Hadron Physics
& QCD's Dyson-Schwinger Equations

Lecture 3: The pion and dynamical chiral symmetry breaking

Ian Cloët

Argonne National Laboratory

HUGS 2015 Summer School

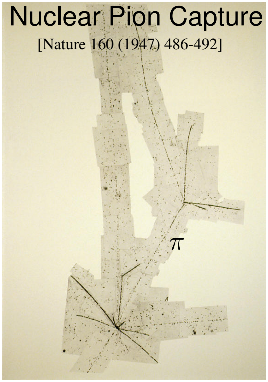
1-19 June 2015

U.S. DEPARTMENT OF ENERGY | Office of Science

Argonne NATIONAL LABORATORY

The Pion – Nature's strong messenger

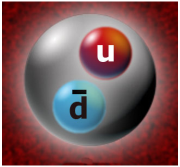
- Hideki Yukawa in 1935 postulated a strongly interacting particle of mass $\sim 100\,\text{MeV}$
  - Yukawa called this particle a "meson"
- Cecil Powell in 1947 discovered the π-meson from cosmic ray tracks in a photographic emulsion – a technique Cecil developed



- Cavendish Lab had said method is incapable of "*reliable and reproducible precision measurements*"
- The measured *pion* mass was: $130 - 150\,\text{MeV}$
- Both Yukawa & Powell received Nobel Prize – in 1949 and 1950 respectively
- Discovery of pion was beginning of particle physics; before long there was the particle *zoo*

The Pion in QCD

- Today the pion is understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD



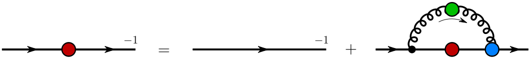
- This dichotomous nature has numerous ramifications, e.g.:

$$m_\rho/2 \sim M_N/3 \sim 350\,\text{MeV} \qquad \text{however} \qquad m_\pi/2 \simeq 0.2 \times 350\,\text{MeV}$$

- The pion is unusually light, the key is dynamical chiral symmetry breaking
- The pion – π^+, π^0, π^- – is stable under the strong interaction, however does decay via the weak interaction: $\tau_{\pi^\pm} \simeq 10^{-8}\text{s}$
- Pion properties are therefore difficult to measure but there have been numerous successes:
  - masses; decay constants and rates; electromagnetic form factor and parton distribution function
- Critical need for modern theory to examine the structure of the pion
  - *the Dyson-Schwinger equations are the ideal tool*

QCD's Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - tractability $\Longrightarrow$ must implement a symmetry preserving truncation
- The most important DSE is QCD's gap equation $\Longrightarrow$ quark propagator

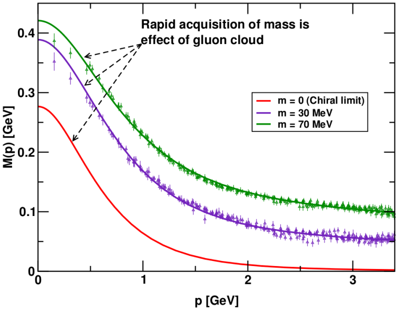


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

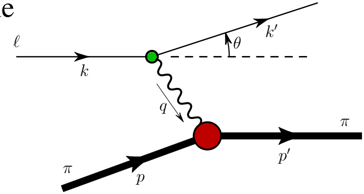
- $S(p)$ has correct perturbative limit
- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell $\Longrightarrow$ confinement

[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]

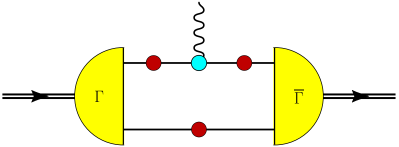


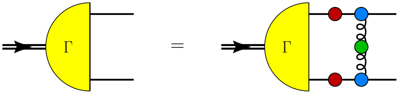
Pion Elastic Form Factor

- Form factors parametrize the interaction of the electromagnetic current with a hadron
  - the pion form factor furnishes information on the distribution of charge inside the pion
- The interaction of the EM current with a pion has the form: $\langle J_\pi^\mu \rangle = (p'^\mu + p^\mu)\, F_\pi(Q^2)$



