QCD and hadron structure

Lecture 3: exclusive processes and GPDs

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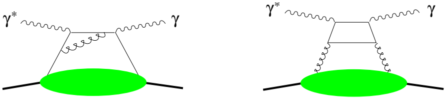
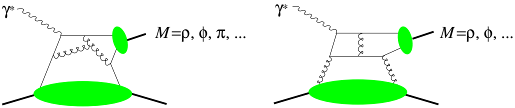
Key processes involving GPDs

- deeply virtual Compton scattering (DVCS)



also: $\gamma p \to \gamma^* p$ with $\gamma^* \to \ell^+ \ell^-$ (timelike CS)
$\gamma^* p \to \gamma^* p$ (double DVCS)

- meson production: large Q^2 or heavy quarks

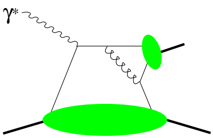


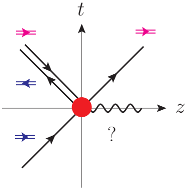
Helicity selection rules

- selection of helcities in hard-scattering part
- ingredients: conservation of angular mom. and of chirality
  - scattering collinear $\to$ ang. mom. J^z = sum of helicities
  - chirality conserved by quark-gluon and quark-photon coupling

chirality	$+1$	-1
q helicity	$+1/2$	$-1/2$
$\bar{q}$ helicity	$-1/2$	$+1/2$

light meson production (not J/Ψ or Υ)





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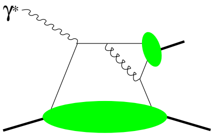

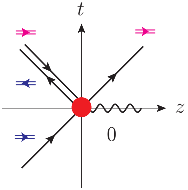

(analogous argument for graphs with gluon GPD)

- dominant transition: $\mathcal{A}(\gamma^*_L \to \text{meson}_L) \sim 1/Q$

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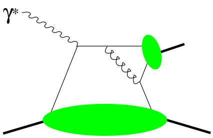
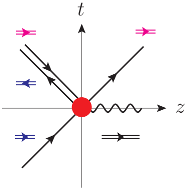
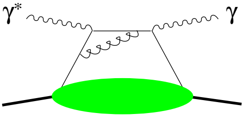


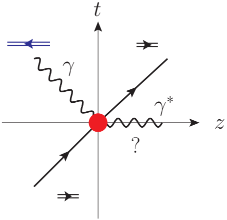


(analogous argument for graphs with gluon GPD)

- $\mathcal{A}(\gamma_T^* \to V_T) \sim 1/Q^2$, but sizeable at $Q^2 \sim$ few GeV^2 (ρ and ϕ data) can describe phenomenologically by keeping k_T finite in hard scattering

Helicity selection rules for the Compton amplitude





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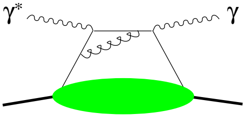

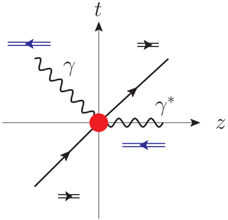

- leading transition: $T \to T$

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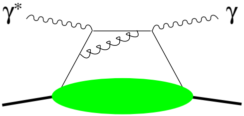

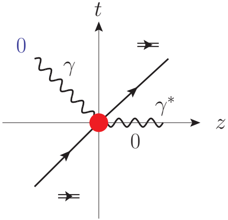

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  (in DIS: correction to Callan-Gross relation)

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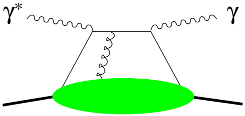

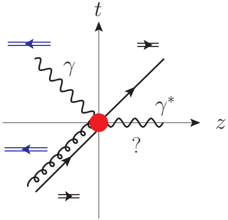

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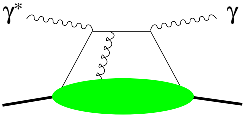

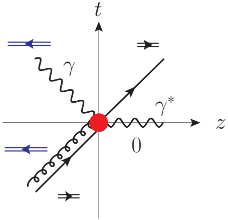

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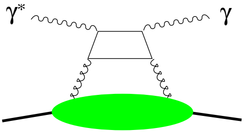

