QCD and hadron structure

Lecture 1: elements of factorization

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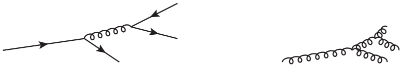
Jefferson Lab, June 2016

Two views of the nucleon:

- three quarks (spectroscopy, quark models): $p = uud, n = ddu, \ldots$
- many quarks, antiquarks, gluons (high-energy processes, $\mathcal{L}_{\text{QCD}}$)

How are these two pictures and the underlying concepts related?

- simple (and often quoted) picture of nucleon:
  - three quarks at low resolution scale
  - gluons and sea quarks generated by perturbative splitting



is too simple:

  - PDF fits of Glück, Reya et al. require gluons and sea quarks at very low scales $(< 1\,\text{GeV})$

- parton densities have tails at large x
  not just three quarks with small relative momenta

General setting

- explore and quantify how quarks, antiquarks, gluons are distributed inside nucleon ("nucleon tomography")
- essential tool: factorization to separate
  - physics at long and short distances
    confinement vs. asymptotic freedom
  - "structure of nucleon" and "probe"

Plan of lectures

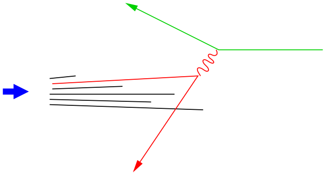
- factorization
- generalized parton distributions (GPDs)
  and the transverse spatial distribution of partons more: J Roche's lectures
- transverse-momentum dependent distributions (TMDs)
  and the limitations of separating "structure" from "probe"
  more: C Aidala's lectures
- multiparton interactions in high-energy pp collisions
  transverse spatial distribution and correlations between partons

Some references for all lectures:

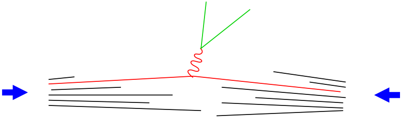
- more on short-distance factorization
  - J Collins, hep-ph/9907513 and hep-ph/0107252
  - J Collins, Foundations of Perturbative QCD, CUP 2011
- short overview of GPDs and TMDS
  - MD, arXiv:1512.01328
- full bibliography for GPDs e.g. in reviews
  - S Boffi and B Pasquini, arXiv:0711.2625
  - A Belitsky and A Radyushkin, hep-ph/0504030
  - MD, hep-ph/0307382
  - K Goeke et al., hep-ph/0106012
- overviews of TMDs
  - A Bacchetta et al., hep-ph/0611265
  - S Mert Aybat and T Rogers, arXiv:1101.5057
  - T Rogers, arXiv:1509.04766
- multiparton interactions
  - MD, summer school lectures (2014)
  - `https://indico.in2p3.fr/event/9917/other-view?view=standard`

The parton model for DIS, Drell-Yan, etc.

- fast-moving hadron
  $\approx$ set of free partons with low transv. momenta
- physical cross section
  $=$ cross section for partonic process $(\gamma^* q \to q,\ q\bar{q} \to \gamma^*)$
  $\times$ parton densities



Deep inelastic scattering: $\ell p \to \ell X$



Drell-Yan: $pp \to \ell^+ \ell^- X$

The parton model for DIS, Drell-Yan, etc.

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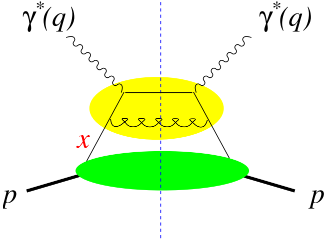
Short-distance factorization in QCD

- implement the parton-model ideas in QCD
  and correct them where necessary
  - identify conditions and limitations of validity
    (kinematics, processes, observables)
  - corrections: partons interact
    α_s small at large scales $\rightsquigarrow$ perturbation theory
  - definition of parton distributions in QCD
    derive their general properties
    make contact with non-perturbative methods
    $\rightsquigarrow$ effective field theories, lattice QCD

Example: inclusive DIS (deep inelastic scattering)

- $\sigma_{\text{tot}}(\gamma^* p \to X)$
  $\xrightarrow{\text{opt. theorem}}$ $\text{Im}\, \mathcal{A}(\gamma^* p \to \gamma^* p)$
  forward amplitude
- measure in $ep \to eX$



