Effective field theories

I] Introduction

Principles: At a given energy scale:
- Degrees of freedom
- Symmetries.

⇒ Formulate the theory

Decoupling:
- To play pool you do not need to know the movement of earth around the sun.
- You do not need to understand nuclear physics to build a bridge.

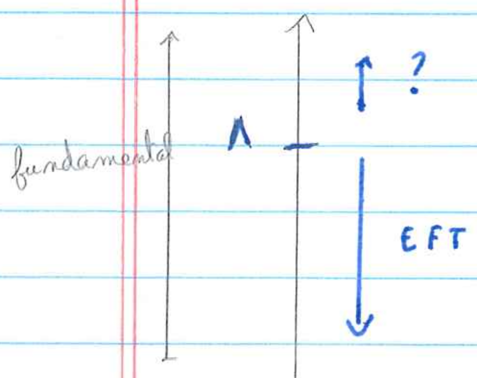
What does it mean? Scales much smaller / bigger should not matter much.

→ Dynamics at long distance does not depend on what is going on at short distance.

→ Or equivalently: low energy interactions do not care about the details of high energy interactions.

Ex:
$v \ll c$: ignore relativity
$E \cdot t \gg \hbar$: ignore quantum effects

Definition: Effective field theory: field theory that describes the physics below some scale Λ



as opposed to a fundamental field theory which should be valid up to arbitrarily high energies.

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What are examples of EFT?

Every field theory we know!

- Standard Model: describe electromagnetic and weak interactions
- QCD: theory of strong interaction.

→ consistent to assume that it is valid up to infinite energies because of asymptotic freedom. but confinement at low energy

⇒ difficult to describe: ChPT.

- QED: consistent up to extremely high but finite energies. Theory of E.M. interactions.

How does it work in practice?

Ingredients to build an EFT:

- scale separation: high / low scale.
- active degrees of freedom: building blocks
- symmetries: how are the interactions constrained by the symmetries?
- power counting: → organize the expansion in low over high.

The theory is only valid up to the scale Λ

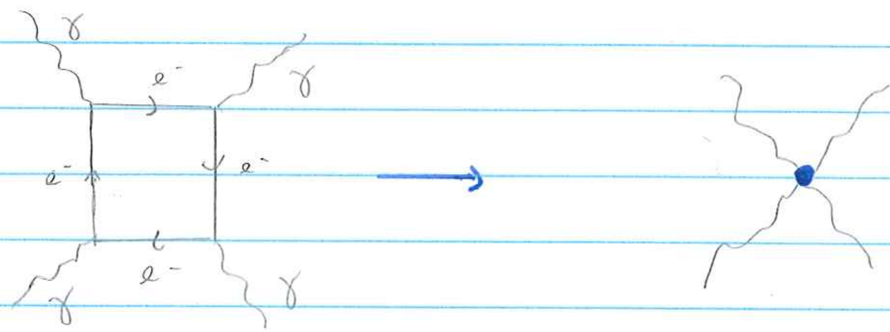
⇒ a finite number of parameters describes the physics up to effects suppressed by $\left(\frac{E}{\Lambda}\right)^n$ with E energy of particles in EFT

The higher n, the more parameters will be needed.

⇒ Renormalizability is lost.

II] Examples of EFT

Example 1: light by light scattering — Euler, Heisenberg, Kockel 1936



* Energy scales:
  - Photon Energy ω
  - Electron mass : m_e

* Consider $\omega \ll m_e$

  → Fermions are massive d.o.f that can be integrated out

$\Lambda = m_e$

$$\mathcal{L}_{QED}(\underbrace{\psi, \bar\psi}_{\text{fermions}}, \overbrace{A_\mu}^{\text{photon}}) = i\bar\psi\gamma^\mu\partial_\mu\psi - e\bar\psi\gamma_\mu A^\mu\psi - m\bar\psi\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

