QCD structure of the nucleon and spin physics

Lecture 2 & 3: QCD collinear factorization and evolution

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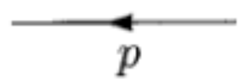
QED: the fundamental theory of electo-magnetic interaction

- QED Lagrangian:

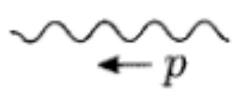
$$\boxed{\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Feynman rule: photon has no charge, thus does not self-interact

Dirac propagator: 

$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

Photon propagator: 

$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}$

QED vertex: 

$= iQe\gamma^\mu$

($Q = -1$ for an electron)

QCD: the fundamental theory of the strong interaction

- As the fundamental theory, QCD describes the interaction between quarks and gluons (not hadrons directly)

$$\boxed{\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + g\bar{\psi}\gamma^\mu t_a \psi G^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a}$$

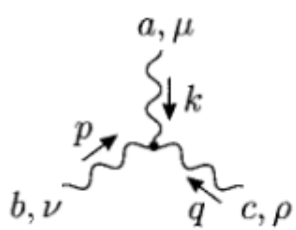
$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu$$

- Feynman rules: gluon carries color, thus can self-interact

Fermion vertex:

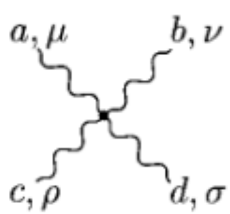
$= ig\gamma^\mu t^a$

3-boson vertex:



$$= g f^{abc}\left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\right]$$

4-boson vertex:

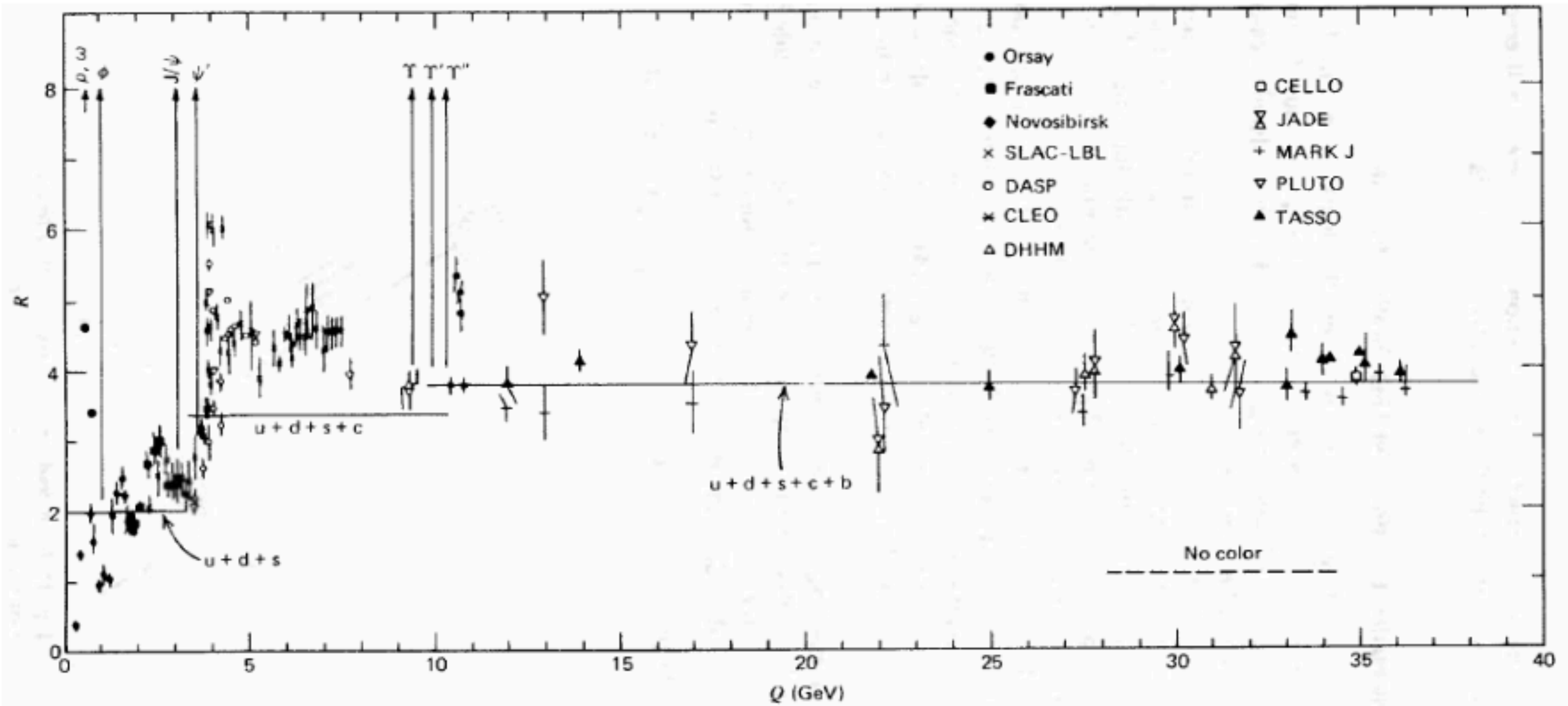


$$= -ig^2\left[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\right]$$

Experimental verification of the color

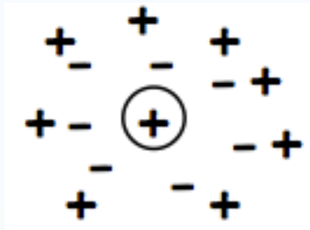
- The color does exist: color of quarks Nc = 3 (low energy R=2/3 v.s. 2)

