- Start with a case when spin "S" is not observed (or spin-0 particle like pions)

Simpler

$$\Gamma = \{ 1,\ \gamma^\mu,\ \gamma^\mu\gamma^5,\ \sigma^{\mu\nu},\ i\gamma^5,\ \sigma^{\mu\nu} i\gamma^5 \}$$

show $\gamma^0 \gamma^\mu \gamma^0 = \gamma_\mu$

$$\left.\begin{aligned} \mu=0:&\ \gamma^0\gamma^0\gamma^0 = \gamma^0 \\ \mu\neq 0:&\ \gamma^0\gamma^i\gamma^0 = \gamma^0(-\gamma^0\gamma^i) = -\gamma^i \end{aligned}\right\} \quad (\gamma^0, -\gamma^i) = \gamma_\mu$$

$$\gamma^0 \{ 1,\ \gamma^\mu,\ \gamma^\mu\gamma^5,\ \sigma^{\mu\nu},\ i\gamma^5,\ \sigma^{\mu\nu} i\gamma^5 \} \gamma^0$$

$$= \{ 1,\ \gamma_\mu,\ -\gamma_\mu\gamma^5,\ \sigma_{\mu\nu},\ -i\gamma^5,\ -\sigma_{\mu\nu} i\gamma^5 \}$$

$$\boxed{\begin{aligned} \Phi^\dagger(P,k) &= \gamma^0\, \Phi(P,k)\,\gamma^0 \\ \Phi(P,k) &= \gamma^0\, \Phi(\bar{P},\bar{k})\,\gamma^0 \\ \Phi^*(P,k) &= J\, \Phi(\bar{P},\bar{k})\, J \qquad \text{where } J = -i\gamma^5 C = i\gamma^1\gamma^3 \end{aligned}}$$

$$k^\mu = x P^\mu + k_T^\mu$$

Two possible momenta : P, K

$\mathbb{1}$: A_1

γ^μ : $\{P_\mu, K_\nu\}$ $\Rightarrow$ A_2, A_3

$\gamma^\mu\gamma^5$: $\{P_\mu, K_\nu\}$ $\Rightarrow$ A_5, A_6

$\sigma^{\mu\nu}$: $K_\mu P_\nu$ $\Rightarrow$ A_4

$i\gamma^5$: A_7

$\sigma^{\mu\nu} i\gamma^5$: $K_\mu P_\nu$ $\Rightarrow$ A_8

$$\Phi(P,K) = \left[M A_1 + A_2 \not{P} + A_3 \not{K} + A_4 \frac{\sigma^{\mu\nu} K_\mu P_\nu}{M} \right]$$
$$+ \left[A_5 \not{P}\gamma^5 + A_6 \not{K}\gamma^5 + M A_7 (i\gamma^5) + A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M} \right]$$

- Hermiticity $\Rightarrow$ All "A" should be real

e.g. $A_5 \not{P}\gamma^5$

$$(A_5 \not{P}\gamma^5)^\dagger = \gamma^0 (A_5 \not{P}\gamma^5)\gamma^0$$

Note $(\gamma^0)^2 = 1$, $\gamma^0 (\gamma^\mu)^\dagger \gamma^0 = \gamma^\mu$

$\Rightarrow$ LHS $= A_5^* (\gamma^\mu\gamma^5)^\dagger P_\mu = A_5^* \gamma^5 (\gamma^\mu)^\dagger P_\mu = A_5^* \gamma^5 \gamma^0\gamma^0 (\gamma^\mu)^\dagger \gamma^0\gamma^0 P_\mu$

$= A_5^* \gamma^5 \gamma^0 \gamma^\mu \gamma^0 P_\mu$

$= A_5^* (-\gamma^0 \gamma^5 \gamma^\mu \gamma^0) P_\mu$

$= A_5^* (\gamma^0 \gamma^\mu \gamma^5 \gamma^0) P_\mu$

used $\{\gamma^\mu, \gamma^5\} = 0$

RHS $= A_5 P_\mu \gamma^0 (\gamma^\mu\gamma^5)\gamma^0$

$\Rightarrow$ $\boxed{A_5^* = A_5}$

• parity $\Rightarrow$ $A_5 = A_6 = A_7 = A_8 = 0$

$$\Phi(P, K) = \gamma^0 \, \Phi(\bar{P}, \bar{K}) \, \gamma^0$$

e.g. $A_8 \dfrac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$

$$LHS = A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$$

$$RHS = \gamma^0 \left[A_8 \frac{\sigma^{\mu\nu} i\gamma^5 \bar{K}_\mu \bar{P}_\nu}{M} \right] \gamma^0$$