→ integrate out $\psi, \bar\psi$

EFT

$$\mathcal{L}_{eff}(A_\mu)$$

* Active d.o.f: photon, A_μ

* Symmetries: $U(1)$ ⇒ invariants: $F_{\mu\nu}F^{\mu\nu} \propto \vec{E}^2 - \vec{B}^2$

  $F_{\mu\nu}\tilde{F}^{\mu\nu} \propto (\vec{E}\cdot\vec{B})^2$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\boxed{\mathcal{L}_{eff} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{16\pi^2 m_e^4}\left[a(\vec{E}^2 - \vec{B}^2)^2 + b(\vec{E}\cdot\vec{B})^2\right] + \dots}$$

* Couplings a and b not fixed by effective theory ⇒ compute with underlying

Theory: $7a = b = \frac{14}{45}$

Calculation of cross section and other observables using $\mathcal{L}_{eff}$ much easier than using $\mathcal{L}_{QED}$.

$$\boxed{\sigma(\omega) = \frac{1}{16\pi^2} \frac{973}{10125\pi} \frac{e^8}{m_e^2} \left(\frac{\omega}{m_e}\right)^6}$$

* Power counting: energy expansion: $\left(\frac{\omega}{m_e}\right)^{2n}$ = small parameter

⇒ Measurement with PW lasers Bragant et al. PRD 78 (2008) 032 006

· 3 examples:

Low Energy EFT	High-energy theory "full theory"	Fundamental scale Λ
SM	?	1 TeV ?
Fermi Theory	SM	$M_W \sim 80$ GeV
ChPT	QCD / SM	$m_p \sim 1$ GeV

Each of these examples ⇒ different aspects why EFTs are useful.

SM : * Energy scales $E < 1$ TeV
* d.o.f : quarks and leptons
* symmetries: $SU(2) \times U(1)$
* Power counting $\left(\frac{E}{\Lambda}\right)^n$ ($n = 1, 2$)

we do not know the fundamental theory

$\rightarrow$ SM an EFT

$\rightarrow$ search for low energy $\left(\frac{E}{\Lambda}\right)^n$ $(n = 1,2)$ effects to find out at which scale new physics enters

$\Rightarrow$ large program of precision measurements

Precision EW suggests $\Lambda \gtrsim 5\,\mathrm{TeV}$

Precision flavour physics suggests $\Lambda \gtrsim 1 - 100\,\mathrm{TeV}$

Theoretical arguments $\Rightarrow$ $\Lambda \sim 200\,\mathrm{GeV}$ $\Rightarrow$ bounds higher

"Little hierarchy problem" and "flavour problem"

Fermi theory: The full theory is known: SM

$\Rightarrow$ can derive the couplings of the EFT from full-theory calculations.

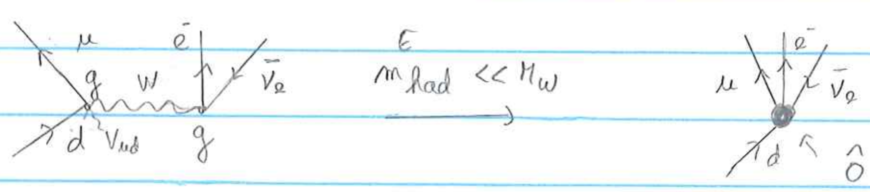
Is EFT useful in this case?

$\rightarrow$ Yes! Calculations become much simpler

Perturbation theory much better behaved.

* Energy scales: weak decays mediated by charged W bosons.

