QCD structure of the nucleon and spin physics

Lecture 4: operator analysis

Zhongbo Kang
Los Alamos National Laboratory

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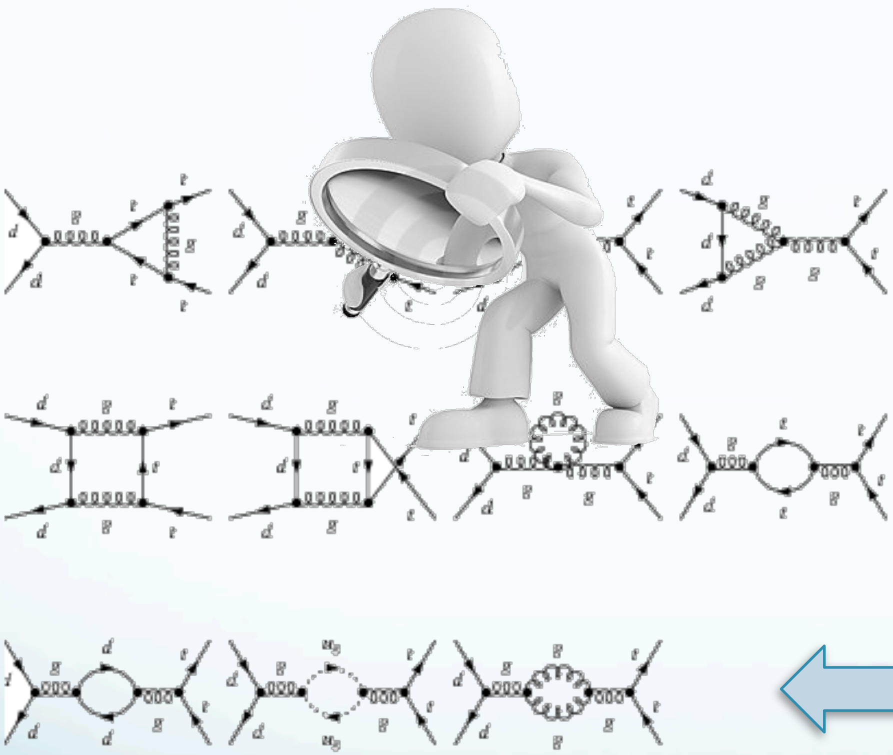
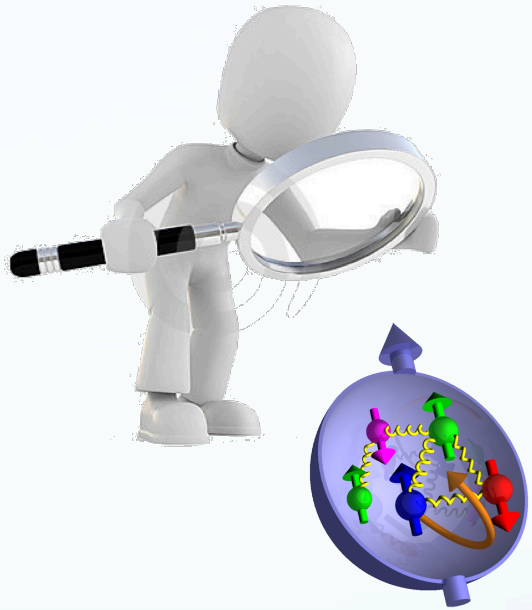
High energy physics and hadron physics

- We are looking into both the partonic dynamics at the short distance, as well as the nucleon structure at long distance