NOTE: $\gamma^0 (\sigma^{\mu\nu} i\gamma^5) \gamma^0 = - \sigma_{\mu\nu} i\gamma^5$

$$\bar{P}_\mu = P^\mu$$

$$\bar{K}_\mu = K^\mu$$

$$= A_8 \left[\frac{- \sigma_{\mu\nu} i\gamma^5 \; K^\mu P^\nu}{M} \right]$$

$$= - A_8 \frac{\sigma_{\mu\nu} i\gamma^5 K^\mu P^\nu}{M}$$

$$= - A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$$

LHS = RHS $\Rightarrow$ $A_8 = - A_8$

$\Rightarrow$ $\boxed{A_8 = 0}$

$$p^\mu = p^+ \bar{n}^\mu \qquad \bar{n}^\mu = [1^+, 0^-, 0_\perp]$$

$$k^\mu = x p^\mu + k_T^\mu$$

$$\Phi(P,k) = M A_1 + A_2 \not{P} + A_3 (x \not{P} + \not{k}_T) + A_4 \frac{\sigma^{\mu\nu}(xP_\mu + k_{T\mu})\, P_\nu}{M}$$

$$\Downarrow \quad \sigma^{\mu\nu} P_\mu P_\nu = 0$$

$$= M A_1 + A_3 \not{k}_T$$

$$+ (A_2 + A_3 x) \not{P}$$

$$+ A_4 \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M}$$

$\Downarrow$ NOTE $\quad P \gg k_T \sim M \quad$ Thus we can drop terms like $M A_1$, $A_3 \not{k}_T$

keep only the largest contribution (twist analysis)

$$= (A_2 + A_3 x) \not{P}$$

$$+ A_4 \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M}$$

$\Downarrow$ give them better names

$$= \frac{1}{2}\left[f_1(x, k_T^2) \not{P} + h_1^\perp(x, k_T^2) \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M} \right]$$

unpolarized PDFs

Boer-Mulders function

transversely polarized quark inside unpolarized nucleon/pion

- what happens to nucleon with spin "S" ?

Three momenta/vector : $k,\ P,\ S$

under parity $\{k,\ P,\ S\} \to \{\ \bar{k},\ \bar{P},\ -\bar{S}\}$

only $k\cdot S$ non-vanishing $(\ P\cdot S = 0 \quad k\cdot P = 0\ )$

$$k\cdot S \xrightarrow{\text{Parity}} \{\ \bar{k}\cdot(-\bar{S}) = -\bar{k}\cdot\bar{S} = -k\cdot S\ \}$$

with parity conserving

$\mathbb{1}$: A_1

$i\gamma^5$: $A_5\ (k\cdot S)\ i\gamma^5$

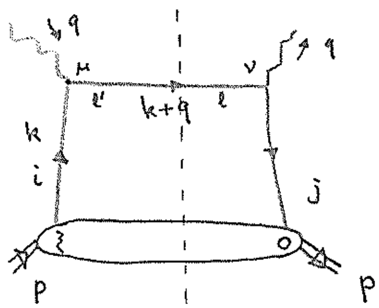
γ^μ : $\{\overset{A_2}{P_\mu}\ ,\ \overset{A_3}{k_\mu}\}\ ,\ \cancel{S_\mu}\ ,\ \overset{\text{Sivers}}{\boxed{\epsilon_{\mu\nu\rho\sigma}\ \gamma^\mu P^\nu k^\rho S^\sigma}}$

$\gamma^\mu\gamma^5$: $\{\ \overset{A_6}{S_\mu}\ ,\ \overset{A_7}{P_\mu\ k\cdot S}\ ,\ \overset{A_8}{k_\mu\ k\cdot S}\ \}$

$i\sigma^{\mu\nu}\gamma^5$: $\{\ \overset{A_9}{S_\mu P_\nu}\ ,\ \overset{A_{10}}{S_\mu k_\nu}\ ,\ \overset{A_{11}}{k\cdot S\ P_\mu k_\nu}\}$

$\sigma^{\mu\nu}$: $\overset{A_4}{P_\mu k_\nu}$

How many correlation function / parton distribution function do we need to characterize the structure of a spin-½ proton ?



$$\sim \langle PS| \tilde{\bar{\psi}}_j \, \gamma^{\nu}_{jl} (\not{k}+\not{q})_{ll'} \, \gamma^{\mu}_{l'i} \, \tilde{\psi}_i |PS\rangle$$

$$\sim \boxed{\langle PS| \tilde{\bar{\psi}}_j \, \tilde{\psi}_i |PS\rangle} * \left[\gamma^{\nu}(\not{k}+\not{q})\gamma^{\mu}\right]_{ji}$$

$\Downarrow$

- generic two-quark correlation function to characterize the nucleon structure
- So far in momentum space
- in Coordinate space we have

$$\Phi_{ij}(k, P, S) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{ik\cdot\xi} \, \langle PS| \bar{\psi}_j(0) \, \psi_i(\xi) |PS\rangle$$

NOTE: In general we also need gauge link to render the above definition gauge invariant, we'll talk about that later

Q: how is Φ_{ij} related to the parton distribution function, as well as many other quantities like transversity, Sivers function, etc ?