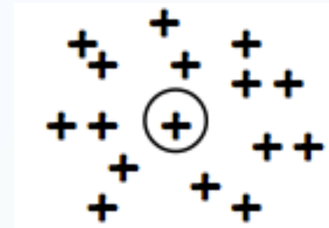
$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to q\bar{q})}{e^+e^- \to \mu^+\mu^-} = N_c \sum_q e_q^2$$



Understanding QCD: running coupling (asymptotic freedom)

- Rough qualitative picture: due to gluon carrying color charges
  - Value of the strong coupling α_s depends on the distance (i.e., energy)





Screening: $\alpha_{em}(r)\ \uparrow\ \ \text{as}\ \ r\ \downarrow$

Anti-screening: $\alpha_s(r)\ \downarrow\ \ \text{as}\ \ r\ \downarrow$

Asymptotic Freedom ⇔ antiscreening

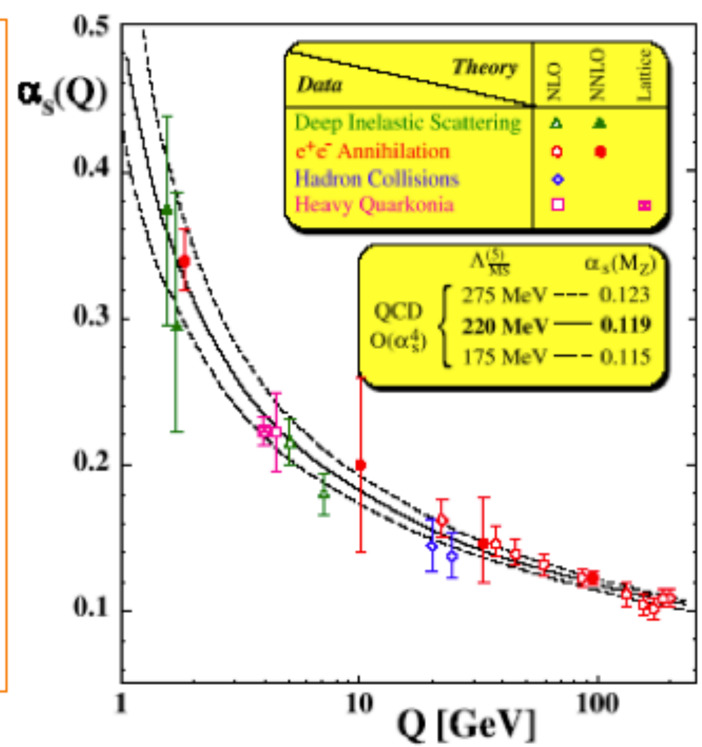
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

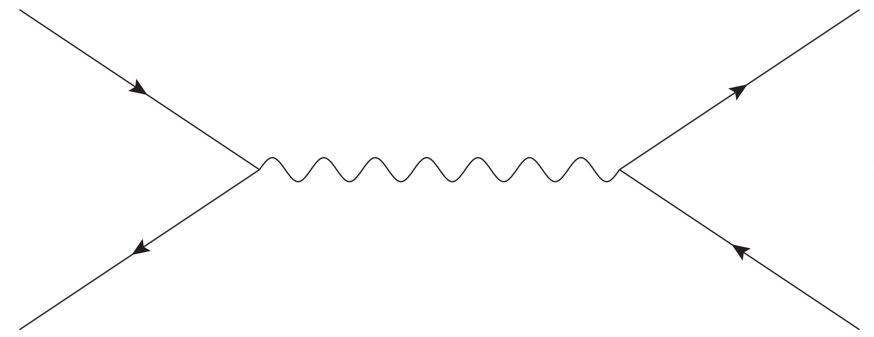
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

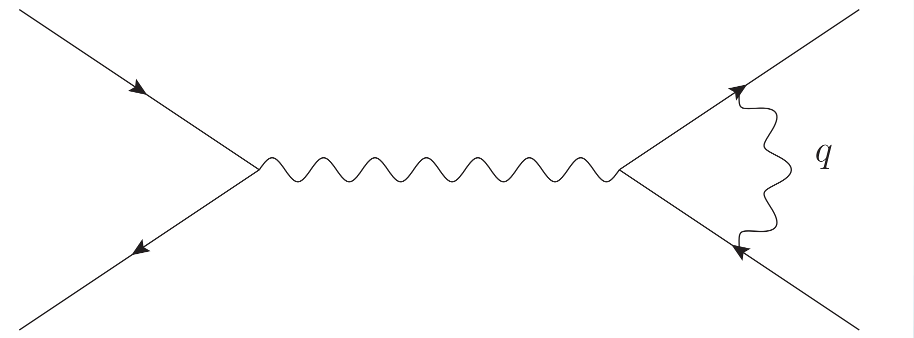


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams – always finite



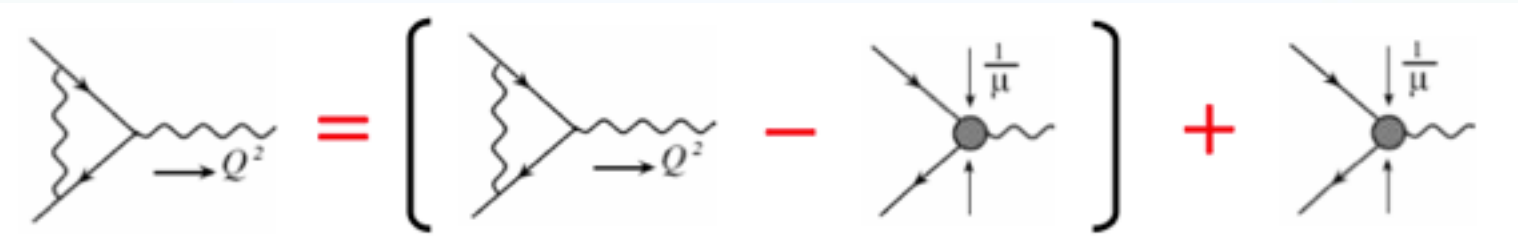
- Study a higher order Feynman diagram: one-loop, the diagram is divergent as $q \to \infty$