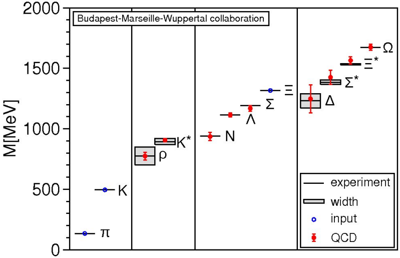
- In the impulse approximation the pion form factor is given by the (triangle) diagram:

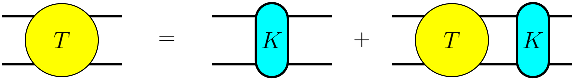




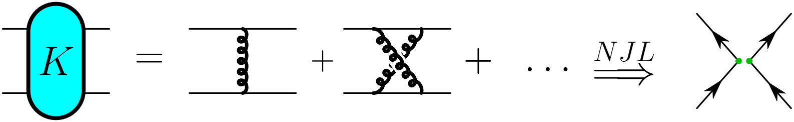
- Ingredients:
  - dressed quark propagators
  - homogeneous Bethe-Salpeter vertices
  - dressed quark-photon vertex
- Measurement and computation of the pion form factor will shed light on DCSB in QCD

- In QFT physical states appear as poles in n-point Green Functions
- That is, the full $\bar{q}q$ scattering matrix or t-matrix, contains poles for all $\bar{q}q$ bound states, that is, the physical mesons
- The $\bar{q}q$ t-matrix is given by the Bethe-Salpeter equation (BSE):





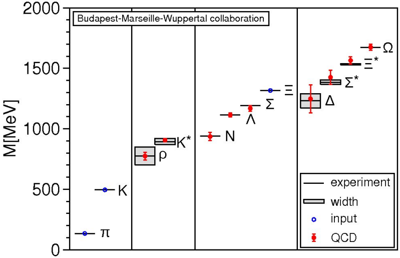
- In principle the kernel, K, contains all possible 2PI diagrams

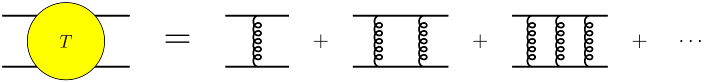


- The BSE is compact and correct – its solution in a particular channel gives the two-body (meson) propagator

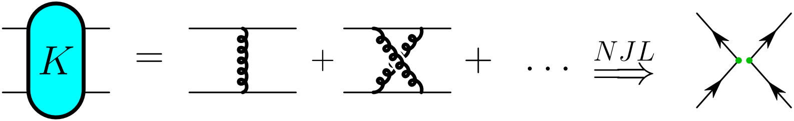
Bethe-Salpeter Equation

- In QFT physical states appear as poles in n-point Green Functions
- That is, the full $\bar{q}q$ scattering matrix or t-matrix, contains poles for all $\bar{q}q$ bound states, that is, the physical mesons
- The $\bar{q}q$ t-matrix is given by the Bethe-Salpeter equation (BSE):



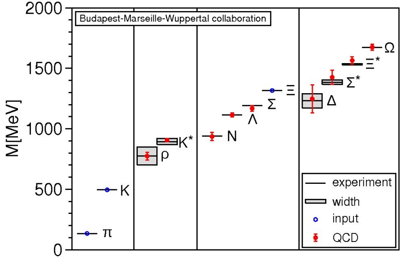


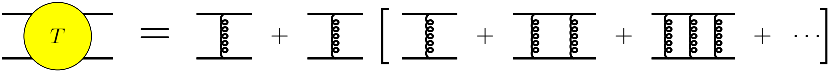
- In principle the kernel, K, contains all possible 2PI diagrams



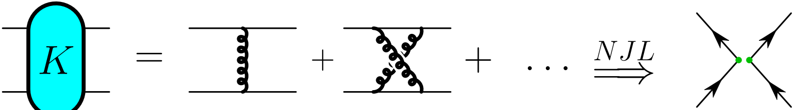
- The BSE is compact and correct – its solution in a particular channel gives the two-body (meson) propagator