- Bjorken limit: $Q^2 = -q^2 \to \infty$ at fixed $x_B = \frac{Q^2}{2p \cdot q}$
- $\text{Im}\, \mathcal{A}(\gamma^* p \to \gamma^* p) =$

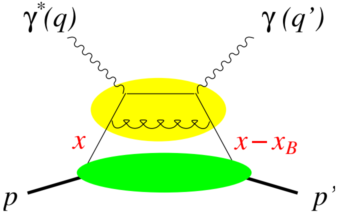
  hard-scattering coefficient $\otimes$ parton distribution

  - hard-scattering coefficient $\sim$ $\text{Im}\, \mathcal{A}(\gamma^* q \to \gamma^* q)$
    small print $\to$ later
  - parton densities (PDFs): process independent
    also appear in $pp \to \ell^+ \ell^- X$, $\gamma^* p \to \text{jet} + X$, ...

$\to$ lectures of A Cooper-Sarkar

Example: DVCS (deeply virtual Compton scattering)

- exclusive cross section
  $\propto \left|\mathcal{A}(\gamma^* p \to \gamma p)\right|^2$
  square of amplitude
- measure in $ep \to ep\gamma$



- Bjorken limit: $Q^2 = -q^2 \to \infty$ at fixed x_B and $t = (p - p')^2$
- $\mathcal{A}(\gamma^* p \to \gamma p) =$
  hard-scattering coefficient $\otimes$ generalized parton distribution
  - GPD depends on x, x_B = momentum fraction lost by proton
    and on t
  - hard-scattering coefficient $\sim$ $\mathcal{A}(\gamma^* q \to \gamma q)$

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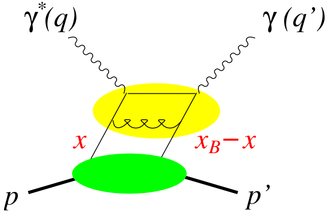

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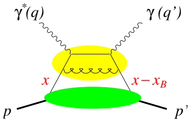
  - GPD depends on x, $x_B =$ momentum fraction lost by proton
    and on t
  - hard-scattering coefficient $\sim$ $\mathcal{A}(\gamma^* q \to \gamma q)$ or $\mathcal{A}(\gamma^* q\bar{q} \to \gamma)$
    both cases included in $\int dx$

Short-distance factorization in QCD: step by step

- specify kinematic limit
  choose suitable reference frame
  identify small and large momentum components
- establish dominant graphs
  and dominant loop momentum regions of these graphs



- simplify resulting expression to obtain factors for
  - short distance $\leftrightarrow$ large virtuality $\leftrightarrow$ parton level
    calculate in perturbation theory
  - long distance $\leftrightarrow$ low virtuality
    transition from hadrons to partons
    matrix elements of quark/gluon operators

Note difference with high-energy/small x factorization

- separate dynamics according to rapidity (not virtuality) of particles
- overlap of two factorization schemes if have strong ordering in rapidity and virtuality

Light-cone coordinates

$\rightsquigarrow$ blackboard

Kinematics of DIS and of DVCS

$\rightsquigarrow$ blackboard

Factorization from Feynman graphs

DIS and DVCS very similar, discuss in parallel

- consider Bjorken limit, choose frame where
  - $p^+ \gg p^-$ (proton fast right-moving)
  - $q^+ \sim q^- \sim p^+$
  - $\boldsymbol{p}_T = \boldsymbol{q}_T = 0$
- for power counting
  - large: $p^+ \sim q^+ \sim q^- \sim Q$
  - small: hadron masses, scales of non-perturbative interact. $\sim m$
  - very small: $p^- \sim m^2/Q$

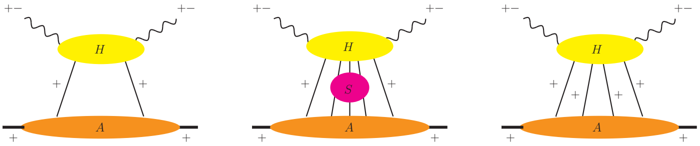
small expansion parameter is m/Q

Dominant momentum regions

Libby-Sterman analysis

- in Bj limit graphs for Compton amplitude dominated by distinct momentum regions:
  - hard: $k^+ \sim k^- \sim k_T \sim Q$, $\quad k^2 \sim Q^2$
  - collinear (to proton): $k^+ \sim Q$, $k_T \sim m$, $k^- \sim m^2/Q$, $\quad k^2 \sim m^2$
  - soft: $k^+, k^-, k_T \ll Q$, $\quad k^2 \ll Q^2$

  proof involves advanced quantum field theory methods
- organize graphs into hard, collinear, and soft subgraphs



