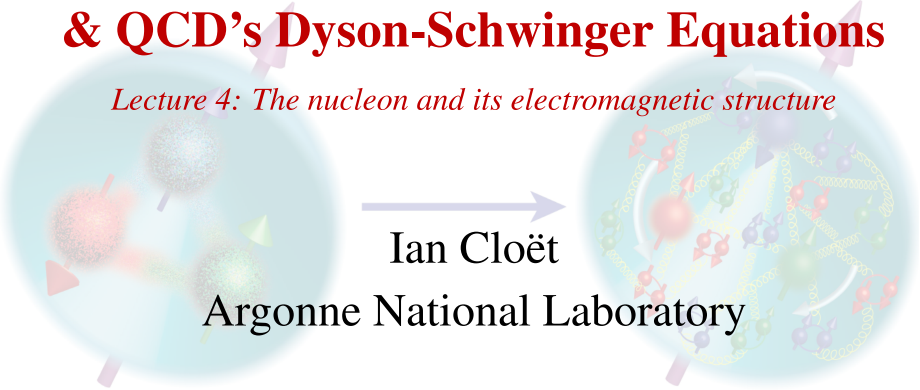
Hadron Physics
& QCD's Dyson-Schwinger Equations

Lecture 4: The nucleon and its electromagnetic structure



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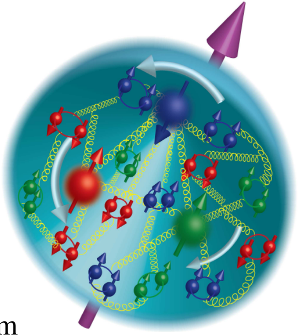
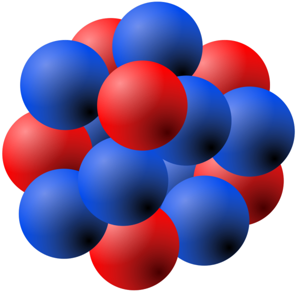
The Beginning of Nuclear Physics

- Hadron physics and ultimately nuclear physics means to chart and compute the distribution of quarks & gluons – even photons, electrons, . . . – within hadrons and nuclei



- The archetype for these studies is the proton (uud) – the only stable composite in the Standard Model

- With the discovery of the neutron in 1932 by James Chadwick, the proton and neutron are known to form an isospin-doublet under the strong interaction: *the nucleon – the building blocks of nuclei*



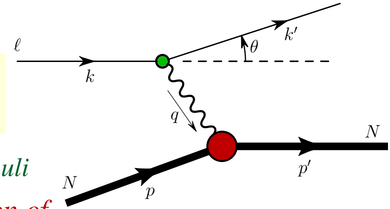
- Key questions in proton structure:
  - how is spin and angular momentum distributed among its constituents
  - how is charge and magnetization distributed among its constituents

Nucleon Electromagnetic Form Factors

- Nucleon electromagnetic current

$$\langle J^\mu \rangle = \bar{u}(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M} F_2(Q^2) \right] u(p)$$

Dirac *Pauli*



- *Provides vital information on the distribution of charge and magnetization within the most basic element of nuclear physics*
  - form factors also directly probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
  - proton radius puzzle
  - $\mu_p\, G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
  - flavour decomposition and evidence for diquark correlations
  - meson-cloud effects
  - seeking verification of perturbative QCD scaling predictions & scaling violations

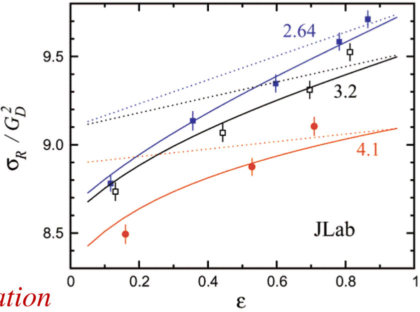
Nucleon Sachs Form Factors

- Experiment gives Sachs form factors:

$$\sigma_R(\varepsilon, Q^2) \equiv \varepsilon\,\frac{d\sigma}{d\Omega}\,\frac{1+\tau}{\sigma_{\text{Mott}}} = \varepsilon\, G_E^2 + \tau\, G_M^2$$

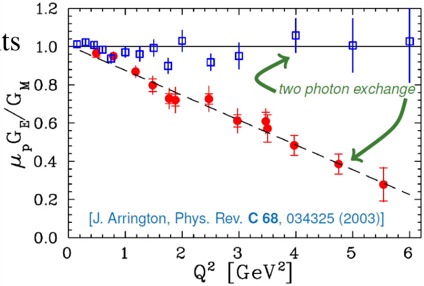
$$G_E = F_1 - \tau\, F_2; \qquad G_M = F_1 + F_2$$

  - at a fixed Q^2 the slope of σ_R gives G_E and the y-axis intercept G_M; $\tau = Q^2/4M^2$



- Until the late 90s these *Rosenbluth separation* experiments found a flat $\mu_p\, G_{Ep}/G_{Mp}$ ratio

- However *polarization transfer* experiments – pioneered at JLab – completely altered our picture of nucleon structure
  - slope indicates that the distribution of charge and magnetization not the same
  - discrepancy likely a consequence of two-photon exchange



Proton Radius Puzzle

- Since the formulation of QED it has been known that muonic atoms are the ideal testing ground
  - however only very recently has it been possible to study the spectroscopy of muonic atoms
- The charge radius of a hadron is defined by:

$$\langle r_E^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_E(Q^2)\Big|_{Q^2=0}$$

