Effective Field Theory for Nuclear Physics

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Overview

• Introduction
• Basic ideas of EFT
• Basic Examples of EFT
• Algorithm of EFT
• Review NN scattering
• NN scattering in Effective Field Theory
• Radiative neutron capture in C-14
• Form factors of one nucleon halo C-15
Introduction

• Basic Building blocks of Nature - Quarks and Leptons

• QCD and QED describes the dynamics of fundamental particles

• Describe Low Energy Physics (Light Nuclei) with QCD is complicated, non-perturbative, and infinite body problem.

• Possible Solution
  • Lattice QCD
  • EFT
Basic ideas of EFT

- Dynamics at long distance do not depend on what happens at short distance.
- Low energy interactions do not care about details of high energy interactions.
- We don't need to understand Nuclear Physics to build a bridge!!
EFT Concepts

- EFT is a systematic approximation to some underlying dynamics, valid in specific regime
- Phenomena at low energy (long wave length) cannot probe detail of high energy (short distance) structure of Physics
- Expand short distance physics in terms of contact interaction
- Coefficients fit to low energy data
Some Basic Examples
Gravity for \( h < R \)

- Gravitational potential energy,

\[ \Delta U = mgh \left(1 - \frac{h}{R} + \frac{h^2}{R^2} + \ldots\right) \]

- \( h \)- Height above the Earth surface
- \( R \)- Radius of the Earth
- Theory is converge for \( h < R \)
Multipole Expansion

- Multipole expansion of electric potentials,

\[ V = q \frac{1}{R} + p \frac{1}{R^2} + Q \frac{1}{R^3} + \ldots \]

- Sum converge for a \( \ll R \)

- Low energy probes are sensitive to only bulk properties.
EFT Algorithm

• Identify low energy scale and high energy scale in the theory. (Ration of scales will form expansion parameter)

• Identify the Symmetry of the theory

• Power Counting

• Write down Effective Lagrangian and interaction, consistent with the Symmetry

• Calculate loops and renormalize them
Nucleon-Nucleon Scattering at Low Energy

- Potential Model with strong interaction
- Asymptotic wave function and the cross section

\[ \psi(r) = \exp(ik_i \cdot r) + f(k, \theta) \frac{\exp(ik_f r)}{r} \]

\[ \frac{d\sigma}{d\Omega} = |f(k, \theta)|^2 \]
• T-Matrix (scattering amplitude)

\[ T = -\frac{4\pi}{M} \frac{1}{k \cot \delta - i k}. \]

• Effective Range Expansion,

\[ k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r_0 k^2 - P r_0^3 k^4 + \ldots. \]

• a-scattering length

• r0-effective range

• P-shape parameter
• Leading ERE amplitude

\[ T_{NN}^{ERE(0)} = \frac{4\pi a}{M} \frac{1}{1 + iak} \]

• Second order ERE amplitude

\[ T_{NN}^{ERE(2)} = -\frac{4\pi}{M} \frac{1}{\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik} \]

• Analytical structure of T(k) must reproduce in Effective Field Theory
EFT for Nucleon Nucleon Scattering

- Build EFT to reproduce ERE Amplitude
- Low energy Degree of freedom - Nucleon
- Separation of Scale

\[ k \ll M_\pi \]
• Symmetry- Translational invariance, rotational invariance

• Non Relativistic-Nucleon Momentum $k \ll M_\pi$
Effective Lagrangian

\[ \mathcal{L} = N^\dagger \left( iD_0 + \frac{D^2}{2M} \right) N + \left( \frac{\mu}{2} \right)^{4-n} C_0 (N^T P N)^\dagger (N^T P N) + \ldots \]

- P - projection operator for spin and isospin
- n - number of space-time dimension
- \( \mu \) - arbitrary mass scale to regulate dimensions
- \( \ldots \) - higher derivative operator for higher body interaction
Feynman Diagrams

- Sum Feynman diagrams which give the amplitude $T$ to the desired order in $k/\Lambda$ expansion.
• Summing Feynman diagrams, scattering amplitude from our effective Lagrangian,

\[ T_{NN} = [ C_0 + C_0 I_0 C_0 + C_0 I_0 C_0 I_0 C_0 + \ldots ] = \frac{1}{1/C_0 - I_0} \]

\( I_0 \) - Momentum independent single loop integral

\[ I_0 = -i \left( \frac{\mu}{2} \right)^{4-n} \int \frac{d^n q}{(2\pi)^n} \left( \frac{i}{E + q_0 - q^2/2M + i\epsilon} \right) \left( \frac{i}{-q_0 - q^2/2M + i\epsilon} \right) \]

• here, Nucleon momentum, \( k = \sqrt{ME} \)

• This integral diverges and needs to be regularized and renormalized
MS Subtraction scheme

- Integral using MS subtraction scheme

\[ I_0 = -M \left( -|k|^2 - i\epsilon \right)^{(n-3)/2} \left( \frac{\mu}{2} \right)^{4-n} \Gamma \left( \frac{3-n}{2} \right) (4\pi)^{(1-n)/2} \]

\[ I_0^{\text{MS}} = - \left( \frac{M}{4\pi} \right) i|k|, \]
Power Divergence Subtraction (PDS)