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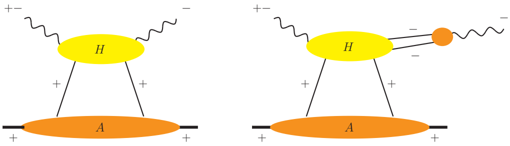
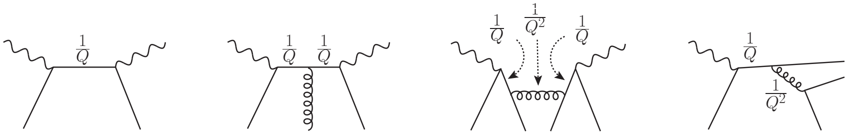


- for real photon can have collinear subgraph:
  $k^- \sim Q,\ k_T \sim m,\ k^+ \sim m^2/Q$
  "hadronic behavior of photon"

Power counting

- power counting
  - hard subgraph $\propto Q^{\dim(H)}$
  - collinear subgraph $\propto m^{\dim(A)}$ complications from spin $\rightarrow$ later
  - collinear lines:
    $$d^4k = dk^+\, dk^-\, d^2k_T \sim Q \times m^2/Q \times m^2 = m^4$$
  - soft subgraph and lines: depends on detailed size of k^μ
- leading term: smallest possible number of lines to H



in tree graphs no large k_T, but $k^+ \sim k^- \sim Q$; in loops $k_T \sim Q$

Power counting

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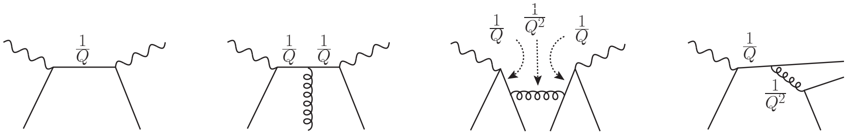


- "twist" of contribution (sometimes called "dynamical twist")
  - twist 2 $\leftrightarrow$ leading term in Bj limit
  - twist 3 $\leftrightarrow$ down by relative factor $1/Q$
  - twist 4 $\leftrightarrow$ down by relative factor $1/Q^2$

Collinear expansion

- in hard graphs neglect small components of external coll. lines
  $\rightsquigarrow$ Taylor expansion

$$H(k^+, k^-, k_T) = H(k^+, 0, 0) + k_T^\mu \left[\frac{\partial H(k^+, 0, k_T)}{\partial k_T^\mu} \right]_{k_T = 0} + \mathcal{O}(m^2)$$

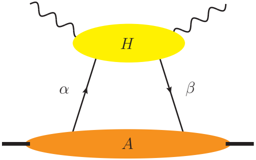
  first term $\rightarrow$ leading twist, second term $\rightarrow$ twist three, ...

- loop integration simplifies:

$$\int d^4k \, H(k) A(k) \approx \int dk^+ \, H(k^+, 0, 0) \, \int dk^- d^2k_T \, A(k^+, k^-, k_T)$$

- in hard scattering (and only there) treat incoming/outgoing partons as exactly collinear $(k_T = 0)$ and on-shell $(k^- = 0)$
- in coll. matrix element integrate over k_T and virtuality
  $\rightsquigarrow$ collinear (or k_T integrated) parton densities
  only depend on k^+

Complication from spin of partons (here: quarks, similar for gluons)



- H and A carry spinor indices:
  $$H_{\beta\alpha} A_{\alpha\beta} = \mathrm{tr}(HA)$$
- use Fierz transformation
  $\rightsquigarrow$ $\mathrm{tr}(\gamma_\mu H)\, \mathrm{tr}(\gamma^\mu A)$ etc.
  $\rightsquigarrow$ blackboard