- $d \rightarrow u\, e\, \nu$
  $n \rightarrow p\, \bar{e}\, \nu$
  $udd \quad uud$



- $E \sim m_{had}$ = energy release in neutron β decays $\sim 1\,\mathrm{MeV}$
  energy release in kaon decays $\sim$ few $100\,\mathrm{MeV}$
  $K \rightarrow \pi\, \ell\, \nu$

- W energy: $M_W \simeq 80\,\mathrm{GeV}$

* Active d.o.f : quarks and
* symmetries :
* Consider $E \sim m_{Kad} << M_W$: W is integrated out

$$A = \frac{g^2}{8} V_{ud} \frac{i}{q^2 - M^2_W} \bar{u} \gamma_\mu (1-\gamma_5) d \; \bar{e} \gamma^\mu (1-\gamma_5) \nu_e$$

$\dfrac{-g_{\mu\nu} + q_\mu q_\nu / M^2_W}{q^2 - M^2_W}$

$q^2 \sim E << M^2_W \longrightarrow$

$$A = -\frac{g^2}{8} V_{ud} \frac{i}{M^2_W\left(1 - \frac{q^2}{M^2_W}\right)} \left(\bar{u} \gamma_\mu (1-\gamma_5) d \; \bar{e} \gamma^\mu (1-\gamma_5) \nu_e\right)$$

$$= -\frac{i g^2}{8 M^2_W} V_{ud} \left(1 + \frac{q^2}{M^2_W} + \dots\right) \hat{O}$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M^2_W}} \Rightarrow \boxed{A = -i \frac{G_F}{\sqrt{2}} V_{ud} \langle \hat{O} \rangle + O\left(\frac{q^2}{M^2_W}\right)}$$

Fermi's current current interaction:

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} J^+_\mu(x) J^\mu(x)$$

Chiral perturbation theory

The full theory QCD so complicated $\Rightarrow$ can't calculate the coupling constants in $\mathcal{L}_{eff}$ from QCD

↳ with lattice QCD became now to some extend possible.

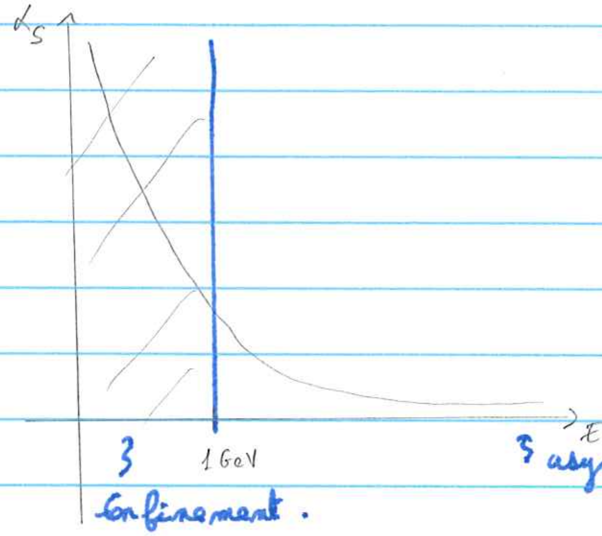
* Energy scales :
  - Dynamics of π, K, η : $m_\pi \sim 139$ MeV
    $m_K \sim 436$ MeV
    $m_\eta \sim 500$ MeV
  - Hadronic scale $\Lambda_H \sim m_p \sim 1$ GeV $\to \rho, K^*, \eta', N$

  Consider $p << \Lambda_H$

* Active d.o.f : π, K, η
* Symmetries : QCD Lagrangian symmetries $SU(3)_L \otimes SU(3)_R$

ChPT very useful to describe the dynamics of the mesons in a region where perturbative QCD can't be applied.

α_s diverges!



confinement.

asymptotic freedom.

* Power counting: Expansion in $\frac{1}{\Lambda_H}$ and $m_u, m_d, m_s / \Lambda$

⇒ At low energies can describe all scattering processes of pions in terms of F_π and meson masses.

Because of complexity of QCD ⇒ many different EFTs taylored to various physics situations developped
Most QCD calculations including lattice calculations rely on EFT:
- χPT: physics of light quarks
- HQET: heavy quark effective field theory
- Heavy baryon χPT
- SCET: soft collinear effective field theory

III] Construction of an effective Lagrangian

Consider a field theory with a characteristic scale M
⇒ we are interested in physics at low energies $E \ll M$
↳ physics situation EFT are designed to analyse.