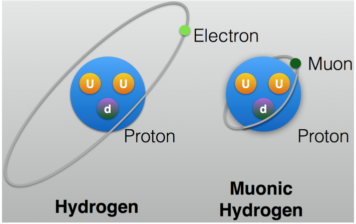
- Transitions between energy levels in electronic or muonic atoms are sensitive to $\langle r_E^2 \rangle$
- Radius from muonic hydrogen [Pohl (2010)]:

$$r_{Ep} = 0.84087 \pm 0.00039 \text{ fm}$$

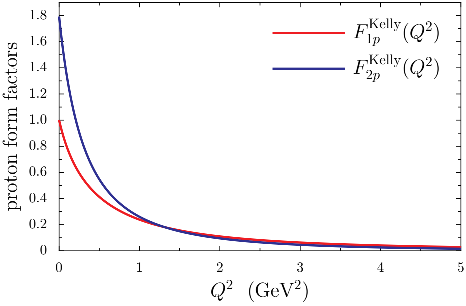
- CODATA: $e\,p$ scattering + e-hydrogen:

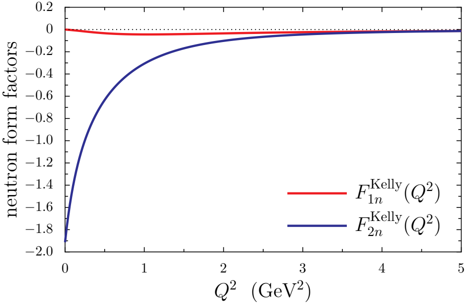
$$r_{Ep} = 0.8775 \pm 0.0051 \text{ fm}$$

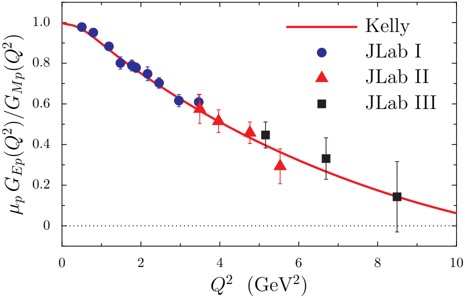
- *There is a 7σ or 4% difference!*
  - one of the most interesting puzzles in physics

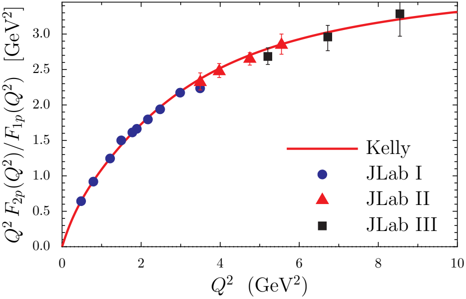


Experimental Status



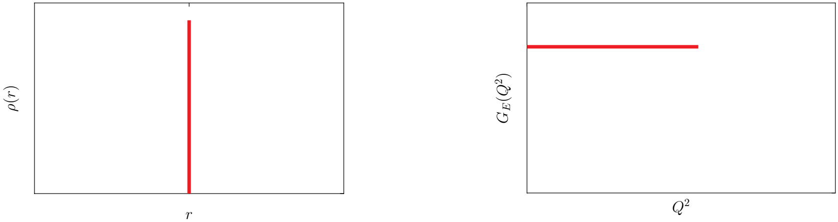




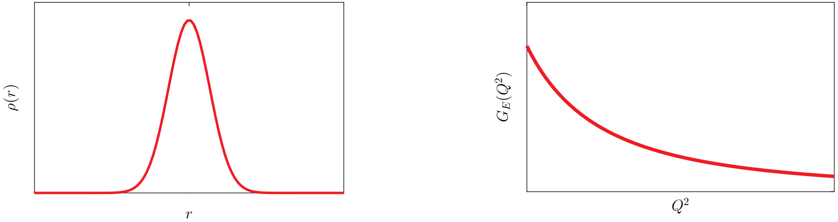


Physical Interpretation of Form Factors

- Textbooks teach that in Breit frame ($\vec{p}' = -\vec{p}$) Sachs form factors can be interpreted as 3-d Fourier transforms of the charge & magnetization densities

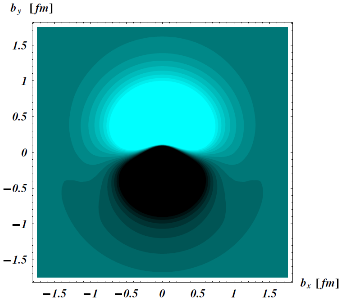


- Deviation from a constant provides information on target structure



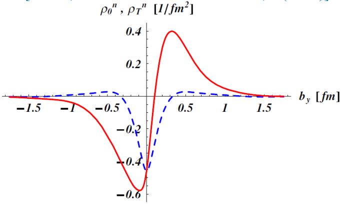
- Interpretation breaks down at small distances – cannot form a density
- Must consider transverse charge densities, given by 2-d Fourier transform

Transverse Charge Densities



Quark transverse charge density for a neutron polarized along the x-axis

[Carlson and Vanderhaeghen, Phys. Rev. Lett. **100**, 032004 (2008)]
[Miller, Ann. Rev. Nucl. Part. Sci. **60**, 1 (2010)]

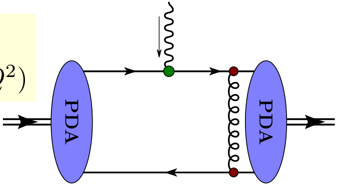


- It is now recognized that care must be taken when interpreting a 3-D Fourier transform of a form factor as a charge or magnetization density
- A rigorous density can be defined via a 2-D Fourier transform
  - these hadronic transverse charge densities are quantities as seen in a reference frame moving with infinite momentum
- Numerous new physical insights for elastic and transition form factors
  - e.g. the negative central neutron charge density, caused by the dominance of d quarks at the center

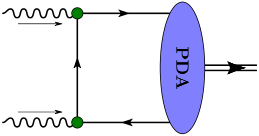
- At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* & a hard scattering kernel [Farrar, Jackson; Lepage, Brodsky]
  - only the valence Fock state ($\bar{q}q$ or qqq) can contribute as $Q^2 \to \infty$
  - both confinement and asymptotic freedom in QCD are important in this limit

- Most is known about $\bar{q}q$ bound states, e.g., for the pion:

$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



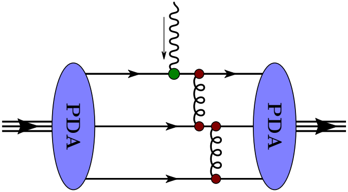
$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$