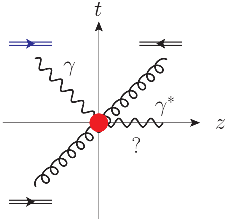

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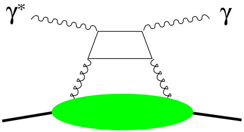

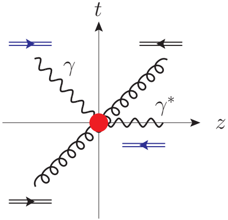

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- double helicity flip in $T \to T$ at twist-two with gluons

DVCS amplitudes and GPDs

- twist-two amplitudes involve 4 four GPDs per parton
  - H, E: unpolarized quark/gluon
  - $\tilde{H}, \tilde{E}$: long. pol. quark/gluon
- twist-three amplitudes:
  - Wandzura-Wilczek part involves same four twist-two GPDs calculated up to NLO
  - genuine twist-three part: matrix elements of $\bar{q} G^{\mu\nu} q$ largely unknown
- photon double helicity-flip amplitudes:
  - at twist two with gluon helicity-flip GPDs, $\mathcal{A} \propto \alpha_s$ distributions very unknown
  - at twist four $\propto 1/Q^2$ start at tree level Wandzura-Wilczek part with usual quark GPDs calculated

DVCS form factors

- for photon helicity conserving amplitudes write

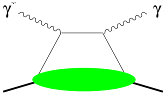
$$e^{-2}\mathcal{A}(\gamma^* p \to \gamma p) = \bar{u}(p')\gamma^+ u(p)\, \mathcal{H} + \bar{u}(p')\, \tfrac{i}{2m_p}\sigma^{+\alpha}(p'-p)_\alpha\, u(p)\, \mathcal{E}$$
$$+ \bar{u}(p')\gamma^+\gamma_5 u(p)\, \widetilde{\mathcal{H}} + \bar{u}(p')\, \tfrac{(p'-p)^+}{2m_p}\gamma_5\, u(p)\, \widetilde{\mathcal{E}}$$

  - Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$ depend on ξ, t, Q^2
  - representation holds for any Q^2, not only at twist two

- at leading twist and LO in α_s

$$\mathcal{H} = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right] H^q(x,\xi,t)$$

same kernels for E, different set for $\widetilde{H}, \widetilde{E}$



Aside: imaginary and absorptive part

- scattering matrix $\mathcal{S}$: $|X\rangle_{\text{in}} = \mathcal{S}|X\rangle_{\text{out}}$
  $\rightsquigarrow$ transition amplitude ${}_{\text{out}}\langle f|i\rangle_{\text{in}} = {}_{\text{out}}\langle f|\mathcal{S}|i\rangle_{\text{out}}$
- $\mathcal{S}$ is unitary: $\mathcal{S}^\dagger S = 1$ $\rightarrow$ blackboard

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- $\mathcal{S}$ is unitary: $\mathcal{S}^\dagger S = 1$ → blackboard
- $\mathcal{S} = 1 + i\mathcal{T}$ ... leave out factors 2π etc.

  $\mathcal{S}$ unitary $\Rightarrow \frac{1}{i}(\mathcal{T} - \mathcal{T}^\dagger) = \mathcal{T}^\dagger\mathcal{T}$
- absorptive part: $\frac{1}{i}\langle f|\mathcal{T} - \mathcal{T}^\dagger|i\rangle = \sum_X \langle f|\mathcal{T}^\dagger|X\rangle\langle X|\mathcal{T}|i\rangle$

  on-shell intermediate states possible between i and f

  in simple cases and with appropriate phase conventions:

  absorptive part $= 2\times$ imaginary part of amplitude
- for $f = i$ get optical theorem

$$2\,\mathrm{Im}\langle i|\mathcal{T}|i\rangle = \sum_X \left|\langle X|\mathcal{T}|i\rangle\right|^2 \propto \sigma_{tot}$$

Real and imaginary part

for brevity suppress $\sum_q e_q^2$ and arguments t, Q^2

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx\, H(x,\xi) \left[\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right]$$

$$\mathrm{Im}\, \mathcal{H}(\xi) = \pi \big[H(\xi,\xi) - H(-\xi,\xi) \big]$$

$$\mathrm{Re}\, \mathcal{H}(\xi) = \mathrm{PV} \int_{-1}^{1} dx\, H(x,\xi) \left[\frac{1}{\xi - x} - \frac{1}{\xi + x} \right]$$

