Global analysis of polarized parton distribution functions

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Overview of JAM Analysis

- Extracting reliable spin PDFs through a $\chi^2$ fit to world polarized scattering data

- **Data:**
  - Fitting ~2880 data points
  - Less stringent kinematic cuts
  - Inclusion of new JLab data

- **Theory:**
  - Finite-$Q^2$ and nuclear corrections implemented
Deep-inelastic Scattering (DIS)

- **Unpolarized**

\[ \sigma = \sigma_{\text{Mott}} \left( \frac{2}{M} \tan^2 \theta \frac{1}{2} F_1(x, Q^2) + \frac{1}{\nu} F_2(x, Q^2) \right) \]

- **Polarized**

\[ \sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow} = \sigma_{\text{Mott}} \frac{1}{M\nu} 4\tan^2 \theta \frac{1}{2} \left( [E + E'\cos\theta] g_1 - 2Mxg_2 \right) \]

\[ \sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\leftarrow} = \sigma_{\text{Mott}} \frac{1}{M\nu} 4E'\tan^2 \theta \frac{1}{2} \sin\theta \left( g_1 - \frac{2E}{\nu} g_2 \right) \]

\[ Q^2 = -q^2 \approx 4EE'\sin^2 \frac{\theta}{2} \]

\[ x = \frac{Q^2}{2M\nu} \]

\[ W^2 = M^2 + \frac{Q^2(1-x)}{x} \]
Polarized DIS Observables

- Experiment measure electron polarization asymmetries

\[ A_{\parallel} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D \left( A_1 + \eta A_2 \right) \]

\[ A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\leftarrow}} = d \left( A_2 + \xi A_1 \right) \]

where \( A_1 \) and \( A_2 \) are defined in terms of polarized structure functions \( g_1 \) and \( g_2 \)

\[ A_1 = \frac{2x}{(1 + \gamma^2)F_2 - F_L} \left( g_1 - \gamma^2 g_2 \right) \]

\[ A_2 = \frac{2x}{(1 + \gamma^2)F_2 - F_L} \gamma \left( g_1 + g_2 \right) \]

\[ g_i = g_i(x, Q^2) \]
Finite-$Q^2$ Corrections

• **Higher Twist** – $1/Q^2$ corrections arise from local (higher twist) operators in QCD matrix elements – can decompose $g_1$ and $g_2$ as a sum of twist terms

$$g_1(x, Q^2) = g_1^{\tau 2+TMC} + g_1^{\tau 3+TMC} + g_1^{\tau 4}$$

$$g_2(x, Q^2) = g_2^{\tau 2+TMC} + g_2^{\tau 3+TMC}$$

• **Leading twist** ($\tau = 2$) at NLO without target mass corrections (TMC)

$$g_1^{\tau 2}(x) = \frac{1}{2} \sum_q e_q^2 [\Delta C_{qq} \otimes \Delta q(x) + \Delta C_{qg} \otimes \Delta g(x)]$$

• Relationship between **twist-3** $g_1$ and $g_2$ (Bluemlein, Tkabladze NPB 553, 427 (1999))

$$g_1^{\tau 3} = \frac{4M^2 x^2}{Q^2} \left[ g_2^{\tau 3} - 2 \int_x^1 \frac{dy}{y} g_2^{\tau 3} \right]$$
Finite-$Q^2$ Corrections

• **Target mass corrections** - $M^2/Q^2$ suppressed contributions to LT structure functions from finite nucleon mass

\[ g_1^{\tau^2+TM^C} (x) = \zeta_1^1 g_1^{\tau^2} (\xi) + \zeta_1^2 \int_{\xi}^{1} \frac{dz}{z} g_1^{\tau^2} (z) + \zeta_1^3 \int_{\xi}^{1} \frac{dz}{z} g_1^{\tau^2} (z) \log \left( \frac{z}{\xi} \right) \]

\[ \zeta = \zeta \left( x, \frac{M^2}{Q^2} \right) \quad \xi = \xi \left( x, \frac{M^2}{Q^2} \right) \]