- For nucleon normalization is unknown

$$G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4$$

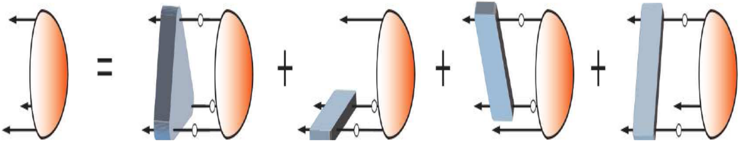
  - orbital angular momentum effects approach



- *Gluons play a critical role – formalism must reflex this!*

Nucleon Structure

- A robust description of the nucleon as a bound state of 3 dressed-quarks can only be obtained within an approach that respects Poincaré covariance
- Such a framework is provided by the Poincaré covariant Faddeev equation



  - sums all possible interactions between three dressed-quarks
  - much of the three-body interaction can be absorbed into renormalized two-body interactions

- A *prediction* of these approaches is that owing to DCSB in QCD – strong diquark correlations exist within baryons
  - any interaction that describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour-$\bar{3}$ channel
  - where *scalar and axial-vector diquarks* are most important for the nucleon

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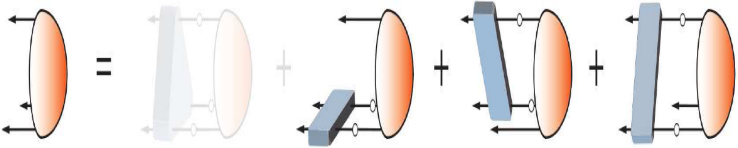

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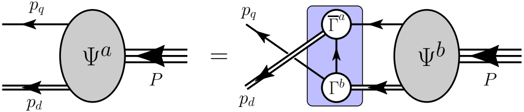
Diquarks

- Diquarks are dynamically generated correlations between quarks inside baryons



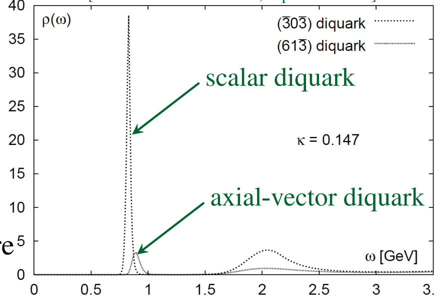
  - typically diquark radii are similar to analogous mesons: $r_{0^+} \sim r_\pi, \; r_{1^+} \sim r_\rho$

- These dynamic qq correlations are not the static diquarks of old
  - all quarks participate in all diquark correlations
  - in a given baryon the Faddeev equation predicts a probability for each diquark cluster
  - for the nucleon: scalar $(0^+) \sim 70\%$
    axial-vector $(1^+) \sim 30\%$

- *Faddeev equation spectrum has significant overlap with constituent quark model and limited relation to Lichtenberg's quark+diquark model*

- Mounting evidence from hadron structure (e.g. PDFs) and lattice QCD

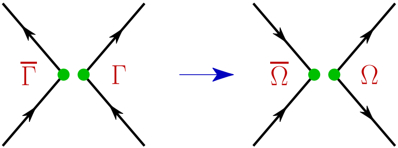
[I. Wetzorke and F. Karsch, hep-lat/0008008]



Diquarks in the NJL model

- To describe diquarks in the NJL model one usually rewrites the $\bar{q}q$ interaction Lagrangian into a qq interaction Lagrangian

$$\left(\bar{\psi}\,\Gamma\,\psi\right)^2 \to \left(\bar{\psi}\,\Omega\,\bar{\psi}^T\right)\left(\psi^T\,\bar{\Omega}\,\psi\right)$$



  - Ω has quantum numbers if interaction channel

- NJL qq Lagrangian in the scalar and axial-vector diquark channels reads

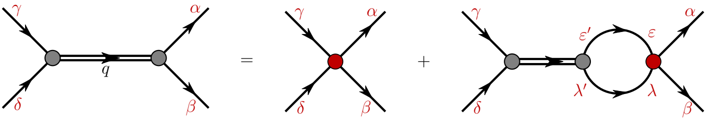
$$\mathcal{L}_I = G_s\left[\overline{\psi}\,\gamma_5\,C\,\tau_2\,\beta^A\,\overline{\psi}^T\right]\left[\psi^T\,C^{-1}\,\gamma_5\,\tau_2\,\beta^{A'}\,\psi\right] + G_a\left[\overline{\psi}\,\gamma_\mu\,C\,\tau_i\tau_2\,\beta^A\,\overline{\psi}^T\right]\left[\psi^T\,C^{-1}\,\gamma^\mu\,\tau_2\tau_j\,\beta^{A'}\,\psi\right] + \ldots.$$

  - the first term is the scalar diquark channel ($J^P = 0^+, T = 0$)
  - τ_2 couples isospin of two quarks to $T = 0$, $C\gamma_5$ couples spin to $J = 0$, $\beta^A = \sqrt{\frac{3}{2}}\,\lambda^A \quad (A = 2, 5, 7)$ couples quarks to colour $\bar{3}$
  - the second the axial-vector diquark channel ($J^P = 1^+, T = 1$)

NJL diquark t-matrices

- Bethe-Salpeter equation for qq scattering matrix reads

$$\mathcal{T}(q)_{\alpha\beta,\gamma\delta} = K_{\alpha\beta,\gamma\delta} + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\, K_{\alpha\beta,\varepsilon\lambda}\, S(k)_{\varepsilon\varepsilon'}\, S(q-k)_{\lambda\lambda'}\, \mathcal{T}(q)_{\varepsilon'\lambda',\gamma\delta},$$



  - note symmetry factor of $\frac{1}{2}$ (c.f. $\bar{q}q$ BSE)