- Lorentz invariance: in proton rest frame all components of
  $\mathrm{tr}\big(\gamma^\mu A(k,p,s)\big),\ \mathrm{tr}\big(\gamma^\mu \gamma_5 A(k,p,s)\big),\ \mathrm{tr}\big(\sigma^{\mu\nu}\gamma_5 A(k,p,s)\big),\ \ldots$
  are $\sim m^{\dim(A)}$ since $k^\mu, p^\mu, m\, s^\mu \sim m$
- boost to Breit frame $\rightsquigarrow$ largest components
  $\mathrm{tr}\big(\gamma^+ A(k,p,s)\big),\ \mathrm{tr}\big(\gamma^+ \gamma_5 A(k,p,s)\big),\ \mathrm{tr}\big(\sigma^{+j}\gamma_5 A(k,p,s)\big)$
  are $\sim Q\, m^{\dim(A)-1}$ $\qquad$ $j = 1, 2$ transverse index
- in Breit frame all components of $\mathrm{tr}\big(\gamma_\mu H\big), \mathrm{tr}\big(\gamma_\mu \gamma_5 H\big), \ldots$
  are $\sim Q^{\dim(B)}$

- up to power corrections have

$$\mathrm{tr}(HA) = \tfrac{1}{4}\Big[\mathrm{tr}(\gamma^- H)\,\mathrm{tr}(\gamma^+ A) + \mathrm{tr}(\gamma_5\gamma^- H)\,\mathrm{tr}(\gamma^+\gamma_5 A) + \tfrac{1}{2}\,\mathrm{tr}(i\sigma^{-j}\gamma_5 H)\,\mathrm{tr}(i\sigma^{+j}\gamma_5 A)\Big]$$

- coll. approx.: in H replace $k \to \bar{k}$
  with $\bar{k}^+ = k^+$, $\bar{k}^- = 0$, $\bar{k}_T = 0$

$$\int d^4k\ \mathrm{tr}(HA)$$

$$= \int dk^+\ \tfrac{1}{4}\,\mathrm{tr}\Big[\gamma^- H(\bar{k})\Big] \int dk^- d^2k_T\ \mathrm{tr}\Big[\gamma^+ A(k)\Big] + \{\text{other terms}\}$$

$$= \int \frac{dk^+}{k^+}\ \mathrm{tr}\Big[\tfrac{1}{2}\bar{k}^+\gamma^- H(\bar{k})\Big] \times \int dk^- d^2k_T\ \mathrm{tr}\Big[\tfrac{1}{2}\gamma^+ A(k)\Big] + \{\text{other terms}\}$$

$$= \int \frac{dx}{x}\ \Big[\tfrac{1}{2}\sum_s \bar{u}(\bar{k},s) H u(\bar{k},s)\Big] \times \int dk^- d^2k_T\ \mathrm{tr}\Big[\tfrac{1}{2}\gamma^+ A(k)\Big] + \{\text{other terms}\}$$

- $x = k^+/p^+$ = plus-momentum fraction of parton
- unpolarized term: average H over parton polarization

Definition of quark densities

- express collinear graph in terms of fields and matrix elements

$$A(k) = \int \frac{d^4 z}{(2\pi)^4}\, e^{ikz}\, \tilde{A}(z), \qquad \tilde{A}(z) = \big\langle p \big| T \bar{q}(0)\, q(z) \big| p \big\rangle$$

- momentum integration

$$\int dk^- d^2 k_T \int \frac{d^4 z}{(2\pi)^4}\, e^{ikz}\, \tilde{A}(z) = \int \frac{dz^-}{2\pi}\, e^{ik^+ z^-}\, \tilde{A}(z) \Big|_{z^+=0,\, z_T=0}$$

$\rightsquigarrow$ z on light cone

- trace over $\frac{1}{2}\gamma^+$ gives

$$\int \frac{dz^-}{2\pi}\, e^{ixp^+ z^-} \big\langle p \big| \bar{q}(0)\, \tfrac{1}{2}\gamma^+\, q(z^-) \big| p \big\rangle$$

Definition of quark densities

- parton distribution

$$f_1(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \langle p \,|\, \bar{q}(0) \tfrac{1}{2}\gamma^+ W(0,z^-)\, q(z^-) \,|\, p \rangle$$

$$= \begin{cases} q(x) & \text{for } x > 0 \qquad \text{from } b^\dagger b \\ -\bar{q}(-x) & \text{for } x < 0 \qquad \text{from } dd^\dagger = -d^\dagger d \end{cases}$$

