

QCD structure of the nucleon and spin physics

Lecture 2 & 3: QCD collinear factorization and evolution

Zhongbo Kang
Los Alamos National Laboratory

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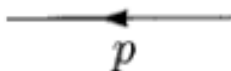
QED: the fundamental theory of electro-magnetic interaction

- QED Lagrangian:


$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Feynman rule: photon has no charge, thus does not self-interact

Dirac propagator:  $= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

Photon propagator:  $= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}$

QED vertex:  $= iQe\gamma^\mu$
($Q = -1$ for an electron)


QCD: the fundamental theory of the strong interaction

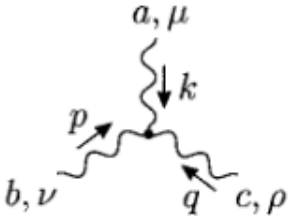
- As the fundamental theory, QCD describes the interaction between quarks and gluons (not hadrons directly)

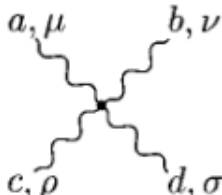
$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + g\bar{\psi}\gamma^\mu t_a \psi G_\mu^a - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}$$

$$G_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf_{abc}G_\mu^b G_\nu^c$$

- Feynman rules: gluon carries color, thus can self-interact

Fermion vertex:  = $ig\gamma^\mu t^a$

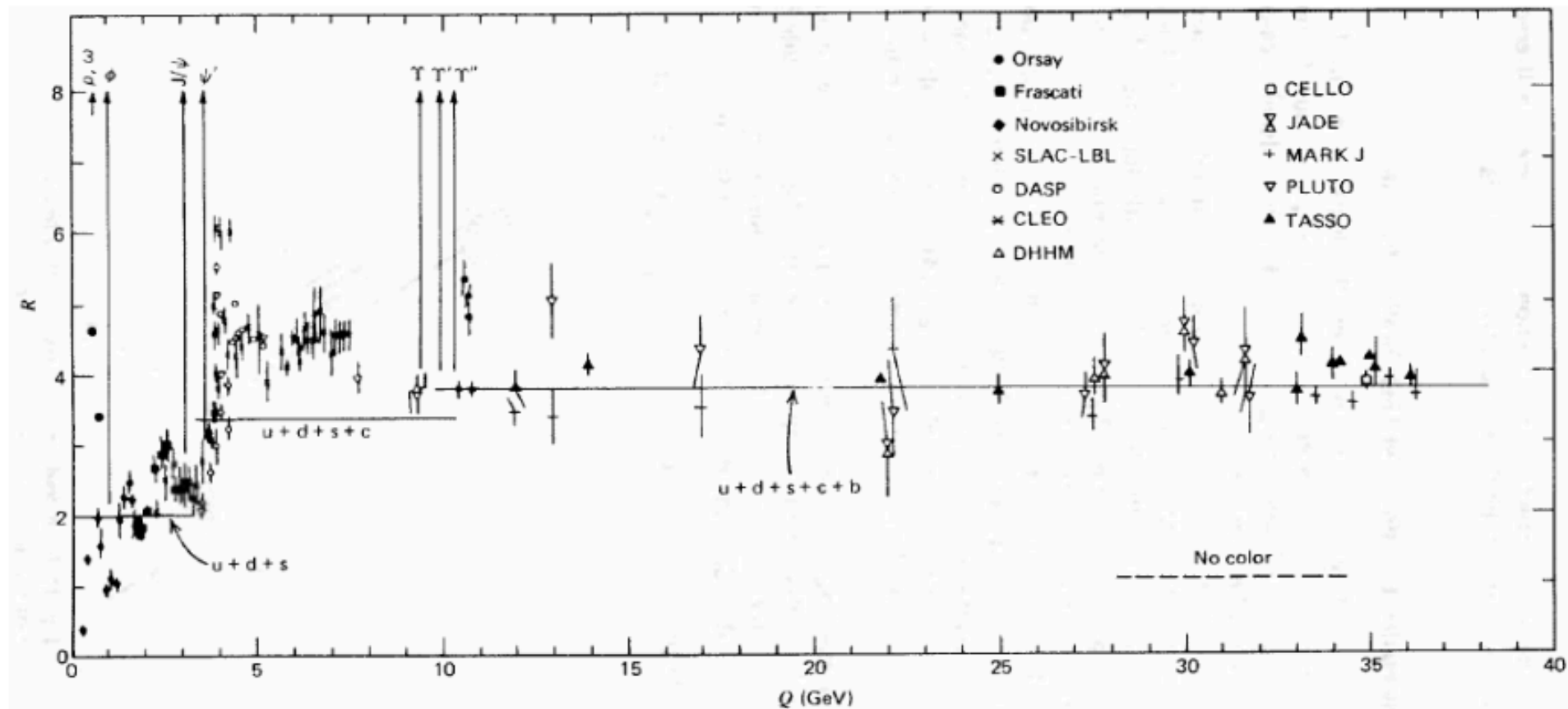
3-boson vertex:  = $gf^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$

