Hadron Physics
& QCD's Dyson-Schwinger Equations

Lecture 1: A Continuum-QCD Primer

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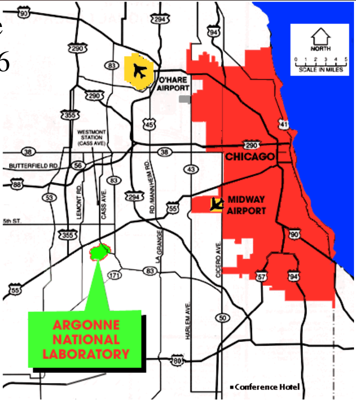
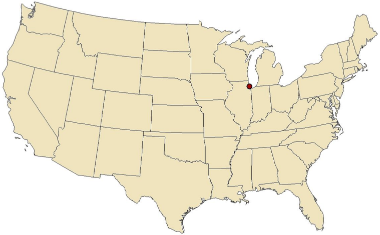
Argonne NATIONAL LABORATORY

Argonne National Laboratory

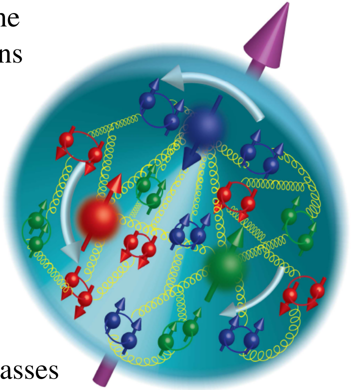
- Argonne was the first national laboratory in the United States and was established 1st July 1946
  - was originally formed to continue Enrico Fermi's work on nuclear reactors as part of the Manhattan Project
- Today Argonne is a multidisciplinary research facility:
  - hadron, nuclear and particle physics; supercomputing; material science; renewable energy; national security; etc
  - ANL has 3350 staff & a budget of $750 million





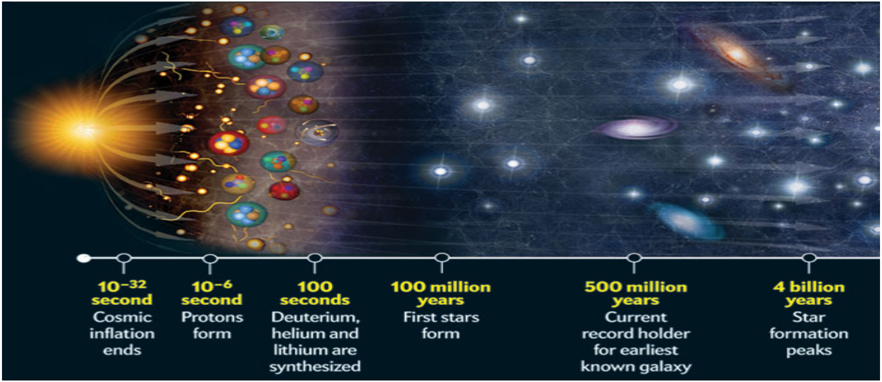
What is Hadron Physics?

- Hadron Physics means to chart and compute the distribution of matter and energy within hadrons (and nuclei)
  - mapping these types of correlations and exposing their influence has been a hallmark of nuclear physics since its inception



- The piece of the Standard Model that is supposed to *largely* describe this matter and energy is Quantum Chromodynamics (QCD)
- However the study of Hadron Physics encompasses all aspects of the Standard Model
  - e.g. electrons, muons and neutrinos are all used to probe hadron properties
- Electroweak forces can also have profound impacts on hadrons; e.g.
  - the Higgs mechanism gives the pion a small mass – *if this was not so the strong force would have infinite range*; similarly the Higgs makes the neutron heavier than the proton – QCD + QED alone gives $M_p > M_n$

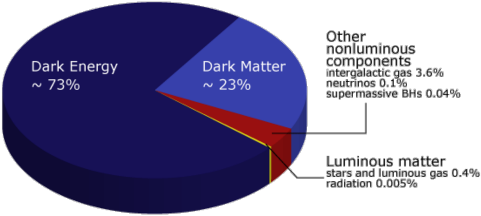
Beginnings . . .



- Hadron physics started about 10^{-6}s after the big bang and understanding the QCD mechanisms behind this will have profound implications
  - e.g. it will explain how *massless gluons* and *light quarks* bind together to form hadrons & thereby explain the origin of $\sim$98% of the mass in the visible universe
  - *the Higgs mechanism is largely irrelevant as a mass generating mechanism; however hadron/nuclear physics would be profoundly different without it!*

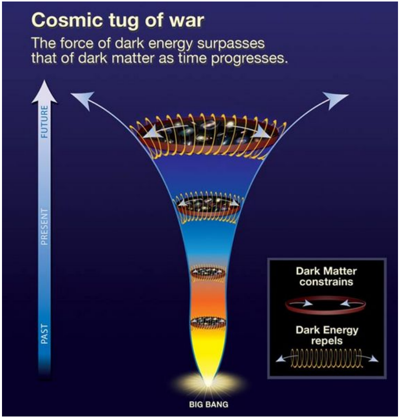
There is Much More . . .