- Im only involves H at $x = \pm\xi$ at LO
  at NLO and higher: only DGLAP region $|x| \ge \xi$
- Re involves both DGLAP and ERBL regions
- deconvolution problem:
  reconstruction of $H(x,\xi;\mu^2)$ from $\mathcal{H}(\xi,Q^2)$ only via Q^2 dep'ce
  i.e. via evolution effects, requires large lever arm in Q^2 at given ξ

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- only quark flavor combination $\frac{4}{9}u + \frac{1}{9}d + \frac{1}{9}s$
  with neutron target in addition $\frac{4}{9}d + \frac{1}{9}u + \frac{1}{9}s$
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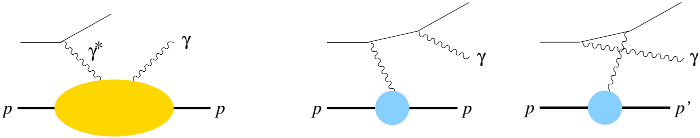
- useful to get information from meson production
  - e.g. $\mathcal{A}_{\rho^0} \propto \frac{2}{3}(u + \bar{u}) + \frac{1}{3}(d + \bar{d}) + \frac{3}{4}\,g$
    $\mathcal{A}_{\phi} \propto \frac{1}{3}(s + \bar{s}) + \frac{1}{4}\,g$
  - but theory description more difficult
    meson wave function, larger corrections in $1/Q^2$ and α_s
  - J/Ψ production: directly sensitive to gluons

Deeply virtual Compton scattering

- competes with Bethe-Heitler process at amplitude level



- analogy with optics:
  - DVCS $\sim$ diffraction experiment
  - BH $\sim$ reference beam with known phase

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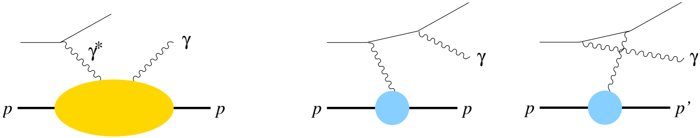
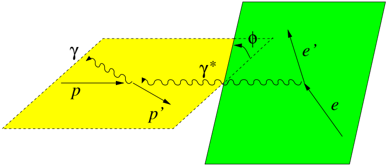


- cross section for $\ell p \to \ell\gamma p$

$$\frac{d\sigma_{\mathrm{VCS}}}{dx_B\, dQ^2\, dt} : \frac{d\sigma_{\mathrm{BH}}}{dx_B\, dQ^2\, dt} \sim \frac{1}{y^2}\frac{1}{Q^2} : \frac{1}{t} \qquad y = \frac{Q^2}{x_B\, s_{\ell p}}$$

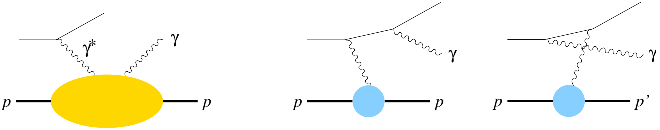
- $1/Q^2$ and $1/t$ from photon propagators
  $1/y^2$ from vertex $e \to e\gamma^*$
- small y: σ_{VCS} dominates $\rightsquigarrow$ high-energy collisions
  moderate to large y: get VCS via interference with BH
  $\rightsquigarrow$ separate $\mathrm{Re}\,\mathcal{A}(\gamma^* p \to \gamma p)$ and $\mathrm{Im}\,\mathcal{A}(\gamma^* p \to \gamma p)$



- filter out interference term using cross section dependence on
  - beam charge e_ℓ
  - azimuth ϕ
  - beam polarization P_ℓ
  - target polarizaton S_L, S_T, ϕ_S

- general structure:

$$d\sigma(\ell p \to \ell\gamma p) \sim d\sigma^{BH} + e_\ell\, d\sigma^{I} + d\sigma^{C}$$