$b, d =$ annihilation operators for quarks, antiquarks
q contains b and $d^\dagger$; $\bar{q}$ contains $b^\dagger$ and d

depends on $x = k^+/p^+$ due to boost invariance

- $W(0, z^-) =$ Wilson line

$$W[0,z^-] = P \exp\Big[-ig t_a \int_0^{z^-} d\xi^- A_a^+(\xi)\Big]$$

  - makes product of fields gauge invariant
  - reduces to 1 in light-cone gauge $A^+ = 0$
  - dynamical origin $\rightarrow$ lecture on TMDs

Definition of quark densities

- parton distribution

$$f_1(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \, \langle p \big| \bar{q}(0) \tfrac{1}{2}\gamma^+ W(0,z^-)\, q(z^-) \big| p \rangle$$

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$b, d =$ annihilation operators for quarks, antiquarks
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- helicity distrib. $g_1(x)$ with $\gamma^+ \to \gamma^+\gamma_5$
- transversity distrib. $h_1(x)$ with $\gamma^+ \to i\sigma^{+j}\gamma_5$
- alternative notation: $f_1 = q,\ g_1 = \Delta q,\ h_1 = \delta q$
- def. for GPDs: same operators, different hadron states

Gluon densities

$$q(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \, \langle p \big| \bar{q}(0) \tfrac{1}{2}\gamma^+ W(0,z^-)\, q(z^-) \big| p \rangle$$

- for gluons replace

$$q(x) \to xg(x) \qquad\qquad \Delta q(x) \to x\Delta g(x)$$

$$\tfrac{1}{2}\bar{q}\gamma^+ q \to F^{+i} F_i{}^{+} \qquad\qquad \tfrac{1}{2}\bar{q}\gamma^+\gamma_5 q \to F^{+i} \widetilde{F}_i{}^{+}$$

  with dual field strength $\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$

- understand extra factors x
  - $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
  - in light-cone gauge $A^+ = 0$ have $F^{+i} = \partial^+ A^i$
  - compare $\frac{1}{2}\bar{q}\gamma^+ q \to k^+$ with $F^{+i}F_i{}^{+} = (\partial^+ A^i)^2 \to (k^+)^2$

Result for DIS and DVCS

- leading twist = twist two ($\sim 1/Q^0$): handbag graphs
  and their radiative corrections
- twist three ($\sim 1/Q$): graphs with extra transverse gluon from proton + subleading parts of handbag graphs

Result for DIS and DVCS

- hard-scattering part of handbag graphs → blackboard

Result for DIS and DVCS

- hard-scattering part of handbag graphs:

$$\frac{1}{x - x_B + i\varepsilon} + \{\text{crossed graph}\} = \mathrm{PV}\,\frac{1}{x - x_B} - i\pi\delta(x - x_B) + \{\text{crossed graph}\}$$

- for DIS:

$$\sigma_{\text{tot}} \propto \mathrm{Im}\,\mathcal{A}(\gamma^* p \to \gamma^* p) = \sum_q (e e_q)^2 \big[q(x_B) + \bar{q}(x_B) \big]$$
$$+ \{\text{helicity distributions}\} + \mathcal{O}(\alpha_s) + \mathcal{O}(m/Q)$$

- no contribution from transversitiy distribution
  is chiral odd: $\sigma^{+j}\gamma_5 =$ even number of Dirac matrices
  need another chiral odd quantity to get a contribution

Result for DIS and DVCS

- hard-scattering part of handbag graphs:

$$\frac{1}{x - x_B + i\varepsilon} + \{\text{crossed graph}\} = \mathrm{PV}\,\frac{1}{x - x_B} - i\pi\delta(x - x_B) + \{\text{crossed graph}\}$$

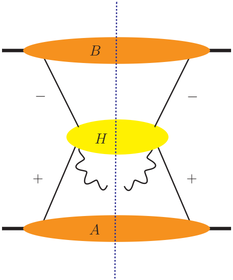
- for DVCS:

$$\mathcal{A}(\gamma^* p \to \gamma p) = \sum_q (e e_q)^2 \left[\mathrm{PV} \int dx\, \frac{\mathsf{GPD}(x, x_B, t)}{x_B - x} + i\pi\, \mathsf{GPD}(x_B, x_B, t) \right] + \{\text{crossed}\}$$
$$+ \{\text{helicity distributions}\} + \mathcal{O}(\alpha_s) + \mathcal{O}(m/Q)$$