- In QFT physical states appear as poles in n-point Green Functions
- That is, the full $\bar{q}q$ scattering matrix or t-matrix, contains poles for all $\bar{q}q$ bound states, that is, the physical mesons
- The $\bar{q}q$ t-matrix is given by the Bethe-Salpeter equation (BSE):





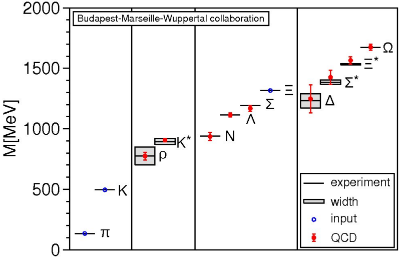
- In principle the kernel, K, contains all possible 2PI diagrams

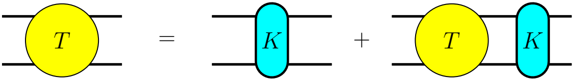


- The BSE is compact and correct – its solution in a particular channel gives the two-body (meson) propagator

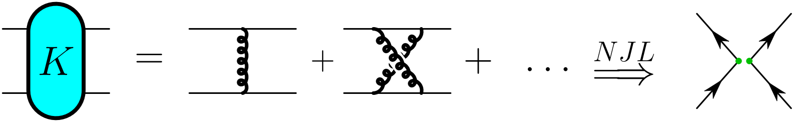
Bethe-Salpeter Equation

- In QFT physical states appear as poles in n-point Green Functions
- That is, the full $\bar{q}q$ scattering matrix or t-matrix, contains poles for all $\bar{q}q$ bound states, that is, the physical mesons
- The $\bar{q}q$ t-matrix is given by the Bethe-Salpeter equation (BSE):



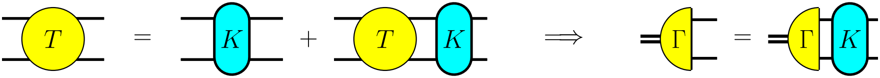


- In principle the kernel, K, contains all possible 2PI diagrams



- The BSE is compact and correct – its solution in a particular channel gives the two-body (meson) propagator

Bethe-Salpeter Vertex Functions



- Near a bound state pole of mass m a two-body T-matrix behaves as

$$\mathcal{T}(p,k) \to \frac{i\,\Gamma(p,k)\,\bar{\Gamma}(p,k)}{p^2 - m^2} \qquad \text{where} \qquad p = p_1 + p_2, \quad k = p_1 - p_2$$

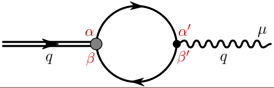
- *$\Gamma(p,k)$ is the homogeneous Bethe-Salpeter vertex & describes relative motion of the quark and anti-quark while they form the bound state*
- Pion BSE vertex has the general form

$$\Gamma_\pi(p,k) = \gamma_5 \Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, \mathcal{G}(p,k) + \sigma^{\mu\nu} k_\mu p_\nu\, \mathcal{H}(p,k) \Big]$$

  - the dominant amplitude is $E_\pi(p,k)$, $F_\pi(p,k)$ becomes important for large Q^2

- Bethe-Salpeter vertex needed for calculations e.g. f_π or $F_\pi(Q^2)$

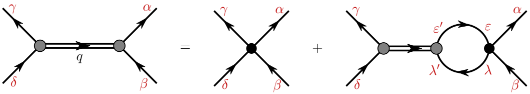
$$i\, f_\pi\, p^\mu\, \delta_{ij} = \int \frac{d^4k}{(2\pi)^4}\, \text{Tr}\left[\tfrac{1}{2}\,\gamma^\mu \gamma_5 \tau_j\, S(k)\, \Gamma_\pi^i(p,k)\, S(k-p)\right]$$