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
  - UV divergence due to “high momentum” states
  - Experiments cannot resolve the details of these states

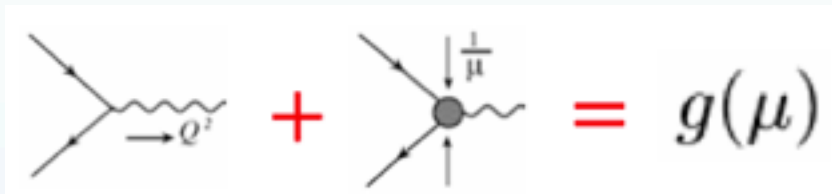


Low momentum state

High momentum state

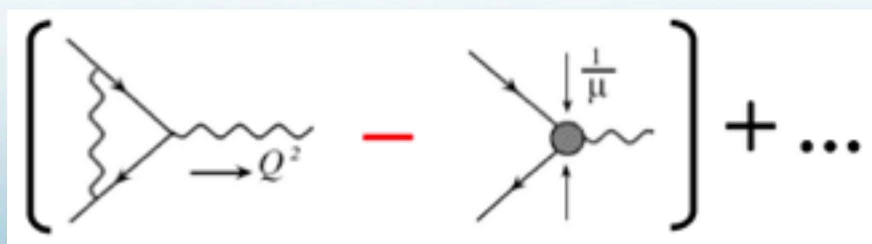
- Combine the “high momentum” states with leading order

LO:



Renormalized coupling

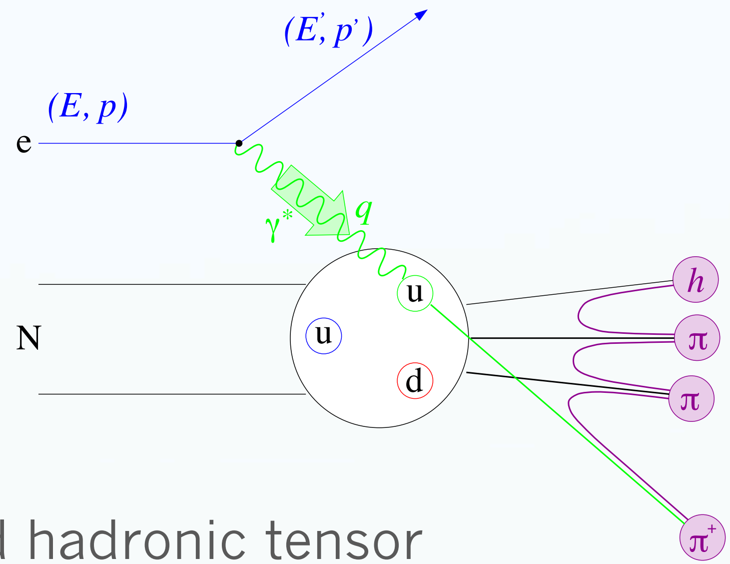
NLO:



No UV divergence!

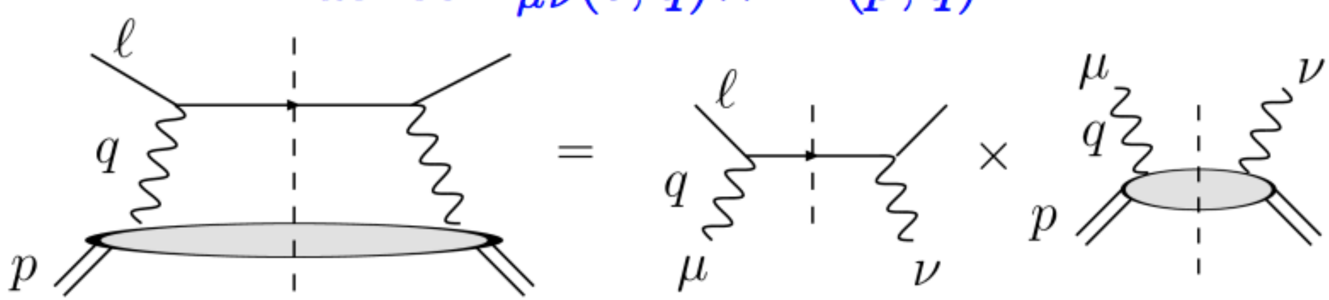
Simple study of Deep Inelastic Scattering (DIS): parton model

- DIS has been used a lot in extracting hadron structure



- Leptonic and hadronic tensor

$$d\sigma \propto L_{\mu\nu}(\ell, q)W^{\mu\nu}(p, q)$$



– Electron is elementary: $L_{\mu\nu}$ can be calculated perturbatively

Structure functions

- Hadronic tensor: Lorentz decomposition + parity invariance (for photon case) + time-reversal invariance + gauge invariance