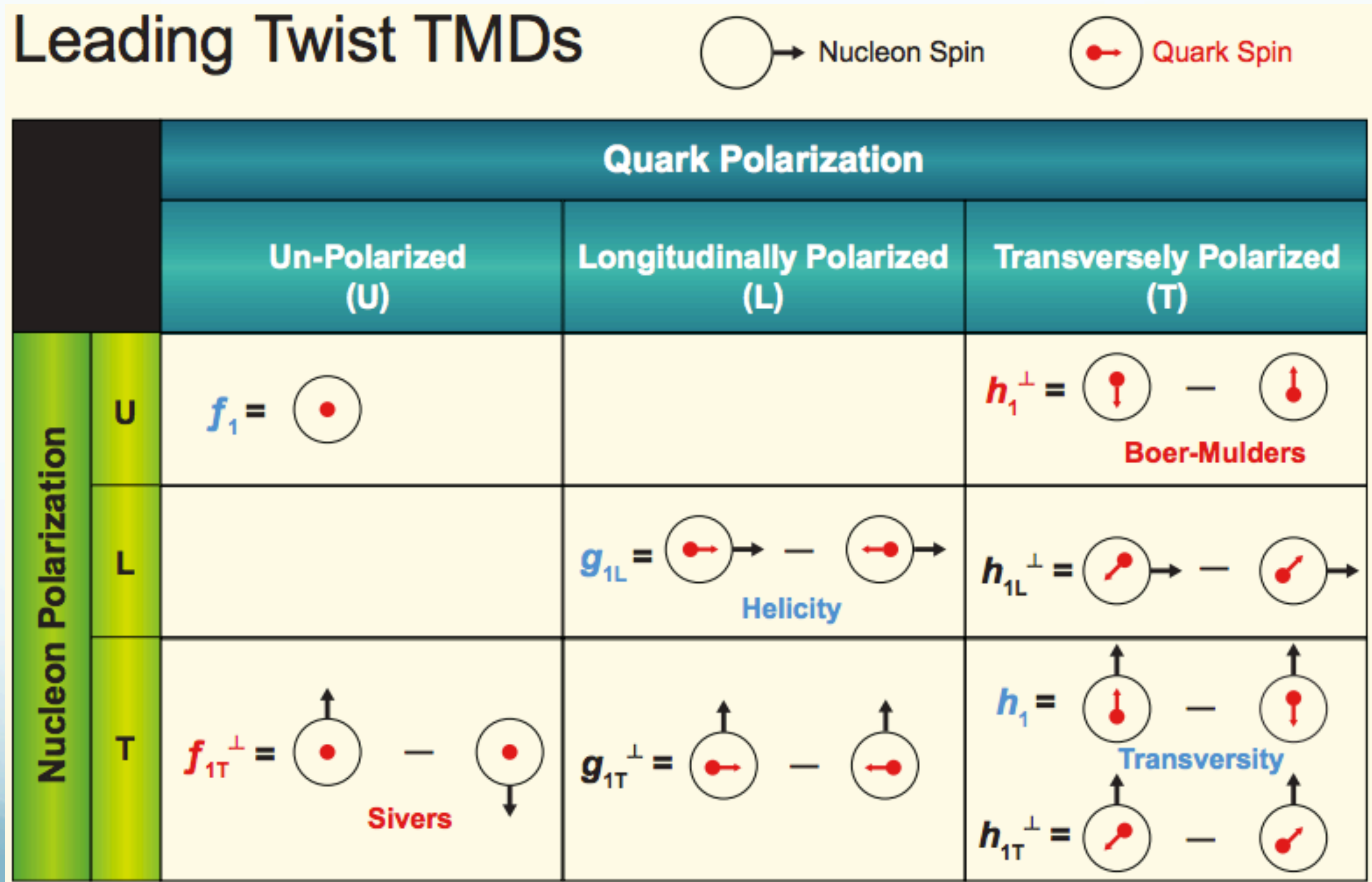




QCD Factorization

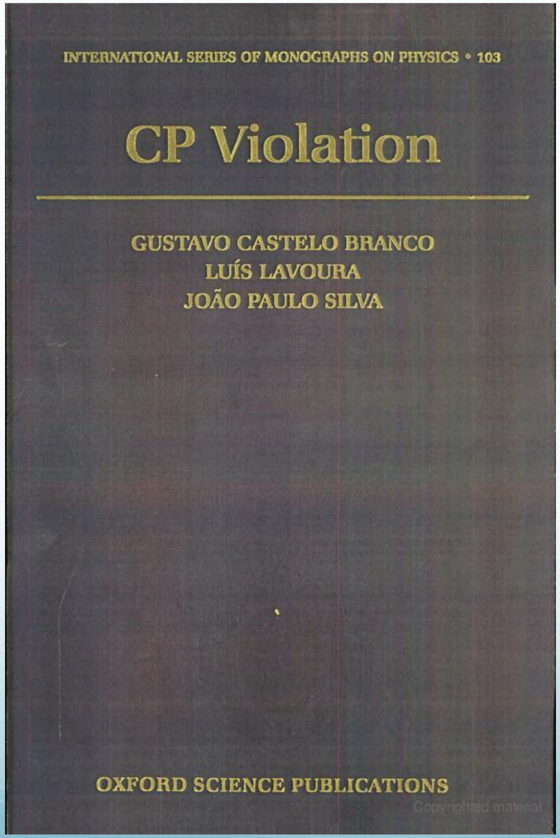
How many distributions are needed

- In order to fully characterize the proton structure, how many parton distribution functions are actually needed



Good textbooks

- Understand C, P, T discrete symmetry properties of the correlation function
  - Most textbooks on quantum field theory will give discussion (somewhat limited) on this topic, such as Peskin, Sterman
  - If you want extensive discussion, see this book



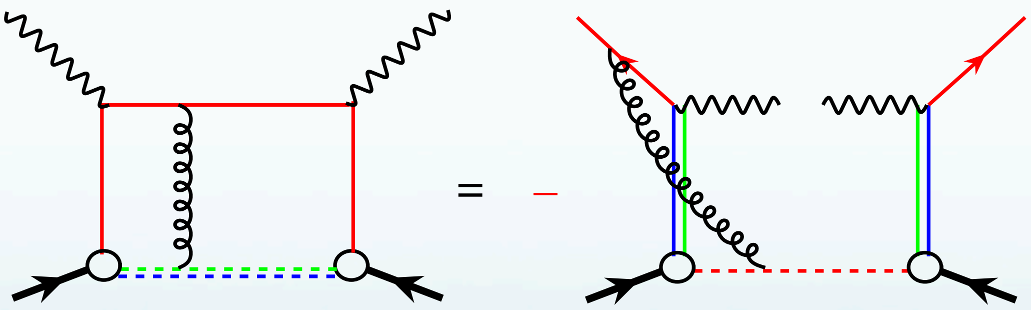
One example: Sivers function

- Sivers function: an asymmetric parton distribution in a transversely polarized nucleon (kt correlated with the spin of the nucleon)

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) - \frac{1}{M} f_{1T}^{\perp q}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\mathbf{k}_\perp)$$

Spin-independent Spin-dependent

- Naïve time-reversal-odd, and its existence requires a phase (generate through interactions)



SIDIS = − DY

$$f_{1T}^{\perp,\mathrm{DIS}}(x,k_\perp) = - f_{1T}^{\perp,\mathrm{DY}}(x,k_\perp)$$

The history of Sivers function

- 1990: Sivers function
  - introduce kt dependence of PDFs, generate the SSA through a correlation between the hadron spin and the parton kt
- 1993: Collins
  - show Sivers function vanishes due to time-reversal invariance
- 2002: Brodsky, Hwang, Schmidt
  - explicit model calculation show the existence of the Sivers function
  - the existence of Sivers function relies on the initial- and final-state interactions between the active parton and the remnant of the polarized hadron
- 2002: Ji, Yuan, Belitsky
  - the initial- and final-state interaction presented by Brodsky, et.al. is equivalent to the color gauge links in the definition of the TMD distribution functions
  - since the details of the initial- and final-state interaction depend on the specific scattering process, the gauge link thus the Sivers function could be process-dependent

C. P. T symmetry and operation

define operation U_c, U_P, U_T are the operation operator for C, P, T

Then

- $\boxed{\text{P operation}}$

P-invariance : $|\alpha_P\rangle = U_P|\alpha\rangle \Rightarrow \langle\alpha|\beta\rangle = \langle\alpha_P|\beta_P\rangle$

