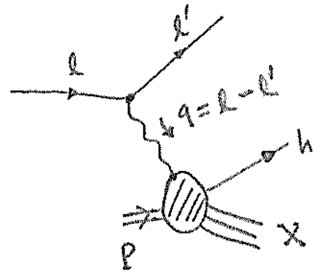
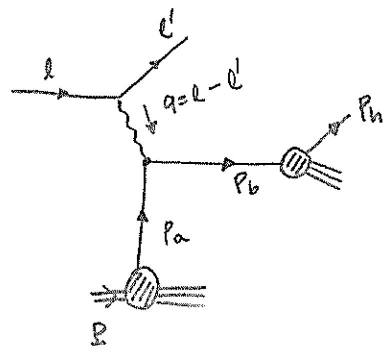
Semi-inclusive deep inelastic scattering (SIDIS)

$$e(l) + P(p) \to e(l') + h(P_h) + X$$



partonic level



$$P_a \approx x P$$

$$P_h \approx z P_b$$

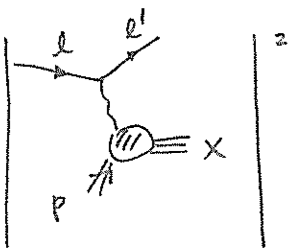
usual SIDIS kinematic variables

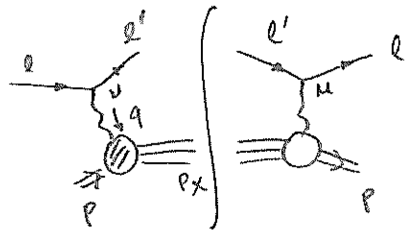
$$S_{ep} = (P + l)^2 = 2P \cdot l$$

$$Q^2 = -q^2 = -(l - l')^2$$

$$x_B = \frac{Q^2}{2P \cdot q} \qquad z_h = \frac{P \cdot P_h}{P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot l} = \frac{Q^2}{x_B S_{ep}}$$

- Normalization of DIS-type process

$d\sigma =$ 

$=$ 

$$= \frac{1}{F} \overline{|M|^2} \, dPS$$

initial-flux $\quad F = 4\sqrt{(P_A \cdot P_B)^2 - m_A^2 m_B^2} = 4\, P\cdot \ell = 2 S_{ep}$

$$\overline{|M|^2}\, dPS = \frac{1}{2} \mathrm{Tr}[\gamma^\mu \not{\ell}' \gamma^\nu \not{\ell}]\, e^2 * \frac{d^3\ell'}{(2\pi)^3 2E'} (2\pi)^4 \delta^4(P+q-P_X) * \left(\frac{1}{Q^2}\right)^2$$

$$* \; e^2 \; \frac{1}{2}\sum_s \left(\frac{d^3 P_X}{(2\pi)^3 2E_X} \sum_{S_X}\right) \langle PS| J_\mu^\dagger(0) |X\rangle\langle X| J_\nu(0) |PS\rangle$$

denote $\displaystyle \sum_X \equiv \left(\frac{d^3 P_X}{(2\pi)^3 2E_X}\right) \sum_{S_X}$

$$L^{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\gamma^\mu \not{\ell}' \gamma^\nu \not{\ell}]$$

$$= L^{\mu\nu} e^4 \left(\frac{1}{Q^2}\right) \frac{d^3\ell'}{(2\pi)^3 2E'} \quad \frac{1}{2}\sum_{S,X} \langle PS| J_\mu^\dagger(0) |X\rangle\langle X| J_\nu(0) |PS\rangle$$

$$* (2\pi)^4 \delta^4(P+q-P_X)$$

$\Downarrow \quad e^4 = (4\pi \alpha_{em})^2$

$$= \frac{\alpha_{em}^2}{Q^4} \left(\frac{d^3\ell'}{E'}\right) L^{\mu\nu} \left(\frac{1}{\pi}\right) * \frac{1}{2}\sum_{S,X} \langle PS| J_\mu^\dagger(0) |X\rangle\langle X| J_\nu(0) |PS\rangle$$

$$* (2\pi)^4 \delta^4(P+q-P_X)$$

define

$$W_{\mu\nu} = \frac{1}{4\pi} \; \frac{1}{2}\sum_{S,X} \langle PS|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|PS\rangle \; (2\pi)^4\delta^4(P+q-P_X)$$

Then we have

$$d\sigma = \frac{1}{2S}\;\frac{\alpha_{em}^2}{Q^4}\;\frac{d^3l'}{E'}\;4W_{\mu\nu}L^{\mu\nu}$$

$$\boxed{E'\frac{d\sigma}{d^3l'} = \frac{2}{S}\;\frac{\alpha_{em}^2}{Q^4}\;L^{\mu\nu}W_{\mu\nu}}$$

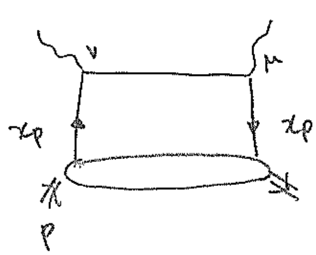
further using $\delta^4(P+q-P_X) = \int \frac{d^4y}{(2\pi)^4}\; e^{i(P+q-P_X)}$

one might show

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4y\; e^{iq\cdot y}\;\frac{1}{2}\sum_S \langle PS|J_\mu^\dagger(y)J_\nu(0)|PS\rangle$$