4-boson vertex:  = $-ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$

Experimental verification of the color

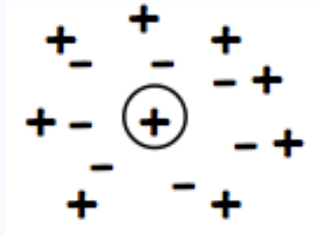
- The color does exist: color of quarks $N_c = 3$ (low energy $R=2/3$ v.s. 2)

$$R_{e^+e^-} = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_q e_q^2$$

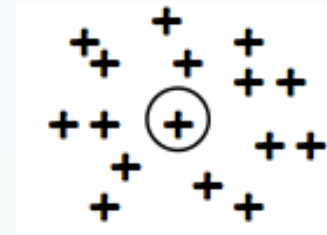


Understanding QCD: running coupling (asymptotic freedom)

- Rough qualitative picture: due to gluon carrying color charges
 - Value of the strong coupling α_s depends on the distance (i.e., energy)



Screening: $\alpha_{em}(r) \uparrow$ as $r \downarrow$



Anti-screening: $\alpha_s(r) \downarrow$ as $r \downarrow$

Asymptotic Freedom \Leftrightarrow antiscreening

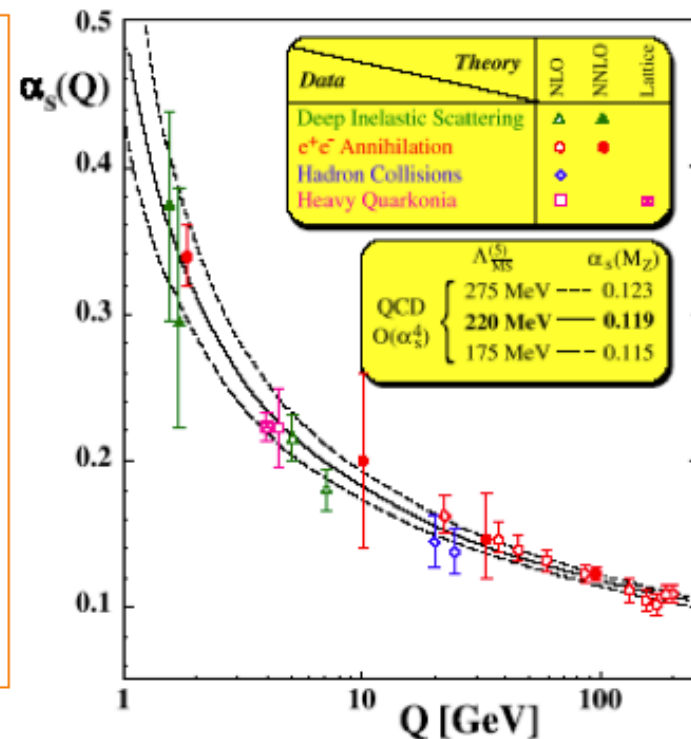
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

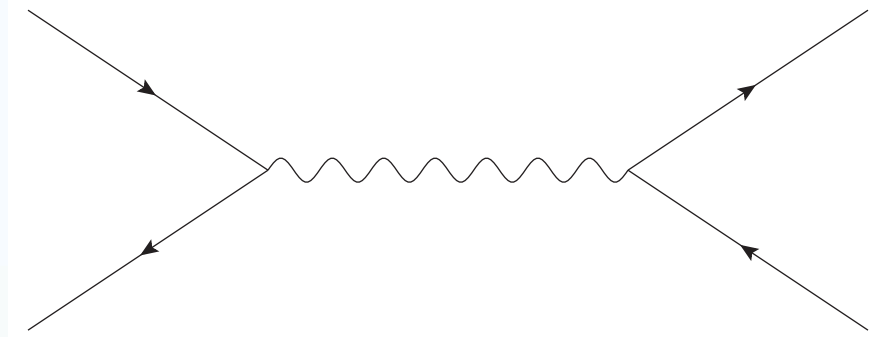
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

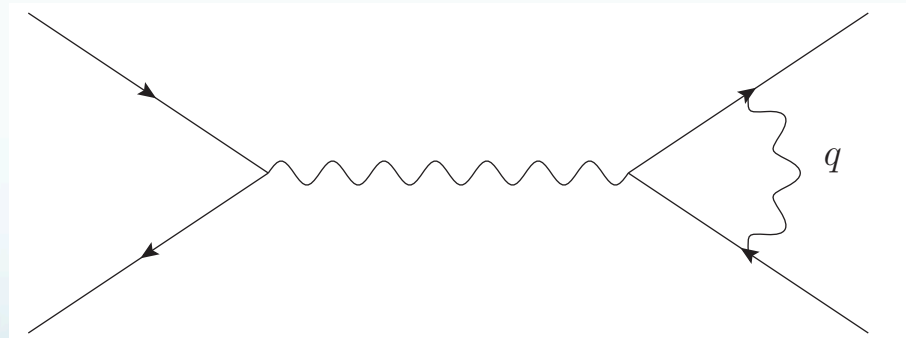


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams – always finite



- Study a higher order Feynman diagram: one-loop, the diagram is divergent as $q \rightarrow \infty$



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
 - UV divergence due to “high momentum” states
 - Experiments cannot resolve the details of these states

The diagram shows an equation: a vertex with two incoming lines and one outgoing wavy line labeled q^2 is equal to the difference between the same vertex and a vertex with a counterterm (a circle with a vertical line and $1/\mu$ label), plus another counterterm vertex.

Low momentum state

High momentum state

- Combine the “high momentum” states with leading order

LO:

The diagram shows the leading order (LO) equation: the original vertex plus the counterterm vertex equals $g(\mu)$.