- Worth noting that hadron physics as we know it concerns as little as 4% of the mass-energy in the Universe; *the nature of the rest is almost completely unknown*



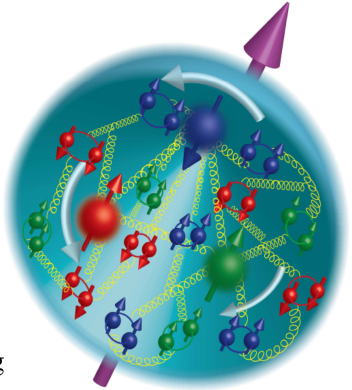
- Dark Matter: There appears to be a halo of mysterious invisible matter engulfing galaxies – inferred from the rotational curves of the stars in the outer regions of these galaxies

- Dark Energy: Discovered in 1998 through observation of distant supernovae, where the expansion of the Universe was found to be accelerating, contrary to expectation: *2011 Nobel Prize in Physics was awarded to the team leaders*



Key Questions in Hadron Physics

- What is confinement?
- How do quarks and gluons bind together to form hadrons?
  - how are they distributed and what are the key correlations
  - what is the origin of the nucleon mass
  - what gives rise to the nucleon anomalous magnetic moment
- How is angular momentum distributed among the quarks and gluons; e.g. *spin crisis*
- How do nuclei emerge from QCD? . . .



Difficult to know what lies beyond the Standard Model unless one first knows what is in the Standard Model;

e.g. NuTeV anomaly; muon $g-2$; proton radius puzzle

Standard Model of Particle Physics

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FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c²	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

- With discovery of the Higgs boson the Standard Model is now complete
- Its formulation and verification are a remarkable and continuing story
- Electromagnetism: Feynman, Schwinger, Tomonaga – 1965 Nobel Prize:
  "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"
- Weak interaction: Glashow, Salam, Weinberg – 1979 Nobel Prize:
  "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

Standard Model of Particle Physics

- Strong Interaction:

  Gell-Mann's and Zweig's constituent quark theory (1964) was a critical step forward

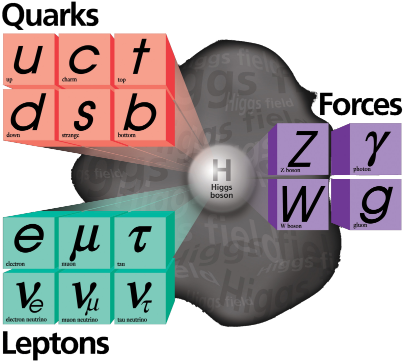
  Gell-Mann – Nobel Prize 1964:

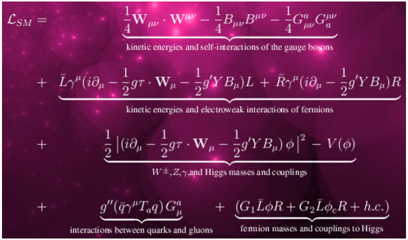
  "for his contributions and discoveries concerning the classification of elementary particles and their interactions"

  Gross, Politzer, Wilczek – Nobel Prize 2004:

  "for the discovery of asymptotic freedom in the theory of the strong interaction"

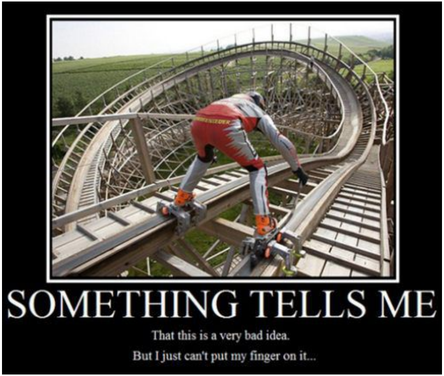
- SM has 19 parameters which need to be determined by experiment
  - 2 intrinsic to QCD: Λ_{QCD} & $\theta_{\text{QCD}} \lesssim 10^{-9}$





Motivation of Lectures

- *Explore the non-perturbative structure of QCD, through the interplay of theory and experiment, as it relates to hadron and nuclear structure*
  - hadron physics is fun because one can perform calculations, and use these results to confront & guide experiment
  - performing some simple but illustrative calculations will be a theme of these lectures



- We will focus on QCD's Dyson-Schwinger Equations (DSEs)
  - other approaches with a direct connection to QCD include lattice-regularized QCD and effective field theories (e.g. χPT)
- Some of the advantages of the DSEs include
  - can explore a wider array of physics problems
  - may provide better insight into important physics mechanisms
  - facilitate a dynamic interplay between experiment and theory

Plan of Lectures

- Lecture 1 – A continuum-QCD primer
- Lecture 2 – The Dyson-Schwinger equations
- Lecture 3 – The pion and dynamical chiral symmetry breaking
- Lecture 4 – The nucleon and its electromagnetic structure
- Lecture 5 – Partonic structure of nucleons
- Lecture 6 – Partonic structure of nuclei