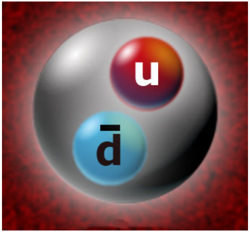


NJL BSE for the Pion

- NJL BSE for the pion: $\mathcal{K}_\pi = -2i\, G_\pi\, (\gamma_5 \boldsymbol{\tau})_{\alpha\beta} (\gamma_5 \boldsymbol{\tau})_{\lambda\epsilon}$

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4}\, \mathcal{K}\, S(q+k)\, S(k)\, \mathcal{T}(q)$$





- Solving for the t-matrix and expanding about the pole:

$$\mathcal{T} = \gamma_5 \tau_i\, \frac{-2i\, G_\pi}{1 + 2\, G_\pi\, \Pi_\pi(q^2)}\, \gamma_5 \tau_i \to \frac{i\, Z_\pi}{q^2 - m_\pi^2} (\gamma_5 \tau_i)(\gamma_5 \tau_i) \implies \Gamma_\pi = \sqrt{Z_\pi}\, \gamma_5 \tau_i$$

  - Z_π is effective pion-quark coupling constant & Γ_π the pion BS vertex

- The pion mass is then given by – $1 + 2\, G_\pi\, \Pi_\pi(q^2 = m_\pi^2) = 0$ – where

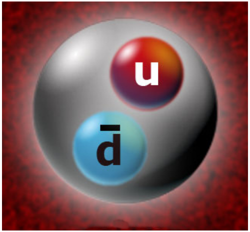
$$\Pi_\pi\,(q^2)\, \delta_{ij} = 3i \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma_5\, \tau_i\, S(k)\, \gamma_5\, \tau_j\, S(k+q)\right]$$

  - this result is straightforward to obtain

The Pion as a Goldstone Boson

- Is the pion a Goldstone boson? NJL gap equation gives:

$$\Pi_\pi(q^2) = \frac{m}{2\,G_\pi\,M} - \frac{1}{2\,G_\pi} - q^2\,I(q^2)$$



- Pole condition – $1 + 2\,G_\pi\,\Pi_\pi(q^2 = m_\pi^2) = 0$ – implies

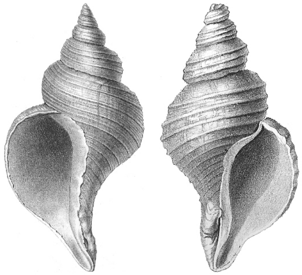
$$m_\pi^2 = \frac{m}{2\,G_\pi\,M\,I(m_\pi^2)}$$

- Therefore as demanded by chiral symmetry we have: $m_\pi^2 \propto m$ (GMOR)
  - also in the chiral limit – $m \to 0$ $(M \neq 0)$ – pion is massless

- The NJL model also satisfies all another relations associated with chiral symmetry; for example
  - $f_\pi\,g_{\pi qq} = M\,g_{Aqq}$ Goldberger–Treiman (GT) relation
  - $f_\pi^2\,m_\pi^2 = \frac{1}{2}\,(m_u + m_d)\,\langle \bar{u}u + \bar{d}d \rangle$ Gell-Mann–Oakes–Renner (GMOR)

Chiral Partners

- *IF* chiral symmetry was *NOT* dynamically broken in nature expect mass degenerate chiral partners; e.g. $m_\sigma \simeq m_\pi \quad \& \quad m_{a_1} \simeq m_\rho$



- The ρ and a_1 are the lowest lying vector ($J^P = 1^-$) and axial-vector ($J^P = 1^+$) $\bar{q}q$ bound states:
  $m_\rho^{\text{exp't}} \simeq 770\,\text{MeV}$ & $m_{a_1}^{\text{exp't}} \simeq 1260\,\text{MeV}$

- The associated NJL BSE pole conditions read:

$$1 + 2\,G_\rho\,\Pi_\rho(q^2 = m_\rho^2) = 0 \;\; \& \;\; 1 + 2\,G_\rho\,\Pi_{a_1}(q^2 = m_{a_1}^2) = 0$$

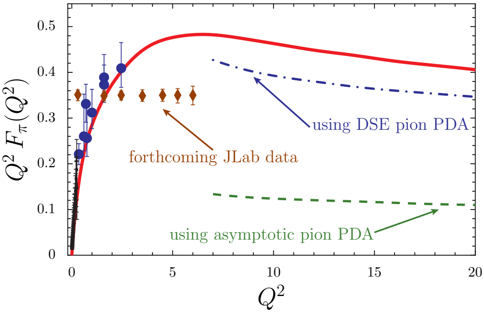
  - where $\Pi_{a_1}(q^2) = M^2\,I(q^2) + \Pi_\rho(q^2)$

- If $m = 0$ and there is *NO* DCSB ($M = 0$) would have: $m_\rho = m_{a_1}$

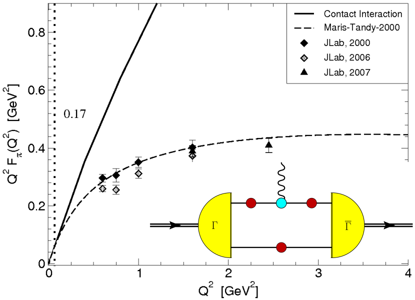
- In nature and NJL, DCSB splits chiral partner masses
  - NJL gives: $m_\rho \equiv 770\,\text{MeV} \quad \& \quad m_{a_1} \simeq 1098\,\text{MeV}$
  - agrees with the Weinberg relation: $m_{a_1} \simeq \sqrt{2}\,m_\rho$; $[m_\sigma^2 \simeq m_\pi^2 + 4\,M^2]$

Consequences of Running Quark Mass

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



[L. X. Gutierrez-Guerrero *el al.*, Phys. Rev. **C81**, 065202 (2010)]



- Full DSE results uses the pion BSE vertex:

$$\Gamma_\pi(p,k) = \gamma_5\Big[E_\pi(p,k) + \not{p}\,F_\pi(p,k) + \not{k}\,k\cdot p\,\mathcal{G}(p,k) + \sigma^{\mu\nu}k_\mu p_\nu\,\mathcal{H}(p,k)\Big]$$

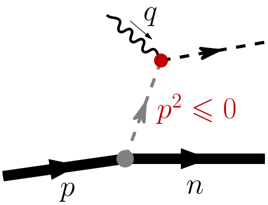
- In gap equation use simple kernel $\Longleftrightarrow$ NJL model with $\pi - a_1$ mixing

$$g^2\,D_{\mu\nu}(p-k)\Gamma^\nu(p,k) \;\to\; \frac{1}{m_G^2}\,g_{\mu\nu}\,\gamma^\nu \;\Longrightarrow\; \Gamma_\pi(p,k) = \gamma_5\left[E_\pi + \not{p}\,F_\pi\right]$$

  - quark no longer has a running mass

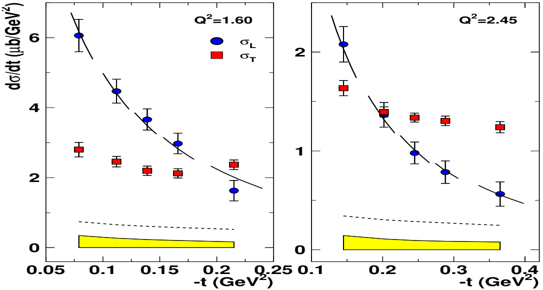
- *Nature of interaction can have observable consequences for $Q^2 > 0$*

Measuring Pion Form Factor



Sullivan Process

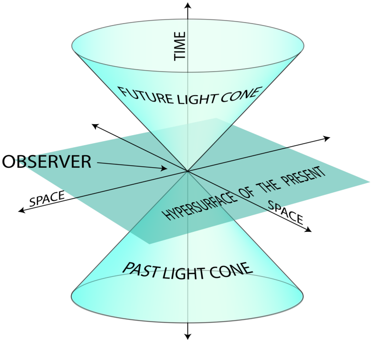
[G. M. Huber *et al.* [Jefferson Lab Collaboration], Phys. Rev. C **78**, 045203 (2008)]