- The Feynman rules for the interaction kernels are

$$\mathcal{K}_s = 4i\, G_s \left(\gamma_5\, C\, \tau_2\, \beta^A\right)_{\alpha\beta}\left(C^{-1}\,\gamma_5\,\tau_2\,\beta^A\right)_{\gamma\delta} \qquad \mathcal{K}_a = 4i\, G_a \left(\gamma_\mu\, C\, \tau_i\tau_2\, \beta^A\right)_{\alpha\beta}\left(C^{-1}\,\gamma^\mu\,\tau_2\tau_i\,\beta^A\right)_{\gamma\delta}$$

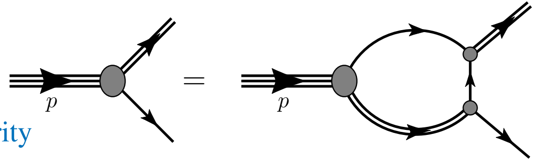
- The solution to the BSE is of the form: $\mathcal{T}(q)_{\alpha\beta,\gamma\delta} = \tau(q^2)\,\Omega_{\alpha\beta}\bar{\Omega}_{\gamma\delta}$

$$\tau_s(q^2) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_s(q^2)} \qquad \tau_a^{\mu\nu}(q) = \frac{4\,i\,G_a}{1+2\,G_a\,\Pi_a(q^2)}\left[g^{\mu\nu} + 2\,G_a\,\Pi_a(q^2)\,\frac{q^\mu q^\nu}{q^2}\right]$$

  - these reduced t-matrices are the diquark propagators

NJL Faddeev Equation

- To describe a nucleon, Faddeev equation kernel must be projected onto colour singlet, spin-$\frac{1}{2}$, isospin-$\frac{1}{2}$ & positive parity



- In NJL common to make the *static approximation* to quark exchange kernel: $S(p) \to -\frac{1}{M}$
  - with this approximation Faddeev amplitude does not depend of relative momentum between the quark and diquark
- The Faddeev equation can then be written in as

$$\Gamma_N(p,s) = K(p)\,\Gamma_N(p,s)$$

- With only scalar and axial-vector diquarks the vertex must have the form

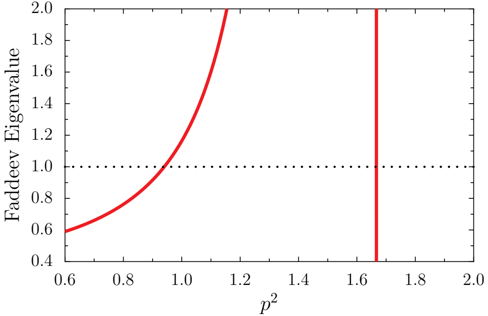
$$\Gamma_N(p,s) = \sqrt{-Z_N}\begin{bmatrix} \alpha_1 \\ \alpha_2\,\frac{p^\mu}{M_N}\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5 \end{bmatrix} u_N(p,s)$$

Faddeev Equation Solutions

- Explicitly the NJL Faddeev equation reads: $\Pi^{\mu\nu}_{Na(s)} = \int \frac{d^4k}{(2\pi)^4}\, \tau^{\mu\nu}_{a(s)}(p-k)\, S(k)$

$$\begin{bmatrix} \Gamma_s \\ \Gamma^\mu_a \end{bmatrix} = \frac{3}{M} \begin{bmatrix} \Pi_{Ns} & \sqrt{3}\gamma_\alpha\gamma_5\, \Pi^{\alpha\beta}_{Na} \\ \sqrt{3}\gamma_5\gamma^\mu\, \Pi_{Ns} & -\gamma_\alpha\gamma^\mu\, \Pi^{\alpha\beta}_{Na} \end{bmatrix} \begin{bmatrix} \Gamma_s \\ \Gamma_{a,\beta} \end{bmatrix}$$

- The Faddeev equation reduces to a linear matrix equation which is a function of p^2 – the mass-squared of the bound state
  - a physical state exists for any p^2 that gives an eigenvalue if one



- First solution is the nucleon $M_N = 940\,\text{MeV}$
- Second solution is 1st excited state of the nucleon $\Longleftrightarrow$ Roper: $M_{\text{Roper}} = 1670\,\text{MeV}$
- and so on

DSE Faddeev Equation

- The DSE Faddeev equation has far more structure than in NJL
- For example the DSE Faddeev equation including scalar and axial-vector diquarks reads

$$\begin{bmatrix} \mathcal{S}(k,P) \\ \mathcal{A}_i^{\mu}(k,P) \end{bmatrix} u_N(p) = \int \frac{d^4\ell}{(2\,\pi)^4}\, \mathcal{M}_{ij}^{\mu\nu}(\ell;k,P) \begin{bmatrix} \mathcal{S}(\ell,P) \\ \mathcal{A}_{\nu}^{j}(\ell,P) \end{bmatrix} u_N(p)$$

  - importantly the vertex function depends on the relative momentum, k, between the quark and diquark
  - the Faddeev kernel is $\mathcal{M}_{ij}^{\mu\nu}(\ell;k,P)$
  - $\mathcal{S}(k,P)$ and $\mathcal{A}_i^{\mu}(k,P)$ describe the momentum space correlation between the quark and diquark in the nucleon
- This equation can be solved numerically on a large grid in k and P
- However standard practice to use an expansion in Chebyshev polynomials for $\mathcal{S}(k,P)$ & $\mathcal{A}_i^{\mu}(k,P)$ and the solve for the coefficients of this expansion

Nucleon EM Form Factors from DSEs

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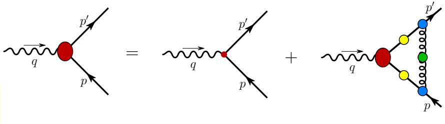
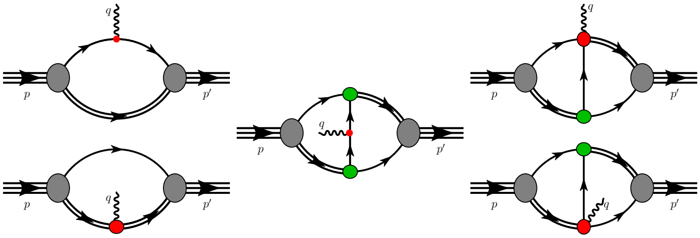
- A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied
- For quark-photon vertex WTI implies:



$$q_\mu\,\Gamma^\mu_{\gamma qq}(p',p) = \hat{Q}_q\left[S_q^{-1}(p') - S_q^{-1}(p)\right]$$

  - transverse structure unconstrained

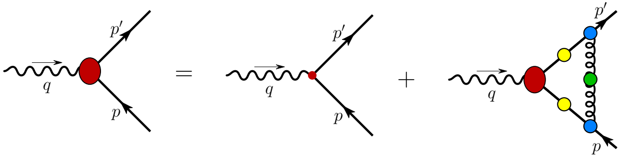
- Diagrams needed for a gauge invariant nucleon EM current in DSEs



- Feedback with experiment can shed light on elements of QCD via DSEs

Dressed Quark EM Form Factors

- Quark-photon vertex is given by the *inhomogeneous Bethe-Salpeter equation* – driving term is an external vector current: $\gamma^\mu \left(\frac{1}{6} + \frac{\tau_3}{2}\right)$



- Lorentz covariance implies that the quark–photon vertex has the structure

$$\Gamma^\mu_{\gamma qq}(p',p) = \sum_{i=1}^{12} \lambda_i^\mu \; f_i(p'^2, p^2, q^2) = \Gamma^\mu_L(p',p) + \Gamma^\mu_T(p',p)$$

- In QCD the properties of the quark–photon vertex are governed by the quark propagator and the quark–gluon vertex

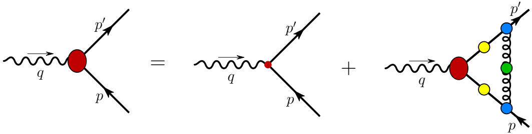
- A Ward-Takahashi identity constrains Γ^μ_L piece of quark–photon vertex

$$q_\mu \, \Gamma^\mu_{\gamma qq} = q_\mu \, \Gamma^\mu_L = \hat{Q} \, \left[S^{-1}(p') - S^{-1}(p)\right], \qquad q_\mu \, \Gamma^\mu_T = 0$$

  - these identities are a consequence of local $U(1)_V$ gauge invariance

DSE Quark Form Factors

$$q_\mu \, \Gamma^\mu_{\gamma qq} = \hat{Q}\left[S^{-1}(p') - S^{-1}(p)\right]$$

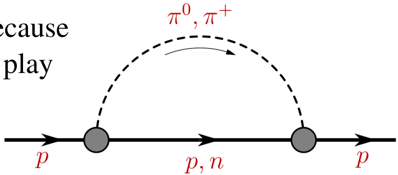


- The longitudinal piece of the quark-photon vertex, $\Gamma^\mu_{\gamma qq} = \Gamma^\mu_L + \Gamma^\mu_T$, is completely determined by the quark propagator
- This result is encapsulated in by Ball-Chiu vertex

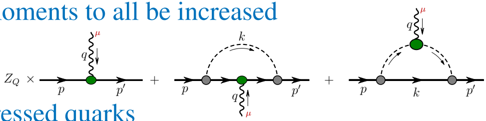
$$\Gamma^\mu_{BC} = \frac{A(p'^2)+A(p^2)}{2}\,\gamma^\mu - \frac{A(p'^2)-A(p^2)}{p'^2-p^2}\,i(p'+p)^\mu + \frac{1}{2}\,\frac{A(p'^2)-A(p^2)}{p'^2-p^2}\,(\not{p}'+\not{p})(p'+p)$$

- Recall: $S^{-1}(p) = i\not{p}\,A(p^2) + B(p^2)$ – it is then straight forward to show Γ^μ_{BC} satisfies the WTI
- The nature of the *quark-photon vertex* is largely controlled by the structure of the *quark-gluon vertex*
  - different quark-gluon vertices can give very similar quark-propagators
  - therefore transverse piece of $\Gamma^\mu_{\gamma qq}$ sensitive to the quark-gluon vertex
- $\Gamma^\mu_{\gamma,T}$ is largely unknown – but expect large anomalous magnetic moment

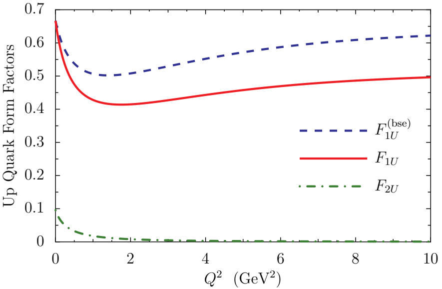
- Pions are the lightest hadrons, therefore, because of quantum fluctuations we expect them to play an important role in many observables

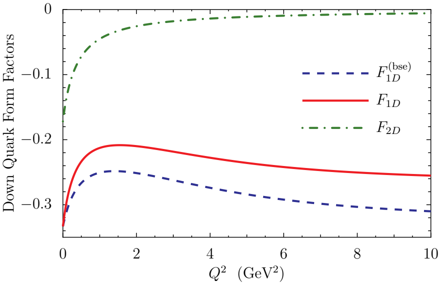


- Because the pion is light it is long range
  - expect nucleon radii, and magnetic moments to all be increased
- Dressed quark with pion cloud:

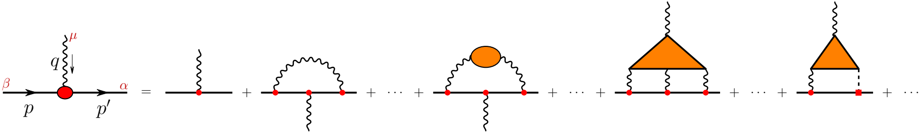


  - key results is that the pion give the dressed quarks and anomalous magnetic moment and an increased size

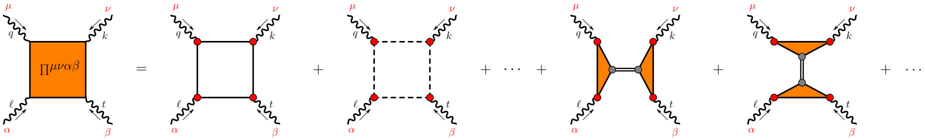




An Aside – Muon Anomalous Magnetic Moment



- $a_\mu^{\text{exp}} = 11659208.0 \pm 6.3 \times 10^{-10}; \quad a_\mu^{\text{theory}} = 11659179.0 \pm 6.5 \times 10^{-10}$
- largest theory error come from HLBL scattering contribution



- Box diagram contribution is least know
  - only γ^μ coupling and VMD has been considered so far
  - we argue that the anomalous magnetic moment term cannot be ignored
- At least error on $a_\mu^{\text{HLBL}} = 8.3 \pm 3.2 \times 10^{-10}$ should be much larger
- Good reference: F. Jegerlehner, A. Nyffeler, Physics Reports 477 (2009)

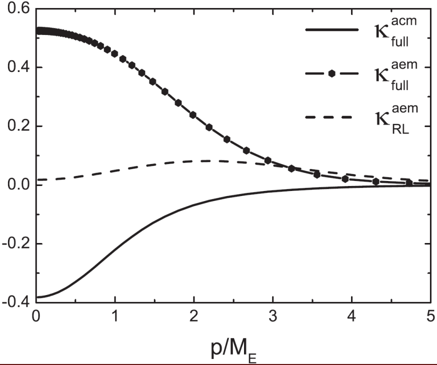
Beyond Rainbow Ladder Truncation

- Include *"anomalous chromomagnetic"* term in quark-gluon vertex

$$\frac{1}{4\pi}\, g^2\, D_{\mu\nu}(\ell)\,\Gamma_\nu(p',p) \;\to\; \alpha_{\text{eff}}(\ell)\, D^{\text{free}}_{\mu\nu}(\ell)\,[\gamma_\nu + i\sigma^{\mu\nu} q_\nu\, \tau_5(p',p)]$$

- In chiral limit *anomalous chromomagnetic* term can only appear through DCSB – since operator flips quark helicity
- EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
  - charge: e & magnetic moment: μ
- Expect strong gluon dressing to produce non-trivial electromagnetic structure for a dressed quark
  - recall dressing produces – from massless quark – a $M \sim 400\,\text{MeV}$ dressed quark
- Large anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*
  - *dressed quarks are not point particles*

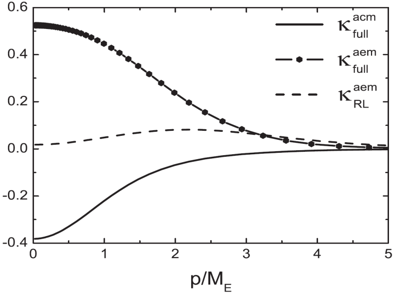
[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]



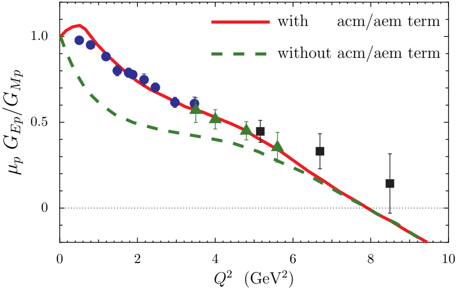
Proton G_E/G_M ***Ratio***

Argonne NATIONAL LABORATORY

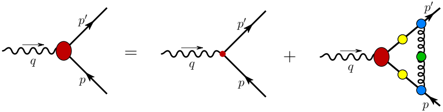
[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]



[I. C. Cloët, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



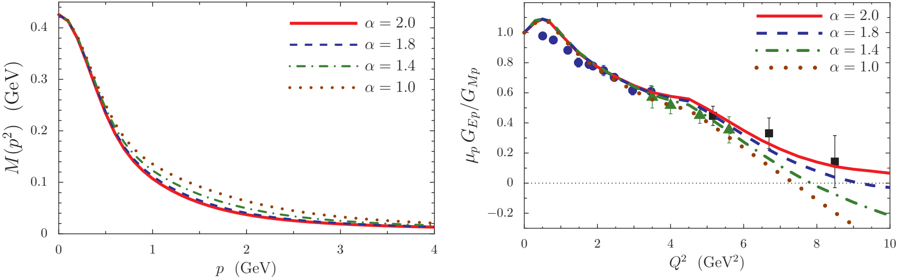
- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate Q^2
  - power law suppressed at large Q^2



- Illustrates how feedback with EM form factor measurements can help constrain the quark–photon vertex and therefore the quark–gluon vertex within the DSE framework
  - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs ⇔ confinement

Proton G_E form factor and DCSB

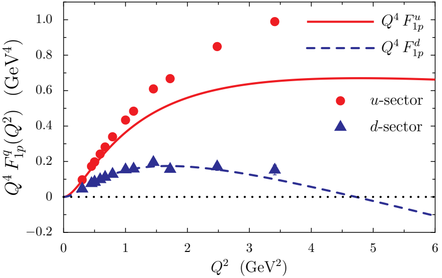
[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]

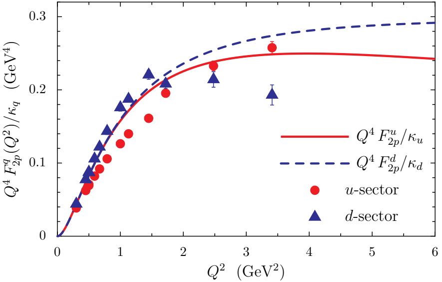


- Find that slight changes in $M(p^2)$ on the domain $1 \lesssim p \lesssim 3\,\text{GeV}$ have a striking effect on the G_E/G_M proton form factor ratio
  - *strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD*
- Zero in $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$ largely determined by evolution of $Q^2\,F_2$
  - F_2 is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – *vanishes in perturbative limit*
  - the quicker the perturbative regime is reached the quicker $F_2 \to 0$

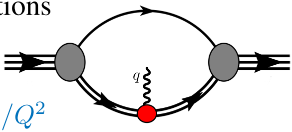
Flavour separated proton form factors

[I. C. Cloët, W. Bentz, A. W. Thomas, Phys. Rev. C **90**, 045202 (2014)]





- Prima facie, these experimental results are remarkable
  - u and d quark sector form factors have very different scaling behaviour
- However, when viewed in context of diquark correlations results are straightforward to understand
  - in proton (uud) the d quark is much more likely to be in a scalar diquark $[ud]$ than a u quark; diquark $\implies 1/Q^2$

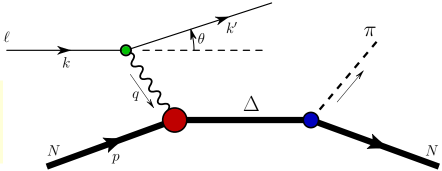


- Zero in F_{1p}^d a result of interference between scalar and axial-vector diquarks
  - location of zero indicates relative strengths – correlated with d/u ratio as $x \to 1$

Nucleon to Resonance Transitions

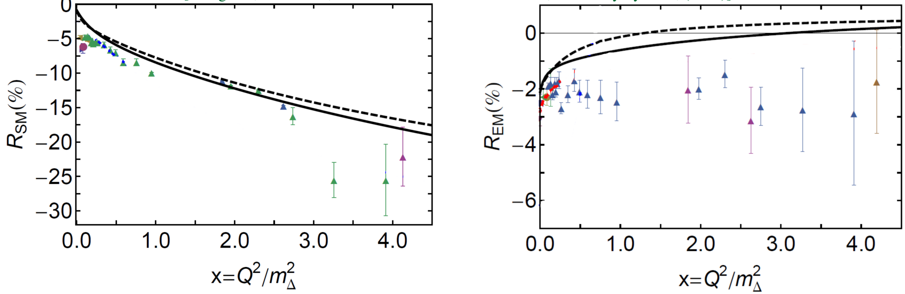
- Given the challenges posed by non-perturbative QCD it is insufficient to study hadron ground-states alone
- Nucleon transition form factors provide a critical extension to elastic form factors – providing more windows into and different perspectives on quark-gluon dynamics
  - e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons
- Important example is $N \to \Delta$ transition – parametrized by three form factors
  - $G_E^*(Q^2), \quad G_M^*(Q^2), \quad G_C^*(Q^2)$
  - if both N and Δ were purely S-wave then $\quad G_E^*(Q^2) = 0 = G_C^*(Q^2)$
- When analyzing the $N \to \Delta$ transition it is common to construct the ratios:

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\,\frac{G_C^*}{G_M^*}$$



$N \to \Delta$ *form factors from the DSEs*

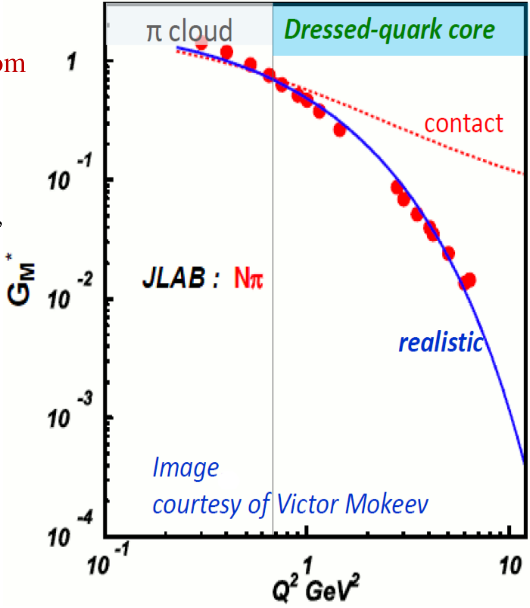
[J. Segovia, I. C. Cloët, C. D. Roberts and S. M. Schmidt, Few Body Syst. **57**, (2014)]



- For $R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\frac{G_C^*}{G_M^*}$ DSEs reproduces rapid fall off with Q^2
- Find that $R_{EM} = -\frac{G_E^*}{G_M^*}$ is a particular sensitive measure of *quark orbital angular momentum* within the nucleon and Δ
- At large Q^2 helicity conservation demands: $R_{SM} \to$ constant, $R_{EM} \to 1$
  - however these asymptotic results are not reached until incredibility large Q^2 – which will not be accessible at any present or foreseeable facility
- Comparison with Argonne-Osaka results suggest that the pion cloud is masking expected zero in R_{EM}

$N \to \Delta$ *magnetic form factor*

- Results are indistinguishable from data for $Q^2 \gtrsim 0.7\,\text{GeV}^2$
- With same set of inputs provide a unified description of nucleon, Delta and $N \to \Delta$ form factors
- For example, same
  - quark propagators
  - diquark masses and amplitudes
  - Faddeev kernel
  - electromagnetic current operator

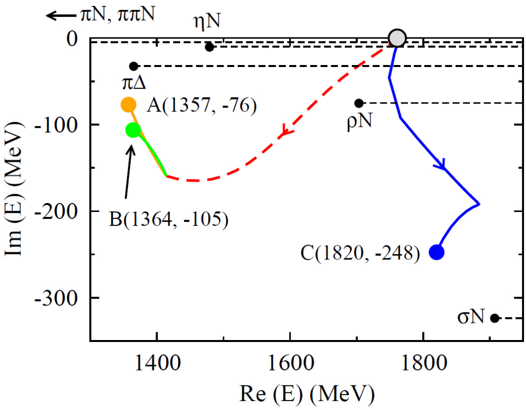


Roper Resonance

Three poles, each seeded by a single dressed quark core:

Two poles associated with Roper resonance and the third with the next higher P_{11} resonance

[H. Kamano, *et al.*, Phys. Rev. C **88**, no. 3, 035209 (2013)]