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

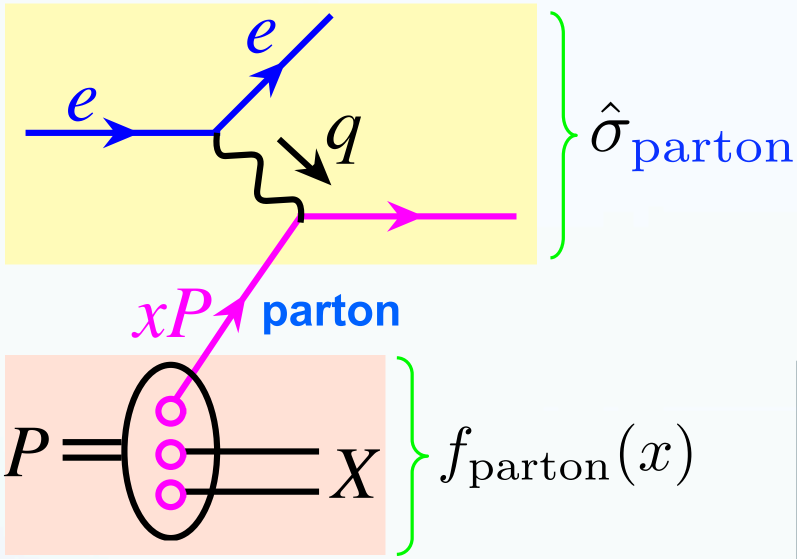
- All the information about hadron structure is contained in the structure functions

$$L_{\mu\nu} = 2\left(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \ell\cdot\ell' g_{\mu\nu}\right)$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{x_B Q^4}\left[(1-y)\, F_2(x_B, Q^2) + y^2 x_B F_1(x_B, Q^2)\right]$$

The paradigm of perturbative QCD

- The common wisdom: to trace back what's inside the proton from the outcome of the collisions, we rely on QCD factorization



Parton Distribution Functions (PDFs):
Probability density for finding a parton in a proton with momentum fraction x

$$\sigma_{\text{proton}}(Q) = f_{\text{parton}}(x) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

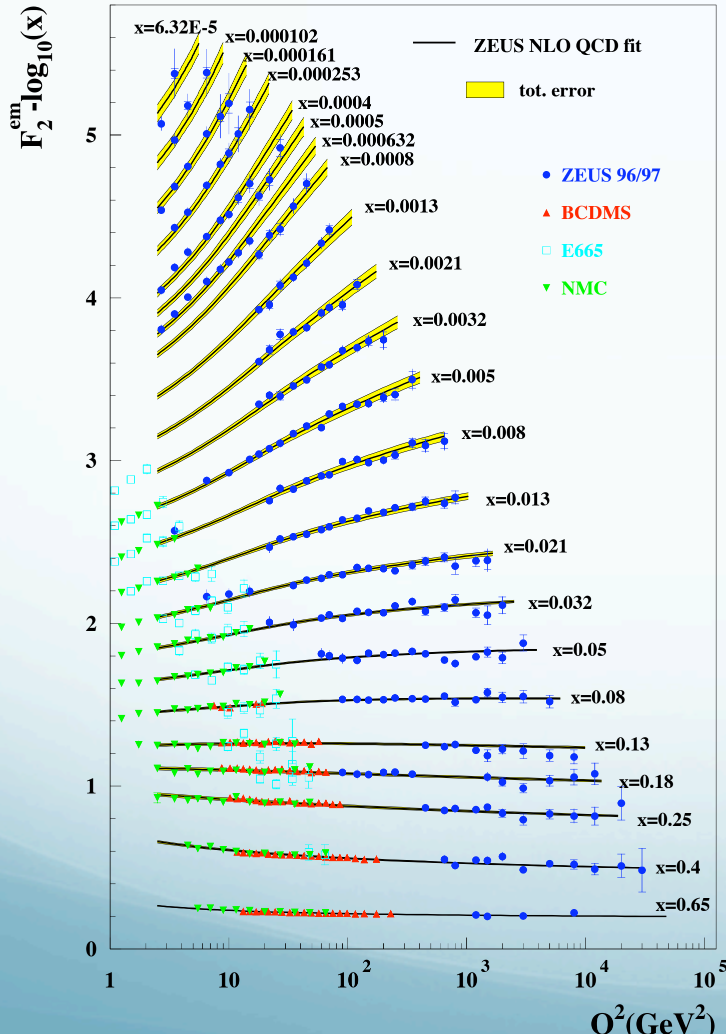
Universal (measured) **calculable**

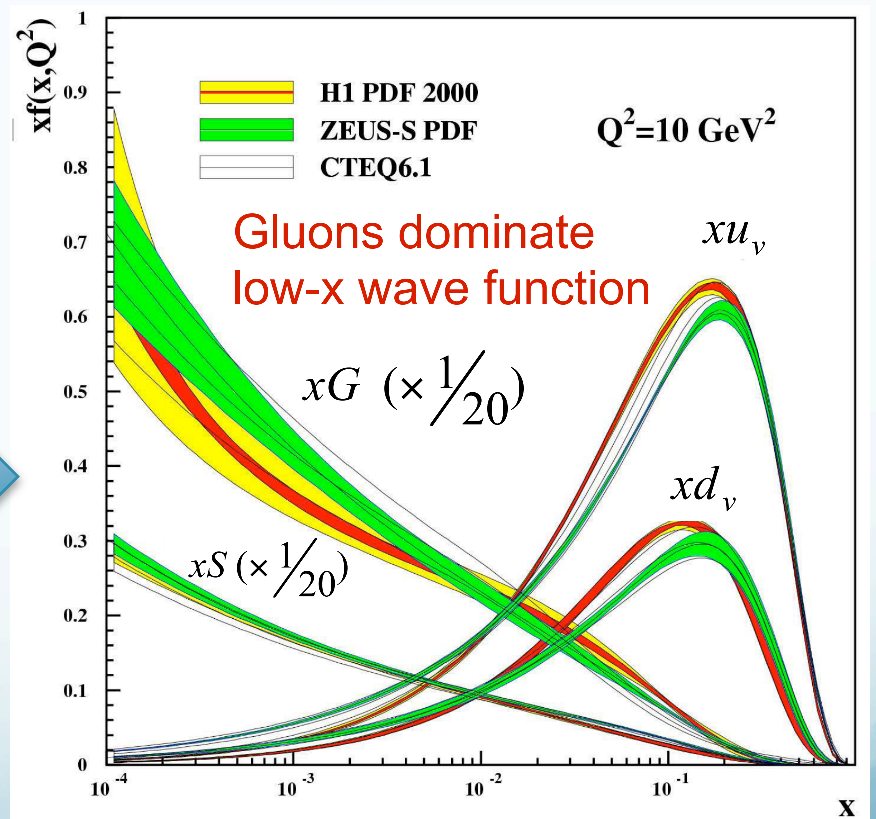
- Hadron structure: encoded in PDFs
- QCD dynamics at short-distance: partonic cross section, perturbatively calculable