Recommended Reading

- DSE review articles:
  - C. D. Roberts and A. G. Williams, "*Dyson-Schwinger equations and their application to hadronic physics*", Prog. Part. Nucl. Phys. **33** (1994) 477;
  - P. Maris and C. D. Roberts, "*Dyson-Schwinger equations: A tool for hadron physics*", Int. J. Mod. Phys. E **12**, 297 (2003);
  - A. Bashir, L. Chang, I. C. Cloët, B. El-Bennich, Y. X. Liu, C. D. Roberts and P. C. Tandy, "*Collective perspective on advances in Dyson-Schwinger Equation QCD*", Commun. Theor. Phys. **58**, 79 (2012);
  - I. C. Cloët and C. D. Roberts,, "*Explanation and Prediction of Observables using Continuum Strong QCD*", Prog. Part. Nucl. Phys. **77**, 1 (2014);
- Key DSE articles:
  - I. C. Cloët, G. Eichmann, B. El-Bennich, T. Klahn and C. D. Roberts, "*Survey of nucleon electromagnetic form factors*", Few Body Syst. **46**, 1 (2009);
  - L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt and P. C. Tandy, "*Pion electromagnetic form factor at spacelike momenta*", PRL **111**, 141802 (2013);
  - G. Eichmann, R. Alkofer, A. Krassnigg and D. Nicmorus, "*Nucleon mass from a covariant three-quark Faddeev equation*", Phys. Rev. Lett. **104**, 201601 (2010).

Recommended Reading

NJL review articles:

- U. Vogl and W. Weise, "*The Nambu and Jona Lasinio model: Its implications for hadrons and nuclei*", Prog. Part. Nucl. Phys. **27**, 195 (1991);
- S. P. Klevansky, "*The Nambu-Jona-Lasinio model of quantum chromodynamics*", Rev. Mod. Phys. **64**, 649 (1992);
- M. Buballa, "*NJL model analysis of quark matter at large density*", Phys. Rept. **407**, 205 (2005);

Key NJL articles:

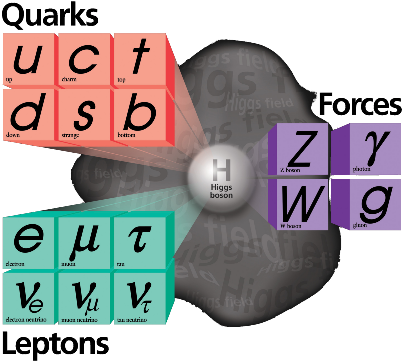
- Y. Nambu and G. Jona-Lasinio, "*Dynamical model of elementary particles based on an analogy with superconductivity I & II*", Phys. Rev. **122**, 345 (1961); Phys. Rev. **124**, 246 (1961);
- N. Ishii, W. Bentz and K. Yazaki, "*Baryons in the NJL model as solutions of the relativistic Faddeev equation*", Nucl. Phys. A **587**, 617 (1995);
- I. C. Cloët, W. Bentz and A. W. Thomas, "*Isovector EMC effect explains the NuTeV anomaly*," Phys. Rev. Lett. **102**, 252301 (2009);
- I. C. Cloët, W. Bentz and A. W. Thomas, "*Role of diquark correlations and the pion cloud in nucleon elastic form factors*", Phys. Rev. C **90**, 045202 (2014).

Recommended Reading

- QCD and Hadron Physics:
  - S. J. Brodsky, A. L. Deshpande, H. Gao, R. D. McKeown, C. A. Meyer *et al.*, "*QCD and Hadron Physics*", arXiv:1502.05728 [hep-ph];
    [**White Paper for the 2015 Long Range Plan**: Drawn from presentations and input derived during and following the QCD Town Meeting, 13-15 September 2014, Temple University. Summary of successes in cold QCD from the last seven years and description of plans for the next ten years.]
  - J. Dudek, R. Ent, R. Essig, K. S. Kumar, C. Meyer, R. D. McKeown *et al.*, "*Physics Opportunities with the 12 GeV Upgrade at Jefferson Lab*", Eur. Phys. J. A **48**, 187 (2012) [arXiv:1208.1244 [hep-ex]];
  - A. Accardi, J. L. Albacete, M. Anselmino, N. Armesto, E. C. Aschenauer *et al.*, "*Electron Ion Collider: The Next QCD Frontier - Understanding the glue that binds us all*", arXiv:1212.1701 [nucl-ex];
  - G. P. Lepage and S. J. Brodsky, "*Exclusive Processes in Perturbative Quantum Chromodynamics*", Phys. Rev. D **22**, 2157 (1980);
  - P. Pascual and R. Tarrach, "*QCD: Renormalization For The Practitioner*", Lect. Notes Phys. **194**, 1 (1984).

What is QCD?

