Key processes involving GPDs

- deeply virtual Compton scattering (DVCS)

\[ e p \rightarrow e p \gamma \]

also:

\[ \gamma p \rightarrow \gamma^* p \text{ with } \gamma^* \rightarrow \ell^+ \ell^- \text{ (timelike CS)} \]

\[ \gamma^* p \rightarrow \gamma^* p \text{ (double DVCS)} \]

- meson production: large \( Q^2 \) or heavy quarks

\[ M = \rho, \phi, \pi, \ldots \]
Helicity selection rules

- selection of helcities in **hard-scattering part**
- ingredients: conservation of angular mom. and of chirality
  - scattering collinear → ang. mom. $J^z = \text{sum of helicities}$
  - chirality conserved by quark-gluon and quark-photon coupling

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<thead>
<tr>
<th>chirality</th>
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<td>$q$ helicity</td>
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light meson production (not $J/\Psi$ or $\Upsilon$)

(analogous argument for graphs with gluon GPD)
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light meson production (not $J/\Psi$ or $\Upsilon$)

![Diagram of helicity selection](image)

- dominant transition: $A(\gamma^*_L \rightarrow \text{meson}_L) \sim 1/Q$

(analogous argument for graphs with gluon GPD)
Helicity selection rules

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- ingredients: conservation of angular mom. and of chirality
  - scattering collinear \( \rightarrow \) ang. mom. \( J^z = \text{sum of helcities} \)
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    \[
    \begin{align*}
    \text{chirality} & \quad +1 & \quad -1 \\
    q \text{ helicity} & \quad +1/2 & \quad -1/2 \\
    \bar{q} \text{ helicity} & \quad -1/2 & \quad +1/2 
    \end{align*}
    \]

light meson production (not \( J/\Psi \) or \( \Upsilon \))

\[ A(\gamma^*_T \rightarrow V_T) \sim 1/Q^2, \text{ but sizeable at } Q^2 \sim \text{few GeV}^2 \ (\rho \text{ and } \phi \text{ data}) \]

\( \text{can describe phenomenologically by keeping } k_T \text{ finite in hard scattering} \)
Helicity selection rules for the Compton amplitude
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- leading transition: $T \rightarrow T$
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- $L \rightarrow T$ at twist-three level ($\propto 1/Q$)
- double helicity flip in $T \rightarrow T$ at twist-two with gluons
DVCS amplitudes and GPDs

- twist-two amplitudes involve 4 four GPDs per parton
  - $H, E$: unpolarized quark/gluon
  - $\tilde{H}, \tilde{E}$: long. pol. quark/gluon

- twist-three amplitudes:
  - Wandzura-Wilczek part involves same four twist-two GPDs calculated up to NLO
  - genuine twist-three part: matrix elements of $\bar{q} G^{\mu\nu} q$
    largely unknown

- photon double helicity-flip amplitudes:
  - at twist two with gluon helicity-flip GPDs, $A \propto \alpha_s$
    distributions very unknown
  - at twist four $\propto 1/Q^2$ start at tree level
    Wandzura-Wilczek part with usual quark GPDs calculated
DVCS form factors

- for photon helicity conserving amplitudes write

\[ e^{-2} A(\gamma^* p \rightarrow \gamma p) = \bar{u}(p') \gamma^+ u(p) \mathcal{H} + \bar{u}(p') \frac{i}{2m_p} \sigma^{+\alpha} (p' - p)_\alpha u(p) \mathcal{E} \]

\[ + \bar{u}(p') \gamma^+ \gamma_5 u(p) \tilde{\mathcal{H}} + \bar{u}(p') \frac{(p' - p)_+}{2m_p} \gamma_5 u(p) \tilde{\mathcal{E}} \]

- Compton form factors \( \mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}} \) depend on \( \xi, t, Q^2 \)
- representation holds for any \( Q^2 \), not only at twist two

- at leading twist and LO in \( \alpha_s \)

\[ \mathcal{H} = \sum_q e_q^2 \int_{-1}^{1} dx \left[ \frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right] H^q(x, \xi, t) \]

same kernels for \( E \), different set for \( \tilde{\mathcal{H}}, \tilde{\mathcal{E}} \)
Aside: imaginary and absorptive part

- scattering matrix $S$: $|X\rangle_{in} = S|X\rangle_{out}$
  $\rightsquigarrow$ transition amplitude $\langle f|i\rangle_{in} = \langle f|S|i\rangle_{out}$
- $S$ is unitary: $S^\dagger S = 1$
Aside: imaginary and absorptive part

- Scattering matrix $S$: $|X\rangle_{\text{in}} = S|X\rangle_{\text{out}}$
  $\rightsquigarrow$ transition amplitude $\langle \text{out} | f | i \rangle_{\text{in}} = \langle \text{out} | f | S | i \rangle_{\text{out}}$

- $S$ is unitary: $S^\dagger S = 1$

- $S = 1 + iT$ ... leave out factors $2\pi$ etc.
  $S$ unitary $\Rightarrow \frac{1}{i}(T - T^\dagger) = T^\dagger T$

- Absorptive part: $\frac{1}{i}\langle f | T - T^\dagger | i \rangle = \sum_X \langle f | T^\dagger | X \rangle \langle X | T | i \rangle$
  on-shell intermediate states possible between $i$ and $f$
  in simple cases and with appropriate phase conventions:
  absorptive part $= 2 \times$ imaginary part of amplitude

- For $f = i$ get optical theorem
  \[ 2 \text{Im} \langle i | T | i \rangle = \sum_X |\langle X | T | i \rangle|^2 \propto \sigma_{\text{tot}} \]
Real and imaginary part for brevity suppress $\sum_q e_q^2$ and arguments $t, Q^2$

$$H(\xi) = \int_{-1}^{1} dx\ H(x, \xi) \left[ \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right]$$

$$\text{Im} H(\xi) = \pi \left[ H(\xi, \xi) - H(-\xi, \xi) \right]$$

$$\text{Re} H(\xi) = \text{PV} \int_{-1}^{1} dx\ H(x, \xi) \left[ \frac{1}{\xi - x} - \frac{1}{\xi + x} \right]$$

- Im only involves $H$ at $x = \pm\xi$ at LO
  - at NLO and higher: only DGLAP region $|x| \geq \xi$
- Re involves both DGLAP and ERBL regions
- deconvolution problem:
  - reconstruction of $H(x, \xi; \mu^2)$ from $H(\xi, Q^2)$ only via $Q^2$ dep’ce
  - i.e. via evolution effects, requires large lever arm in $Q^2$ at given $\xi$
Why DVCS?

- theoretical accuracy at NNLO
- very close to inclusive DIS
  power corrections empirically not too large, in part computed
Why not only DVCS?

- theoretical accuracy at NNLO
- very close to inclusive DIS
  power corrections empirically not too large, in part computed
- only quark flavor combination $\frac{4}{9}u + \frac{1}{9}d + \frac{1}{9}s$
  with neutron target in addition $\frac{4}{9}d + \frac{1}{9}u + \frac{1}{9}s$
- gluons only through $Q^2$ dependence
  via LO evolution, NLO hard scattering
  most prominent at small $x, \xi$
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- useful to get information from meson production
  - e.g. $A_{\rho^0} \propto \frac{2}{3}(u + \bar{u}) + \frac{1}{3}(d + \bar{d}) + \frac{3}{4}g$
    $A_{\phi} \propto \frac{1}{3}(s + \bar{s}) + \frac{1}{4}g$
  - but theory description more difficult
    meson wave function, larger corrections in $1/Q^2$ and $\alpha_s$
  - $J/\Psi$ production: directly sensitive to gluons
Deeply virtual Compton scattering

- competes with Bethe-Heitler process at amplitude level

- analogy with optics:
  - DVCS $\sim$ diffraction experiment
  - BH $\sim$ reference beam with known phase
Deeply virtual Compton scattering

- competes with Bethe-Heitler process at amplitude level

\[ \frac{d\sigma_{\text{VCS}}}{dx_B dQ^2 dt} : \frac{d\sigma_{\text{BH}}}{dx_B dQ^2 dt} \sim \frac{1}{y^2} \frac{1}{Q^2} : \frac{1}{t} \]

- cross section for $\ell p \rightarrow \ell \gamma p$

\[ y = \frac{Q^2}{x_B s_{\ell p}} \]

- $1/Q^2$ and $1/t$ from photon propagators
- $1/y^2$ from vertex $e \rightarrow e\gamma^*$

- small $y$: $\sigma_{\text{VCS}}$ dominates $\sim$ high-energy collisions
- moderate to large $y$: get VCS via interference with BH
- $\sim$ separate $\text{Re} \ A(\gamma^* p \rightarrow \gamma p)$ and $\text{Im} \ A(\gamma^* p \rightarrow \gamma p)$
filter out interference term using cross section dependence on

- beam charge $e_\ell$
- azimuth $\phi$
- beam polarization $P_\ell$
- target polarization $S_L, S_T, \phi_S$

general structure:

$$d\sigma(\ell p \rightarrow \ell \gamma p) \sim d\sigma^{BH} + e_\ell d\sigma^I + d\sigma^C$$
in more detail:

\[ d\sigma(\ell p \rightarrow \ell' \gamma p) \sim d\sigma_{UU}^{BH} + e_\ell d\sigma_{UU}^I + d\sigma_{UU}^C + e_\ell P_\ell d\sigma_{LU}^I + P_\ell d\sigma_{LU}^C + e_\ell S_L d\sigma_{UL}^I + S_L d\sigma_{UL}^C + e_\ell S_T d\sigma_{UT}^I + S_T d\sigma_{UT}^C + P_\ell S_L d\sigma_{LL}^{BH} + e_\ell P_\ell S_L d\sigma_{LL}^I + P_\ell S_L d\sigma_{LL}^C + P_\ell S_T d\sigma_{LT}^{BH} + e_\ell P_\ell S_T d\sigma_{LT}^I + P_\ell S_T d\sigma_{LT}^C \]

with \( d\sigma \) even and \( d\sigma \) odd under \( \phi \rightarrow -\phi, \phi_S \rightarrow -\phi_S \)

single spin terms \( LU, UL, UT \)
- \( d\sigma^I \propto \text{Im} \mathcal{A}, \quad d\sigma^C \propto \text{Im} (\mathcal{A}^* \mathcal{A}') \)
  \( \mathcal{A}, \mathcal{A}' = \gamma^* p \rightarrow \gamma p \) helicity amplitudes \( \sim \) Compton form factors
- no Bethe-Heitler contribution

unpolarized and double spin terms \( UU, LL, LT \)
- \( d\sigma^I \propto \text{Re} \mathcal{A}, \quad d\sigma^C \propto \text{Re} (\mathcal{A}^* \mathcal{A}') \)
general consequence of \textit{parity} and \textit{time reversal} invariance

\textbf{transformation properties}

<table>
<thead>
<tr>
<th></th>
<th>parity $P$</th>
<th>time reversal $T$</th>
<th>$PT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>spin 1/2 vector</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>momentum vector</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>azimuths $\phi$, $\phi_S$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$P_\ell$, $S_L$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
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\begin{itemize}
  \item parity inv. $\rightsquigarrow$ single spin term \textit{odd} in $\phi$, $\phi_S$
  \item “time reversal odd”
  \item time reversal and parity inv. $\rightsquigarrow$ $\langle f|\mathcal{T}|i \rangle = \langle i_T|\mathcal{T}|f_T \rangle = \langle i_{PT}|\mathcal{T}|f_{PT} \rangle$
  \hfill $i_{PT}, f_{PT} =$ spins reversed, momenta unchanged
  \begin{align*}
  \text{single spin asy. } \propto & \left| \langle f|\mathcal{T}|i \rangle \right|^2 - \left| \langle f_{PT}|\mathcal{T}|i_{PT} \rangle \right|^2 \\
  & = \left| \langle f|\mathcal{T}|i \rangle \right|^2 - \left| \langle i|\mathcal{T}|f \rangle \right|^2
  
  \text{requires nonzero \textit{absorptive part}}
  \end{align*}
  \item $\langle f|\mathcal{T}|i \rangle - \langle i|\mathcal{T}|f \rangle^* = \langle f|\mathcal{T} - \mathcal{T}^\dagger|i \rangle = i \sum_X \langle f|\mathcal{T}|X \rangle \langle X|\mathcal{T}|i \rangle$
\end{itemize}
\[ \langle f | T | i \rangle - \langle i | T | f \rangle^* = \langle f | T - T^\dagger | i \rangle = i \sum_X \langle f | T | X \rangle \langle X | T | i \rangle \]

- Bethe-Heitler has no absorptive part ("is purely real")
- absorpt. part from \( O(\alpha_{em}) \) corrections, i.e. two-photon exchange
- single-spin asymmetries only from DVCS or from interference
The $\phi$ dependence

- reflects helicity structure of $\gamma^*$ in DVCS process
  $\phi \leftrightarrow$ rotation about $\gamma^*$ momentum
  with ang. mom. operator $L^z = -i \frac{\partial}{\partial \phi}$ have $L^z e^{-i\lambda \phi} = \lambda e^{i\lambda \phi}$

- in $\sigma^I$ and $\sigma^C$ have correspondence
  $\cos(n\phi), \sin(n\phi) \leftrightarrow \gamma^*$ helicity in $A$
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  $\cos(n\phi), \sin(n\phi) \leftrightarrow \gamma^*$ helicity in $A$

- $\phi$ has no simple meaning in BH process
  variables $Q^2, t, x_B, \phi$ chosen to make DVCS simple
  $\phi$ dependence from Bethe-Heitler propagators known

$$s'u' = -\text{const.} \left[ 1 - \cos \phi \mathcal{O} \left( \frac{\sqrt{t_0-t}}{Q} \right) + \cos(2\phi) \mathcal{O} \left( \frac{t_0-t}{Q^2} \right) \right]$$
Access to GPDs

- DVCS and meson production at LO in $\alpha_s$: GPDs appear as
  \[ \mathcal{F} \propto \int dx \frac{F(x, \xi, t)}{x - \xi + i\epsilon} \pm \{\xi \to -\xi\} \]

- DVCS: many independent observables at leading twist ($\gamma^*_T$) in interference term can separate all 4 GPDs:

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<td>$U$</td>
<td>$F_1 \mathcal{H} + \xi (F_1 + F_2) \tilde{\mathcal{H}} + \frac{t}{4m^2} F_2 \mathcal{E}$</td>
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<tr>
<td>$L$</td>
<td>$F_1 \tilde{\mathcal{H}} + \xi (F_1 + F_2) \mathcal{H} - \frac{\xi}{1+\xi} F_1 \xi \tilde{\mathcal{E}} + \ldots$</td>
</tr>
<tr>
<td>$T_{\cos(\phi - \phi_S)}$</td>
<td>$F_2 \tilde{\mathcal{H}} - F_1 \xi \tilde{\mathcal{E}} + \ldots$</td>
</tr>
<tr>
<td>$T_{\sin(\phi - \phi_S)}$</td>
<td>$F_2 \mathcal{H} - F_1 \mathcal{E} + \ldots$</td>
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with unpolarized or polarized lepton beam

$F_1, F_2 = $ Dirac and Pauli form factors at mom. transfer $t$
Access to GPDs

- DVCS and meson production at LO in $\alpha_s$: GPDs appear as

$$F \propto \int dx \frac{F(x, \xi, t)}{x - \xi + i\epsilon} \pm \{\xi \to -\xi\}$$

- meson production: two leading-twist observables ($\gamma_L^*$)

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<td>$\text{Im}(\mathcal{E}^*\mathcal{H})$</td>
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<td>$U$</td>
<td>$(1 - \xi^2)</td>
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<tr>
<td>scalar</td>
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<td>$\text{Im}(\xi \tilde{\mathcal{E}}^*\tilde{\mathcal{H}})$</td>
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with unpolarized lepton beam

note: $\text{Im}(\mathcal{E}^*\mathcal{H})$ can be small even if $\mathcal{E}$ and $\mathcal{H}$ are large