$J_P = \gamma^0$

$U_P\,\psi(x_0,\vec{x})\,U_P^{-1} = \gamma_0\,\psi(x_0,-\vec{x})$

$J_P\,\gamma^\mu J_P^{-1} = -(-1)^{\delta_{\mu,0}}\gamma^\mu$

$x^\mu \xrightarrow{P} x_\mu$

$U_P\,A_\mu(x_0,\vec{x})\,U_P^{-1} = A^\mu(x_0,-\vec{x})$

$p^\mu \xrightarrow{P} p_\mu$

$\partial^\mu \xrightarrow{P} \partial_\mu$

$\vec{p} \xrightarrow{U_P} -\vec{p}$

$\vec{s} \xrightarrow{U_P} \vec{s}$ (spin)

$U_P|\vec{p},\vec{s}\rangle = |-\vec{p},\vec{s}\rangle$ $\qquad U_P^\dagger = U_P^{-1}$

$F^{\mu\nu} \xrightarrow{P} F_{\mu\nu}$

- $\boxed{\text{T operation}}$

T invariance: $|\alpha_T\rangle = U_T|\alpha\rangle \Rightarrow \langle\alpha|\beta\rangle = \langle\beta_T|\alpha_T\rangle$

$U_T\,\psi(x_0,\vec{x})\,U_T^{-1} = J^\dagger\,\psi(-x_0,\vec{x})$

$J^\dagger = J = J^{-1} = i\gamma^1\gamma^3$

$J(\gamma^\mu)^* J^{-1} = \gamma_\mu$

$U_T\,A_\mu(x_0,\vec{x})\,U_T^{-1} = A^\mu(-x_0,\vec{x})$

$x^\mu \xrightarrow{T} -x_\mu$

$U_T|\vec{p},\vec{s}\rangle = |-\vec{p},-\vec{s}\rangle$

$\partial^\mu \xrightarrow{T} -\partial_\mu$

$p^\mu \xrightarrow{T} p_\mu$

$\vec{p} \xrightarrow{U_T} -\vec{p}$

$\vec{s} \xrightarrow{U_T} -\vec{s}$ (spin)

$U_T\,(c\,\#)\,U_T^{-1} = (c\,\#)^*$

$F^{\mu\nu} \xrightarrow{T} -F_{\mu\nu}$

- $\boxed{\text{C operation}}$

$U_c^\dagger = U_c^{-1}$

$J_c = i\gamma^2\gamma^0$ $\qquad$ transpose $\qquad [J_c\,\gamma_\mu\,J_c^{-1} = -(\gamma_\mu)^t$

$U_c\,\bar{\psi}(x_0,\vec{x})\,U_c^{-1} = -\psi(x_0,\vec{x})\,J_c^{-1}$

$J_c^{-1} = J_c^\dagger = J_c^{t} = -J_c$

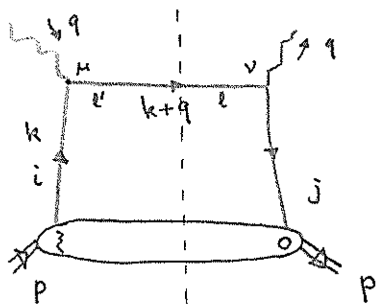
$U_c\,\psi(x_0,\vec{x})\,U_c^{-1} = J_c\,\bar{\psi}(x_0,\vec{x})$

$U_c\,A_\mu(x_0,\vec{x})\,U_c^{-1} = -A_\mu(x_0,\vec{x})$

$U_c|(\vec{p},\vec{s})^{(-)}\rangle = |(\vec{p},\vec{s})^{(+)}\rangle$

[The operation of charge conjugation
reverses particle and anti particle states,
while leaving spins and momenta unchanged]

How many correlation function / parton distribution function do we need to characterize the structure of a spin-½ proton ?



$$\sim \langle PS| \tilde{\bar{\psi}}_j \gamma^{\nu}_{jl} (\not{k}+\not{q})_{ll'} \gamma^{\mu}_{l'i} \tilde{\psi}_i |PS\rangle$$

$$\sim \boxed{\langle PS| \tilde{\bar{\psi}}_j \tilde{\psi}_i |PS\rangle} * [\gamma^{\nu}(\not{k}+\not{q})\gamma^{\mu}]_{ji}$$

$$\Downarrow$$

- generic two-quark correlation function to characterize the nucleon structure
- So far in momentum space
- in Coordinate space we have

$$\Phi_{ij}(k, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0) \psi_i(\xi) |PS\rangle$$

NOTE: In general we also need gauge link to render the above definition gauge invariant, we'll talk about that later

Q: how is Φ_{ij} related to the parton distribution function, as well as many other quantities like transversity, Sivers function, etc ?