Universality of PDFs: extraction from DIS

$$\sigma_{\text{proton}}(Q) = f_{\text{parton}}(x) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

Universal (measured) **calculable**

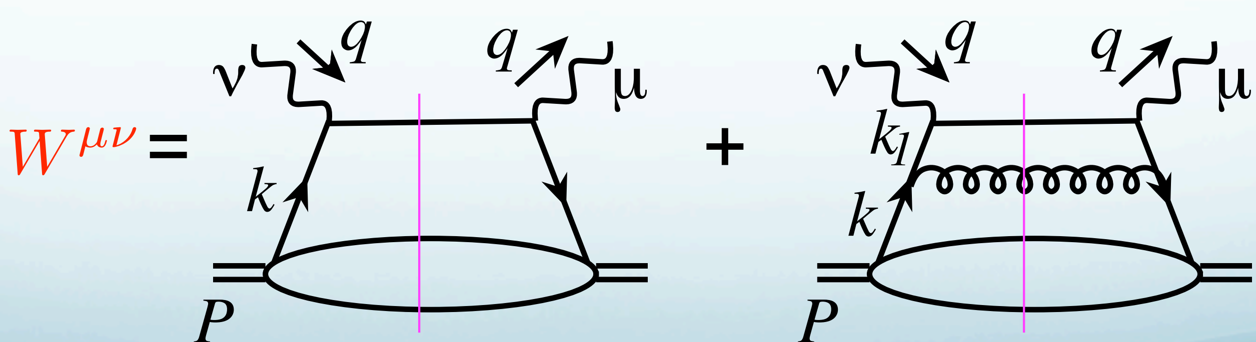




What about higher order?

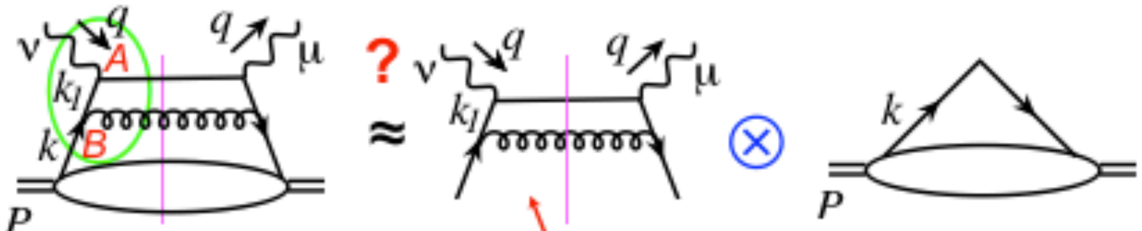
- pQCD calculations: understand and make sense of all kinds of divergences
  - Ultraviolet (UV) divergence $k \to \infty$: renormalization (redefine coupling constant)
  - Collinear divergence $k//P$: redefine the PDFs and FFs
  - Soft divergence $k \to 0$: usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations

- Going beyond the leading order of the DIS, we face another divergence



QCD dynamics beyond tree level

- Going beyond leading order calculation



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k+k_g)^2 = 2EE_g(1-\cos\theta)$$