Renormalized coupling

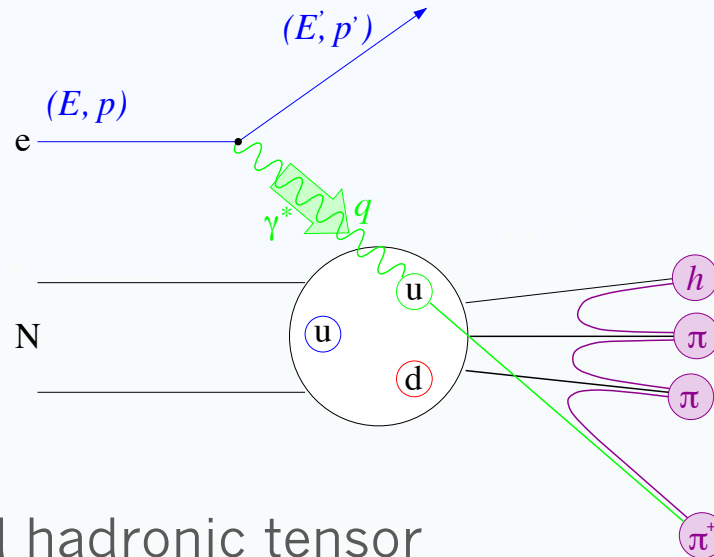
NLO:

The diagram shows the next-to-leading order (NLO) equation: the difference between the original vertex and the counterterm vertex, followed by an ellipsis.

No UV divergence!

Simple study of Deep Inelastic Scattering (DIS): parton model

- DIS has been used a lot in extracting hadron structure



- Leptonic and hadronic tensor

$d\sigma \propto L_{\mu\nu}(\ell, q)W^{\mu\nu}(p, q)$

The diagram shows the factorization of the DIS cross-section. On the left, a lepton ℓ with momentum ℓ and a hadron p with momentum p interact via a virtual photon q . The interaction is shown as a lepton line and a hadron line connected by a wavy line representing the photon. On the right, the cross-section is expressed as the product of a leptonic tensor $L_{\mu\nu}$ and a hadronic tensor $W^{\mu\nu}$. The leptonic tensor $L_{\mu\nu}$ is shown as a lepton line with incoming momentum ℓ and outgoing momentum μ , and a virtual photon q with momentum q . The hadronic tensor $W^{\mu\nu}$ is shown as a hadron line with incoming momentum p and outgoing momentum ν , and a virtual photon q with momentum q .

– Electron is elementary: $L_{\mu\nu}$ can be calculated perturbatively

Structure functions

- Hadronic tensor: Lorentz decomposition + parity invariance (for photon case) + time-reversal invariance + gauge invariance

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x_B, Q^2) + \frac{1}{p \cdot q} \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) F_2(x_B, Q^2)$$

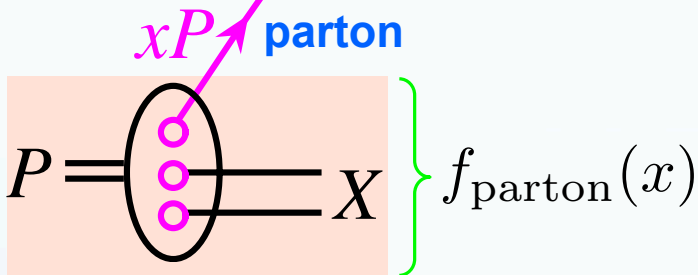
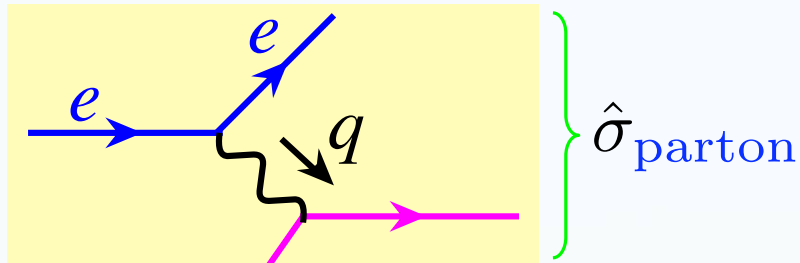
- All the information about hadron structure is contained in the structure functions

$$L_{\mu\nu} = 2 (\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \ell \cdot \ell' g_{\mu\nu})$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha_{\text{em}}^2}{x_B Q^4} [(1-y) F_2(x_B, Q^2) + y^2 x_B F_1(x_B, Q^2)]$$

The paradigm of perturbative QCD

- The common wisdom: to trace back what's inside the proton from the outcome of the collisions, we rely on QCD factorization



Parton Distribution Functions (PDFs):
Probability density for finding a parton in a proton with momentum fraction x

$$\sigma_{\text{proton}}(Q) = f_{\text{parton}}(x) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

Universal (measured) **calculable**

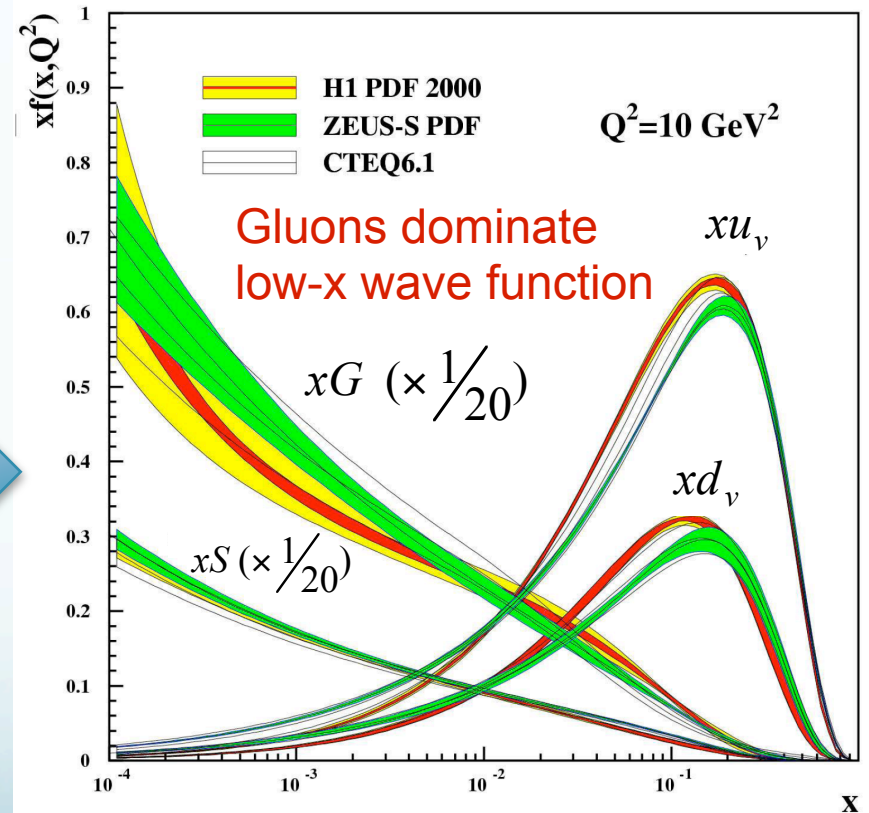
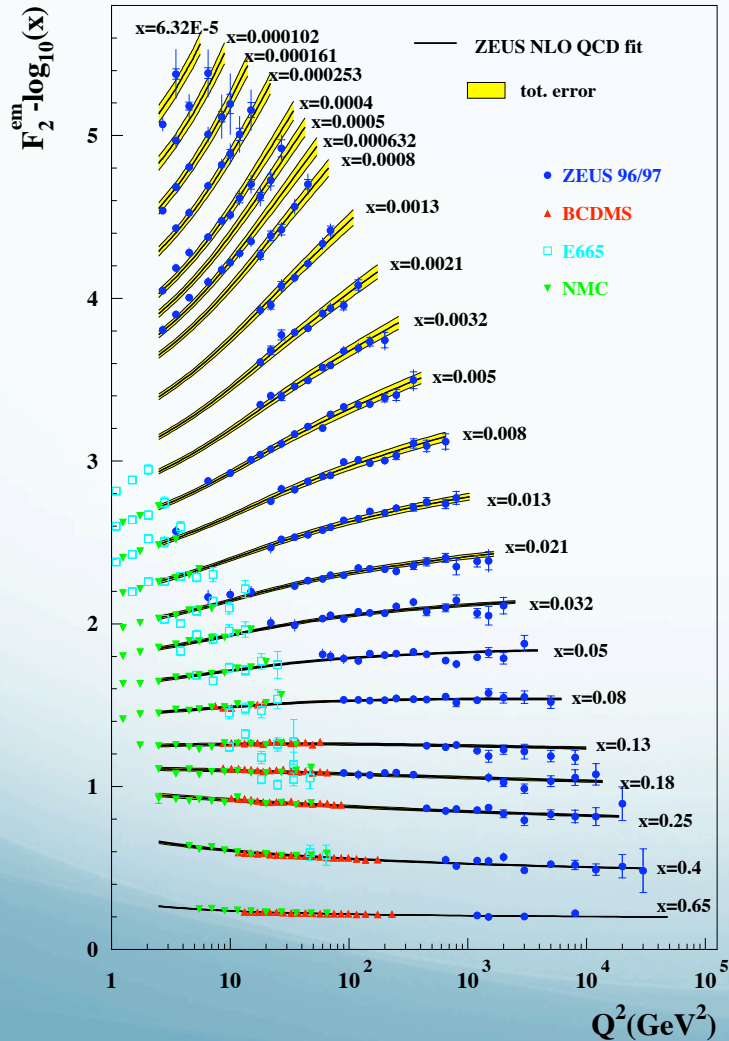
- Hadron structure: encoded in PDFs
- QCD dynamics at short-distance: partonic cross section, perturbatively calculable

Universality of PDFs: extraction from DIS

$$\sigma_{\text{proton}}(Q) = f_{\text{parton}}(x) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

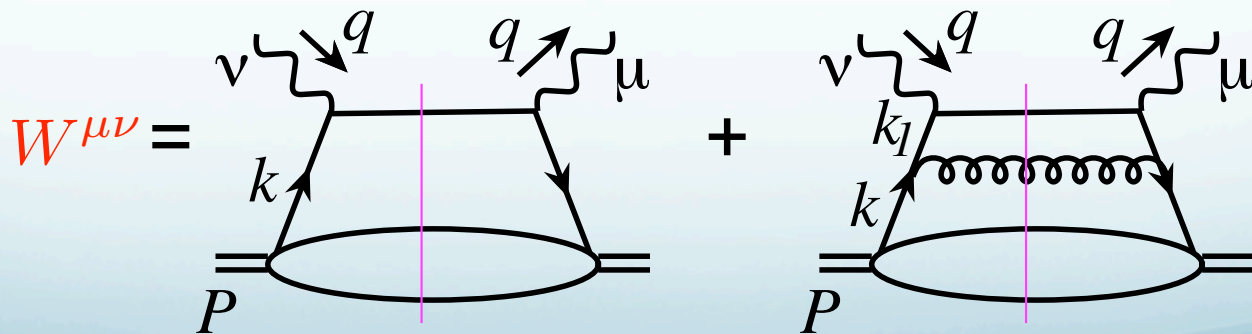
Universal (measured)

calculable



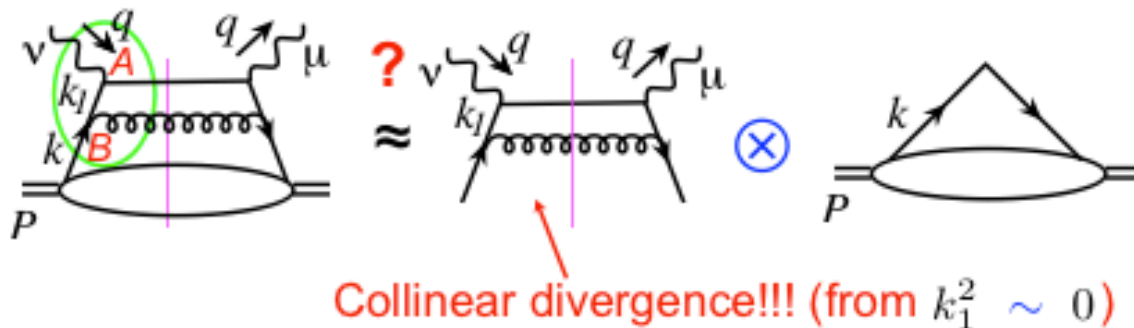
What about higher order?

- pQCD calculations: understand and make sense of all kinds of divergences
 - Ultraviolet (UV) divergence $k \rightarrow \infty$: renormalization (redefine coupling constant)
 - Collinear divergence $k // P$: redefine the PDFs and FFs
 - Soft divergence $k \rightarrow 0$: usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations
- Going beyond the leading order of the DIS, we face another divergence



QCD dynamics beyond tree level

- Going beyond leading order calculation



$$\Rightarrow \int d^4 k_1 \frac{i}{k_1^2 + i\epsilon} \frac{-i}{k_1^2 - i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k + k_g)^2 = 2EE_g(1 - \cos \theta)$$

❖ $k_1^2 \sim 0$ intermediate quark is on-shell

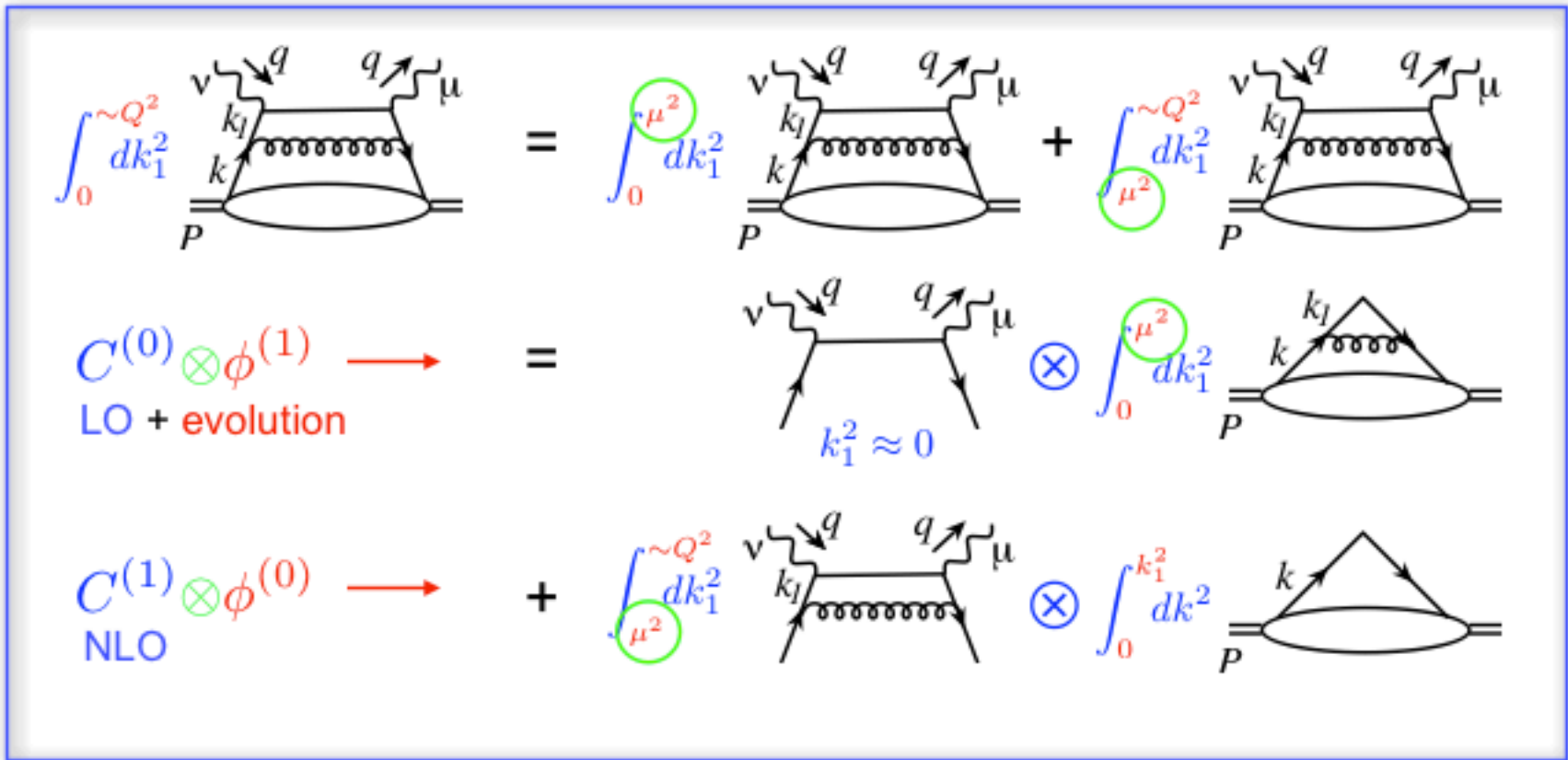
$$t_{AB} \rightarrow \infty$$

❖ gluon radiation takes place long before the photon-quark interaction
 \Rightarrow a part of PDF

Partonic diagram has both long- and short-distance physics

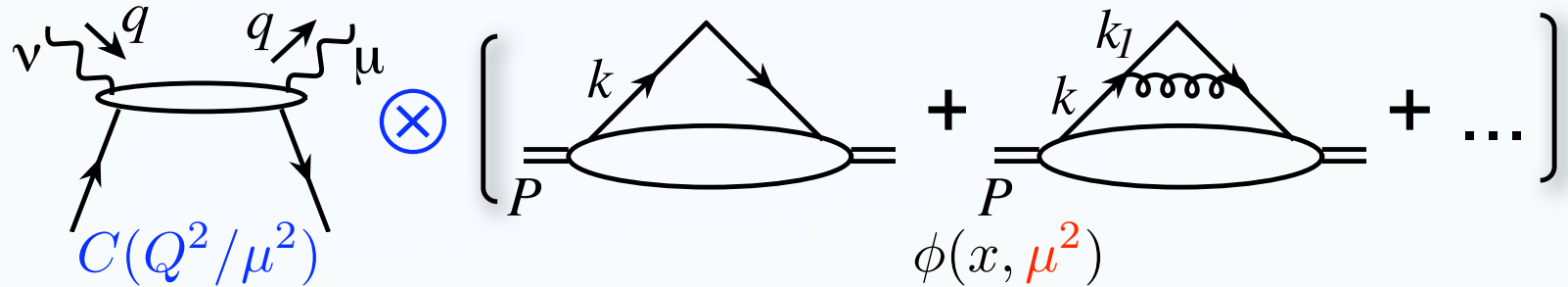
QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs

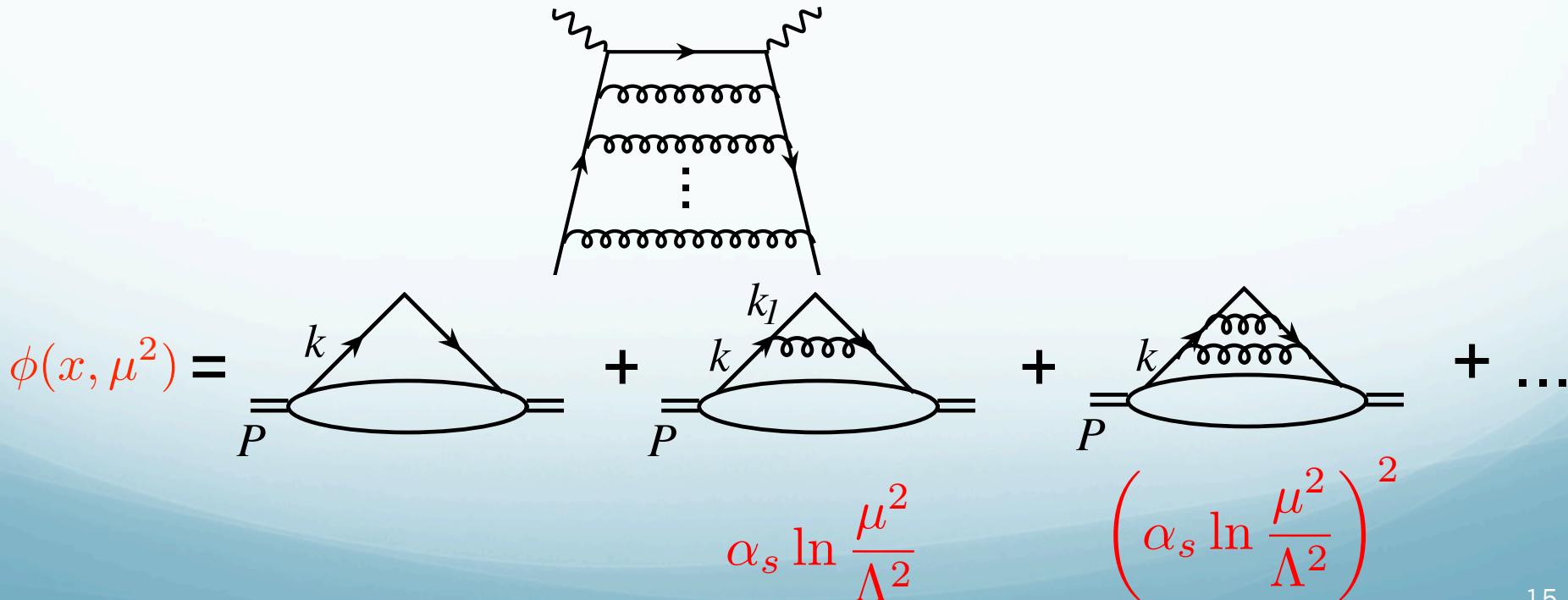


Scale-dependence of PDFs

- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs



DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$$\phi(x, \mu^2) = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$

The diagrams show a series of terms in a sum. Each term consists of a triangle loop (representing a quark) on top of a horizontal oval (representing a parton distribution function). The first diagram has a single gluon line (curly) connecting the two vertices of the triangle, with momentum \$k\$ labeled. The second diagram has a gluon line with a self-energy correction (a loop) on it, with momentum \$k\$ and \$k_1\$ labeled. The third diagram has a more complex gluon radiation structure. All diagrams have a double line labeled \$P\$ entering from the left and exiting to the right.

$$\phi(x, \mu^2) - \text{Diagram 1} = \text{Evolution kernel} \otimes \left(\text{Diagram 1} + \text{Diagram 2} + \dots \right)$$

The evolution kernel is shown as a triangle loop with a gluon line and a self-energy correction, enclosed in a red box. It is multiplied by a tensor product symbol (\$\otimes\$) and the sum of diagrams from the previous equation.

DGLAP Equation

Evolution kernel splitting function

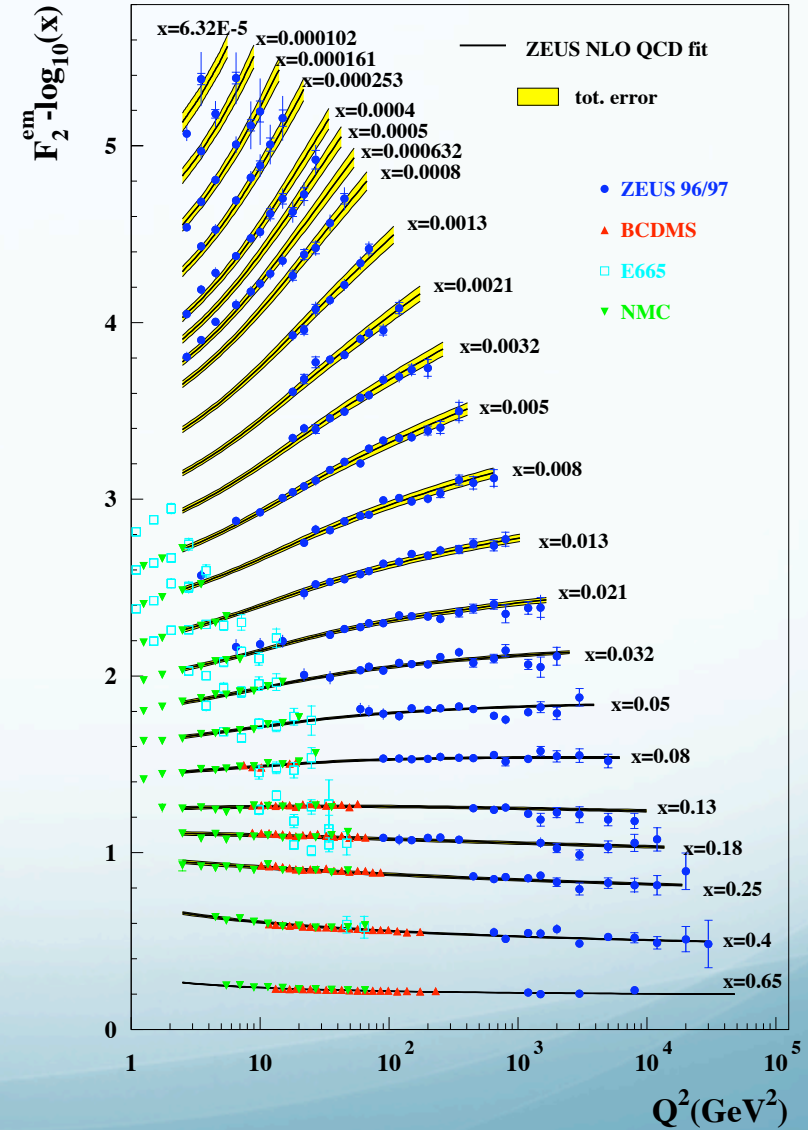
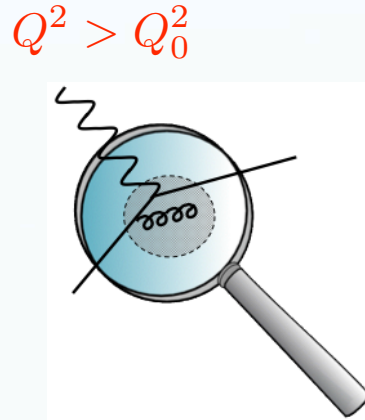
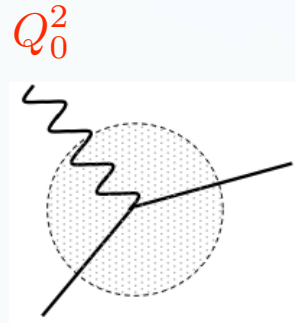
$$\frac{\partial}{\partial \ln \mu^2} \phi_i(x, \mu^2) = \sum_j P_{ij}\left(\frac{x}{x'}\right) \otimes \phi_j(x', \mu^2)$$

The splitting function \$P_{ij}\$ is highlighted in a red box in the original image.

- By solving the evolution equation, one resums all the single logarithms of type $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2} \right)^n$

PDFs also depends on the scale of the probe

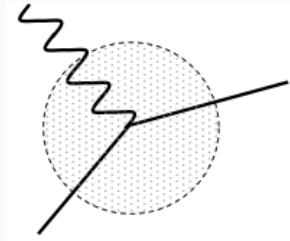
- Increase the energy scale, one sees parton picture differently



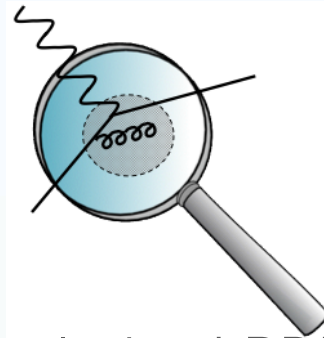
Evolutions of PDFs

- Perturbative change:

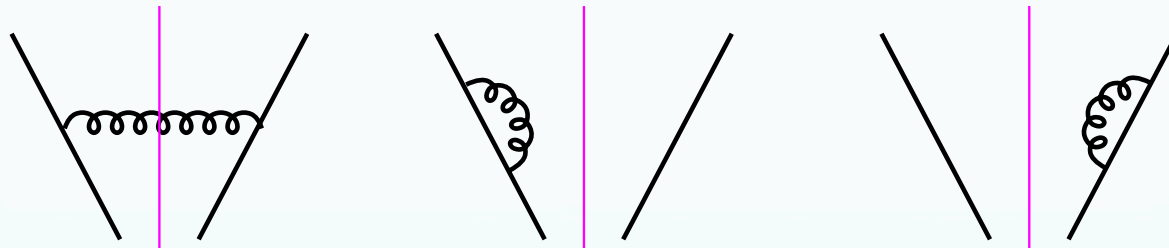
Q_0^2



$Q^2 > Q_0^2$



- Feynman diagrams for unpolarized PDFs



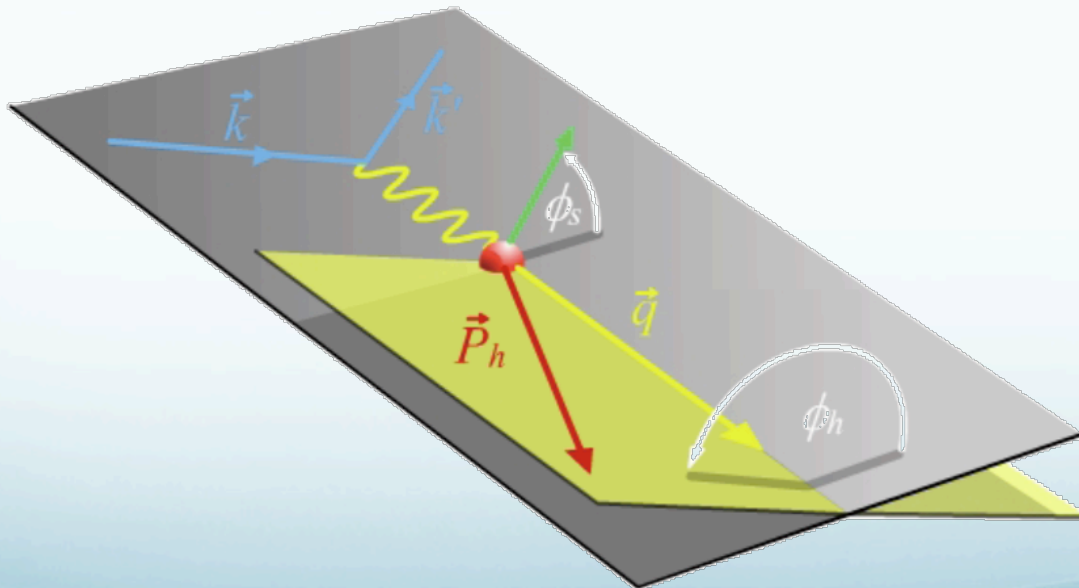
$$\frac{q(x, \mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left[P_{qq}(z) q(\xi, \mu_F) \right]$$

$$P_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right]$$

Example

- We will now study a detailed example
 - SIDIS at both LO and NLO
 - We can understand QCD collinear factorization and evolution of PDFs and FFs
- SIDIS and hadron frame

$$\ell + p^\uparrow \rightarrow \ell' + h(P_h) + X$$



Computer program

- A little bit on the Dirac gamma matrix computation
 - FeynCalc (mathematica): <http://www.feyncalc.org>
 - Tracer (mathematica):
<http://library.wolfram.com/infocenter/MathSource/2987/>
 - Reduce: <http://reduce-algebra.sourceforge.net/>
 - FORM: <http://www.nikhef.nl/~form/>
- The detailed calculations are provided on scanned notes as well as white board presentation

Summary

- Asymptotic freedom: allow one to calculate partonic cross sections
- Parton distribution functions
- Renormalization scale and factorization scale