- QCD is the only known example in nature of a fundamental quantum field theory that is innately non-perturbative
  - *a priori* no idea what such a theory can produce



- QCD is *likely* a perfect theory – nothing needs to be added or changed
  - validated over an incredible energy range: $0 < E < 8000\,\text{GeV}$
  - unlikely to break down at any energy scale: *asymptotic freedom*
  - QCD has basically no intrinsic parameters: *all dimensionless ratios of observables can be determined without a single parameter* [θ_{QCD} likely zero]
  - just need one observable to define the scale, e.g. proton mass, $\Lambda_{\text{QCD}} \simeq 200\,\text{MeV}$
  - *QCD is NOT an effective theory - it is a theory*
- Possible that future extensions of the Standard Model will be based on the paradigm established by QCD: e.g. extended technicolor

- QCD is the fundamental theory of the strong interaction, where the *quarks* and *gluons* are the basic degrees of freedom

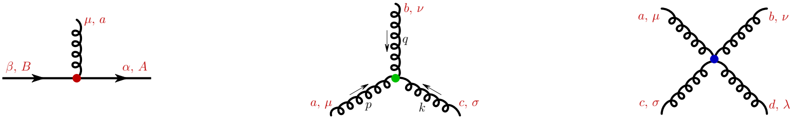
$$(q_\alpha)^A_f \quad \begin{cases} \text{colour} & A = 1,\, 2,\, 3 \\ \text{spin} & \alpha = \uparrow,\, \downarrow \\ \text{flavour} & f = u,\, d,\, s,\, c,\, b,\, t \end{cases} \qquad A^a_\mu \quad \begin{cases} \text{colour} & a = 1, \ldots, 8 \\ \text{spin} & \varepsilon^{\pm,0}_\mu \end{cases}$$

- QCD is a non-abelian gauge theory whose dynamics are governed by the Lagrangian

$$\mathcal{L} = \bar{q}_f \left(i\not{D} + m_f\right) q_f - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a;$$

$$i\not{D} = \gamma^\mu \left(i\partial_\mu + g_s\, A^a_\mu\, T^a\right)$$

$$F^a_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + g_s\, f_{abc}\, A^b_\mu\, A^c_\nu$$



- Gluon self-interactions have many profound consequences

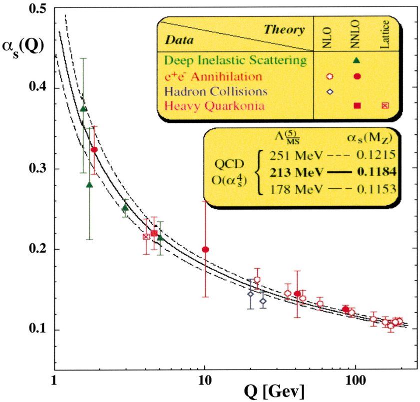
Asymptotic Freedom in QCD

- At *large energies* or *small distances* QCD interactions vanish logarithmically $\Longleftrightarrow$ asymptotic freedom
  - perturbation theory is a useful tool in this domain $\Longleftarrow$ factorization

$$\alpha_{\text{QCD}}^{\text{LO}}(Q^2) = \frac{4\,\pi}{\left(11 - \frac{2}{3}\,N_f\right)\ln\left(Q^2/\Lambda_{\text{QCD}}^2\right)}$$

- Λ_{QCD} most important parameter in QCD [dimenional transmutation of g_s]
  - $\Lambda_{\text{QCD}} \simeq 200 - 300\,\text{MeV} \quad @ \quad 1\,\text{GeV}$ – sets scale, QCD's "standard kilogram"
- Gluon self-interactions make QCD profoundly different from QED

2004 Nobel Prize in Physics



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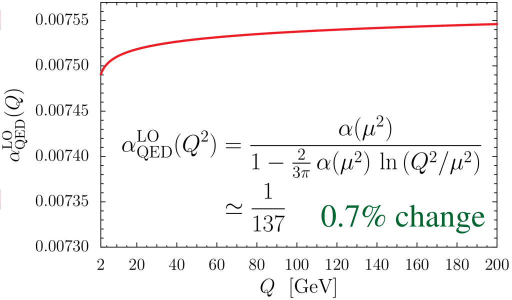
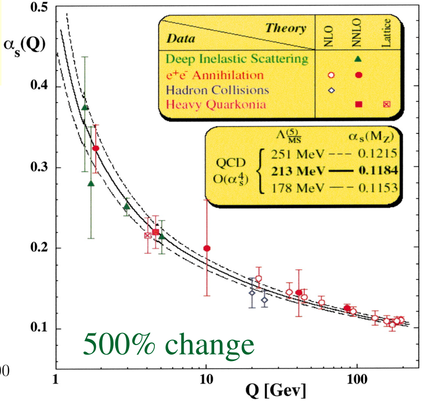
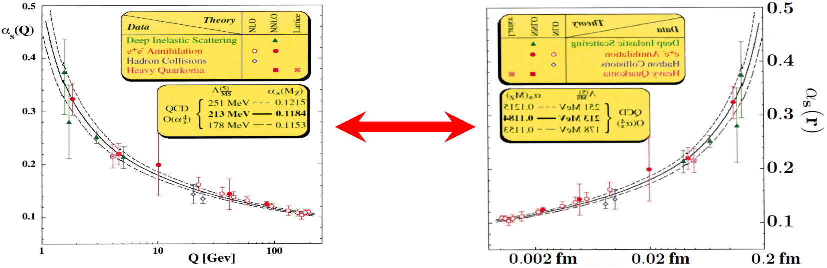




QCD is non-perturbative

- Momentum-dependent coupling $\Longleftrightarrow$ coupling depends on separation
  - interaction strength between quarks and gluons grows with separation
- At distance scales typical of hadron physics: $r \sim 0.2\,\text{fm} \simeq \frac{1}{4}\,r_\text{p} \Longrightarrow \alpha_s \sim 0.5$
  - perturbation theory completely breaks down in this domain
  - QCD interaction is non-perturbative over 98% of the proton's volume
- The truly interesting aspects of QCD – i.e. hadron physics – are in non-perturbative domain
- Intimately tied to the coupling in the infrared are two emergent phenomena that characterize QCD and hadron physics: Confinement & *DCSB*

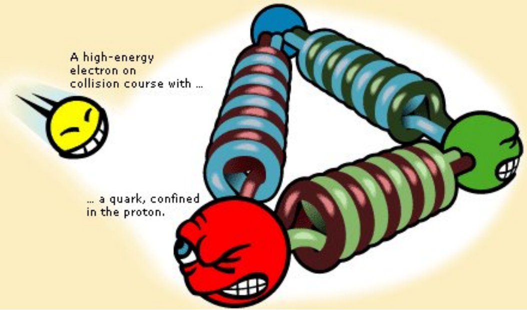


Discover the meaning of *confinement* and its relation to *dynamical chiral symmetry breaking*

– origin of visible mass –

Colour Confinement

- All known hadrons are colour singlets, even though they are composed of coloured quarks and gluons:
  baryons (qqq) & mesons ($\bar{q}q$)



- Confinement conjecture: *particles that carry the colour charge cannot be isolated and can therefore not be directly observed*

- Related to \$1 million Millennium Prize – Clay Mathematics Institute:

Yang-Mills Existence And Mass Gap: *Prove that for any compact simple gauge group G, quantum Yang-Mills theory on* $\mathbf{R}^4$ *exists and has a mass gap* $\Delta > 0$.

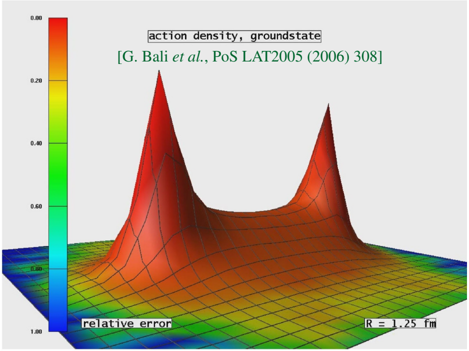
  - for $SU(3)_c$ must prove that glueballs have a lower bound on their mass
  - partial explanation as to why strong force is short ranged

- Understanding confinement should be intimately related to the infrared properties of $\alpha_s(Q^2)$ or QCD's β-function: $\beta(g_s) = \mu \frac{\partial g_s}{\partial \mu}$ & $\alpha_s = \frac{g_s^2}{4\pi}$

Confinement Folklore

- Confinement is an empirical fact:
  the cross-section for the production of any combination of asymptotic coloured states from a colour singlet state must vanish
- Must be associated with a failure of the cluster decomposition property of quantum field theory
- **Folklore:** "The color field lines between a quark and an anti-quark form flux tubes. A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons." **Hall-D Conceptual Design Report(5)**



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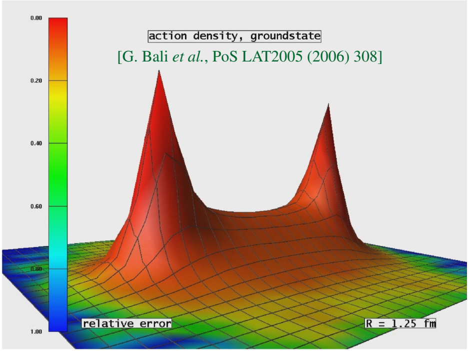

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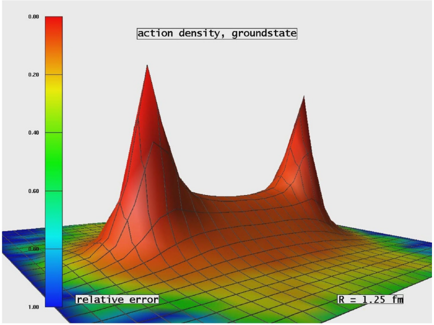
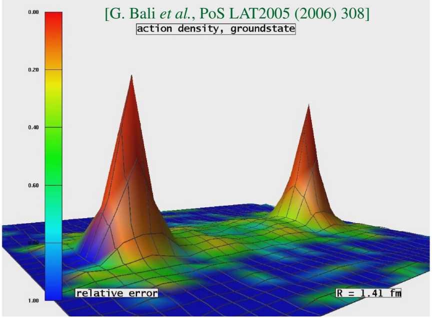
Confinement in QCD with light-quarks

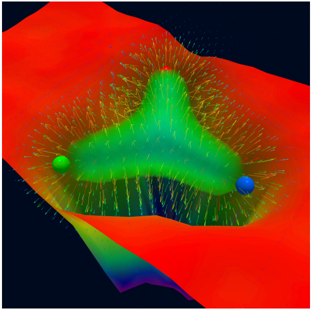




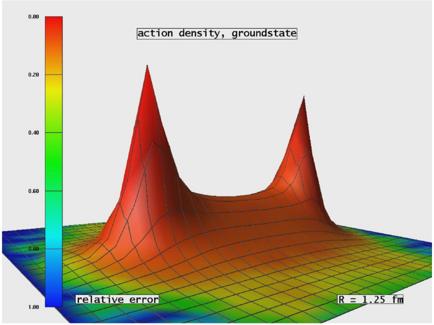
[G. Bali *et al.*, PoS LAT2005 (2006) 308]

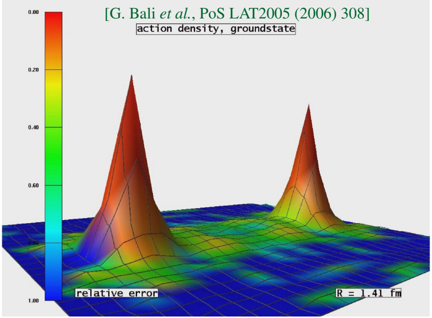
- In the presence of *light quarks* the breaking of the flux tube appears to be an instantaneous non-localized process
  - no evidence of localization of the $\bar{q}q$ pair creation as expected in flux tube picture

- *Paradigm for confinement in hadron physics is likely different from the flux tube picture*



Confinement in QCD with light-quarks





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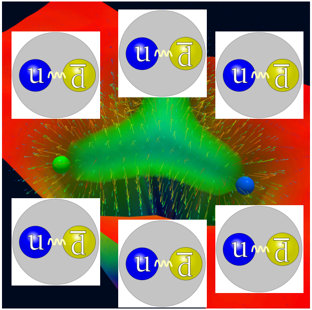
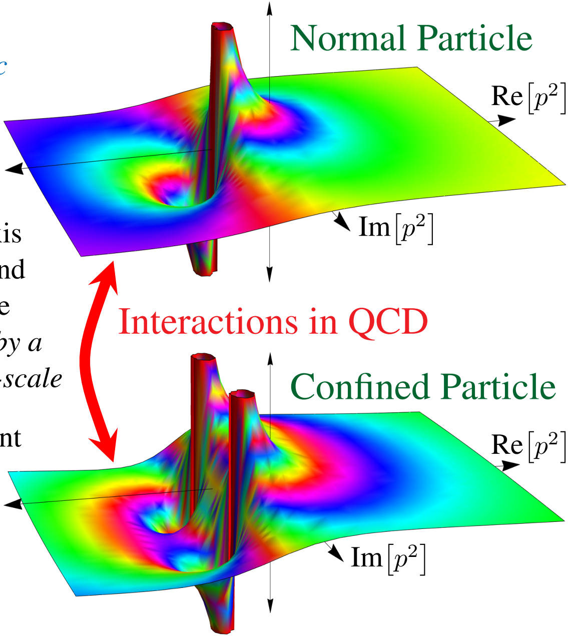


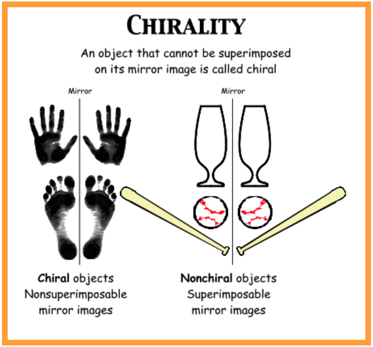
- *In QCD confinement must be expressed through a dramatic change in the analytic structure of propagators for coloured states*
- Interactions cause the real-axis mass-pole to move off axis and form, e.g., complex conjugate singularities– *characterized by a dynamically generated mass-scale*
- Is it plausible that confinement and DCSB – *two phenomena so important in the Standard Model* – can have different origins and fates?