To proceed, let's study what requirements do we have from QCD

- Hermiticity

$$\Phi^\dagger(k,P,S) = \gamma^0\, \Phi(k,P,S)\, \gamma^0$$

- Parity

$$\Phi(k,P,S) = \gamma^0\, \Phi(\bar{k},\bar{P},-\bar{S})\, \gamma^0$$

- Time reversal

$$\Phi^*(k,P,S) = (-i\gamma^5 C)\, \Phi(\bar{k},\bar{P},\bar{S})\, (-i\gamma^5 C)$$

where $C = i\gamma^2\gamma^0$, $-i\gamma^5 C = i\gamma^1\gamma^3$, and $\bar{k} = (k^0, -\vec{k})$

To show this, we need some background how fields transform under "C, P, T", which you might find some limited information from any standard textbook on Quantum Field Theory; for extended discussion, see

CP violation by Branco, Lavoura, Silva

- Hermiticity

$$(\Phi^\dagger)_{ij} = \Phi^*_{ji} = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\,\langle PS|\,\bar{\psi}_i(0)\,\psi_j(\xi)\,|PS\rangle^*$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\,\langle PS|\,\psi^\dagger_\ell(0)\,\gamma^0_{\ell i}\,\psi_j(\xi)\,|PS\rangle^*$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\,\langle PS|\,\psi^\dagger_j(\xi)\,\gamma^0_{i\ell}\,\psi_\ell(0)\,|PS\rangle$$

$\Downarrow$ Note

$$\gamma^0_{j'\ell'}\gamma^0_{\ell'j} = \delta_{jj'}$$

$$\psi^\dagger_j(\xi) = \psi^\dagger_{j'}(\xi)\,\delta_{jj'}$$

$$= \underbrace{\psi^\dagger_{j'}(\xi)\,\gamma^0_{j'\ell'}}\,\gamma^0_{\ell'j}$$

$$= \bar{\psi}_{\ell'}(\xi)\;\gamma^0_{\ell'j}$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\,\langle PS|\,\bar{\psi}_{\ell'}(\xi)\,\gamma^0_{\ell'j}\,\gamma^0_{i\ell}\,\psi_\ell(0)\,|PS\rangle$$

$\Downarrow$ Change variable $\xi \to -\xi$

$$d^4\xi = d^4(-\xi)$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ik\cdot\xi}\,\langle PS|\,\bar{\psi}_{\ell'}(-\xi)\,\gamma^0_{\ell'j}\,\gamma^0_{i\ell}\,\psi_\ell(0)\,|PS\rangle$$

translational invariance

$\Downarrow$ $\langle PS|\,\bar{\psi}(-\xi)\cdots\psi(0)\,|PS\rangle = \langle PS|\,\bar{\psi}(0)\cdots\psi(\xi)\,|PS\rangle$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ik\cdot\xi}\,\langle PS|\,\bar{\psi}_{\ell'}(0)\,\gamma^0_{\ell'j}\,\gamma^0_{i\ell}\,\psi_\ell(\xi)\,|PS\rangle$$

$$= \gamma^0_{i\ell}\,\Phi_{\ell\ell'}\,\gamma^0_{\ell'j} = [\gamma^0\,\Phi\,\gamma^0]_{ij}$$

$$\Rightarrow \quad \boxed{\Phi^\dagger(k,P,S) = \gamma^0\,\Phi(k,P,S)\,\gamma^0}$$