- At low Q^2 pion form factor is measured by scattering a pion from the electron cloud of an atom $[t \equiv p^2]$
  - small mass of electron limits this to $Q^2 < 0.5\,\text{GeV}^2$
- Higher Q^2 experiments have been performed at Jefferson Lab where a virtual photon scatters from a virtual pion that is part of the nucleon
- Initial pion is off its mass shell – $p^2 \leqslant 0$ – on mass shell $p^2 = m_\pi^2$
  - need to extrapolate to the pion pole $p^2 = m_\pi^2$

Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
  - boost operators are dynamical, that is, they are interaction dependent

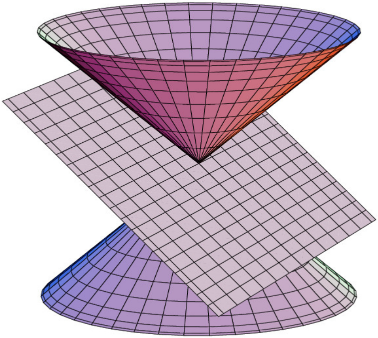


- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front WFs; many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

$$\varphi(x) = \int d^2\vec{k}_\perp \, \psi(x, \vec{k}_\perp)$$

Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
  - boost operators are dynamical, that is, they are interaction dependent

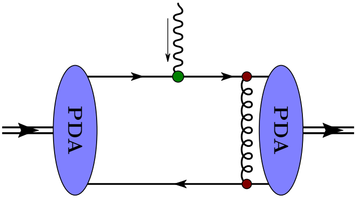


- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t + z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front WFs; many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

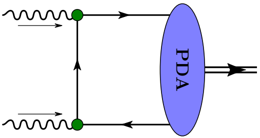
$$\varphi(x) = \int d^2\vec{k}_\perp \; \psi(x, \vec{k}_\perp)$$

Pion's Parton Distribution Amplitude

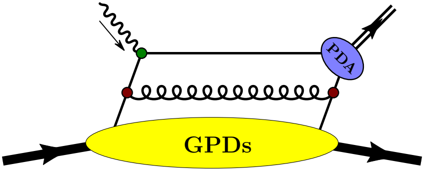
- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2

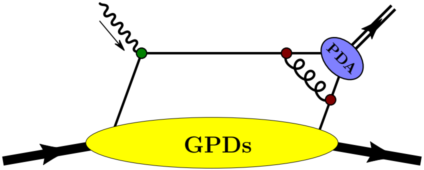


$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$





- PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi \, \varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \, \delta\left(k^+ - x\, p^+\right) \mathrm{Tr}\left[\gamma^+ \gamma_5 \, S(k) \, \Gamma_\pi(k,p) \, S(k-p)\right]$$

- $S(k)\, \Gamma_\pi(k,p)\, S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [at twist-2 also pseudoscalar projection]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q^2 dependence of pion form factor

$$Q^2 \, F_\pi(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} 16\, \pi \, f_\pi^2 \, \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\text{asy}}(x) = 6\, x\,(1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA   [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu}\,\varphi(x,\mu) = \int_0^1 dy\, V(x,y)\,\varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

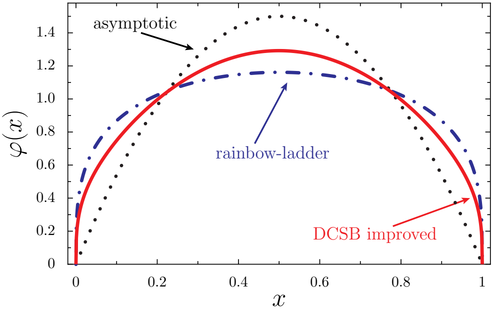
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

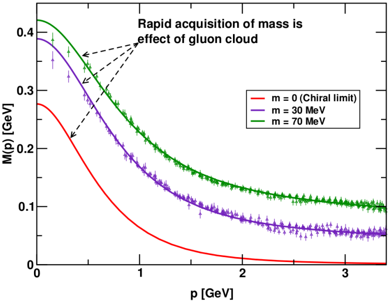
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\mathrm{GeV}$
- A realization of DCSB on the light-front
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

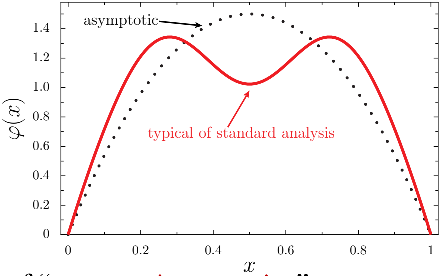
Pion PDA from lattice QCD

- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^\alpha(1-x)^\alpha\left[1 + \sum_{n=2,\,4,\dots} a_n^{\alpha+1/2}(Q^2)\,C_n^{\alpha+1/2}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

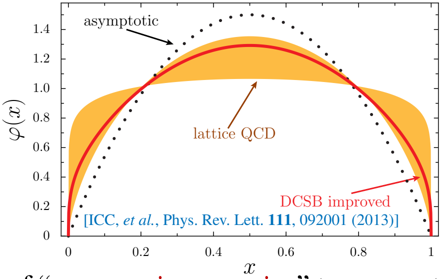
Pion PDA from lattice QCD

- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum\nolimits_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

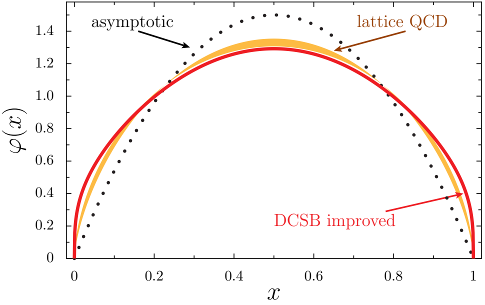
$$\varphi_\pi(x,Q^2) = N_\alpha\,x^\alpha(1-x)^\alpha\left[1+\sum\nolimits_{n=2,\,4,\dots} a_n^{\alpha+1/2}(Q^2)\,C_n^{\alpha+1/2}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

Updated Pion PDA from lattice QCD

- Generalized expansion

$$\varphi_\pi(x) = N_\alpha\, x^\alpha (1-x)^\alpha \left[1 + \sum a_n^{\alpha+}(Q^2)\, C_n^{\alpha+}(2x-1)\right]$$



- Updated lattice QCD moment: [V. Braun *et al.*, arXiv:1503.03656 [hep-lat]]

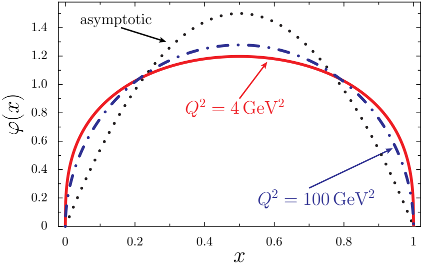
$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.2361\,(41)\,(39)\,(?)$$

- DSE prediction:

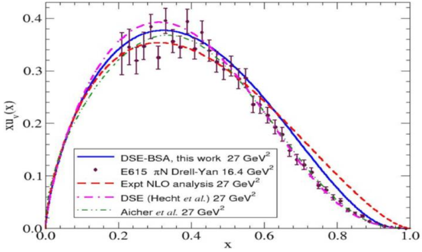
$$\int_0^1 dx\,(2\,x-1)^2 \varphi_\pi(x) = 0.251$$

When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

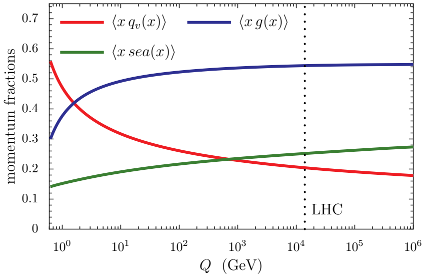


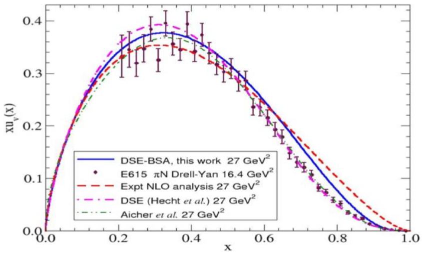
[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\text{GeV}^2$, compared with $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





- LO QCD evolution of momentum fraction carried by valence quarks

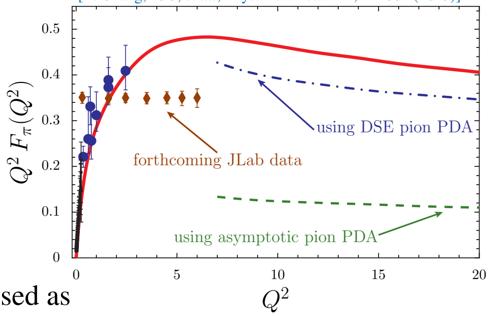
$$\langle x\, q_v(x)\rangle (Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) \propto \delta(x)$
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$
- *Asymptotia is a long way away!*

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2\,F_\pi(Q^2)$ at $Q^2 \approx 6\,\text{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



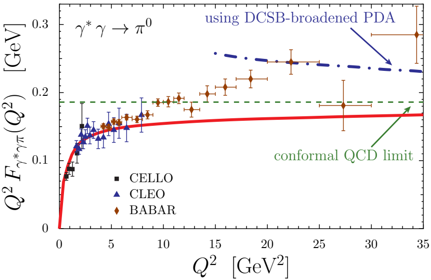
- The QCD prediction can be expressed as

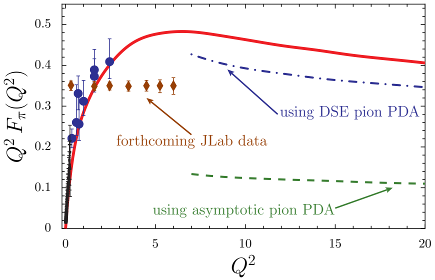
$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
  - 15% disagreement explained by higher order/higher-twist corrections

- *We predict that QCD power law behaviour – with QCD's scaling law violations – sets in at* $Q^2 \sim 8\,\text{GeV}^2$

Pion Transition Form Factor





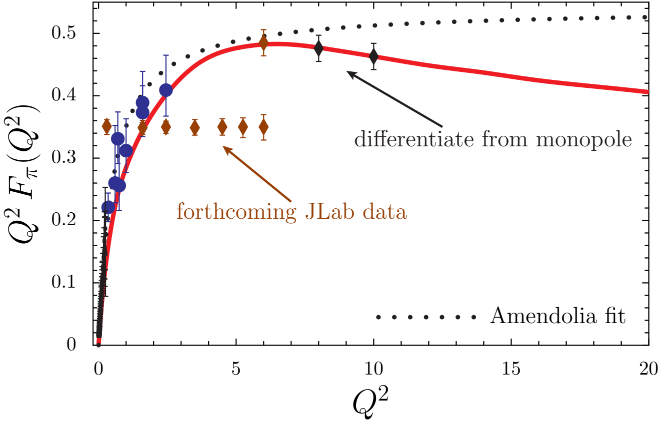
- At large Q^2 the hard gluon exchange in the $\gamma^* + \pi \to \pi$ form factor – needed to keep the pion intact – results in distinctly different behaviour to the pion transition form factor $\gamma^* + \pi \to \gamma$

$$Q^2 F_{\gamma^*\pi\gamma}(Q^2) \to 2\, f_\pi\, w_\pi^2 \qquad \text{c.f.} \quad Q^2 F_\pi(Q^2) \to 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2$$

- Therefore approach to asymptotic limit gives *inter alia* a unique window into quark-gluon dynamics in QCD

- In full DSE calculation of $\gamma^*\pi \to \gamma$ conformal limit approached from below

Measuring onset of Perturbative scaling



- To observe onset of perturbative power law behaviour – *to differentiate from a monopole* – optimistically need data at $8\,\mathrm{GeV}^2$ but likely also at $10\,\mathrm{GeV}^2$
  - this is a very challenging task experimentally
- Scaling predictions are valid for both spacelike and timelike momenta
  - timelike data show promise as the means of verifying modern predictions