Chiral Symmetry

- For massless quarks the QCD Lagrangian has a chiral symmetry
  - define left- and right-handed fields: $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\,\psi$
- The QCD Lagrangian then takes the form

$$\mathcal{L} = \bar{\psi}_L\, i\not{D}\,\psi_L + \bar{\psi}_R\, i\not{D}\,\psi_R - \bar{\psi}_R\,\mathbf{m}\,\psi_L - \bar{\psi}_L\,\mathbf{m}\,\psi_R - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$



- Therefore for $\mathbf{m} = 0$ QCD Lagrangian has a chiral symmetry

$$SU(N_f)_L \otimes SU(N_f)_R \implies \psi_{L,R} \to e^{-i\,\omega^a_{L,R}\,T^a}\,\psi_{L,R}$$
$$SU(N_f)_V \otimes SU(N_f)_A \implies \psi \to e^{-i\,\omega^a_V\,T^a}\psi, \quad \psi \to e^{-i\,\omega^a_A\,T^a\,\gamma_5}\psi$$

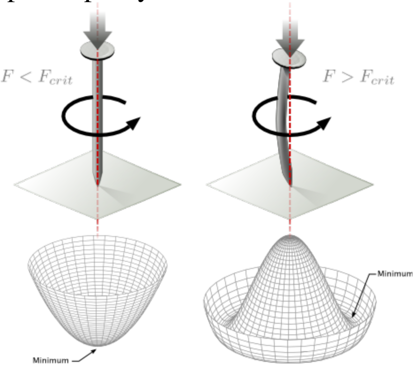
- Chiral symmetry can be realized in two ways:
  - Wigner-Weyl mode: vacuum/interactions respects symmetry
  - Nambu-Goldstone mode: vacuum/interactions breaks symmetry

Dynamical Chiral Symmetry Breaking

- $\mathcal{L}_{\rm QCD}$ invariant under: $SU(N_f)_L \otimes SU(N_f)_R \Longleftrightarrow SU(N_f)_V \otimes SU(N_f)_A$
- For $N_f = 2$, $SU(N_f)_V$ transformations correspond to the isospin
  - hadronic mass spectrum tells us nature largely respects isospin symmetry
  - $m_{\pi^-} \simeq m_{\pi^0} \simeq m_{\pi^+}, \quad m_p \simeq m_n, \quad m_{\Sigma^-} \simeq m_{\Sigma^0} \simeq m_{\Sigma^+}$
  - therefore $SU(N_f)_V$ is realized in the Wigner-Weyl mode
- $SU(N_f)_A$ transformations mix states of opposite parity
  - expect hadronic mass spectrum to exhibit parity degeneracy

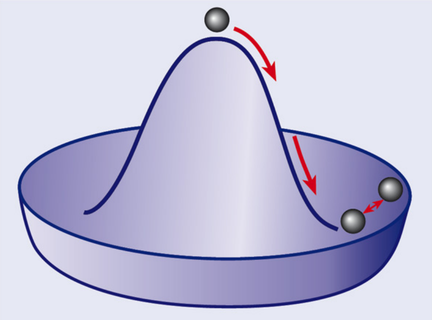
    $m_{a_1} - m_\rho; \; m_N - m_{N^*} \sim 500\,\text{MeV}$
  - $m_u \simeq m_d \simeq 5\,\text{MeV}$, cannot produce such large mass splittings
  - therefore $SU(N_f)_A$ must be realized in the Nambu-Goldstone mode
- Chiral symmetry broken dynamically

$$SU(N_f)_L \otimes SU(N_f)_R \Longrightarrow SU(N_f)_V$$



Goldstone's Theorem

- Goldstone's theorem:
  if a continuous global symmetry is broken dynamically, then for each broken group generator there must appear in the theory a massless spinless particle



- QCD's chiral symmetry is explicitly broken by small current quark masses

$$m_u \simeq 2\,\text{MeV};\; m_d \simeq 4\,\text{MeV} \;\; [\ll \Lambda_{QCD}]$$

- For $N_f = 2$ expect $N_f^2 - 1 = 3$ Goldstone bosons: $\pi^+,\ \pi^0,\ \pi^-$
  - physical particle masses are not zero – $m_\pi \sim 140\,\text{MeV}$ – because of explicit chiral symmetry breaking: $m_{u,d} \neq 0$, however $m_\pi / M_N \simeq 0.15$
- As we will see, in coming to understand the pion's lepton-like mass, DCSB has been exposed as the most important mass generating mechanism for visible matter in the Universe
- Not apparent from $\mathcal{L}_{\text{QCD}}$ and is an innately non-perturbative phenomena

Chiral Condensate; GT & GMOR Relations

- If a symmetry is dynamically broken some operator must acquire a vacuum expectation value, that is, $\langle 0 | \Theta | 0 \rangle \neq 0$ ❁
  - operator must be Lorentz scalar and a colour singlet: QCD $\Longrightarrow$ composite operator

- Simplest candidate for the DSCB order parameter is $\langle \bar{\psi}\psi \rangle = \langle \bar{u}u + \bar{d}d \rangle$

$$\langle 0 | \bar{\psi}\psi | 0 \rangle^{\mu = 2\,\text{GeV}}_{\overline{\text{MS}}} \simeq -\,(230\,\text{MeV})^3$$

- Some important non-trivial consequences of DCSB
  - $f_\pi \, g_{\pi NN} = M_N \, g_A$ Goldberger–Treiman (GT) relation
  - $f_\pi^2 \, m_\pi^2 = \frac{1}{2}\,(m_u + m_d)\,\langle \bar{u}u + \bar{d}d \rangle$ Gell-Mann–Oakes–Renner (GMOR)