• **Nuclear smearing** - effects from bound nucleons in $^3$He and deuterium nuclei - convolute nucleon structure functions with momentum distribution functions

\[ g_1^A (x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N (y) g_j^N \left( \frac{x}{y}, Q^2 \right) \]
Parameterization

- Parton plus distributions parameterized as

\[ x \Delta q^+ (x) = N x^\alpha (1 - x)^\beta (1 + \gamma x) \]

\[ \Delta q^+ \equiv \Delta q + \Delta \bar{q} \]

- Twist-3 structure function parameterization enters at the parton level

\[ D^{\tau 3} (x) = N' x'^\alpha' (1 - x')^{\beta'} (1 + \gamma' x') \]

- Twist-4 structure function parameterized by

\[ g_1^{\tau 4} = \frac{1}{Q^2} N'' x''^{\alpha''} (1 - x'')^{\beta''} (1 + \gamma'' x'') \]

- Total # of parameters = 10 shape parameters for spin PDFS + 8 x 2 higher twist parameters = 26
Fitting Procedure

- Standard $\chi^2$ fit is defined

$$\chi^2(\vec{p}) = \sum_{i=1}^{N_{exp}} \left[ \sum_{j=1}^{N_{data}} \frac{(D_j - T_j(\vec{p}))^2}{\sigma} \right]$$

- Uncertainties added in quadrature

- JAM analysis uses modified $\chi^2$ to account for correlated errors (e.g. overall normalization)

- PDF uncertainties computed using Hessian method

$$H_{ij} = \left. \frac{1}{2} \frac{\partial \chi^2(\vec{p})}{\partial p_i \partial p_j} \right|_{\vec{p}=best}$$

$$C = H^{-1}$$

Covariance matrix
Uncertainties

• In the eigenbasis \( \{ \hat{e}_i \} \) of the covariance matrix, the shift of parameters from best value is defined by scale factors \( \{ t_i \} \)

\[
\Delta \vec{p} = \sum_{i} t_i \hat{e}_i
\]

• Uncertainties on observables are defined by

\[
\delta \mathcal{O}_+^2 \simeq \sum_{i} \max \left[ \mathcal{O}(t_i^+) - \mathcal{O}(0), \mathcal{O}(t_i^-) - \mathcal{O}(0), 0 \right]^2
\]

\[
\delta \mathcal{O}_-^2 \simeq \sum_{i} \max \left[ \mathcal{O}(0) - \mathcal{O}(t_i^+), \mathcal{O}(0) - \mathcal{O}(t_i^-), 0 \right]^2
\]

Observable at best parameter values

Edges of confidence regions (e.g. 68% or 98%)
Inclusive DIS Data

- **EMC** \((npts = 10)\)
- **SMC** \((npts = 28)\)
- **SLAC** \((npts = 853)\)
- **HERMES** \((npts = 103)\)
- **COMPASS** \((npts = 81)\)
- **JLab** \((npts = 1798)\)

soon to be extended to SIDIS & polarized pp data
Preliminary Results

Total of 2887 data points, $\chi^2_{\text{dof}} = 1.18$

$Q^2 > 1.0 \text{ GeV}^2$

$W^2 > 3.5 \text{ GeV}^2$
Preliminary Results

- Consistent with DSSV
- Reduction of uncertainties with JLab data
Preliminary Results

- Neutron higher twist set to zero (not well constrained)
Preliminary Results

- Reduced uncertainties with the inclusion of JLab data
Summary

• Finalized results will reveal impact of JLab 6 GeV data on LT PDFs and higher twist matrix elements

• Future work:
  – Universal fit to extract unpolarized and polarized PDFs from DIS data simultaneously
  – Inclusion of SIDIS and polarized pp data
    • Constrains gluon and sea distributions
  – Extension to transverse momentum dependent PDFs

• www.jlab.org/JAM
Thank You!