- in more detail:

$$
\begin{aligned}
d\sigma(\ell p \to \ell\gamma p) \sim d\sigma_{UU}^{BH} &+ e_\ell \, d\sigma_{UU}^{I} + d\sigma_{UU}^{C} \\
&+ e_\ell P_\ell \, d\sigma_{LU}^{I} + P_\ell \, d\sigma_{LU}^{C} \\
&+ e_\ell S_L \, d\sigma_{UL}^{I} + S_L \, d\sigma_{UL}^{C} \\
&+ e_\ell S_T \, d\sigma_{UT}^{I} + S_T \, d\sigma_{UT}^{C} \\
+P_\ell S_L \, d\sigma_{LL}^{BH} &+ e_\ell P_\ell S_L \, d\sigma_{LL}^{I} + P_\ell S_L \, d\sigma_{LL}^{C} \\
+P_\ell S_T \, d\sigma_{LT}^{BH} &+ e_\ell P_\ell S_T \, d\sigma_{LT}^{I} + P_\ell S_T \, d\sigma_{LT}^{C}
\end{aligned}
$$

  with $d\sigma$ even and $d\sigma$ odd under $\phi \to -\phi$, $\phi_S \to -\phi_S$

- single spin terms LU, UL, UT
  - $d\sigma^{I} \propto \mathrm{Im}\,\mathcal{A}$, $\quad d\sigma^{C} \propto \mathrm{Im}\,(\mathcal{A}^* \mathcal{A}')$
    $\mathcal{A}, \mathcal{A}' = \gamma^* p \to \gamma p$ helicity amplitudes $\rightsquigarrow$ Compton form factors
  - no Bethe-Heitler contribution
- unpolarized and double spin terms UU, LL, LT
  - $d\sigma^{I} \propto \mathrm{Re}\,\mathcal{A}$, $\quad d\sigma^{C} \propto \mathrm{Re}\,(\mathcal{A}^* \mathcal{A}')$

general consequence of parity and time reversal invariance

transformation properties

	parity P	time reversal T	PT
spin $1/2$ vector	$+$	$-$	$-$
momentum vector	$-$	$-$	$+$
azimuths ϕ, ϕ_S	$-$	$-$	$+$
P_ℓ, S_L	$-$	$+$	$-$

- parity inv. $\rightsquigarrow$ single spin term odd in ϕ, ϕ_S
  $\rightsquigarrow$ "time reversal odd"
- time reversal and parity inv. $\rightsquigarrow \langle f|\mathcal{T}|i\rangle = \langle i_T|\mathcal{T}|f_T\rangle = \langle i_{PT}|\mathcal{T}|f_{PT}\rangle$
  $i_{PT}, f_{PT} =$ spins reversed, momenta unchanged

single spin asy. $$\propto \big|\langle f|\mathcal{T}|i\rangle\big|^2 - \big|\langle f_{PT}|\mathcal{T}|i_{PT}\rangle\big|^2 = \big|\langle f|\mathcal{T}|i\rangle\big|^2 - \big|\langle i|\mathcal{T}|f\rangle\big|^2$$

- requires nonzero absorptive part

$$\langle f|\mathcal{T}|i\rangle - \langle i|\mathcal{T}|f\rangle^* = \langle f|\mathcal{T} - \mathcal{T}^\dagger|i\rangle = i \sum_X \langle f|\mathcal{T}|X\rangle\langle X|\mathcal{T}|i\rangle$$

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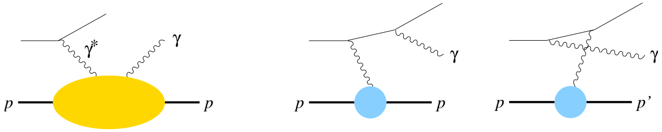
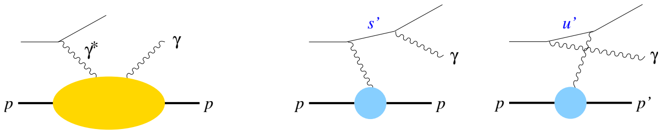


- Bethe-Heitler has no absorptive part ("is purely real")
  absorpt. part from $O(\alpha_{em})$ corrections, i.e. two-photon exchange
- single-spin asymmetries only from DVCS or from interference