• Parity

① under parity $\qquad p^\mu \longrightarrow p_\mu$

$$p^\mu = (p^0, \vec{p}) \qquad p_\mu = (p^0, -\vec{p}) \Rightarrow \bar{p}$$

• momentum change

• spin does not change

$$S^\mu = (0, \vec{S}) \longrightarrow S^\mu = (0, \vec{S})$$

use notation $\quad \bar{S} \Rightarrow S_\mu = (0, -\vec{S})$

$$-\bar{S} = (0, \vec{S})$$

$$(k, P, S) \rightarrow (\bar{k}, \bar{P}, -\bar{S})$$

$$\Phi_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0) \; \psi_i(\xi) |PS\rangle$$

$$\Downarrow \quad U_P U_P^{-1} = \mathbb{1}$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{ik\cdot\xi} \langle PS| U_P^{-1} U_P \bar{\psi}_j(0) U_P^{-1} U_P \psi_i(\xi) U_P^{-1} U_P |PS\rangle$$

$$\Downarrow \quad U_P |PS\rangle = |\bar{P}, -\bar{S}\rangle$$

$$\langle PS| U_P^{-1} = \langle \bar{P}, -\bar{S}|$$

$$U_P \psi(\xi) U_P^{-1} = \gamma^0 \psi(\bar{\xi})$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{ik\cdot\xi} \langle \bar{P}, -\bar{S}| \bar{\psi}_l(0) \gamma^0_{lj} \quad \gamma^0_{ii'} \psi_{i'}(\bar{\xi}) |\bar{P}, -\bar{S}\rangle$$

NOTE: $k \cdot \xi = k^0 \xi^0 - \vec{k} \cdot \vec{\xi}$ $\quad k = (k^0, \vec{k}) \quad \xi = (\xi^0, \vec{\xi})$

$\bar{k} \cdot \bar{\xi} = k^0 \xi^0 - \vec{k} \cdot \vec{\xi}$ $\quad \bar{k} = (k^0, -\vec{k}) \quad \bar{\xi} = (\xi^0, -\vec{\xi})$

Thus $k \cdot \xi = \bar{k} \cdot \bar{\xi}$

$$\Phi_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{i\bar{k}\cdot\bar{\xi}} \langle \bar{P}, -\bar{S} | \bar{\psi}_\ell(0) \gamma^0_{\ell j} \; \gamma^0_{ii'} \psi_{i'}(\xi) | \bar{P}, -\bar{S} \rangle$$

$$\Downarrow \quad d^4\xi = d^4\bar{\xi}$$

$$= \frac{1}{(2\pi)^4} \int d^4\bar{\xi} \; e^{i\bar{k}\cdot\bar{\xi}} \langle \bar{P}, -\bar{S} | \bar{\psi}_\ell(0) \gamma^0_{\ell j} \; \gamma^0_{ii'} \psi_{i'}(\bar{\xi}) | \bar{P}, -\bar{S} \rangle$$

$$= \gamma^0_{ii'} \; \Phi_{i'\ell}(\bar{k}, \bar{P}, -\bar{S}) \; \gamma^0_{\ell j}$$

$$\boxed{\Phi(k, P, S) = \gamma^0 \, \Phi(\bar{k}, \bar{P}, -\bar{S}) \, \gamma^0}$$

- Time reversal

anti-unitary operator

$$\langle \alpha | = \langle p, s | \hat{O}^\dagger \quad \Rightarrow |\alpha\rangle = \hat{O} |p, s\rangle$$

$$|\beta\rangle = |p, s\rangle \quad \Rightarrow \langle \beta | = \langle p, s |$$

under time

$p \to \bar{p}$

$s \to \bar{s}$

$\xi \to -\bar{\xi}$

$\hookrightarrow (t, \vec{\xi})$

Then time-reversal invariance indicates

$$\langle \alpha | \beta \rangle = \langle \beta_T | \alpha_T \rangle$$

"state" after T-operation

$$|\beta_T\rangle = |\bar{p}, \bar{s}\rangle \quad \Rightarrow \quad \langle \beta_T | = \langle \bar{p}, \bar{s} |$$

$$|\alpha_T\rangle = U_T \, \hat{O} \, U_T^{-1} \, |\bar{p}, \bar{s}\rangle$$

Thus we have

$$\langle p\, s | [\bar{\psi}_j(0)\, \psi_i(\xi)]^\dagger | p s\rangle = \langle \bar{p}\, \bar{s} | U_T \, [\bar{\psi}_j(0)\, \psi_i(\xi)] \, U_T^{-1} | \bar{p} \bar{s}\rangle$$