Full theory is defined in terms of a path integral

Everything we wish to know obtained by calculating the expectation values:

$$\boxed{\langle 0 | T\{\phi(x_1) \dots \phi(x_n)\} | 0 \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, e^{iS(\phi)} \phi(x_1) \dots \phi(x_n)}$$

$$\mathcal{D}\phi = \prod_{x^\mu} \int d\phi(x) \text{ or } \prod_{p^\mu} \int d\tilde{\phi}(p)$$

$$Z = \int \mathcal{D}\phi \, e^{iS(\phi)}$$

→ To obtain the low-energy effective action split the field:

$\phi = \phi_L + \phi_H$ → ϕ_H : contains all Fourier modes with $\omega \geq \Lambda$

ϕ_L : contains the LE modes $\omega < \Lambda$

We are only interested in LE physics: we only want the correlation functions. $\langle 0 | T\{\phi_L(x_1) \dots \phi_L(x_n)\} | 0 \rangle$

$$= \int d\phi_L \underbrace{\int d\phi_H \, e^{iS(\phi_L + \phi_H)}} \phi_L(x_1) \dots \phi_L(x_n)$$

$$= \int d\phi_L \, e^{iS_\Lambda(\phi_L)} \phi_L(x_1) \, \phi_L(x_2) \dots \phi_L(x_n)$$

$S_\Lambda(\phi_L)$: the Wilsonian effective action

We choose $\Lambda \leq M$ to integrate out the physics associated with M.

$S_\Lambda(\phi_L)$ non local on scales $\Delta x^\mu \sim \frac{1}{\Lambda}$ because HE fluctuations have been integrated out.

Final step: expand the non local action as a series of local operators

→ Expansion possible because $E << \Lambda$

$$\Rightarrow \boxed{S_\Lambda(\phi_L) = \int d^d x \, \mathcal{L}_\Lambda^{\text{eff}}(x)}$$

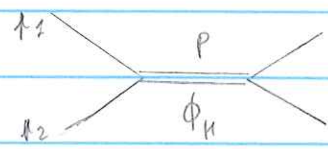
$$\boxed{\mathcal{L}_\Lambda^{\text{eff}}(x) = \sum_i g_i \, O_i(x)}$$ = Effective Lagrangian

$O_i(x)$: local operators allowed by symmetries constructed with the light fields.

g_i : Wilson coefficients or couplings

The information on heavy d.o.f hidden in these couplings.

Ex: Integrate out a heavy particle with mass M (ex M_W in Fermi theory)



$$\sim \frac{1}{p^2 - M^2}$$

$p_1, p_2 << M$ => expand

$$\frac{1}{p^2 - M^2} = -\frac{1}{M^2\left(1 - \frac{p^2}{M^2}\right)}$$

$$= -\frac{1}{M^2} + \frac{p^2}{M^4} + \dots$$

$$= -\frac{1}{M^2}\delta^{(4)}(x) - \frac{\Box}{M^4}\delta^{(4)}(x) \dots$$

$\mathcal{L}_{eff}$ will contain terms such as $\phi_L^4(x)$ and $\partial_\mu \phi_L \partial^\mu \phi_L \, \phi_L^2(x)$ etc...

In general very hard to compute the coefficients g_i (couplings or WC)

+ in principle infinitely many terms in $\mathcal{L}_{eff}$ but power counting! => guide us to keep the relevant terms!
  ↳ dimensional analysis.

The operators O_i are organized according to their mass dimension d_i

$[O_i] = d_i$ $\quad [\mathcal{L}^{\text{eff}}] = 4$ $\quad [m] = [E] = [x^{-1}] = [t^{-1}]$

$g_i = \frac{c_i}{M^{d_i - 4}}$ $\quad c_i$ = dimensionless coefficient.

g_i comes from integrating out the physics associated with M

=> natural to assume $c_i \sim 1$

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Very large c_i e.g. $c_i \sim 10^6$ or very small c_i e.g. $c_i \sim 10^{-6}$ would need to be explained.