❁ *This is the standard interpretation, however some people e.g. Stan Brodsky, Craig Roberts and Peter Tandy have argued that in fact the vacuum condensate equals zero and the order parameters for DCSB are the in-hadron condensates, for example,* $\langle \pi | \bar{q}q | \pi \rangle$

Proof of Gell-Mann–Oakes–Renner

- The axial-vector and pseudoscalar currents are

$$A_a^\mu(x) = \overline{\psi}(x)\,\gamma^\mu\gamma_5\,t_a\,\psi(x) \quad \& \quad P_a(x) = \overline{\psi}(x)\,i\gamma_5\,t_a\,\psi(x).$$

- Pion to vacuum matrix elements of these operators are

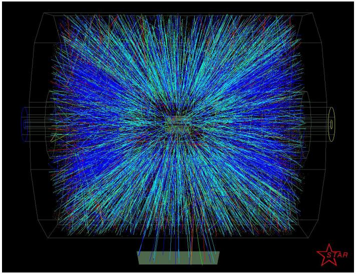
$$\langle 0\,|A_a^\mu(0)|\,\pi_b(p)\rangle = \delta_{ab}\,i\,f_\pi\,p^\mu \quad \& \quad \langle 0\,|P_a(0)|\,\pi_b(p)\rangle = \delta_{ab}\,g_\pi\,.$$

- PCAC: $\partial_\mu A_a^\mu = \overline{\psi}\,i\gamma_5\,\{m, t_a\}\,\psi \quad \Longrightarrow \quad \partial_\mu A_a^\mu = (m_u + m_d)\,P_a,$
- $\langle 0\,|\partial_\mu A_a^\mu|\,\pi_b(p)\rangle = \delta_{ab}\,f_\pi\,p^2 = (m_u + m_d)\,\langle 0\,|P_a|\,\pi_b(p)\rangle = (m_u + m_d)\,\delta_{ab}\,g_\pi$
- *This gives the exact relation in QCD:* $\boxed{f_\pi\,m_\pi^2 = (m_u + m_d)\,g_\pi}$
- In chiral limit – $f_\pi\,m_\pi^2 = 0$ – important consequences
  - ground state: $m_\pi = 0 \implies f_\pi \neq 0$; excited states: $m_\pi \neq 0 \implies f_\pi = 0$
  - decay constants for pseudoscalar excited states are zero
- To complete the proof: $[Q_A^a,\, P_b] = -\delta_{ab}\,\frac{i}{2}\,\overline{\psi}\psi;\quad \int \frac{d^3p}{2\,p^0(2\,\pi^3)^3}\;|\pi_a\rangle\,\langle\pi_a| = 1$

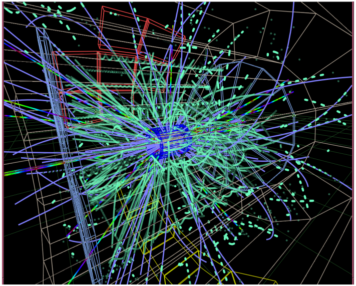
QCD/Hadron Physics Facilities

- Nuclear Science Advisory Council 2007 Long Range Plan
  - "A central goal of (the DOE Office of) Nuclear Physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD."
- Internationally, this is approximately a \$1-billion/year effort in experiment & theory, with roughly \$375-million/year in USA
  - experiment receives about 90% of these funds
  - \$1-billion/year is a similiar size to the operating budget of CERN
- Facilities in: China, Germany, Japan & CERN
- 2015 Long Range Plan expected to make an *Electron Ion Collider* number one priority for new construction in the USA

STAR event [Au-Au] @ RHIC

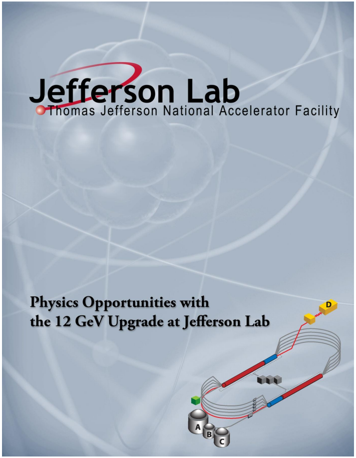


ALICE event [Pb-Pb] @ CERN



QCD/Hadron Physics Machines in the USA

- In the USA Fermilab has a hadron physics program – MINERνA and SeaQuest experiments – and RHIC is purpose build QCD machine
- However, Jefferson Lab (JLab) is the preeminent *hadron physics* facility in the USA & worldwide [polarized e^- beam]
  - *"Explore the fundamental nature of confined states of quarks and gluons"*
  - *"Discover evidence for physics beyond the standard model"*
- Completion of the JLab 12 GeV upgrade promises a new era in hadron physics
  - \$370-million in additional investment and will lead the worldwide hadron physics program for at least the next decade
- Numerous key experiments:
  - measurement of pion form factor to 6 GeV^2
  - nucleon sea-quark and gluon distributions; *etc*



[J. Dudek *et al.*, arXiv:1208.1244 [hep-ex]]