NOTE:

$$\Phi^*_{ij}(k, p, s) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle p\, s | [\bar{\psi}_j(0)\, \psi_i(\xi)]^\dagger | p s\rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle \bar{p}\, \bar{s} | U_T [\bar{\psi}_j(0)\, \psi_i(\xi)] U_T^{-1} | \bar{p}\, \bar{s}\rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle \bar{p}\, \bar{s} | U_T \bar{\psi}_j(0) U_T^{-1} \, U_T \psi_i(\xi) U_T^{-1} | \bar{p}\, \bar{s}\rangle$$

$$U_T \psi(\xi) U_T^{-1} = -i\gamma^5 C\, \psi(-\bar{\xi})$$

$$(-i\gamma^5 C)^\dagger = -i\gamma^5 C$$

$$\Phi^*_{ij}(K,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{-iK\cdot\xi} \langle \bar{P}\bar{S}|\, \bar{\psi}_l(0)(-i\gamma^5 C)_{lj} \quad (-i\gamma^5 C)_{ii'}\ \psi_{i'}(-\bar{\xi})\,|\bar{P}\bar{S}\rangle$$

$$\Downarrow \qquad K\cdot\xi = \bar{K}\cdot\bar{\xi}$$

$$= \frac{1}{(2\pi)^4}\int d^4(-\bar{\xi})\, e^{i\bar{K}\cdot(-\bar{\xi})} \langle \bar{P}\bar{S}|\, \bar{\psi}_l(0)(-i\gamma^5 C)_{lj}\, (-i\gamma^5 C)_{ii'}\, \psi_{i'}(-\bar{\xi})\,|\bar{P}$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\, e^{i\bar{K}\cdot\xi} \langle \bar{P}\bar{S}|\, \bar{\psi}_l(0)(-i\gamma^5 C)_{lj}(-i\gamma^5 C)_{ii'}\, \psi_{i'}(\xi)\,|\bar{P}\bar{S}\rangle$$

$$= (-i\gamma^5 C)_{ii'}\ \Phi_{i'l}(\bar{K},\bar{P},\bar{S})\ (-i\gamma^5 C)_{lj}$$

Independent 4X4 matrix basis

1

γ^μ

$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$\gamma^\mu \gamma^5$

$i\gamma^5$

Now perform expansion

assume proton is moving in +z direction

$$p^\mu = [p^0, 0, 0, p^z]$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^z)$$

$$p^\mu = p^+ \bar{n}^\mu \qquad \bar{n}^\mu = [1^+, 0^-, 0_\perp]$$

$$n^\mu = [0^+, 1^-, 0_\perp]$$

spin of proton

$$S^\mu = \underbrace{\lambda}_{\text{helicity}} \frac{p^+}{m} \bar{n}^\mu + S_T^\mu$$

$$\Phi_{ij}(k, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0) \psi_i(\xi) |PS\rangle$$

- consider purely collinear case

In other words, integrate over k_T, k^- components and set $k^+ = xp^+$

$$\Phi_{ij}(x) = \int d^2k_T \, dk^- \, \Phi_{ij}(k, P, S)\Big|_{k^+ = xp^+}$$

$$= \int \frac{d\xi^-}{2\pi} e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0) \psi_i(\xi) |PS\rangle_{\xi^+ = \xi_T = 0}$$

In other words $\Phi_{ij}(x)$ should depend on $\not{P}$ only (as well as spin "S" vector) since $k \approx xP$