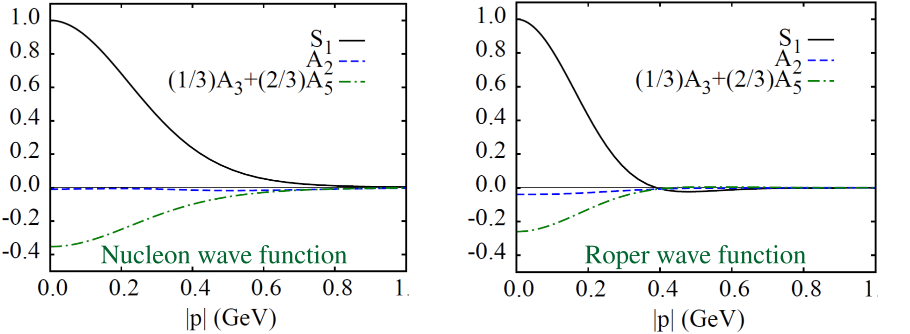


- The Excited Baryon Analysis Center (EBAC), resolved a fifty-year puzzle by demonstrating that the Roper resonance is the proton's first radial excitation
  - its lower-than-expected mass owes to a dressed-quark core shielded by a dense cloud of pions and other mesons

  [Decadal Report on Nuclear Physics: Exploring the Heart of Matter]

Roper Resonance from the DSEs

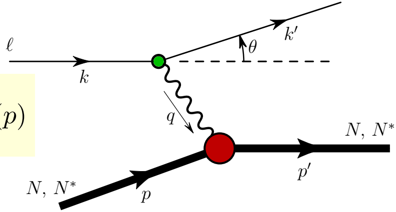
[J. Segovia, B. El-Bennich, E. Rojas, I. C. Cloët, C. D. Roberts, S. S. Xu and H. S. Zong, arXiv:1504.04386 [nucl-th]]



- The Faddeev equation that produces the nucleon also gives its excited states
  - amplitudes for the lightest excited state typically possess a zero
  - therefore lightest nucleon excited state is a radial excitation $\Longleftrightarrow$ Roper resonance
  - "quark core" mass: $M_R = 1.73\,\text{GeV}$; c.f. Argonne-Osaka group $M_R = 1.76\,\text{GeV}$
- Now have a unified description of the nucleon, Delta and Roper baryons
- Find e.g. that the Roper charge radius is 80% larger than the nucleon's

Nucleon and Roper Form Factors

- Recall that the nucleon electromagnetic current has the form



$$\langle J^\mu \rangle = u_N(p') \left[\gamma^\mu\, F_{1N} + \frac{i\sigma^{\mu\nu} q_\nu}{2\, M_N}\, F_{2N} \right] u_N(p)$$

- The Roper $[N^*(1440)]$ is likely the first radial excitation of the nucleon and has the same quantum numbers
- Therefore the Roper electromagnetic current has the form

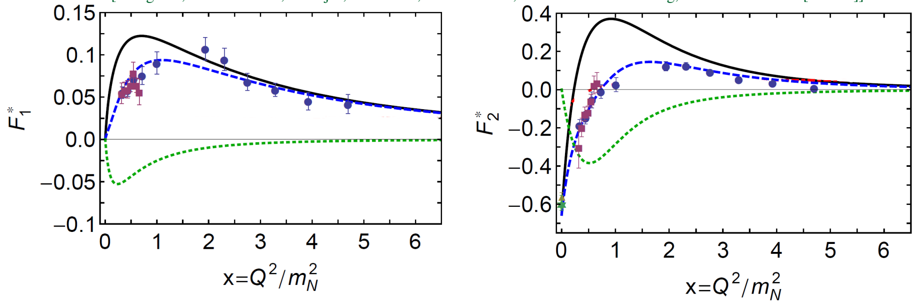
$$\langle J^\mu \rangle = u_{N^*}(p') \left[\gamma^\mu\, F_{1N^*}(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, M_N^*}\, F_{2N^*}(Q^2) \right] u_{N^*}(p)$$

- The electromagnetic current can cause a transition between the nucleon and Roper: $[N \to N^*]$
- Gauge invariance implies this transition current must satisfy: $q_\mu\, J^\mu = 0$

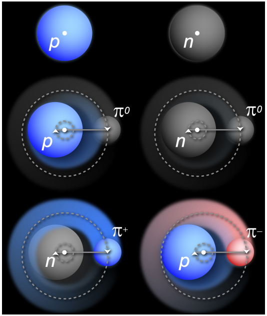
$$\langle J^\mu \rangle = u_{N^*}(p') \left[\left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2} \right) F_{1NR}(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{M_N + M_{N^*}}\, F_{2NR}(Q^2) \right] u_N(p)$$

  - orthogonality of the N and N^* wave functions implies $F_{1NR}(0) = 0$

[J. Segovia, B. El-Bennich, E. Rojas, I. C. Cloët, C. D. Roberts, S. S. Xu and H. S. Zong, arXiv:1504.04386 [nucl-th]]



- Results agree well with data for $Q^2 \gtrsim 2\, m_N^2$ & at the real photon point
- However contemporary kernels just produce a hadron's *dressed-quark core*
  - pion cloud contributions are absent from our calculation, however these are inferred from the deviation with data
  - on domain $0 < Q^2 \lesssim 2\, m_N^2$ pion cloud contributions should be negative and deplete the transition form factors



Conclusion

- QCD will only be solved by deploying a diverse array of experimental and theoretical methods
  - must define and solve the problems of confinement and its relationship with DCSB
- These are two of the most important challenges in fundamental Science
- Nucleon elastic and transition form factors provide an important avenue with which to address these critical questions
- We have provided a unified treatment of the nucleon, Delta and Roper elastic and transition form factors
  - demonstrating e.g. that the location of zero's in form factors – e.g. G_{Ep}, F_{1p}^{d} – provide tight constraints on QCD dynamics
- *Continuum-QCD approaches are essential; are at the forefront of guiding experiment & provide rapid feedback; building intuition & understanding*