- $k_1^2 \sim 0$ intermediate quark is on-shell

  $t_{AB} \rightarrow \infty$

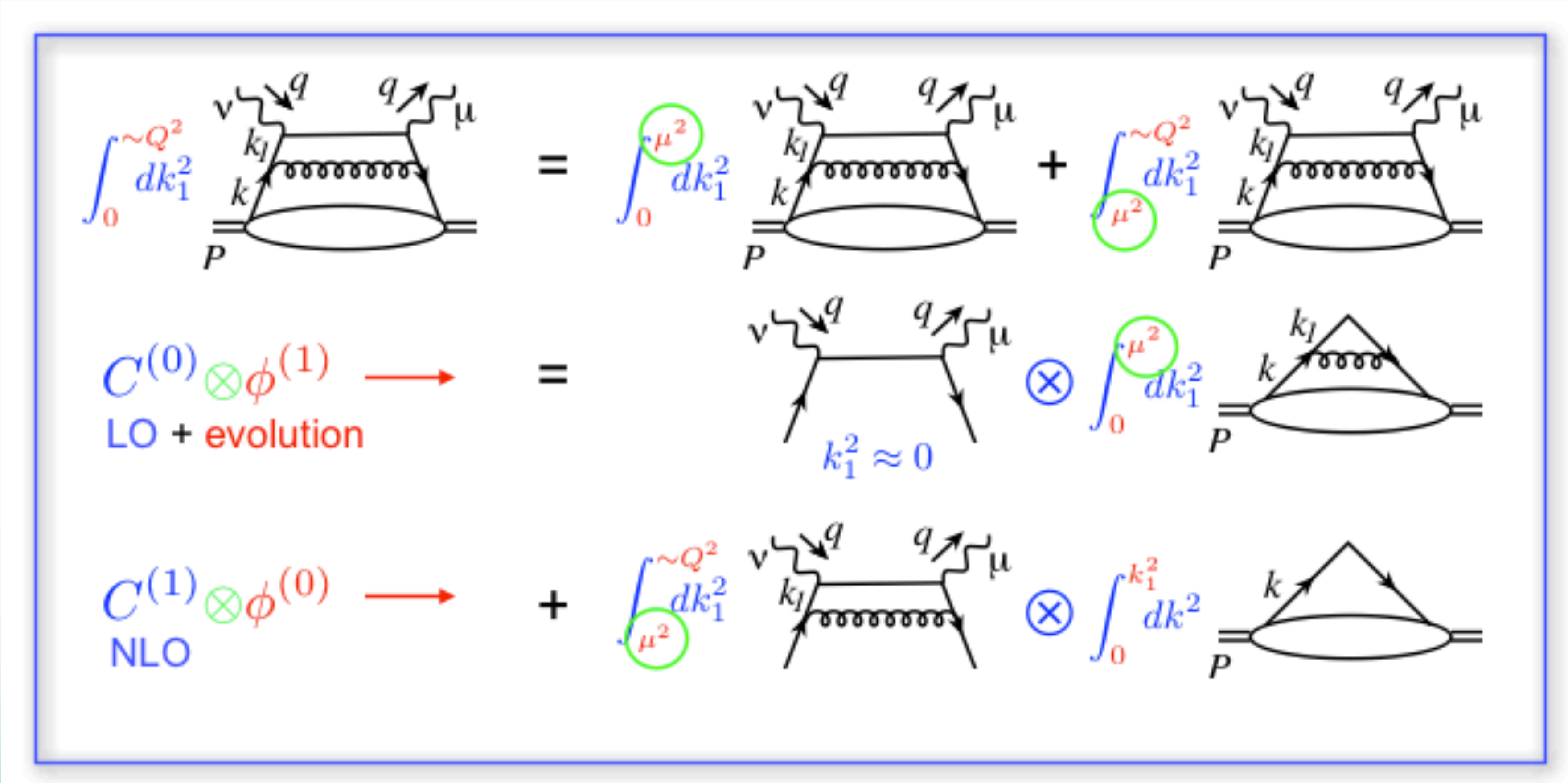
- gluon radiation takes place long before the photon-quark interaction

  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

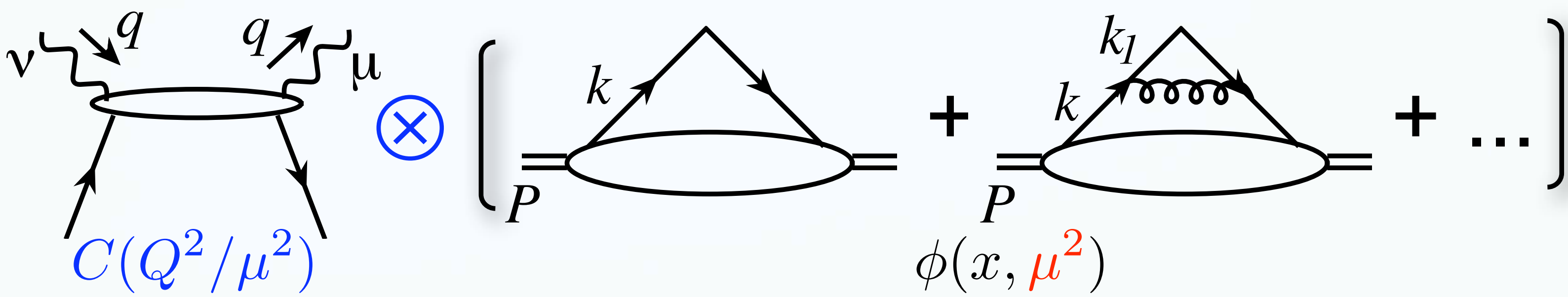
QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs

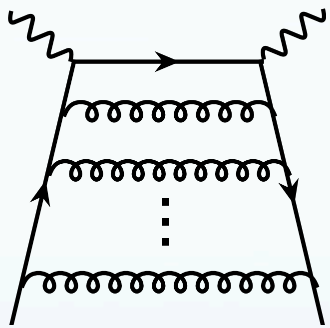


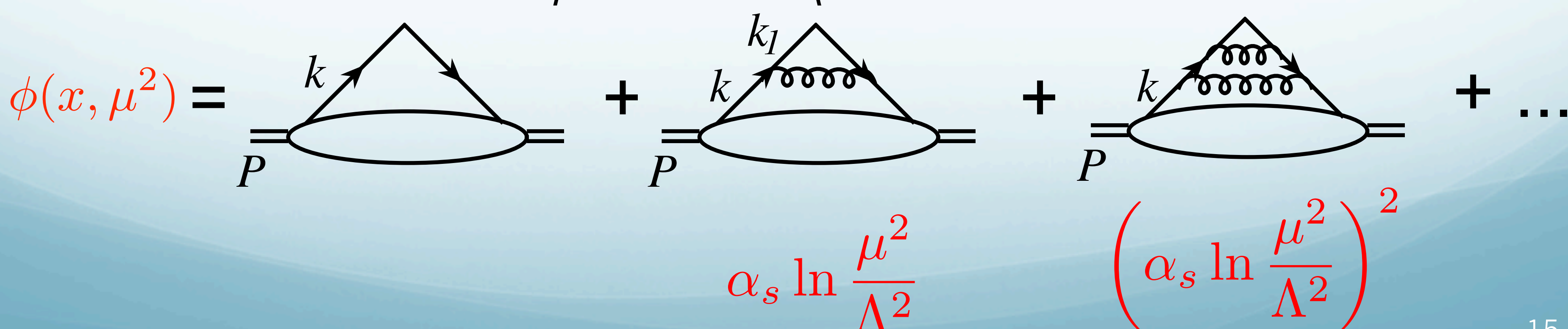
Scale-dependence of PDFs

- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs

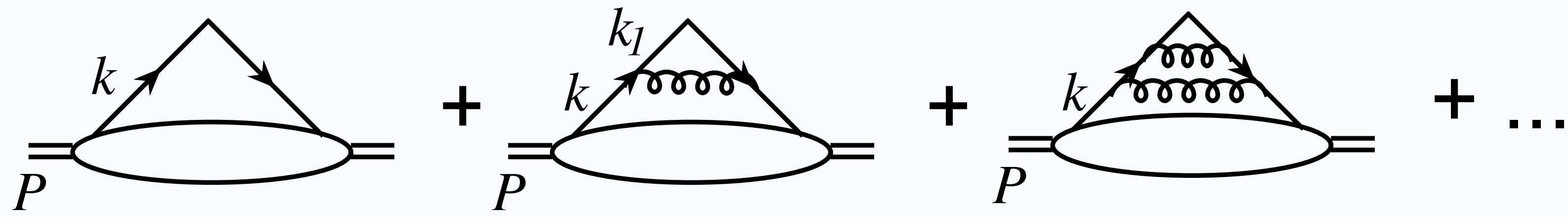




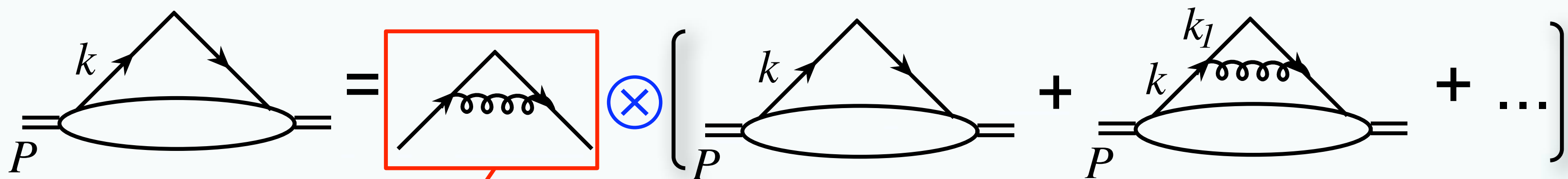
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x,\mu^2) =$



$\phi(x,\mu^2)$ -



Evolution kernel
splitting function

DGLAP Equation

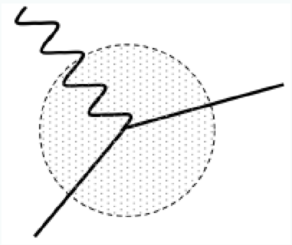
$$\frac{\partial}{\partial \ln \mu^2}\phi_i(x,\mu^2) = \sum_j \boxed{P_{ij}\left(\frac{x}{x'}\right)} \otimes \phi_j(x',\mu^2)$$

- By solving the evolution equation, one resums all the single logarithms of type $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$

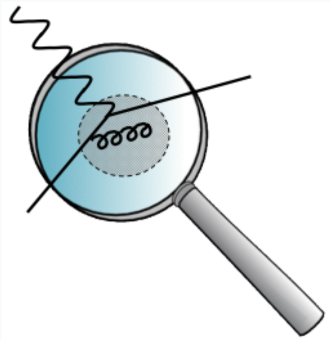
PDFs also depends on the scale of the probe

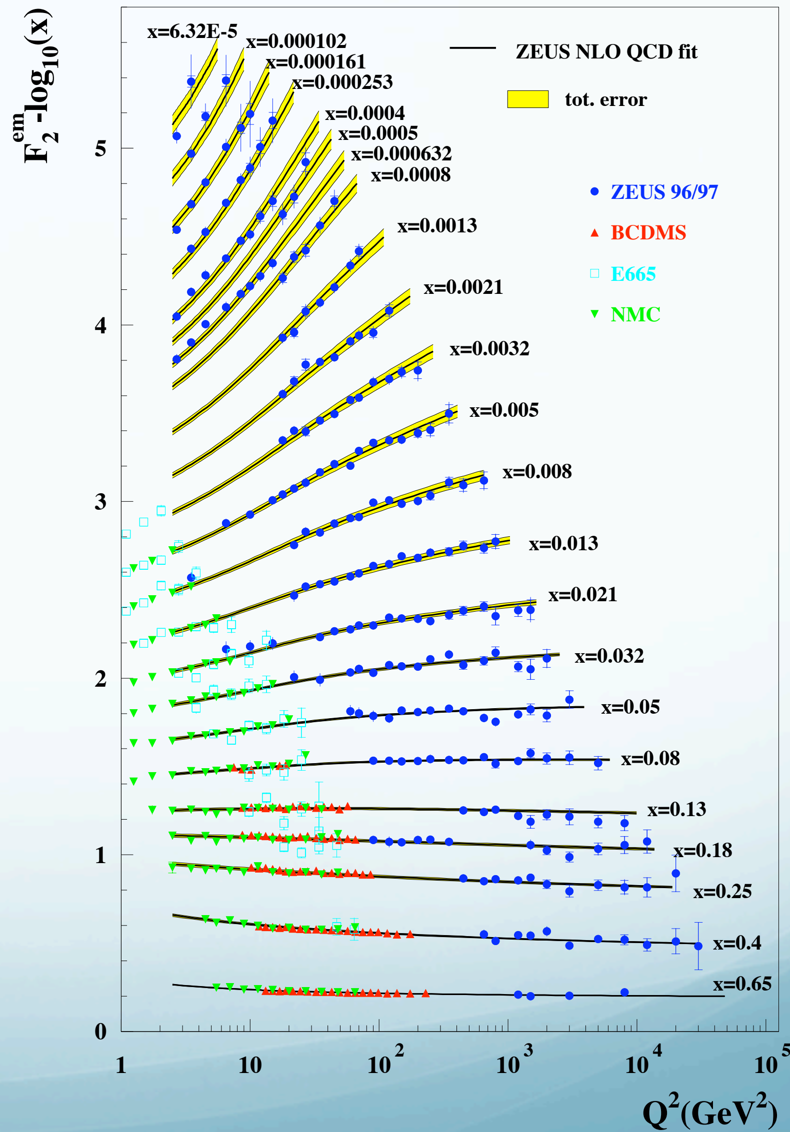
- Increase the energy scale, one sees parton picture differently

Q_0^2



$Q^2 > Q_0^2$

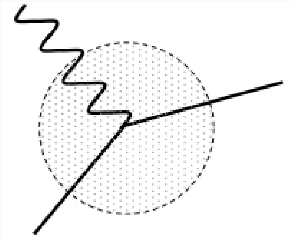




Evolutions of PDFs

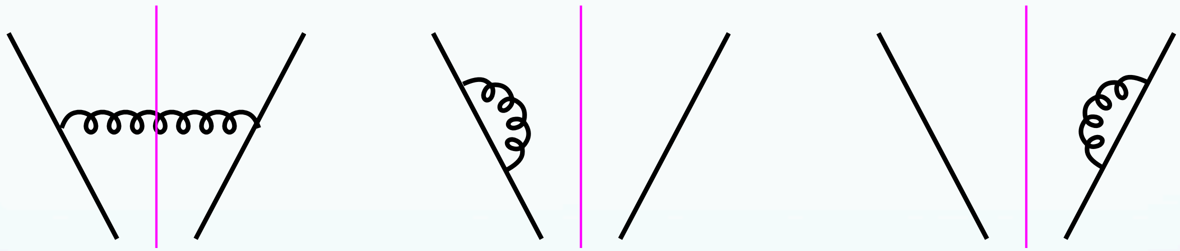
- Perturbative change:

Q_0^2



$Q^2 > Q_0^2$

- Feynman diagrams for unpolarized PDFs



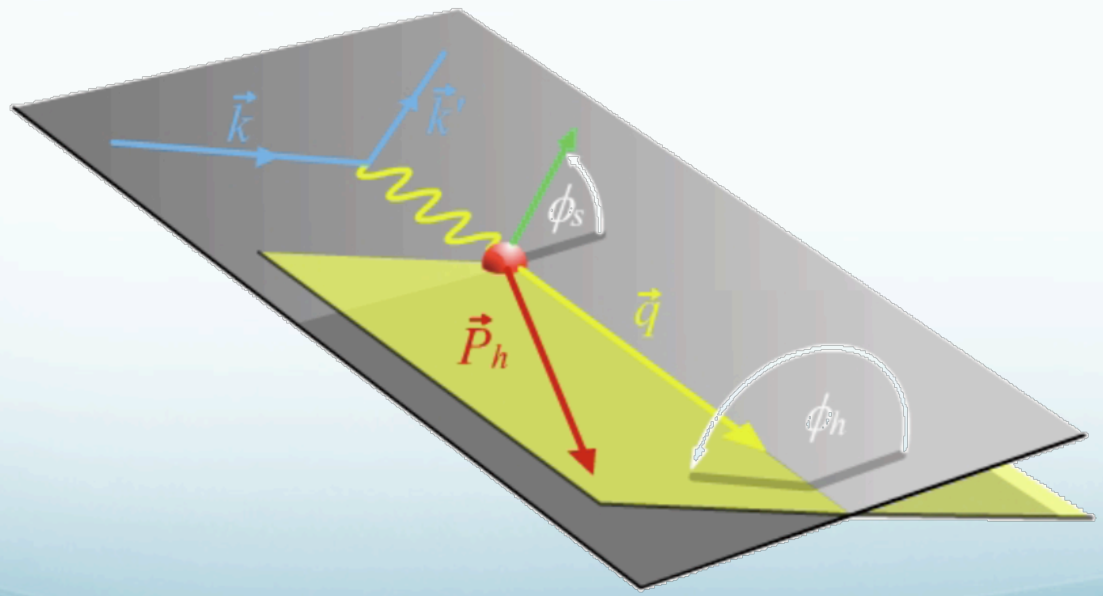
$$\frac{q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}(z)\, q(\xi,\mu_F)\right]$$

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

Example

- We will now study a detailed example
  - SIDIS at both LO and NLO
  - We can understand QCD collinear factorization and evolution of PDFs and FFs
- SIDIS and hadron frame

$$\ell + p^{\uparrow} \to \ell' + h(P_h) + X$$



Computer program

- A little bit on the Dirac gamma matrix computation
  - FeynCalc (mathematica): http://www.feyncalc.org
  - Tracer (mathematica): http://library.wolfram.com/infocenter/MathSource/2987/
  - Reduce: http://reduce-algebra.sourceforge.net/
  - FORM: http://www.nikhef.nl/~form/
- The detailed calculations are provided on scanned notes as well as white board presentation

Summary

- Asymptotic freedom: allow one to calculate partonic cross sections
- Parton distribution functions
- Renormalization scale and factorization scale