Recap:

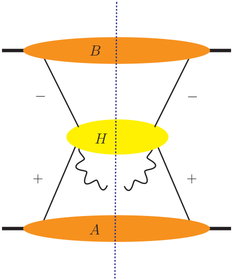
- derive factorization from analysis of Feynman graphs
- main ingredients:
  - dominance of hard, collinear or soft momenta for internal lines
  - kinematic analysis and approximations
- apply to processes totally inclusive (DIS) or totally exclusive (DVCS)
- operator definitions for parton distributions (PDFs)
  and their generalization to finite momentum transfer (GPDs)
- analysis yields non-trival spin dependence

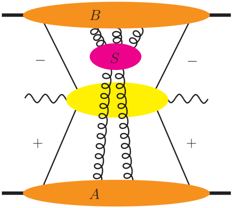
From DIS to Drell-Yan



- two collinear subgraphs for right- and for left-moving particles
- collinear factorization if
  - integrate over q_T of photon or
  - take $q_T \gg m$ large ($q_T \sim m$ in later lecture)
- contribution from transversity: $\delta q \times \delta \bar{q}$

From DIS to Drell-Yan





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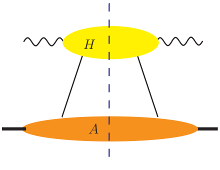
  - take $q_T \gg m$ large ($q_T \sim m$ in later lecture)
- contribution from transversity: $\delta q \times \delta \bar{q}$
- soft interactions between right- and left- moving spectators power suppr. only if sum over details of hadronic final state

More complicated final states

- production of W, Z or other colorless particle (Higgs, etc) same treatment as Drell-Yan
- jet production in ep or pp: hard scale provided by p_T
- heavy quark production: hard scale is m_c, m_b, m_t

Fragmentation

- cross DIS $eh \to e + X$ to $e^+e^- \to \bar{h} + X$
  i.e., $\gamma^* h \to X$ to $\gamma^* \to \bar{h} + X$



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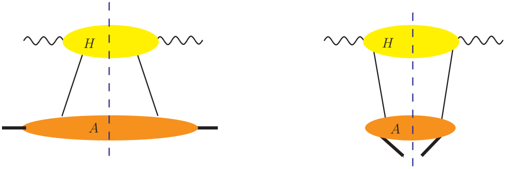
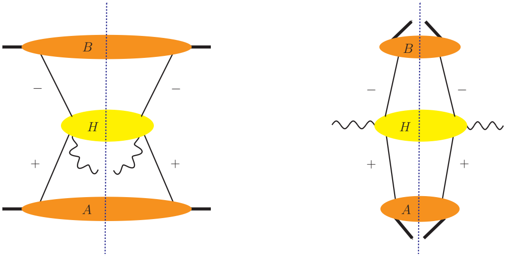


- or Drell-Yan $h_1 h_2 \to \gamma^* + X$ to $\gamma^* \to \bar{h}_1 \bar{h}_2 + X$



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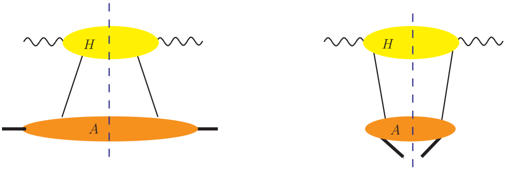
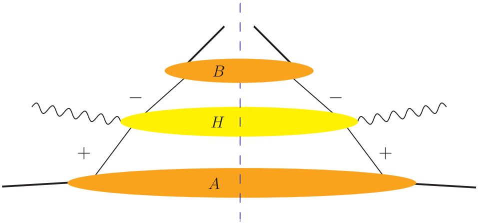
- cross DIS $eh \to e + X$ to $e^+e^- \to \bar{h} + X$
  i.e., $\gamma^* h \to X$ to $\gamma^* \to \bar{h} + X$