- what about TMD — k_T-dependent parton distribution

$$\Phi_{ij}(x, k_T) = \int dk^- \, \Phi_{ij}(k, P, S)\Big|_{k^+ = xp^+}$$

$$= \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_T}{(2\pi)^2} e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0) \psi_i(\xi) |PS\rangle_{\xi^+ = 0}$$

famous mistake — Sivers function vanishes ?!

$$f_{q/p\uparrow}(x, k_T, S_T) = f_{q/p}(x, k_T^2) + \vec{S}_T \cdot (\vec{k}_T \times \hat{p}) \frac{1}{M} f_{1T}^{\perp}(x, k_T)$$

$$\rightarrow \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_T}{(2\pi)^2} e^{ik\cdot\xi} \langle PS | \bar{\psi}(0) \frac{\gamma^+}{2} \psi(\xi) | PS \rangle_{\xi^+=0}$$

$$f_{1T}^{\perp}(x, k_T) \propto \boxed{f_{q/p\uparrow}(x, k_T, S_T) - f_{q/p\uparrow}(x, k_T, -S_T)}$$

Apply both P and T invariance, see what happens

you'll find

$$f_{q/p\uparrow}(x, k_T, S_T) = f_{q/p\uparrow}(x, k_T, -S_T)$$

$\Rightarrow$ Vanish ?!

$\Rightarrow$ gauge link !

$\langle\alpha| = \langle\vec{p},\vec{s}|\,\hat{O}$

$|\beta\rangle = |\vec{p},\vec{s}\rangle$

T-invariance $\Rightarrow$ $\langle\alpha|\beta\rangle = \langle\beta_T|\alpha_T\rangle$

$$\langle\vec{p},\vec{s}|\hat{O}|\vec{p},\vec{s}\rangle = \langle-\vec{p},-\vec{s}|\,U_T\,\hat{O}^+\,U_T^{-1}\,|-\vec{p},-\vec{s}\rangle$$

$$= \langle-\vec{p},-\vec{s}|\,U_P^{-1}U_P\,U_T\,\hat{O}^+\,U_T^{-1}\,U_P^{-1}U_P\,|-\vec{p},-\vec{s}\rangle$$

$$= \langle\vec{p},-\vec{s}|\,U_P\,U_T\,\hat{O}^+\,U_T^{-1}\,U_P^{-1}\,|\vec{p},-\vec{s}\rangle$$

$$\hat{O} = \bar{\psi}(0)\,\Gamma\,\psi(\xi) \qquad \text{with} \quad \Gamma = \frac{\gamma^+}{2}$$

$$\hat{O}^+ = \psi^+(\xi)\,\Gamma^+\,\gamma^0\,\psi(0)$$

$$U_P\,U_T\,\hat{O}^+\,U_T^{-1}\,U_P^{-1} = U_P\,U_T(\psi^+(\xi))\,U_T^{-1}\,U_P^{-1}\,\Gamma^+\gamma^0\,U_P\,U_T\,\psi(0)\,U_T^{-1}\,U_P^{-1}$$

$$= U_P\,\psi^+(-\xi^0,\vec{\xi})\,J\,U_P^{-1}\,\Gamma^+\gamma^0\,U_P\,J\,\psi(0)\,U_P^{-1}$$

$$= \psi^+(-\xi^0,-\vec{\xi})\,\gamma^0 J\,\Gamma^+\gamma^0\,J\,\gamma^0\,\psi(0)$$

$$= \bar{\psi}(-\xi)\,J\,\Gamma^+\,\gamma^0 J\,\gamma^0\,\psi(0)$$

$$J = -i\gamma^5 C = i\gamma^1\gamma^3$$

$$\Downarrow \quad \gamma^0 J = J\gamma^0$$

$$= \bar{\psi}(-\xi)\,J\,\Gamma^+\,J\,\psi(0)$$

$$= \bar{\psi}(-\xi)\,\Gamma^+\,\psi(0)$$

Thus with translational invariance we'll have

$$f_{q/p\uparrow}(x, k_T, \vec{S}_T) = f_{q/p\uparrow}(x, k_T, -\vec{S}_T)$$

$\Rightarrow$ Sivers function vanish ?!

Not Really