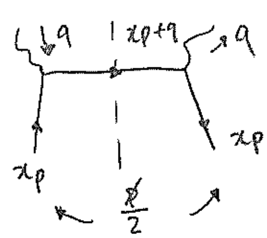
- lowest order of inclusive DIS

$W^{\mu\nu}$ =



$\ast\;\frac{1}{4\pi}$

$= \int dx\; f_{q/p}(x)\;\ast$



$\ast\;\frac{1}{4\pi}$

$$= \int dx \; f_{q/p}(x) \; \frac{1}{4\pi} \; \mathrm{Tr}\left[\frac{\not{p}}{2} \gamma^\mu (x\not{p} + \not{q}) \gamma^\nu \right] e_q^2 \times 2\pi \, \delta\left[(xp+q)^2 \right]$$

$$= \int dx \; f_{q/p}(x) \left[p^\mu (xp+q)^\nu + p^\nu (xp+q)^\mu - g^{\mu\nu} (xp+q) \cdot p \right] e_q^2 \; \delta\left[(xp+q)^2 \right]$$

Choose a frame $\quad p^\mu = p^+ \bar{n}^\mu$

$$q^\mu = -x_B p^\mu + \frac{Q^2}{2x_B p^+} n^\mu \qquad \bar{n}^\mu = [1, 0, 0_\perp]$$

$$n^\mu = [0, 1, 0_\perp]$$

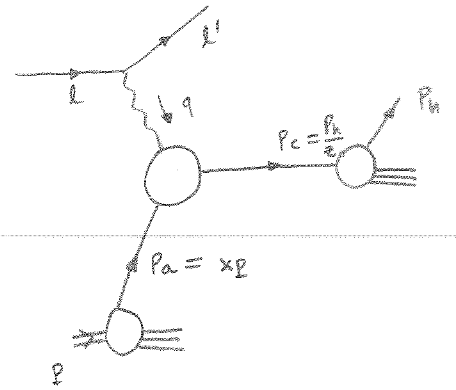
$$v^+ = \frac{1}{\sqrt{2}}(v^0 + v^z)$$

$$v^- = \frac{1}{\sqrt{2}}(v^0 - v^z)$$

$$W^{\mu\nu} = \int dx \; f_{q/p}(x) \; e_q^2 \; \frac{1}{2} d^{\mu\nu} \; \delta(x - x_B)$$

$$= \frac{1}{2} d^{\mu\nu} \sum_q e_q^2 \; f_{q/p}(x_B)$$

Where $\quad d^{\mu\nu} = -g^{\mu\nu} + \bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu$



$$S = (P+\ell)^2 \approx 2P\cdot\ell$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \qquad z_h = \frac{P\cdot P_h}{P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot \ell} = \frac{Q^2}{x_B S}$$

define $\quad \hat{x} = \frac{x_B}{x} \qquad \hat{z} = \frac{z_h}{z}$

work in the so-called hadron frame $\qquad \bar{n}^\mu = [1^+, 0^-, 0_\perp] \qquad n^\mu = [0^+, 1^-, 0_\perp]$

$$P^\mu = P^+ \bar{n}^\mu$$

$$q^\mu = -x_B P^+ \bar{n}^\mu + \frac{Q^2}{2x_B P^+} n^\mu$$

from $\quad z_h = \frac{P\cdot P_h}{P\cdot q} = \frac{P^+ P_h^-}{Q^2/(2x_B)} \Rightarrow P_h^- = z_h \frac{Q^2}{2x_B P^+}$

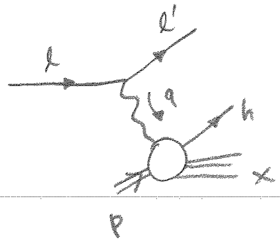
$$P_h^2 = 2P_h^+ P_h^- - \vec{P}_{h\perp}^{\,2} \Rightarrow P_h^+ = \frac{\vec{P}_{h\perp}^{\,2}}{2P_h^-} = \frac{x_B \vec{P}_{h\perp}^{\,2}}{z_h Q^2} P^+$$

thus $\quad P_h^\mu = \frac{x_B \vec{P}_{h\perp}^{\,2}}{z_h Q^2} P^+ \bar{n}^\mu + \frac{z_h Q^2}{2x_B P^+} n^\mu + P_{hT}^\mu \qquad (P_{hT}^\mu P_{hT\mu} = -\vec{P}_{h\perp}^{\,2})$

$$P_c^\mu = \frac{1}{z} P_h^\mu \qquad \left(P_{c\perp} = \frac{P_{h\perp}}{z} \right)$$

$$= \frac{x_B P_{c\perp}^2}{\hat{z} Q^2} P^+ \bar{n}^\mu + \frac{\hat{z} Q^2}{2x_B P^+} n^\mu + P_{cT}^\mu$$

$\boxed{\text{DIS normalization}}$



from CTEQ handbook

$$E' \frac{d\sigma}{d^3 l'} = \left(\frac{2}{S}\right) \left(\frac{\alpha_{em}}{Q^2}\right)^2 L^{\mu\nu} W_{\mu\nu}$$

where $L^{\mu\nu} = \frac{1}{2} \mathrm{Tr}[\not{l} \gamma^\mu \not{l}' \gamma^\nu]$

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4y \; e^{iq\cdot y} \quad \frac{1}{2} \sum_s \langle PS | J_\mu^\dagger(y) \; J_\nu(0) | PS \rangle$$

Note $\quad \frac{d^3 l'}{E'} = \frac{\pi Q^2}{x_B^2 S} \; dx_B \, dQ^2$

$\Downarrow$ define $y \equiv \frac{Q^2}{x_B S}$

$$= \pi S \, y \, dx_B \, dy$$

$$\frac{d\sigma}{dx_B \, dy} = \frac{2\pi \alpha_{em}^2 y}{(Q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

$\Downarrow$ take $\frac{1}{4\pi}$ out from $W_{\mu\nu}$

$$= \frac{\alpha_{em}^2 y}{2(Q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

In a so-called hadron frame, one could write

$$\frac{2}{Q^2} L^{\mu\nu} = (1 + \cosh^2\psi)(X^\mu X^\nu + Y^\mu Y^\nu) + 2\sinh^2\psi \; T^\mu T^\nu$$

$$\cosh\psi = \frac{2}{y} - 1$$

$$X^\mu X^\nu + Y^\mu Y^\nu = -g^{\mu\nu} + T^\mu T^\nu - Z^\mu Z^\nu$$

$$\Downarrow \qquad T^\mu = \frac{1}{Q}(q^\mu + 2x_B p^\mu)$$

$$Z^\mu = -\frac{q^\mu}{Q}$$

drop all q^μ, q^ν since $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$= -g^{\mu\nu} + \frac{4x_B^2}{Q^2} p^\mu p^\nu$$

Thus

$$\frac{2}{Q^2} L^{\mu\nu} \Rightarrow \frac{2}{y^2}\Big[\underbrace{\Big(-g^{\mu\nu} + \frac{4x_B^2}{Q^2} p^\mu p^\nu\Big)}_{\text{called transverse projection}} (1+(1-y)^2) + 2 \underbrace{\frac{4x_B^2}{Q^2} p^\mu p^\nu}_{\text{longitudinal projection}} (2(1-y))\Big]$$

$$= \frac{2}{y^2}\Big[(-g^{\mu\nu})(1+(1-y)^2) + \Big(\frac{4x_B^2}{Q^2} p^\mu p^\nu\Big)(1+4(1-y)+(1-y)^2)\Big]$$

refer to "Metric" contribution

longitudinal contribution

See, eg. hep-th/0411212

If we're only interested in Metric contribution, then we'll have

$$\frac{2}{Q^2} L^{\mu\nu} \to \frac{2}{y^2}\left[1+(1-y)^2\right](-g^{\mu\nu})$$

Then

$$\frac{d\sigma}{dx_B dy} = \frac{\alpha_{em}^2 y}{2(Q^2)^2} \frac{Q^2}{2} \frac{2}{y^2}\left[1+(1-y)^2\right](-g^{\mu\nu}) W_{\mu\nu}$$

$$= \frac{\alpha_{em}^2}{Q^2} \frac{1+(1-y)^2}{2y} (-g^{\mu\nu}) W_{\mu\nu}$$

At the partonic level

$$\frac{d\sigma}{dx_B\,dy} = \frac{\alpha_{em}^2}{Q^2}\,\frac{1+(1-y)^2}{2y}\int \frac{dx}{x}\,dz\; f_{q/p}(x)\, D_{q\to h}(z)\left[-g^{\mu\nu} H_{\mu\nu}\right] dps^{(n)}$$

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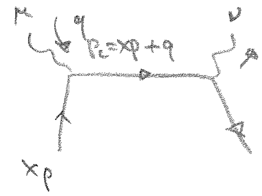
for example, at leading order

$$-g^{\mu\nu} H_{\mu\nu} =$$



$$= \frac{1}{2}\,\mathrm{Tr}\left[(x\not{p})\,\gamma^\nu\,(x\not{p}+\not{q})\,\gamma^\mu\right](-g_{\mu\nu})$$

$$= 4(1-\epsilon)\; xp\cdot q$$

$$= (1-\epsilon)\; 2\,\frac{x}{x_B}\,Q^2$$

$$dps^{(1)} = \frac{d^{n-1}p_c}{(2\pi)^{n-1}2E_c}\,(2\pi)^n\,\delta^n(xp+q-p_c)$$

$$\Downarrow \quad p_c = \frac{1}{z}P_h$$

$$= \frac{1}{z^{n-2}}\,\frac{d^{n-1}P_h}{(2\pi)^{n-1}2E_h}\,(2\pi)^n\,\delta^n(xp+q-p_c)$$

$$= \frac{1}{z^{n-2}}\,\frac{d^nP_h}{\cancel{(2\pi)^n}}\,2\pi\,\delta(P_h^2)\,\cancel{(2\pi)^n}\,\delta^n(xp+q-p_c)$$

$$= \frac{1}{z^{n-2}}\,dP_h^+\,dP_h^-\,d^{n-2}P_{hT}\;2\pi\,\underbrace{\delta\left(2P_h^+P_h^- - \vec{P}_{hT}^{\,2}\right)}_{\frac{1}{2P_h^-}\delta\left(P_h^+ - \frac{\vec{P}_{h\perp}^{\,2}}{2P_h^-}\right)}\;\delta(xp^+ + q^+)\;\delta(q^- - p_c^-)\;\delta^{n-2}(p_{c\perp})$$

$$\Downarrow \quad \frac{dP_h^-}{P_h^-} = \frac{dz_h}{z_h} \qquad \delta^{n-2}(p_{cT}) = \delta^{n-2}(P_{hT}/z) = z^{n-2}\,\delta^{n-2}(P_{hT})$$

$$= \frac{1}{z^{n-2}}\,\frac{dz_h}{2z_h}\,d^{n-2}P_{hT}\;z^{n-2}\,\delta^{n-2}(P_{hT})\,\frac{1}{p^+}\,\delta(x-x_B)\,\frac{1}{q^-}\,\delta(1-\hat{z}) \;*\,2\pi$$

$$dPS^{(1)} = \frac{dz_h}{2z_h} \frac{1}{p^+q^-} \frac{1}{x} \delta(1-\hat{x})\,\delta(1-\hat{z}) \ * 2\pi$$

$$\Downarrow \qquad 2p^+q^- = 2p\cdot q = \frac{Q^2}{x_B}$$

$$= \frac{dz_h}{z_h} \frac{x_B}{x} \frac{1}{Q^2} \delta(1-\hat{x})\,\delta(1-\hat{z}) \ * 2\pi = dz_h \ * \frac{x_B}{z\,x\,Q^2} \delta(1-\hat{x})\,\delta(1-\hat{z}) \ * 2\pi$$

$$\Downarrow \quad z_h = z$$

$$(-g^{\mu\nu}) H_{\mu\nu}\, dPS^{(1)} = 2(1-\epsilon) \cancel{\frac{x}{x_B}} Q^2 \ * \frac{dz_h}{z} \cancel{\frac{x_B}{x}} \cancel{\frac{1}{Q^2}} \delta(1-\hat{x})\,\delta(1-\hat{z}) \ * 2\pi$$

Thus

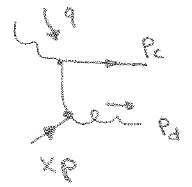
$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \frac{2\pi \alpha_{em}^2}{Q^2} \frac{1+(1-y)^2}{2y} \ * 2(1-\epsilon) \int \frac{dx}{x} \frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z) \ * \delta(1-\hat{x})\,\delta(1-\hat{z})$$

define $\sigma_0 = \frac{2\pi \alpha_{em}^2}{Q^2} \frac{1+(1-y)^2}{y} (1-\epsilon)$, then

$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \sigma_0 \int \frac{dx}{x} \frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z) \ * \delta(1-\hat{x})\,\delta(1-\hat{z})$$

Study higher order

Now for real diagram, we have $dps^{(2)}$



$$dps^{(2)} = \frac{d^{n-1}p_c}{(2\pi)^{n-1}2E_c} \frac{d^{n-1}p_d}{(2\pi)^{n-1}2E_d} (2\pi)^n \delta^n(xp+q-p_c-p_d)$$

$$= \frac{d^{n-1}p_h}{(2\pi)^{n-1}2E_h} \frac{1}{z^{n-2}} \cancel{\frac{d^n p_d}{(2\pi)^d}}\, 2\pi\,\delta(p_d^2) * \cancel{(2\pi)^n \delta^n}(xp+q-p_c-p_d)$$

$$= \frac{d^n p_h}{(2\pi)^n}\, 2\pi\,\delta(p_h^2)\, \frac{1}{z^{n-2}}\, 2\pi\,\delta(p_d^2)$$

$$= dp_h^+\, dp_h^-\, d^{n-2}p_{h\perp}\, \underbrace{\delta(2p_h^+p_h^- - \vec{p}_{h\perp}^{\,2})}_{\frac{1}{2p_h^-}\delta\left(p_h^+ - \frac{\vec{p}_{h\perp}^{\,2}}{2p_h^-}\right)} \frac{1}{z^{n-2}(2\pi)^{n-2}}\, \delta\left[(xp+q-p_c)^2\right]$$

$$\Downarrow \quad \frac{dp_h^-}{p_h^-} = \frac{dz_h}{z_h}$$

$$= \frac{dz_h}{2z_h}\, d^{n-2}p_{h\perp}\, \frac{1}{(2\pi z)^{n-2}}\, \delta\left[(xp+q-p_c)^2\right]$$

$$(xp+q-p_c)^2 = (xp+q)^2 - 2p_c\cdot(xp+q)$$

$$= -Q^2 + x\,2p\cdot q - x\,2p_c\cdot p - 2p_c\cdot q$$

define $\hat{s} = (xp+q)^2 = -Q^2 + x\,2p\cdot q = -Q^2 + x\frac{Q^2}{x_B} = \frac{Q^2(1-\hat{z})}{\hat{z}}$ $\quad \hat{x} = \frac{x_B}{x}$

$$\hat{t} = (p_c-q)^2 = -Q^2 - 2p_c\cdot q = -Q^2 - \left[2p_c^+q^- + 2p_c^-q^+\right]$$

$$= -Q^2 - \left[2\frac{x_B\, p_{c\perp}^2}{\hat{z}\,Q^2} p^+ \frac{Q^2}{2x_Bp^+} + 2\frac{\hat{z}\,Q^2}{2x_Bp^+}(-x_Bp^+)\right]$$

$$= -Q^2 - \left[\frac{p_{c\perp}^2}{\hat{z}} - \hat{z}\,Q^2\right]$$

$$= -\left[(1-\hat{z})Q^2 + \frac{p_{c\perp}^2}{\hat{z}}\right]$$

$$\hat u = (xp - p_c)^2 = x\,(-2p\cdot p_c) = x\,(-2)\,p^+ \,\frac{\hat z\, Q^2}{2x_B\, p^+} = -\frac{\hat z}{\hat x}\, Q^2$$

Thus from $0 = (xp+q-p_c)^2$

$$\Rightarrow \quad \delta\left[(xp+q-p_c)^2\right] = \delta\left[\hat s+\hat t+\hat u + Q^2\right]$$

$$= \delta\left[\frac{Q^2(1-\hat x)}{\hat x} - (1-\hat z)\,Q^2 - \frac{p_{c\perp}^2}{\hat z} - \frac{\hat z}{\hat x}\,Q^2 + Q^2\right]$$

$$= \delta\left[\frac{p_{c\perp}^2}{\hat z} - \frac{Q^2}{\hat x}(1-\hat x)(1-\hat z)\right]$$

$$= \hat z\,\delta\left[p_{c\perp}^2 - Q^2\,\frac{\hat z(1-\hat x)(1-\hat z)}{\hat z}\right]$$

Thus $\quad p_{c\perp}^2 = \dfrac{Q^2\,\hat z\,(1-\hat z)(1-\hat x)}{\hat x}$

$\Rightarrow$ Thus $\quad \hat t = -\left[(1-\hat z)\,Q^2 + \dfrac{\vec p_{c\perp}^{\,2}}{\hat z}\right] = -\left[(1-\hat z)\,Q^2 + Q^2(1-\hat z)\,\dfrac{(1-\hat x)}{\hat x}\right]$

$$= -\left[Q^2(1-\hat z)\,\frac{1}{\hat x}\right]$$

$$\boxed{\begin{aligned} \hat s &= \frac{1-\hat x}{\hat x}\,Q^2 \\ \hat t &= -\frac{1-\hat z}{\hat x}\,Q^2 \\ \hat u &= -\frac{\hat z}{\hat x}\,Q^2 \end{aligned}}$$

$$\frac{\hat t\,\hat u\,\hat s}{(\hat s+Q^2)^2} = \frac{\frac{1-\hat z}{\hat x}\,Q^2 * \frac{\hat z}{\hat x}\,Q^2 * \frac{1-\hat x}{\hat x}\,Q^2}{\left(\frac{Q^2}{\hat x}\right)^2}$$

$$= \frac{\hat z(1-\hat z)(1-\hat x)}{\hat x}\,Q^2 = p_{c\perp}^2$$

$$\Rightarrow \quad \boxed{p_{c\perp}^2 = \frac{\hat t\,\hat u\,\hat s}{(\hat s + Q^2)^2}}$$

$$\boxed{P_{h\perp}^2 = z^2\, p_{c\perp}^2 = z^2\,\frac{\hat t\,\hat u\,\hat s}{(\hat s+Q^2)^2}}$$

$$\delta[(xp+q-p_c)^2] = \hat{z}\,\delta\left[\vec{p}_{c\perp}^{\,2} - \frac{Q^2\hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

$$\Downarrow \quad p_{c\perp}^2 = \frac{p_{h\perp}^2}{z^2}$$

$$= \hat{z}\, z^2\, \delta\left[p_{h\perp}^2 - \frac{z^2 Q^2 \hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

Thus

$$dps^{(2)} = \frac{dz_h}{2z_h}\, d^{n-2}p_{h\perp}\, \frac{1}{(2\pi z)^{n-2}}\quad \hat{z}\, z^2\, \delta\left[p_{h\perp}^2 - \frac{z^2 Q^2 \hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

Note $\int d^d p_{h\perp} = \int p_{h\perp}^{d-1}\, dp_{h\perp} * \Omega_d$

$$= \frac{1}{2}(p_{h\perp}^2)^{\frac{d-2}{2}}\, dp_{h\perp}^2 * \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

$$= \frac{\pi^{d/2}}{\Gamma(d/2)}\,(p_{h\perp}^2)^{\frac{d-2}{2}}\, dp_{h\perp}^2$$

$$\Downarrow \quad d = n-2 = 2-2\epsilon$$

$$= \frac{\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}\,(p_{h\perp}^2)^{-\epsilon}\quad dp_{h\perp}^2$$

$$dps^{(2)} = \frac{dz_h}{2z_h} * \frac{\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}\,(p_{h\perp}^2)^{-\epsilon}\, dp_{h\perp}^2\, \frac{1}{(2\pi z)^{2-2\epsilon}} * \hat{z}\, z^2$$

$$* \,\delta\left[p_{h\perp}^2 - \frac{z^2 Q^2 \hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

$$= \left(dz_h\, \frac{1}{z}\right) * \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\left[\hat{z}(1-\hat{z})\right]^{-\epsilon}\quad \left[(1-\hat{x})^{-\epsilon}\,\hat{x}^{\epsilon}\right]$$

Thus for spin-averaged one

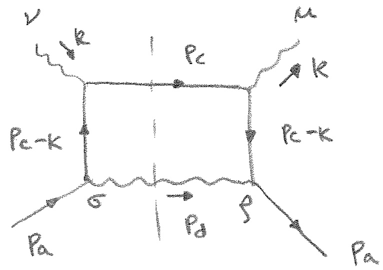
$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \frac{\alpha_{em}^2}{Q^2} \frac{1+(1-y)^2}{2y} \int \frac{dx}{x} \frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z) \left[-g^{\mu\nu} H_{\mu\nu}\right]$$

$$* \; \frac{1}{8\pi} \left(\frac{4\pi}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} \; \hat{x}^{\epsilon} (1-\hat{x})^{-\epsilon}$$

Let's study unpolarized cross-section first

$k^2 = -Q^2$

①

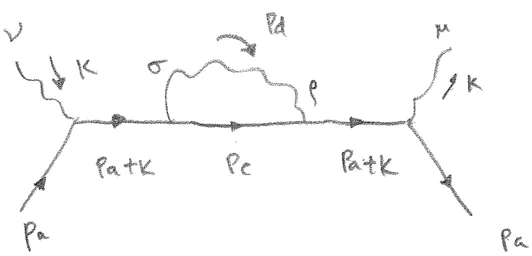


$P_d = k + P_a - P_c$

define
$$\hat{s} = (P_a + k)^2 = -Q^2 + 2P_a\cdot k$$
$$\hat{t} = (P_c - k)^2 = -Q^2 - 2P_c\cdot k$$
$$\hat{u} = (P_a - P_c)^2 = -2P_a\cdot P_c$$

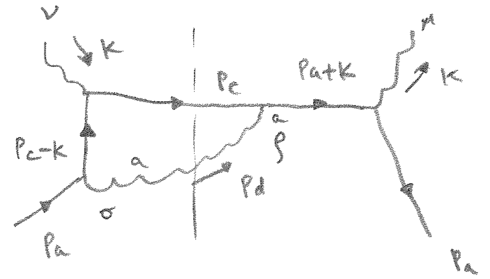
$$\text{Fig1} = \frac{1}{2}\,\mathrm{Tr}\left[\not{P}_a\,\gamma^\rho(\not{P}_c - \not{k})\gamma^\mu \not{P}_c\,\gamma^\nu(\not{P}_c - \not{k})\gamma^\sigma\right](-g_{\mu\nu})\,d_{\rho\sigma}(P_d)$$
$$*\left[\frac{1}{(P_c-k)^2}\right]^2 * g_s^2$$

②



$$\text{Fig2} = \frac{1}{2}\,\mathrm{Tr}\left[\not{P}_a\,\gamma^\mu(\not{P}_a + \not{k})\gamma^\rho \not{P}_c\,\gamma^\sigma(\not{P}_a + \not{k})\gamma^\nu\right](-g_{\mu\nu})\,d_{\rho\sigma}(P_d)$$
$$*\left[\frac{1}{(P_a+k)^2}\right]^2 * g_s^2$$

③

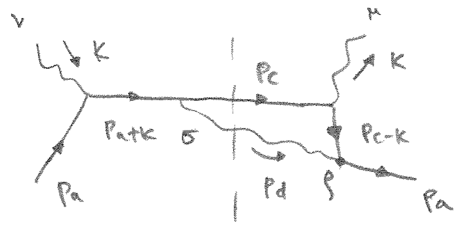


$\text{color} = \frac{1}{N} \mathrm{Tr}[T^a T^a] = C_F$

$$\text{Fig3} = \frac{1}{2} \mathrm{Tr}[\not{p}_a \gamma^\mu (\not{p}_a + \not{k}) \gamma^\rho \not{p}_c \gamma^\nu (\not{p}_c - \not{k}) \gamma^\sigma] (-g_{\mu\nu})\, d_{\rho\sigma}(P_d)$$

$$* \frac{1}{(P_c - K)^2} \frac{1}{(P_a + K)^2}$$

④



$$\text{Fig4} = \frac{1}{2} \mathrm{Tr}[\not{p}_a \gamma^\rho (\not{p}_c - \not{k}) \gamma^\mu \not{p}_c \gamma^\sigma (\not{p}_a + \not{k}) \gamma^\nu] (-g_{\mu\nu})\, d_{\rho\sigma}(P_d)$$

$$* \frac{1}{(P_c - K)^2} \frac{1}{(P_a + K)^2}$$

$$\text{Fig1 + 2 + 3 + 4} = 4(1-\epsilon)\ \frac{1}{\hat{s}\hat{t}}\left[-(1-\epsilon)(\hat{s}^2+\hat{t}^2) + 2\epsilon\hat{s}\hat{t} - 2Q^2(\underbrace{Q^2+\hat{s}+\hat{t}}_{-\hat{u}})\right]$$

$$= 4(1-\epsilon)\left[(1-\epsilon)\left(\frac{\hat{s}}{-\hat{t}} + \frac{-\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

Eventually we have

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \frac{\alpha_{em}^2}{Q^2}\ \frac{1+(1-y)^2}{2y} \int \frac{dx}{x}\ \frac{dz}{z}\ f_{q/p}(x)\ D_{q\to h}(z)$$

$$* (g_s\mu^\epsilon)^2 * 4(1-\epsilon)\left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

$$* \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}\quad \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\quad \hat{x}^\epsilon(1-\hat{x})^{-\epsilon}$$

$$= \frac{2\pi\alpha_{em}^2}{Q^2}\ \frac{1+(1-y)^2}{y} * \frac{\alpha_s}{2\pi}\int \frac{dx}{x}\ \frac{dz}{z}\ f_{q/p}(x)\ D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}\quad \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\quad \hat{x}^\epsilon(1-\hat{x})^{-\epsilon}$$

$$* (1-\epsilon)\left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

define (like before) $\qquad \sigma_0 = \frac{2\pi\alpha_{em}^2}{Q^2}\ \frac{1+(1-y)^2}{y}\ (1-\epsilon)$

$$\frac{dx}{x} = \frac{d\hat{x}}{\hat{x}} \qquad \frac{dz}{z} = \frac{d\hat{z}}{\hat{z}}$$

$\boxed{\text{Color} = C_F}$

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0\ \frac{\alpha_s}{2\pi}\int \frac{dx}{x}\ \frac{dz}{z}\ f_{q/p}(x)\ D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}\quad \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\quad \hat{x}^\epsilon(1-\hat{x})^{-\epsilon}$$

$$* \left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

$$\hat{s} = \frac{1-\hat{x}}{\hat{x}} Q^2 \qquad \hat{t} = -\frac{1-\hat{z}}{\hat{x}} Q^2 \qquad \hat{u} = -\frac{\hat{z}}{\hat{x}} Q^2$$

$$[\cdots] = \left\{ (1-\epsilon)\left[\frac{1-\hat{x}}{1-\hat{z}} + \frac{1-\hat{z}}{1-\hat{x}}\right] + \frac{2\hat{x}}{1-\hat{x}}\frac{\hat{z}}{1-\hat{z}} + 2\epsilon \right\}$$

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x}\frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} \hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$

$$* \left[(1-\epsilon)\left(\frac{1-\hat{x}}{1-\hat{z}} + \frac{1-\hat{z}}{1-\hat{x}}\right) + \frac{2\hat{x}}{1-\hat{x}}\frac{\hat{z}}{1-\hat{z}} + 2\epsilon\right]$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\frac{\ln\hat{z}}{1-\hat{z}} + O(\epsilon^2)$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{1-\epsilon} = (1-\hat{x})\left[1 + \epsilon\ln\frac{\hat{x}}{1-\hat{x}}\right]$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{1-\epsilon} = (1-\hat{z})\left[1 - \epsilon\left(\ln\hat{z} + \ln(1-\hat{z})\right)\right]$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \epsilon\frac{\ln\hat{x}}{1-\hat{x}} + O(\epsilon^2)$$

$$\hat{z}^{1-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+} - \epsilon\,\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\frac{\hat{z}}{1-\hat{z}}\ln\hat{z}$$

$$\hat{x}^{1+\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{\hat{x}}{(1-\hat{x})_+} - \epsilon\,\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\frac{\hat{x}}{1-\hat{x}}\ln\hat{x}$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} = 1 - \epsilon\left(\ln\hat{z} + \ln(1-\hat{z})\right)$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon} = 1 + \epsilon\ln\frac{\hat{x}}{1-\hat{x}}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \frac{dz}{z} \; f_{q/p}(x)\; D_{q\to h}(z) \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)}$$

$$* \Big\{ (1-\epsilon)\left[-\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+}\right]\left[1+\epsilon \ln\frac{\hat{x}}{1-\hat{x}}\right](1-\hat{x})$$

$$+(1-\epsilon)(1-\hat{z})\left[1-\epsilon \ln \hat{z}(1-\hat{z})\right]\left[-\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+}\right]$$

$$+2\left[-\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{\hat{x}}{(1-\hat{x})_+} - \epsilon\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right]$$

$$*\left[-\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+} - \epsilon\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\,\hat{z}\,\frac{\ln\hat{z}}{1-\hat{z}}\right]$$

$$+ 2\epsilon \Big\}$$

$$\{\dots\} = (1-\hat{x})\left[-\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} + \left(1-\ln\frac{\hat{x}}{1-\hat{x}}\right)\delta(1-\hat{z})\right]$$

$$+(1-\hat{z})\left[-\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} + \left(1+\ln\hat{z}(1-\hat{z})\right)\delta(1-\hat{x})\right]$$

$$+2\Big[\frac{1}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z}) - \frac{1}{\epsilon}\delta(1-\hat{x})\frac{\hat{z}}{(1-\hat{z})_+} - \frac{1}{\epsilon}\delta(1-\hat{z})\frac{\hat{x}}{(1-\hat{x})_+}$$

$$+\frac{\hat{x}\,\hat{z}}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z})\left(\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right)$$

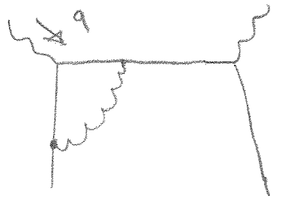
$$+\delta(1-\hat{x})\left(\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + \hat{z}\frac{\ln\hat{z}}{1-\hat{z}}\right)\Big]$$

$$= \frac{2}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z}) - \frac{1}{\epsilon}\delta(1-\hat{x})\frac{1+\hat{z}^2}{(1-\hat{z})_+} - \frac{1}{\epsilon}\delta(1-\hat{z})\frac{1+\hat{x}^2}{(1-\hat{x})_+}$$

$$+\frac{1+(1-\hat{x}-\hat{z})^2}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z})\left[(1-\hat{x})\left(1-\ln\frac{\hat{x}}{1-\hat{x}}\right) + 2\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - 2\hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right]$$

$$+\delta(1-\hat{x})\left[(1-\hat{z})\left(1+\ln\hat{z}(1-\hat{z})\right) + 2\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + 2\hat{z}\frac{\ln\hat{z}}{1-\hat{z}}\right]$$

Now for virtual diagram



$+ \cdots$

$$\Gamma^\mu(q) = \gamma^\mu \left\{ 1 + \frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{-q^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right) \right\}$$

2*Re (virtual * lowest order)

$$\Rightarrow \quad \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$* \quad \underbrace{\frac{\Gamma(1+\epsilon)\Gamma^3(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right)}_{= \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right)}$$

$$\Downarrow$$

$$= \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right]$$

Note $2\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ = \left[1+\hat{x}^2-(1-\hat{x})^2\right]\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+$

$$= (1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - (1-\hat{x})\ln(1-\hat{x})$$

---

likewise for $\hat{z}$, we thus have (Real + virtual)

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0 \frac{\alpha_s}{2\pi}\int\frac{dx}{x}\frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$* \Bigg[\Big\{ -\frac{1}{\epsilon}\delta(1-\hat{x})\, C_F\left[\frac{1+\hat{z}^2}{(1-\hat{z})_+}+\frac{3}{2}\delta(1-\hat{z})\right]$$

$$-\frac{1}{\epsilon}\delta(1-\hat{z})\, C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+}+\frac{3}{2}\delta(1-\hat{x})\right]\Big\}$$

$$+ C_F\Big\{ \frac{1+(1-\hat{x}-\hat{z})^2}{(1-\hat{x})_+(1-\hat{z})_+}$$

$$+\delta(1-\hat{z})\left[(1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \frac{1+\hat{x}^2}{1-\hat{x}}\ln\hat{x} + (1-\hat{x})\right]$$

$$+\delta(1-\hat{x})\left[(1+\hat{z}^2)\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1+\hat{z}^2}{1-\hat{z}}\ln\hat{z} + (1-\hat{z})\right]$$

$$-8\,\delta(1-\hat{x})\,\delta(1-\hat{z})\Big\}\Bigg]$$

This result is consistent with NPB160 (1979) 301

Altarelli - Ellis - Martinelli - Pi

(after convert DIS scheme to $\overline{MS}$ scheme)

Expansion

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\frac{\ln\hat{z}}{1-\hat{z}}$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\frac{\ln\hat{x}}{1-\hat{x}}$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon} = 1 + \epsilon\ln\frac{\hat{x}}{1-\hat{x}}$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} = 1 - \epsilon\ln\hat{z} - \epsilon\ln(1-\hat{z})$$

$$I_0 = z^{-\epsilon}(1-z)^{-\epsilon-1}$$

$$I = \int_0^1 dz \; z^{-\epsilon}(1-z)^{-\epsilon-1} \; f(z)$$

$$= \int_0^1 dz \; z^{-\epsilon}(1-z)^{-\epsilon-1} \left[f(z) - f(1) + f(1) \right]$$

$$= \int_0^1 dz \; z^{-\epsilon}(1-z)^{-\epsilon-1} f(1) + \int_0^1 dz \; z^{-\epsilon}(1-z)^{-\epsilon-1} \left[f(z) - f(1) \right]$$

$$= \frac{\Gamma(1-\epsilon)\,\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)} f(1) + \int_0^1 dz \left[f(z) - f(1) \right] \left[\frac{1}{(1-z)} - \epsilon \frac{\ln(1-z)}{1-z} - \epsilon \frac{\ln z}{1-z} + O(\epsilon^2) \right]$$

$$= \left[-\frac{1}{\epsilon} + \frac{\pi^2}{6}\epsilon + O(\epsilon^2) \right] f(1) + \int_0^1 dz \; \frac{1}{(1-z)} \left(f(z) - f(1) \right]$$

$$- \epsilon \int_0^1 dz \; \frac{\ln(1-z)}{1-z} \left[f(z) - f(1) \right]$$

$$- \epsilon \int_0^1 dz \; \frac{\ln z}{1-z} \left(f(z) - f(1) \right]$$

$$+ O(\epsilon^2)$$

Note $\frac{\ln z}{1-z} \to -1$ when $z \to 1$ thus finite

$$\int_0^1 dz \; \frac{\ln z}{1-z} = -\frac{\pi^2}{6}$$

also $\int_0^1 dz \; \left[w(z) \right]_+ f(z) = \int_0^1 dz \; w(z) \left(f(z) - f(1) \right]$

$$= \left[-\frac{1}{\epsilon} + \cancel{\frac{\pi^2}{6}\epsilon} \right] f(1) + \int_0^1 dz \; \frac{1}{(1-z)_+} f(z) - \epsilon \int_0^1 dz \left(\frac{\ln(1-z)}{1-z} \right)_+ f(z)$$

$$- \epsilon \int_0^1 dz \; \frac{\ln z}{1-z} f(z) - \cancel{\frac{\pi^2}{6} f(1)\,\epsilon} + O(\epsilon^2)$$

$$= \int_0^1 dz \, f(z) \left[-\frac{1}{\epsilon}\delta(1-z) + \frac{1}{(1-z)_+} - \epsilon \left(\frac{\ln(1-z)}{1-z} \right)_+ - \epsilon \frac{\ln z}{1-z} + O(\epsilon^2) \right]$$

Thus $\boxed{z^{-\epsilon}(1-z)^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-z) + \frac{1}{(1-z)_+} - \epsilon \left(\frac{\ln(1-z)}{1-z} \right)_+ - \epsilon \frac{\ln z}{1-z} + O(\epsilon^2)}$

Make sense the result

$$\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\left(-\frac{1}{\epsilon}\right) = \overbrace{-\frac{1}{\epsilon}+\gamma_E-\ln(4\pi)}^{-\frac{1}{\bar{\epsilon}}\quad(\overline{MS}\ \text{scheme})} + \ln\left(\frac{Q^2}{\mu^2}\right) + O(\epsilon)$$

$$= -\frac{1}{\bar{\epsilon}} + \ln\frac{\mu_f^2}{\mu^2} + \ln\left(\frac{Q^2}{\mu_f^2}\right) + O(\epsilon)$$

then $$f_{q/p}(x_B, \mu_f^2) = f^{(0)}_{q/p}(x_B) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\bar{\epsilon}} + \ln\frac{\mu_f^2}{\mu^2}\right)\int_{x_B}^{1}\frac{dx}{x}\, f_{q/p}(x_B)\, P_{qq}(\hat{x})$$

where $$P_{qq}(\hat{x}) = C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{3}{2}\delta(1-\hat{x})\right]$$

Thus $$\frac{\partial}{\partial \ln\mu_f^2} f_{q/p}(x_B, \mu_f^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^{1}\frac{dx}{x}\, f_{q/p}(x_B, \mu_f^2)\, P_{qq}(\hat{x})$$

In other words, we "reabsorb" the divergence (collinear)

into the redefinition of parton distribution function

(similar) for fragmentation function

to become "renormalized" PDFs and FFs

finally we have NLO result

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} \, f_{q/p}(x,\mu_f^2)\, D_{q\to h}(z,\mu_f^2)$$

$$* \Big\{ \ln\frac{Q^2}{\mu_f^2} \left[P_{qq}(\hat{x})\,\delta(1-\hat{z}) + P_{qq}(\hat{z})\,\delta(1-\hat{x}) \right]$$

$$+ C_F \left[\frac{1+(1-\hat{x}-\hat{z})^2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\,\delta(1-\hat{x})\,\delta(1-\hat{z}) \right]$$

$$+ \delta(1-\hat{z})\, C_F \left[(1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \frac{1+\hat{x}^2}{1-\hat{x}}\ln\hat{x} + (1-\hat{x}) \right]$$

$$+ \delta(1-\hat{x})\, C_F \left[(1+\hat{z}^2)\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1+\hat{z}^2}{1-\hat{z}}\ln\hat{z} + (1-\hat{z}) \right] \Big\}$$