- gauge link

$$\Phi(k, P, S) = \frac{1}{(2\pi)^4} \int d^4\xi \, e^{ik\cdot\xi} \quad \langle PS| \, \bar{\psi}(0) \quad \psi(\xi) \, |PS\rangle$$

under gauge transformation

$$\psi(\xi) \to \psi(\xi) \, U(\xi)$$

$$\bar{\psi}(0) \to \bar{\psi}(0) \, U^{+}(0)$$

$$\bar{\psi}(0) \; \psi(\xi) \to \bar{\psi}(0) \, \underbrace{U^{+}(0) \, U(\xi)}_{\neq 1} \, \psi(\xi) \quad \neq \quad \bar{\psi}(0) \, \psi(\xi)$$

Need $\quad W(0,\xi) = P \; \exp\left[-ig \int_0^{\xi} dz \cdot A(z)\right]$

$$W(0,\xi) \to U(0) \; W(0,\xi) \; U^{+}(\xi)$$

$$\bar{\psi}(0) \, W(0,\xi) \; \psi(\xi) \to \bar{\psi}(0) \, \underbrace{U^{+}(0) \; U(0)}_{1} \; W(0,\xi) \; \underbrace{U^{+}(\xi) \; U(\xi)}_{1} \; \psi(\xi)$$

$$= \bar{\psi}(0) \; W(0,\xi) \; \psi(\xi)$$

SIDIS process: $e(l) + P(p) \to e'(l') + h(P_h) + X$

- Since Sivers and Collins function are defined in the CM frame of p and p' where

$$p^\mu = \frac{\sqrt{S_h}}{2}(1,0,0,1) = \sqrt{\frac{S_h}{2}}\,\bar{n}^\mu_{CM}$$

$$P_h^\mu = \frac{\sqrt{S_h}}{2}(1,0,0,-1) = \sqrt{\frac{S_h}{2}}\,n^\mu_{CM}$$

$$S_h = (p+P_h)^2 = 2p\cdot P_h$$

$$\bar{n}^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,1) = [1^+, 0^-, 0_\perp]$$

$$n^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,-1) = [0^+, 1^-, 0_\perp]$$

In this frame, q^μ has transverse momentum, without loss of generality, we could choose the transverse momentum along x-direction, i.e.,

$$q^\mu = (q_0, q_T, q_L) \qquad q^2 = -Q^2 = q_0^2 - q_T^2 - q_L^2$$

This frame will be called CM frame

We'll assume $q_0 < 0$, $q_T < 0$, $q_L < 0$

- Hadron frame is the frame where experiments (close to) try to present the results

In hadron frame, we should have

$$p^\mu \propto (1,0,0,1)$$

$$P_h^\mu \propto (a,b,0,c)$$

$$q^\mu = (0,0,0,-Q)$$

How could one transform from CM frame to hadron frame

① Boost along x-direction, such that $q^\mu_{CM} \Rightarrow$ no transverse component

② Boost along z-direction, such that q^μ has no "zero"-component ($q^0 \to 0$)

after above two boosts, both p^μ and P_h^μ will have transverse component

③ Boost along x-direction again such that p^μ has no transverse component

The result can be found in mathematica file, we only write the final result here!

$$P^\mu = \frac{\sqrt{S_h}}{2Q}\,(q_{0,CM} - q_{L,CM})\,(1,0,0,1)$$

$$P_h^\mu = \frac{\sqrt{S_h}}{2Q}\,\frac{1}{q_{0,CM} - q_{L,CM}}\,(Q^2+q_T^2,\ -2q_T Q,\ 0,\ -Q^2+q_T^2)$$

$$q^\mu = (0,0,0,-Q)$$

NOTE:

$$x_B = \frac{Q^2}{2P\cdot q} \Rightarrow \frac{Q^2}{x_B} = 2\underset{\substack{\uparrow \\ \text{CM frame}}}{P\cdot q} = 2\frac{\sqrt{S_h}}{2}(q_{0,CM} - q_{L,CM}) = \sqrt{S_h}\,(q_{0,CM} - q_{L,CM}) \Rightarrow$$

$$z_h = \frac{P\cdot P_h}{P\cdot q} \Rightarrow S_h = 2P\cdot P_h = z_h\, 2P\cdot q = \frac{z_h}{x_B}Q^2$$

$$q_{0,CM} - q_{L,CM} = \frac{Q^2}{x_B\sqrt{S_h}} = \frac{Q^2}{x_B\sqrt{\frac{z_h}{x_B}}\,Q} = \frac{Q}{\sqrt{z_h x_B}}$$