- new power counting
- this allows for fine tuning between coefficient $C_0$ and linear divergence.

$$I_0^{PDS} = - \left( \frac{M}{4\pi} \right) (\mu + i|k|),$$

- $\mu$ is subtraction scale
- PDS isolate linear divergence from integral
• ERE amplitude,
\[ T_{\text{ERE}}^{(2)} = -\frac{4\pi}{M} \frac{1}{\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}. \]

• EFT amplitude,
\[ T_{\text{NN}} = [ C_0 + C_0 I_0 C_0 + C_0 I_0 C_0 I_0 C_0 + \ldots ] = \frac{1}{1/C_0 - I_0} \]

• Matching leading orders,
\[ \frac{4\pi}{MC_0} = \frac{1}{a} - \mu. \]

• \( c_0 \) depends on both ‘a’ and meaning assigned to divergent part of ‘I_0’

• works with both \(^3S_1, ^1S_0\) channel
RADIATIVE NEUTRON CAPTURE ON CARBON-14
Motivation

- $^{14}\text{C} (\text{n,}\gamma)^{15}\text{C}$ is important in Astrophysics
- Reaction is part of CNO cycle in helium burning stars
- Fundamental reaction in Big Bang Nucleosynthesis for production of nuclei with mass number $> 14$
- Theoretical method involved in radiative capture is important for future experimental efforts
Radiative Neutron capture on Carbon-14

- In EFT, short distance physics is not relevant at low energy
- Ground state of $^{15}\text{C}$ has angular momentum $J^\pi = \left(\frac{1}{2}\right)^+$
- Neutron separation energy 1.218 MeV
- Carbon-15 has a single Neutron halo bound to Carbon-145
• Final state $^{15}\text{C} : \text{S-wave}$

• Incoming state $n + ^{14}\text{C} : \text{P-wave}$

• Transition from P-wave to S-wave

• Parity conservation - E1 (electric dipole transition) transition
Formalism

- EFT Lagrangian
- Feynman diagrams and scattering amplitude
- Cross section
EFT Lagrangian for final S Channel

\[ \mathcal{L}_s = \phi_\alpha^\dagger [\Delta^{(0)} + i\partial_0 + \frac{\nabla^2}{2M}] \phi_\alpha + h^{(0)}[\phi_\alpha^\dagger (N_\alpha C) + \text{h.c.}] \]

- Auxiliary Field - C15 Field - \( \phi_\alpha \)
- Neutron Field - \( N_\alpha \)
- Carbon-14 Field - C
- \( M = M_n + M_c \)
EFT Lagrangian for P Channel

\[ \mathcal{L}_p = \chi_i^{\alpha, \eta \dagger} [\Delta^{(\eta)} + i \partial_0 + \frac{\nabla^2}{2M}] \chi_i^{\alpha, \eta} + \sqrt{3} h^{(\eta)} [\chi_i^{\alpha, \eta \dagger} P_{ik}^{\alpha, \gamma} N_{\gamma} \left( \frac{\overleftarrow{\nabla}}{M_c} - \frac{\overrightarrow{\nabla}}{M_n} \right)_k C + \text{h.c.}] , \]

- Here, projection operator,

\[ P_{ij}^{\alpha, \beta, 1} = \frac{1}{3} (\sigma_i \sigma_j)^{\alpha, \beta} \rightarrow 2P_{\frac{1}{2}} \text{ channel} \]

\[ P_{ij}^{\alpha, \beta, 2} = \delta_{ij} \delta^{\alpha, \beta} - \frac{1}{3} (\sigma_i \sigma_j)^{\alpha, \beta} \rightarrow 2P_{\frac{3}{2}} \text{ channel} \]

- Auxiliary field for p-channel - \( \chi_i^{\alpha} \)
Feynman Diagrams

- E1 Capture
- Single dashed line - dimer field - p-wave interaction - $\chi_i^\alpha$
- Double dashed line - dimer field - final state of C15 - $\phi_\alpha$
Scattering Amplitude

- **$2p_{1/2}$-Channel**

\[
|M^{2p_{1/2}}|^2 = \left| \frac{12e\hbar_0 \sqrt{Z} \phi}{M_c} \right|^2 \frac{32M_n M_c M p^2}{9} \left| g^{2p_{1/2}}(p) \right|^2
\]

\[
g^{2p_{1/2}}(p) = \frac{\mu}{p^2 + \gamma^2} + \frac{6\pi \mu}{-1/a_1^{(1)} + r_1^{(1)} p^2/2 - ip^3} \left[ \frac{\gamma}{4\pi} + \frac{ip^3 - \gamma^3}{6\pi (p^2 + \gamma^2)} \right]
\]

- **$2p_{3/2}$-Channel**

\[
|M^{2p_{3/2}}|^2 = \left| \frac{12e\hbar_0 \sqrt{Z} \phi}{M_c} \right|^2 \frac{16M_n M_o M p^2}{9} \left| g^{2p_{3/2}}(p) \right|^2 (5 - 3\cos^2 \theta),
\]

\[
g^{2p_{3/2}}(p) = \frac{\mu}{p^2 + \gamma^2} + \frac{6\pi \mu}{-1/a_1^{(3)} + r_1^{(3)} p^2/2 - ip^3} \left[ \frac{\gamma}{4\pi} + \frac{ip^3 - \gamma^3}{6\pi (p^2 + \gamma^2)} \right]
\]
Scattering Cross Section

- Total cross section,

\[
\sigma(p) = \frac{1}{2} \frac{64\pi\alpha}{M_c^2 \mu^2} \frac{p\gamma(p^2 + \gamma^2)}{1 - \rho\gamma} \left[ 2|g^{2P_{1/2}}(p)|^2 + 4|g^{2P_{3/2}}(p)|^2 \right]
\]

- Depends on unknown EFT couplings that expressed in terms of \( \rho \), \( a_1 \) and \( r_1 \)
Results

- Parameterize EFT coupling

- ERE parameters, $a_1 = -n_1 / (Q^3)$, $r_1 = 2n_2 Q$

- $n_1, n_2$ can estimate from coulomb dissociation and direct capture data
Form factors of single neutron halo system (C15)

- Halo nuclei play an important role in heavy element synthesis in nuclear astrophysics.

- Clear separation of energy scales:
  weakly bound valance nucleons and tightly bound core.

- Ideal to construct low energy EFT.

- Form factor is important to determine internal properties like charge density, size, magnetic properties.
- Probe internal structure of C15
- $^{15}\text{C}$ - spin-1/2 single neutron halo
- $^{14}\text{C}$-core and single valence neutron
- Analyzed similar like electron scattering to Proton target
Elastic scattering and matching parameter

- EFT coupling relates with ERE parameter

\[
\frac{2\pi \Delta}{\mu h^2} + \lambda = \gamma - \frac{1}{2} \rho \gamma^2,
\]

\[-\frac{2\pi}{h^2 \mu^2} = \rho\]

- Binding momentum \( \gamma = \sqrt{2\mu B} \)

- \( \rho \) - effective range
EFT Calculation of $\Gamma^0$ Photon

- Hadronic current - Electric Coupling
EFT Calculation of $\Gamma^i$ Photon

- Hadronic current - Magnetic coupling

\[ i\Gamma^i = i e Z_c Z_\phi \bar{u}_\phi(p, a) \left\{ \frac{p_i + p'_i}{2M} \left[ \frac{\hbar^2 \mu M_c}{\pi |q|} \right] \tan^{-1} \left( \frac{\mu |q|}{2M c \gamma} \right) + 1 \right\} + i \frac{\mu N}{e Z_c} \left[ i^2 \kappa_n \frac{\mu M_n}{\pi |q|} \right] \tan^{-1} \left( \frac{\mu |q|}{2M_n \gamma} \right) + L_M \epsilon^{ijk} \sigma_j q_k \right\} u_\phi(p', a') \]
Electric Form factor and charge radius

\[ G_E(|q|^2) = Z_\phi \left[ \frac{\hbar^2 \mu M_c}{\pi |q|} \tan^{-1} \left( \frac{\mu |q|}{2M_c \gamma} \right) + 1 \right] \approx 1 - \frac{\mu^2}{12M_c^2 \gamma^2} \frac{1}{1 - \rho \gamma} |q|^2 + \ldots, \]

\[ \langle r_E^2 \rangle = \frac{\mu^2}{2M_c^2 \gamma^2} \frac{1}{1 - \rho \gamma} \langle r_c^2 \rangle + \langle r_c^2 \rangle, \]

- determine charge radius entirely from binding energy
Magnetic Form Factor and Magnetic Moment

\[
\frac{eZ_c}{2M} G_M(|q|^2) = \mu_N Z_\phi \left[ \frac{h^2 g_n \mu M_n}{2 \pi |q|} \tan^{-1} \left( \frac{\mu |q|}{2M_n \gamma} \right) + L_M \right]
\]

\[
\approx (\kappa_n - L_M \rho \gamma) \mu_N \frac{1}{1 - \rho \gamma} - \kappa_n \mu_N \frac{\mu^2}{12M_n^2 \gamma^2} \frac{q^2}{1 - \rho \gamma},
\]

- Magnetic moment of halo nucleus,

\[
\kappa_\phi = (\kappa_n - L_M \rho \gamma) \frac{1}{1 - \rho \gamma} \approx k_n + (\kappa_n - L_M) \gamma \rho
\]
Conclusion

- EFT describes experimental data consistently

- More accurate measurements needed near the resonance energy to verify the interference effect of resonance and non-resonance

- The form factor calculation of Carbon-15 will be useful for future form factor calculation of Beryllium 11 (spin 1/2 halo)
Thank you