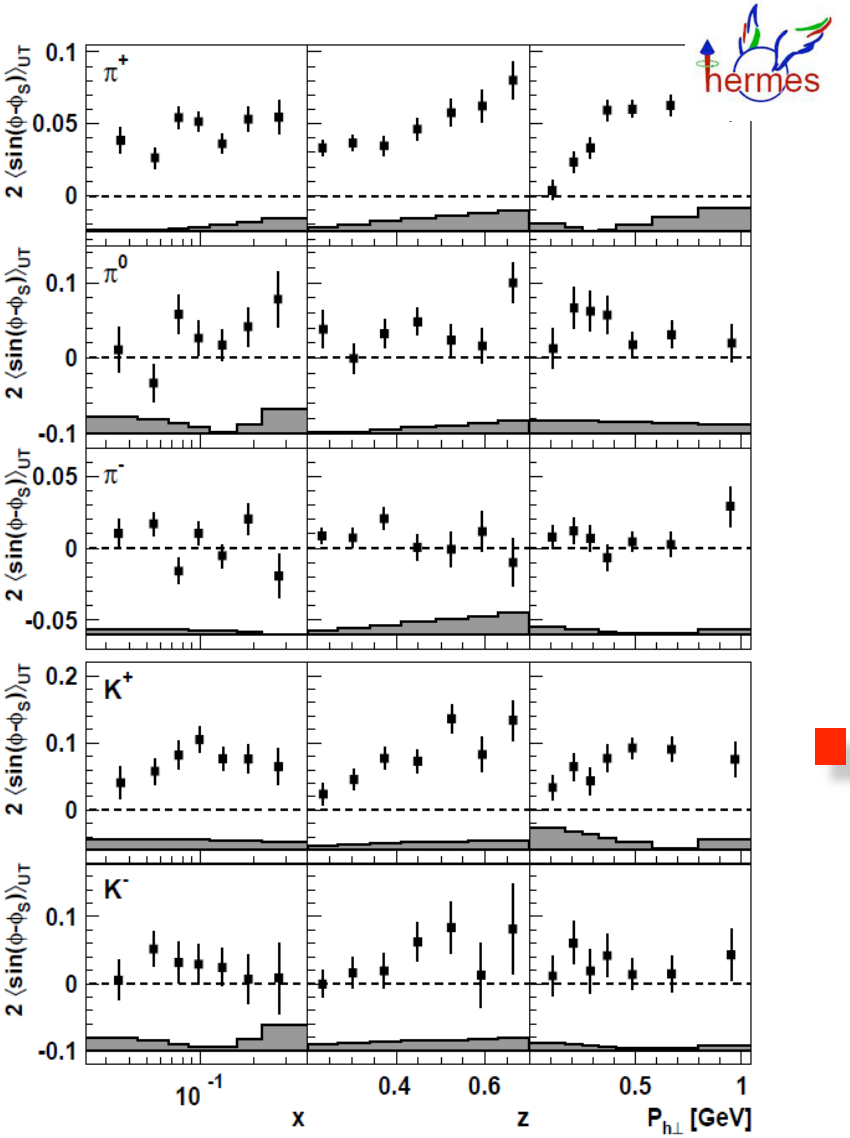
Largely consistent with the predictions of phenomenological fits and quark model calculations.

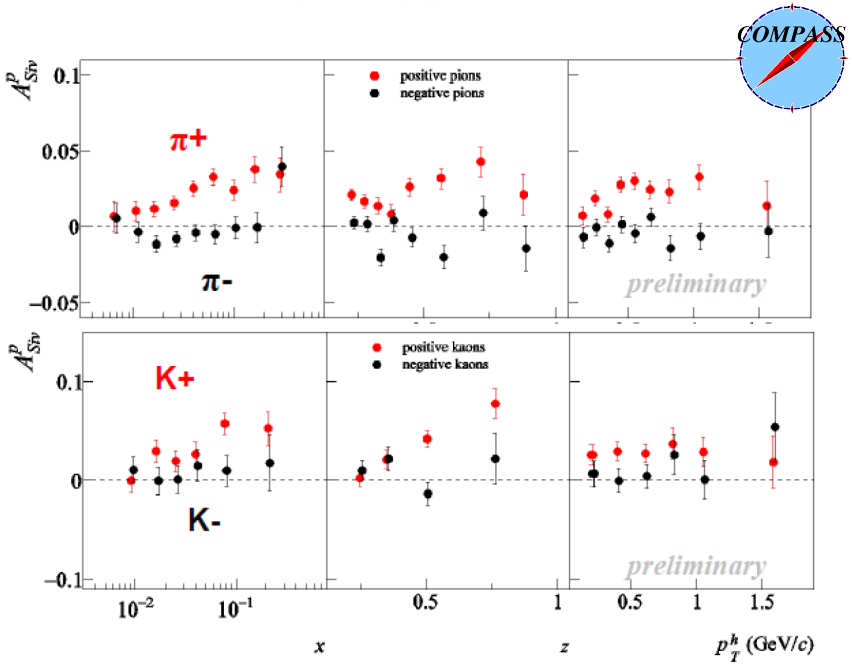
The Kaon Puzzle



The Sivers Amplitude

$f_{1T}^{\perp} \otimes D_1$





syst error ~ 0.6 stat error

- Puzzle in kaon signals:

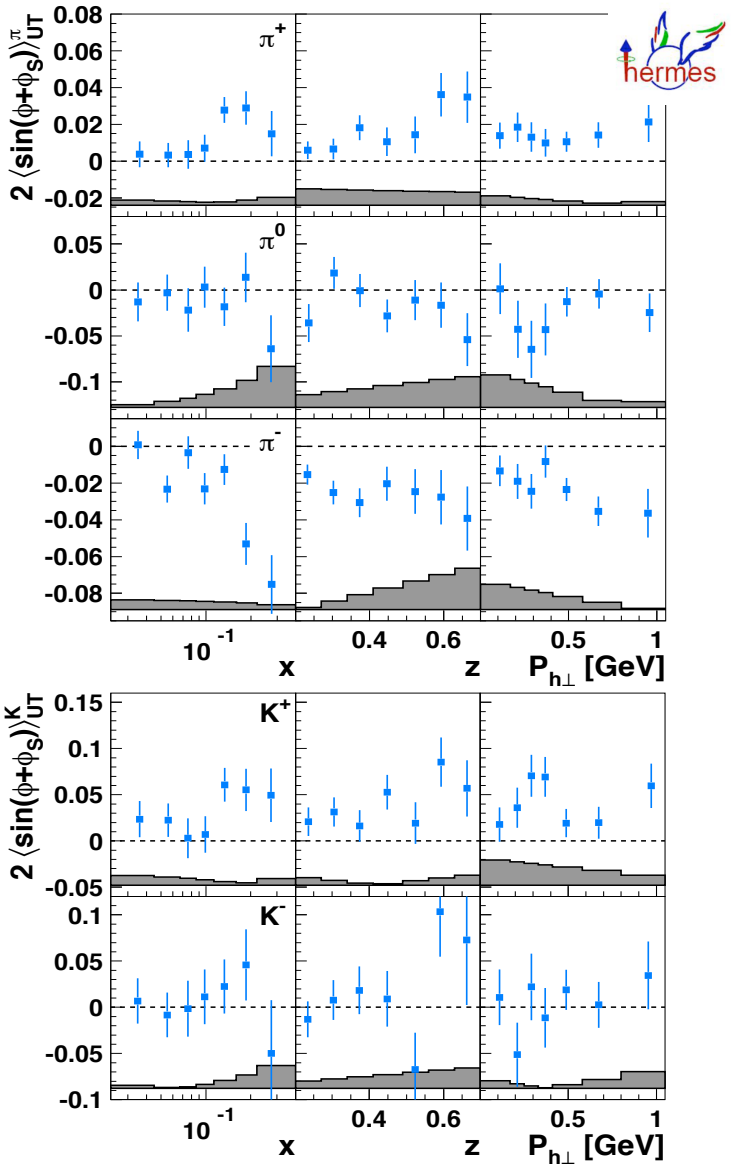
K^+ amplitude larger than π^+

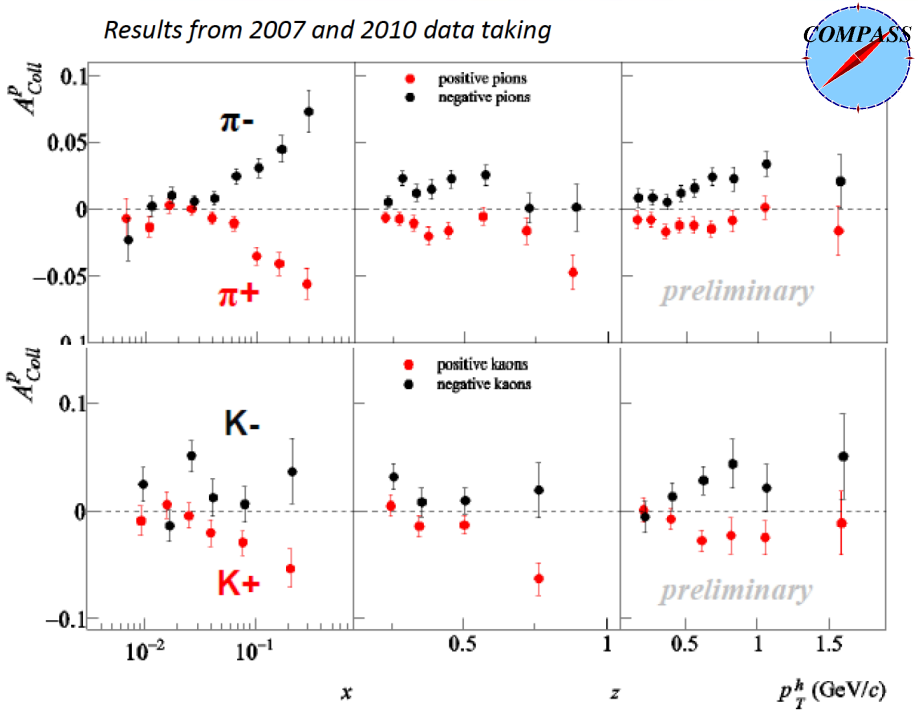
➔Non trivial role of see quarks

$$\pi^+ \equiv \left|u\bar{d}\right\rangle,\ K^+ \equiv \left|u\bar{s}\right\rangle$$

The Collins Amplitude

$h_1 \otimes H_1^{\perp}$





- Puzzle in kaon signals:
  - K^+ amplitude larger than π^+
  - K^+ amplitudes are not in agreement