Thus

$$P^\mu = \frac{Q}{2x_B}(1,0,0,1) = \frac{Q}{\sqrt{2}\,x_B}\bar{n}^\mu \qquad \left(\text{here } \bar{n}^\mu \equiv \bar{n}^\mu_{CM} = \frac{1}{\sqrt{2}}(1,0,0,1)\right)$$

$$P_h^\mu = \frac{z_h}{2Q}(Q^2+q_T^2,\ -2q_T Q,\ 0,\ -Q^2+q_T^2)$$

denote $\quad q_\perp = |q_{T,CM}| =: -q_T \quad$ (note: from beginning we assume $q_{T,CM} < 0$)

thus

$$P_h^\mu = \frac{z_h}{2Q}(Q^2+q_\perp^2,\ 2q_\perp Q,\ 0,\ -Q^2+q_\perp^2)$$

$\Rightarrow$ in hadron frame

$$P_{h\perp} = \frac{z_h}{2Q}\,2q_\perp Q = z_h\, q_\perp = z_h\,|q_{T,CM}|$$

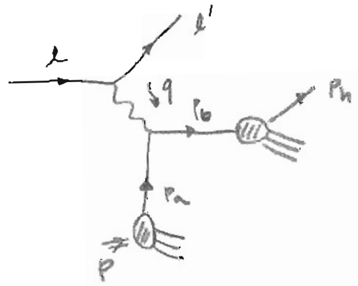
$\uparrow$ transverse momentum of q^μ in CM-frame!!

$$P_h^\mu = \frac{z_h}{2Q}\left(Q^2+q_\perp^2 ,\ 2q_\perp Q,\ 0,\ -Q^2+q_\perp^2 \right)$$

$$= \frac{z_h}{2Q}\left\{ (Q^2,0,0,-Q^2) + (q_\perp^2,0,0,q_\perp^2) + (0,2q_\perp Q,0,0) \right\}$$

$$= \frac{z_h}{2Q}\left\{ \sqrt{2}Q^2 n^\mu + \sqrt{2}q_\perp^2 \bar{n}^\mu + 2Q q_\perp^\mu \right\}$$

$$P_h^\mu = \frac{z_h q_\perp^2}{\sqrt{2}Q}\bar{n}^\mu + \frac{z_h Q}{\sqrt{2}} n^\mu + z_h q_\perp^\mu$$



$$P_h = z p_b + p_\perp \Rightarrow p_b = \frac{P_h - p_\perp}{z}$$

$$(2\pi)^4 \delta^4(p_a+q-p_b)$$

$$p_a + q = p_b$$

$$p_a^\mu = x P^\mu + k_\perp^\mu = \frac{x}{x_B}\frac{Q}{\sqrt{2}}\bar{n}^\mu + k_\perp^\mu$$

$$p_b^\mu = \frac{P_h^\mu - p_\perp^\mu}{z} = \frac{z_h}{z}\frac{q_\perp^2}{\sqrt{2}Q}\bar{n}^\mu + \frac{z_h}{z}\frac{Q}{\sqrt{2}}n^\mu + \left(\frac{z_h}{z}q_\perp^\mu - \frac{p_\perp^\mu}{z}\right)$$

$$q^\mu = (0,0,0,-Q) = \frac{1}{\sqrt{2}}(-Q)\bar{n}^\mu + \frac{Q}{\sqrt{2}}n^\mu$$

$$\delta^4(p_a+q-p_b) = \delta(p_a^+ + q^+ - p_b^+)\,\delta(p_a^- + q^- - p_b^-)\,\delta^2(\perp\text{-part})$$

$$= \delta\left(\frac{x}{x_B}\frac{Q}{\sqrt{2}} - \frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{q_\perp^2}{\sqrt{2}Q}\right)\delta\left(0 + \frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{Q}{\sqrt{2}}\right)$$

$$* \delta^2\left(\vec{k}_\perp - \left(\frac{z_h}{z}q_\perp - \frac{p_\perp}{z}\right)\right)$$

★ only neglect $\frac{q_\perp^2}{Q^2}$, recover usual $x \to x_B$

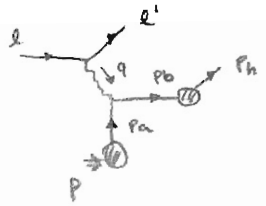
$$\approx \delta\left(\frac{x}{x_B}\frac{Q}{\sqrt{2}} - \frac{Q}{\sqrt{2}}\right)\delta\left(\frac{Q}{\sqrt{2}} - \frac{z_h}{z}\frac{Q}{\sqrt{2}}\right)\delta^2\left(k_\perp - \frac{P_{h\perp} - p_\perp}{z}\right)$$

$$= \frac{x_B}{Q/\sqrt{2}}\delta(x-x_B) * \frac{z}{Q/\sqrt{2}}\delta(z-z_h) * z^2\,\delta^2(z k_\perp + p_\perp - P_{h\perp})$$

$$= \frac{2x_B z^3}{Q^2}\delta(x-x_B)\,\delta(z-z_h)\,\delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

$$= \frac{2x_B z_h^3}{Q^2}\delta(x-x_B)\,\delta(z-z_h)\,\delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

Normalization of cross-section



$$S_{ep} = (P+l)^2 = 2P\cdot l$$

$$Q^2 = -q^2 = -(l-l')^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \qquad z_h = \frac{P\cdot P_h}{P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l} = \frac{Q^2}{x_B S_{ep}}$$

$$d\sigma = \int dx\, d^2k_\perp\, f_1(x, k_\perp^2) \int dz\, d^2p_\perp\, D_1(z, p_\perp^2) \quad d\hat{\sigma}$$

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \overline{|M|^2}\, dPS \qquad\qquad \hat{s} = (xP+l)^2 \approx x S_{ep}$$

$$dps = \frac{d^3l'}{(2\pi)^3 2E'} \frac{d^3p_b}{(2\pi)^3 2E_b} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$\Downarrow \qquad p_b \approx \frac{P_h}{z}$$

$$= \frac{d^3l'}{(2\pi)^3 2E'} \frac{d^3P_h}{(2\pi)^3 2E_h} \frac{1}{z^2} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$\frac{d^3l'}{2E'} = d^4l'\, \delta(l'^2) = \frac{\pi Q^2}{2x_B^2 S_{ep}} dx_B\, dQ^2 = \frac{\pi Q^2}{2x_B} dx_B\, dy$$

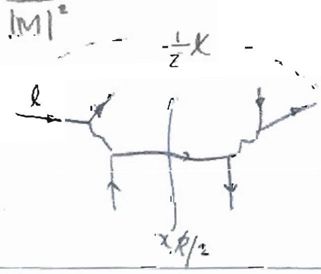
$$\frac{d^3P_h}{2E_h} = d^4P_h\, \delta(P_h^2) = dP_h^+ dP_h^-\, d^2P_{h\perp}\, \underbrace{\delta(2P_h^+P_h^- - P_{h\perp}^2)}_{\frac{1}{2P_h^-}\delta\left(P_h^+ - \frac{P_{h\perp}^2}{2P_h^-}\right)} = \frac{dP_h^-}{2P_h^-} d^2P_{h\perp} = \frac{dz_h}{2z_h} d^2P_{h\perp}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \int dx\, d^2k_\perp\, f_1(x, k_\perp^2) \int dz\, d^2p_\perp\, D_1(z, p_\perp^2)$$

$$* \left(\frac{\pi Q^2}{2x_B} \frac{1}{2z_h}\right) \frac{1}{(2\pi)^6} \frac{1}{z^2} (2\pi)^4 \delta^4(p_a+q-p_b)$$

$$* \frac{1}{2\hat{s}} \overline{|M|^2}$$

$$\overline{|M|^2} \sim$$



$\left(\frac{1}{2}\right)$ from spin-average

In the frame where $q^\mu = (0, 0, 0, -Q)$, one could always express

$$l^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1)$$

$$l'^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, +1) \qquad q^\mu = l^\mu - l'^\mu$$

where $\cosh\psi = \frac{2x_B S_{ep}}{Q^2} - 1 = \frac{2}{y} - 1$

$$l^\mu = \frac{Q}{2}\left[\cosh\psi\, T^\mu + \sinh\psi\cos\phi\, X^\mu + \sinh\psi\sin\phi\, Y^\mu - Z^\mu\right]$$

$$l'^\mu = \frac{Q}{2}\left[\cosh\psi\, T^\mu + \sinh\psi\cos\phi\, X^\mu + \sinh\psi\sin\phi\, Y^\mu + Z^\mu\right]$$

$$L^{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\not{l}\,\gamma^\mu \not{l}'\,\gamma^\nu]$$

$$= \frac{1}{2}\times 4\,(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}\, l\cdot l')$$

$$= 2\,(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}\, l\cdot l')$$

$$\frac{2L^{\mu\nu}}{Q^2} = (1+\cosh^2\psi)\,\overbrace{(X^\mu X^\nu + Y^\mu Y^\nu)}^{-g_\perp^{\mu\nu}} + 2\sinh^2\psi\, T^\mu T^\nu + \sinh 2\psi\cos\phi\,(T^\mu X^\nu + T^\nu X^\mu)$$

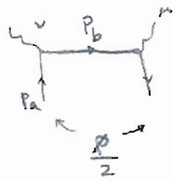
$$+ \sinh 2\psi\sin\phi\,(T^\mu Y^\nu + T^\nu Y^\mu) + \sinh^2\psi\sin 2\phi\,(X^\mu Y^\nu + X^\nu Y^\mu) + \sinh^2\psi\cos 2\phi\,(X^\mu X^\nu - Y^\mu Y^\nu)$$

$$\cosh\psi = \frac{2}{y} - 1$$

$$1 + \cosh^2\psi = 1 + \left(\frac{2}{y} - 1\right)^2 = \frac{4}{y^2}\left(1 - y + y^2/2\right)$$

$$\sinh^2\psi = \cosh^2\psi - 1 = \left(\frac{2}{y} - 1\right)^2 - 1 = \frac{4}{y^2}(1-y)$$

hadronic tensor



should be $\mathrm{Tr}[\frac{\not{x}_a}{2} \gamma^\mu \not{x}_b \gamma^\nu] \sim \underline{\underline{\frac{x}{z}}}\, \mathrm{Tr}[\frac{\not{P}}{2} \gamma^\mu \not{P}_h \gamma^\nu]$

$$W^{\mu\nu}_{UU} = \mathrm{Tr}[\frac{\not{P}}{2}\gamma^\mu \not{P}_h \gamma^\nu] \quad f_1 * D_1$$

$$W^{\mu\nu}_{UT} = \mathrm{Tr}[\gamma_\alpha \gamma^\mu \not{P}_h \gamma^\nu]\, \epsilon^{\alpha\beta\rho\sigma}\, P_\beta k_{\perp\rho} S_{\perp\sigma} \quad \frac{1}{M} f_{1T}^\perp * D_1$$

NOTE: to project D_1, we need $\not{P}_h$, NOT "$\not{n}$" $\hookrightarrow (1,0,0,-1)$

However, $P_h^\mu = \frac{z_h Q}{\sqrt{2}} n^\mu + \frac{z_h q_\perp^2}{\sqrt{2}Q}\bar{n}^\mu + z_h q_\perp^\mu$

$= \frac{z_h Q}{\sqrt{2}} n^\mu + O(q_\perp)$

again, keep the leading contribution
(consistent with TMD)

$$W^{\mu\nu}_{UU} = \mathrm{Tr}[\frac{\not{P}}{2}\gamma^\mu \not{n} \gamma^\nu] * \frac{z_h Q}{\sqrt{2}} \quad f_1 * D_1$$

$$W^{\mu\nu}_{UT} = \mathrm{Tr}[\gamma_\alpha \gamma^\mu \not{n} \gamma^\nu] * \frac{z_h Q}{\sqrt{2}}\, \epsilon^{\alpha\beta\rho\sigma} P_\beta k_{\perp\rho} S_{\perp\sigma} \quad \frac{1}{M} f_{1T}^\perp * D_1$$

Calculate in detail

$$W^{\mu\nu}_{UU} = \frac{Q}{\sqrt{2}x_B}\frac{z_h Q}{\sqrt{2}}\, \mathrm{Tr}[\not{\bar{n}}\gamma^\mu \not{n}\gamma^\nu] \quad f_1 * D_1$$

$$= \frac{z_h}{x_B}\, 2Q^2 \underbrace{(-g^{\mu\nu} + \bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu)}_{-g_\perp^{\mu\nu}} \quad f_1 * D_1$$

$$= \frac{z_h}{x_B}\, 2Q^2 (-g_\perp^{\mu\nu}) \quad f_1 * D_1$$

$$W_{UT}^{\mu\nu} = 4(g_\alpha^{\ \mu} n^\nu - n_\alpha g^{\mu\nu} + g_\alpha^{\ \nu} n^\mu) * \frac{z_h Q}{\sqrt{2}} \frac{Q}{\sqrt{2}x_B} \epsilon^{\alpha \bar{n} k_\perp s_\perp} \frac{1}{M} f_{1T}^\perp * D_1$$

$$= \frac{z_h}{x_B} 2Q^2 \Big(\epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) + \underbrace{\epsilon^{\mu \bar{n} k_\perp s_\perp} n^\nu + \epsilon^{\nu \bar{n} k_\perp s_\perp} n^\mu}_{I^{\mu\nu}} \Big) \frac{1}{M} f_{1T}^\perp D_1$$

NOTE: $I^{\mu\nu} * \begin{pmatrix} X^\mu X^\nu + Y^\mu Y^\nu \\ T^\mu X^\nu + T^\nu X^\mu \\ T^\mu Y^\nu + T^\nu Y^\mu \\ X^\mu Y^\nu + X^\nu Y^\mu \\ X^\mu X^\nu - Y^\mu Y^\nu \end{pmatrix} \Rightarrow$ "zero"

only worry about $T^\mu T^\nu = \frac{1}{\sqrt{2}}(\bar{n}^\mu + n^\mu)(\bar{n}^\nu + n^\nu)\frac{1}{\sqrt{2}}$

$$= \frac{1}{2}(\bar{n}^\mu \bar{n}^\nu + n^\mu n^\nu + \bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu)$$

$$I^{\mu\nu} * T^\mu T^\nu = \frac{1}{2} * [2\epsilon^{n \bar{n} k_\perp s_\perp}] = \epsilon^{\bar{n} n s_\perp k_\perp}$$

$$\epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) T^\mu T^\nu = -\epsilon^{\bar{n} n s_\perp k_\perp}$$

Thus $(-g^{\mu\nu} \epsilon^{\bar{n} n s_\perp k_\perp} + I^{\mu\nu}) T^\mu T^\nu = 0 \Rightarrow W_{UT}^{\mu\nu} T_\mu T_\nu = 0$

only need to care about $(-g_{\mu\nu})$ - term, in other words

$$W_{UT}^{\mu\nu} \rightarrow \frac{z_h}{x_B} 2Q^2 \epsilon^{\bar{n} n s_\perp k_\perp} (-g^{\mu\nu}) * \frac{1}{M} f_{1T}^\perp D_1$$

We thus have

$$L_{\mu\nu} W^{\mu\nu}_{UU} = \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2 \quad f_1 * D_1 \quad [1+\cosh^2\psi]$$

$$L_{\mu\nu} W^{\mu\nu}_{UT} = \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * \epsilon^{\bar{n} n S_\perp k_\perp} * 2 * \frac{1}{M} f_{1T}^{\perp} D_1 \; [1+\cosh^2\psi]$$

also photon propagator $\left(\frac{1}{Q^2}\right)^2$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \int dx\, d^2k_\perp \int dz\, d^2p_\perp \begin{pmatrix} f_1\, D_1 \\ \frac{1}{M} f_{1T}^{\perp} D_1 \end{pmatrix} * e^4 e_q^2$$

$$* \left(\frac{\pi Q^2}{2x_B} \frac{1}{2z_h}\right) \frac{1}{(2\pi)^6} \frac{1}{z^2} (2\pi)^4\, \delta^4(p_a+q-p_b)$$

$$* \frac{1}{2x S_{ep}} * \left(\frac{1}{Q^2}\right)^2 * \begin{pmatrix} \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2 \\ \frac{Q^2}{2} * \frac{z_h}{x_B} 2Q^2 * 2\, \epsilon^{\bar{n} n S_\perp k_\perp} \end{pmatrix} * \frac{x}{z} * \underbrace{[1+\cosh^2\psi]}_{\uparrow\, \frac{4}{y^2}(1-y+y^2/2)}$$

$$\Downarrow$$

$$\delta^4(p_a+q-p_b) = \frac{2x_B z_h^3}{Q^2} \delta(x-x_B)\, \delta(z-z_h)\, \delta^2(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

$$= e_q^2 \frac{4\pi \alpha_{em}^2}{Q^2 y} \left[1-y+y^2/2\right] * \left[f_1 * D_1 + \epsilon^{\bar{n} n S_\perp k_\perp} \frac{f_{1T}^{\perp}}{M} D_1 \right]$$

Summary:

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left[1-y+y^2/2\right]\left[f_1 \otimes D_1 + \epsilon^{\bar{n} n S_\perp k_\perp} \frac{1}{M} f_{1T}^\perp \otimes D_1\right]$$

where $f_1 \otimes D_1 = \int d^2k_\perp\, d^2p_\perp\; \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\; f_1(x_B, k_\perp^2)\; D_1(z_h, p_\perp^2)$

$\epsilon^{\bar{n} n S_\perp k_\perp} \frac{1}{M} f_{1T}^\perp \otimes D_1 = \int d^2k_\perp\, d^2p_\perp\; \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\; \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\; D_1(z_h, p_\perp^2)\; \epsilon^{\bar{n} n S_\perp k_\perp}$

$$\epsilon^{\bar{n} n S_\perp k_\perp} = \epsilon^{\bar{n} n \alpha\beta}\, S_{\perp\alpha}\, k_{\perp\beta}$$

$$= \epsilon_\perp^{\alpha\beta}\, S_{\perp\alpha}\, k_{\perp\beta}$$

where $\epsilon^{\bar{n} n \alpha\beta} \equiv \epsilon_\perp^{\alpha\beta} = \begin{cases} +1 & \alpha=1,\ \beta=2 \\ -1 & \alpha=2,\ \beta=1 \end{cases}$

define

$$I^\beta = \int d^2k_\perp\, d^2p_\perp\; \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\; \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\; D_1(z_h, p_\perp^2)\; k_\perp^\beta$$

after integration over $d^2k_\perp\, d^2p_\perp$, only left vector $P_{h\perp}$

$$= I * P_{h\perp}^\beta$$

$$I = \frac{I^\beta P_{h\perp\beta}}{-\vec{P}_{h\perp}^2} = \frac{\overbrace{k_\perp \cdot P_{h\perp}}^{-\vec{k}_\perp \cdot \vec{P}_{h\perp}}}{-\vec{P}_{h\perp}^2}(\cdots) = \frac{\vec{k}_\perp \cdot \vec{P}_{h\perp}}{\vec{P}_{h\perp}^2}$$

thus $$I^\beta = \frac{P_{h\perp}^\beta}{\vec{P}_{h\perp}^2}\int d^2\vec{k}_\perp\, d^2\vec{p}_\perp\, (\vec{k}_\perp \cdot \vec{P}_{h\perp})\; \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\; \frac{1}{M} f_{1T}^\perp D_1$$

$\Downarrow$ define $\frac{\vec{P}_{h\perp}^\beta}{|\vec{P}_{h\perp}|} = \hat{P}_{h\perp}^\beta$

$$= \hat{P}_{h\perp}^\beta \int d^2k_\perp\, d^2p_\perp\; \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})\; (\vec{k}_\perp \cdot \hat{P}_{h\perp})\; \frac{1}{M} f_{1T}^\perp D_1$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left(1-y+y^2/2\right)\left[f_1 \otimes D_1 + \epsilon^{\bar{n} n S_\perp \hat{P}_{h\perp}} \frac{1}{M} f_{1T}^\perp \otimes D_1\right]$$

$$\begin{aligned}
\epsilon^{\bar{n} n S_\perp \hat{P}_{h\perp}} &= \epsilon_\perp^{\alpha\beta} S_{\perp\alpha} \hat{P}_{h\perp\beta} \\
&= S_{\perp 1} \hat{P}_{\perp 2} - S_{\perp 2} \hat{P}_{\perp 1} \\
&= \cos\phi_s \sin\phi_h - \sin\phi_s \cos\phi_h \\
&= \sin(\phi_h - \phi_s)
\end{aligned}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}\left(1-y+y^2/2\right)\left[f_1 \otimes D_1 + \sin(\phi_h-\phi_s) \frac{1}{M} f_{1T}^\perp \otimes D_1\right]$$

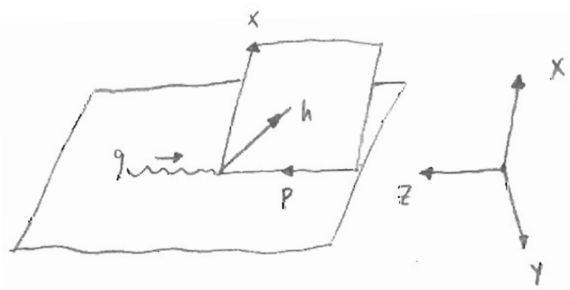
$$f_1 \otimes D_1 = \int f_1(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)$$

$$\frac{1}{M} f_{1T}^\perp \otimes D_1 = \int \frac{1}{M} f_{1T}^\perp(x_B, k_\perp^2)\, D_1(z_h, p_\perp^2)\, \vec{k}_\perp \cdot \hat{P}_{h\perp}$$

$$\int = \int d^2k_\perp\, d^2p_\perp\, \delta^2(z_h \vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp})$$

In fact, in this "hadron frame" : $\phi_h = 0$

our reference frame : hadron frame!



Trento convention: $\tilde{\phi}_h$, $\tilde{\phi}_S$ counted w.r.t leptonic plane!

$$\underbrace{\tilde{\phi}_h - \tilde{\phi}_S}_{\text{lepton plane}} = -[\underbrace{\phi_h - \phi_S}_{\text{hadron plane}}]$$

In Trento convention,

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sum_q e_q^2 \frac{4\pi\alpha_{em}^2}{Q^2 y}(1-y+y^2/2)\left[f_1 \otimes D_1 + \sin(\tilde{\phi}_h - \tilde{\phi}_S)\underline{\underline{(-1)}}\frac{1}{M} f_{1T}^{\perp} \otimes D_1\right]$$