The operators O_i have dimension d_i → fixes the dimension of their coefficient

$$[O_i] = d_i \rightarrow g_i \sim \frac{1}{\underbrace{M}_{\text{heavy scale of the system}}{}^{d_i - 4}} = \frac{1}{\Lambda^{d_i - 4}}$$

At energies $E < M$ → behaviour of operators determined by their dimension.

→ 3 types of operators:

- relevant: $d_i < 4$
- marginal: $d_i = 4$
- irrelevant: $d_i > 4$

The operator in Fermi theory $\bar{u}\, \gamma_\mu (1-\gamma_5)\, d\; \bar{e}\, \gamma^\mu (1-\gamma_5)\, \nu_e = O_F$

$[O_F] = 6$ $\qquad$ $[m \bar{\psi} \psi] = 4$

$d_{O_F} > 4$ → called irrelevant $\qquad$ $[m] = 1 \rightarrow [\psi] = 3/2$

→ effects suppressed by $\frac{E}{M}$ ⇒ small at LE

This does not mean they are not important

⇒ interesting to search for effects mediated by irrelevant operators provide info on the underlying dynamics at higher scales on the physics at high energy.

Fermi interactions: $[O] = 6$ → Operators suppressed by 2 powers of M_W $\quad g_i = \frac{c_i}{M_W^2}$

The weak interaction is so weak at LE because mediated by irrelevant operators.

From the form of the interaction Oscar Klein predicted the existence of massive particles with $M_W \geq 60$ GeV already in 1938.

In spite of being weak the four O_F = four fermion interactions important because they generate the leading contribution to FCNC or to low energy neutrino scattering.
If $M_W, M_Z \sim 10^{16}$ GeV $\Rightarrow$ no sign of weak interaction!

In any situation where there is a large mass gap between the energy scale being analysed and the scale of any heavier state $m, E \ll M$ the effects induced by irrelevant operators are always suppressed by $\frac{E}{M}$ $\Rightarrow$ can usually be neglected.

The resulting EFT containing only relevant + marginal operators = renormalizable.
Predictions valid up to $\frac{E}{M}$ corrections...

Ex: QED: most general renormalizable $(d \leq 4)$ $\mathcal{L}$ with $U(1)$ gauge symmetry
$\Rightarrow$ other interactions exist (Z exchange)
contributes to $e^+e^- \to e^+e^-$
For $E \ll M_Z$ $\Rightarrow$ non renormalizable local couplings of higher dimension: $O_F = \bar{e}\Gamma e \, \bar{e}\Gamma e$
QED is so successful to describe scattering of electrons and positrons at LE not because renormalizability but because M_Z is very heavy

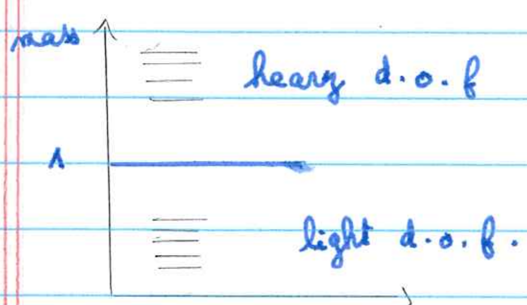
$E \ll M_Z \to$ leading non renormalizable contributions suppressed by $\frac{E^2}{M_Z^2}$

Georgi 1989: In this picture the presence of infinities in QFT is neither a disaster nor an asset.
It is simply a reminder of practical limitation: we do not know what happens at distances much smaller than those we can look at directly

IV] Summary of principles of an EFT

* Dynamics at low energy (large distances) does not depend on details of dynamics at high energy (short distances)
* low energy dynamics in terms of relevant d.o.f's energies or momenta p.

$$0 \leftarrow m \ll E \ll M \to \infty$$



Finite corrections induced by heavy d.o.f can be incorporated as perturbations.

* Expansion in powers of energy / momentum