The ϕ dependence



- reflects helicity structure of γ^* in DVCS process
  $\phi \leftrightarrow$ rotation about γ^* momentum
  with ang. mom. operator $L^z = -i\frac{\partial}{\partial\phi}$ have $L^z e^{-i\lambda\phi} = \lambda\, e^{i\lambda\phi}$
- in σ^I and σ^C have correspondence

$$\cos(n\phi), \sin(n\phi) \;\leftrightarrow\; \gamma^* \text{ helicity in } \mathcal{A}$$

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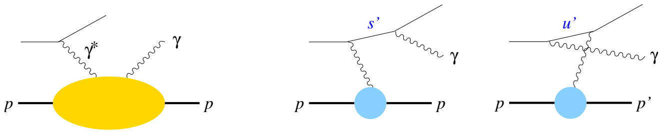

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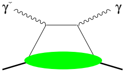
- in σ^I and σ^C have correspondence
  $\cos(n\phi), \sin(n\phi) \leftrightarrow \gamma^*$ helicity in $\mathcal{A}$
- ϕ has no simple meaning in BH process
  variables Q^2, t, x_B, ϕ chosen to make DVCS simple
  ϕ dependence from Bethe-Heitler propagators known

$$s'u' = -\text{const.}\left[1 - \cos\phi\, O\!\left(\tfrac{\sqrt{t_0 - t}}{Q}\right) + \cos(2\phi)\, O\!\left(\tfrac{t_0 - t}{Q^2}\right)\right]$$

Access to GPDs

- DVCS and meson production at LO in α_s : GPDs appear as

$$\mathcal{F} \propto \int dx \, \frac{F(x,\xi,t)}{x - \xi + i\epsilon} \pm \{\xi \to -\xi\}$$



- DVCS: many independent observables at leading twist (γ_T^*) in interference term can separate all 4 GPDs:

target pol.	GPD combination
U	$F_1 \mathcal{H} + \xi(F_1 + F_2)\tilde{\mathcal{H}} + \frac{t}{4m^2} F_2 \, \mathcal{E}$
L	$F_1 \tilde{\mathcal{H}} + \xi(F_1 + F_2)\mathcal{H} - \frac{\xi}{1+\xi} F_1 \, \xi\tilde{\mathcal{E}} + \ldots$
$T_{\cos(\phi-\phi_S)}$	$F_2 \tilde{\mathcal{H}} - F_1 \, \xi\tilde{\mathcal{E}} + \ldots$
$T_{\sin(\phi-\phi_S)}$	$F_2 \mathcal{H} - F_1 \, \mathcal{E} + \ldots$

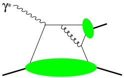
with unpolarized or polarized lepton beam

$F_1, F_2 =$ Dirac and Pauli form factors at mom. transfer t

Access to GPDs

- DVCS and meson production at LO in α_s : GPDs appear as

$$\mathcal{F} \propto \int dx \, \frac{F(x,\xi,t)}{x-\xi+i\epsilon} \pm \{\xi \to -\xi\}$$



- meson production: two leading-twist observables (γ_L^*)

meson	target pol.	GPD combination
vector	U	$\lvert\mathcal{H}\rvert^2 - \frac{t}{4m^2}\lvert\mathcal{E}\rvert^2 - \xi^2\,\lvert\mathcal{H}+\mathcal{E}\rvert^2$
	$T_{\sin(\phi-\phi_S)}$	$\mathrm{Im}(\mathcal{E}^*\mathcal{H})$
pseudo-	U	$(1-\xi^2)\,\lvert\tilde{\mathcal{H}}\rvert^2 - \frac{t}{4m^2}\,\xi^2\lvert\tilde{\mathcal{E}}\rvert^2 - 2\xi^2\,\mathrm{Re}(\tilde{\mathcal{E}}^*\tilde{\mathcal{H}})$
scalar	$T_{\sin(\phi-\phi_S)}$	$\mathrm{Im}(\xi\,\tilde{\mathcal{E}}^*\tilde{\mathcal{H}})$

with unpolarized lepton beam

note: $\mathrm{Im}(\mathcal{E}^*\mathcal{H})$ can be small even if $\mathcal{E}$ and $\mathcal{H}$ are large