- or SIDIS $eh_1 \to eh_2 + X$



Fragmentation functions

- replace parton density $k^+ = xp^+$

$$f(x) = \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x} \,\langle h|\bar{q}(0)\Gamma^+ W(0,\xi^-)\, q(\xi^-)|h\rangle$$

$$= \sum_X \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x}$$

$$\times \sum_X \big\langle h\big|(\bar{q}(0)\Gamma^+)_\alpha W(0,\infty)\big|X\big\rangle \,\big\langle X\big|W(\infty,\xi^-)\, q_\alpha(\xi^-)\big|h\big\rangle$$

by fragmentation function $p^+ = zk^+$

$$D(z) = \frac{1}{2N_c\, z} \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+/z}$$

$$\times \sum_X \big\langle 0\big|W(\infty,\xi^-)\, q_\alpha(\xi^-)\big|\bar{h}X\big\rangle \,\big\langle \bar{h}X\big|(\bar{q}(0)\Gamma^+)_\alpha W(0,\infty)\big|0\big\rangle$$

A closer look at one-loop corrections

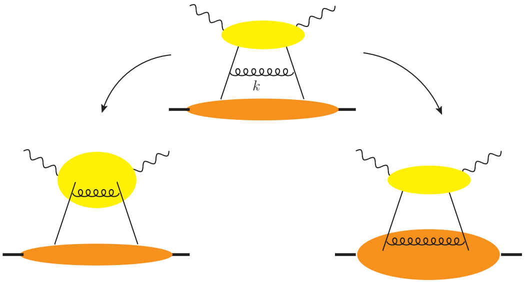
- example: DIS

- ultraviolet divergences $(k^\mu \to \infty)$ removed by standard counterterms
- soft divergences $(k^\mu \to 0)$ cancel in sum over graphs
- collinear div. $(k^\mu \propto p^\mu)$ do not cancel, have integrals

$$\int_0^{k^2_{\text{max}}} \frac{dk_T^2}{k_T^2}$$

what went wrong?

- hard graph should not contain internal collinear lines
  collinear graph should not contain hard lines
- must not double count $\rightsquigarrow$ factorization scale μ



- with cutoff: take $k_T > \mu$
  $1/\mu \sim$ transverse resolution

take $k_T < \mu$

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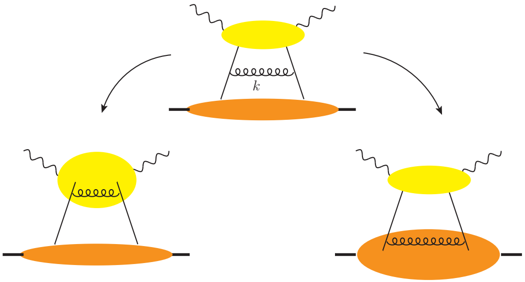
  collinear graph should not contain hard lines
- must not double count $\rightsquigarrow$ factorization scale μ



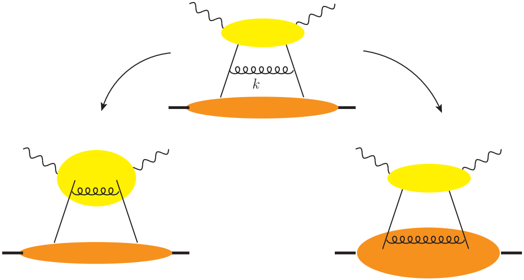
- with cutoff: take $k_T > \mu$
  $1/\mu \sim$ transverse resolution

  take $k_T < \mu$
- avoiding cutoffs:
  in $D = 4 - 2\epsilon$ dimensions
  subtract collinear pole $1/\epsilon$

  subtract ultraviolet pole $1/\epsilon$

Evolution

- μ dependence of parton distr's $\rightarrow$ evolution equations
- μ dependence of parton distr's $\leftrightarrow$ μ dependence of hard scattering
  physical amplitude is μ independent if calculated to all orders in α_s
- choice of μ $\leftrightarrow$ separation of "structure" and "dynamics"



Evolution

- μ dependence of parton distr's $\rightarrow$ evolution equations
- μ dependence of parton distr's $\leftrightarrow$ μ dependence of hard scattering
  physical amplitude is μ independent if calculated to all orders in α_s
- choice of μ $\leftrightarrow$ separation of "structure" and "dynamics"
- quark and gluon densities mix under evolution: