Neutron Skins in Nuclei

31st Annual HUGS Program

(Jorge Piekarewicz - FSU)

The Liquid Drop Model

Bethe-Weizsäcker Mass Formula *(circa 1935-36)*

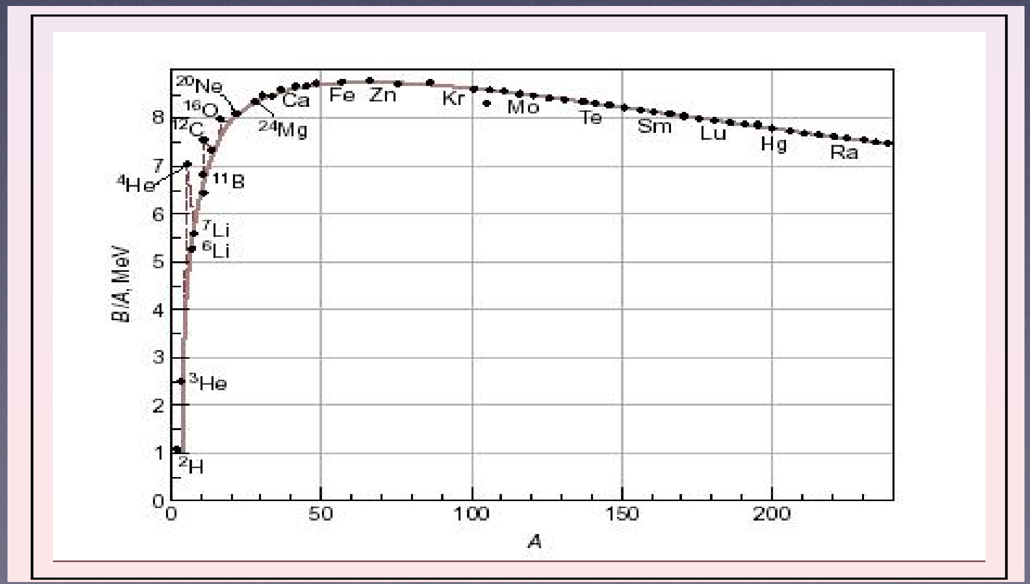
- Nuclear forces saturate $\xrightarrow{R = r_0 A^{1/3}}$ equilibrium density
- Nuclei penalized for developing a surface
- Nuclei penalized by Coulomb repulsion
- Nuclei penalized for isospin imbalance (N≠Z)

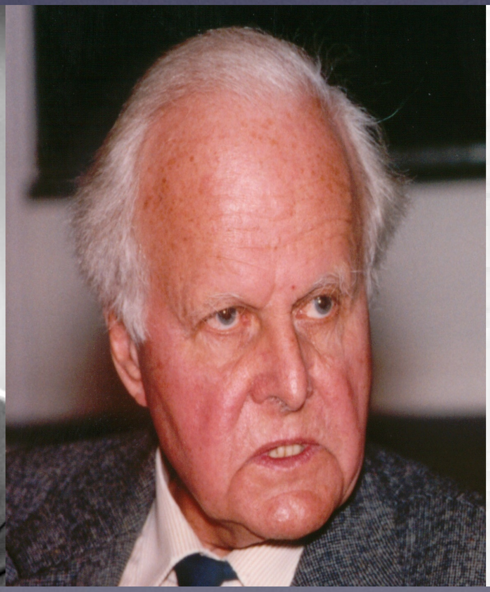
- $B(Z,N) = -a_v A + a_s A^{2/3} + a_c Z^2/A^{1/3} + a_a (N-Z)^2/A + \ldots$

  $+$ **shell corrections** (2, 8, 20, 28, 50, 82, 126, ...)

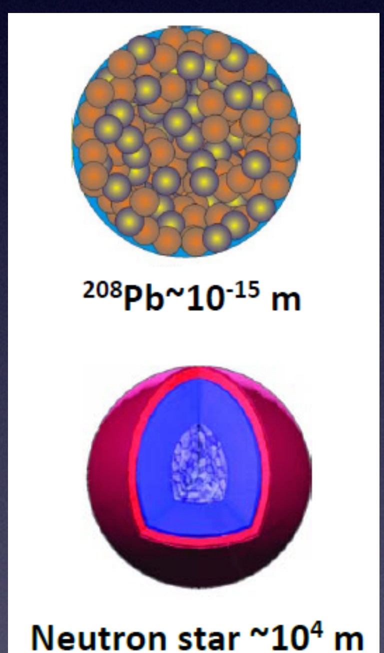
$$\boxed{a_v \simeq 16.0,\ a_s \simeq 17.2,\ a_c \simeq 0.7,\ a_a \simeq 23.3 \text{ (in MeV)}}$$

Neutron stars are gravitationally bound!

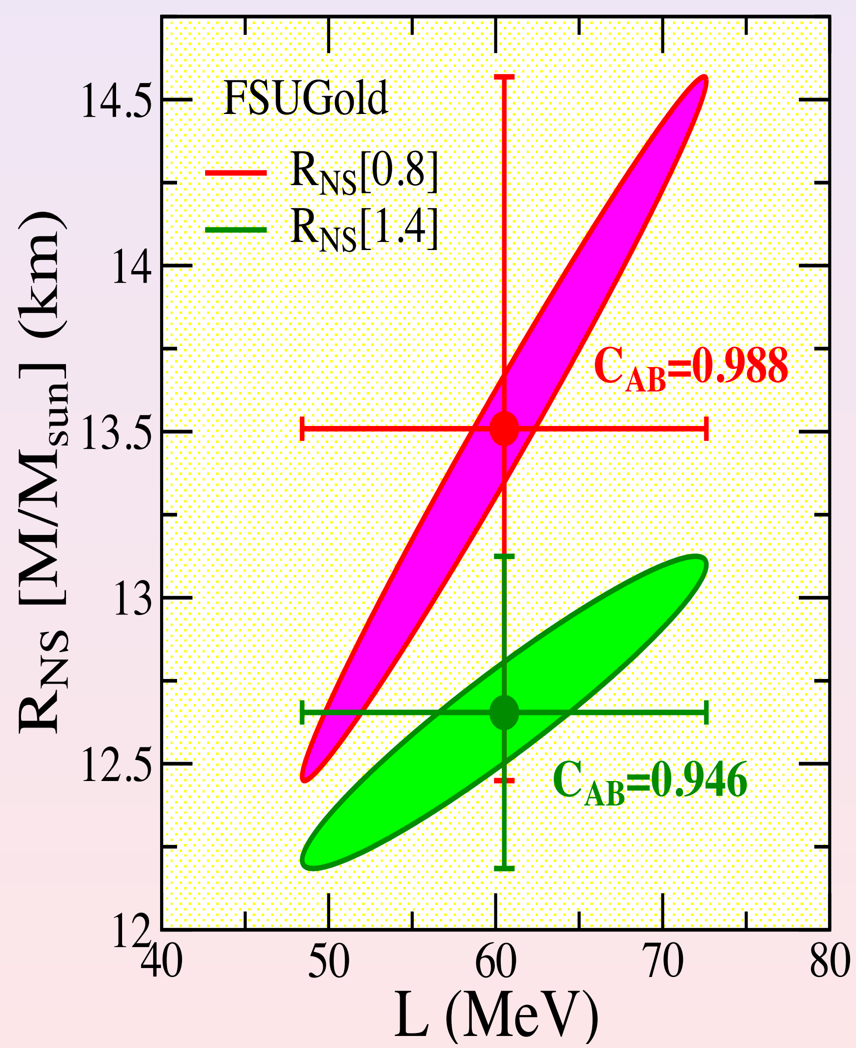


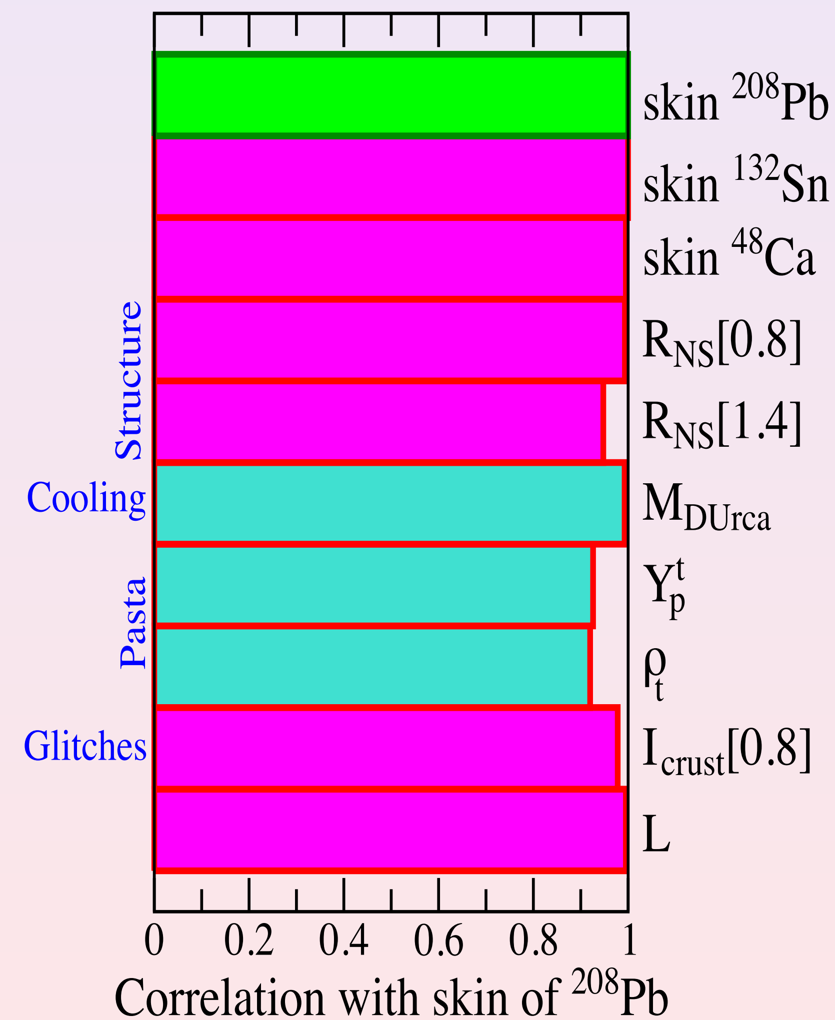




Heaven on Earth!



- Neutron skin as proxy for neutron-star radii . . . and more!
- Calibration of nuclear functional from optimization of a quality measure
- New era: predictability typical – uncertainty quantification demanded
- Neutron skin strongly correlated to a myriad of neutron star properties:
  Radii, Enhanced Cooling, Moment of Inertia, . . .

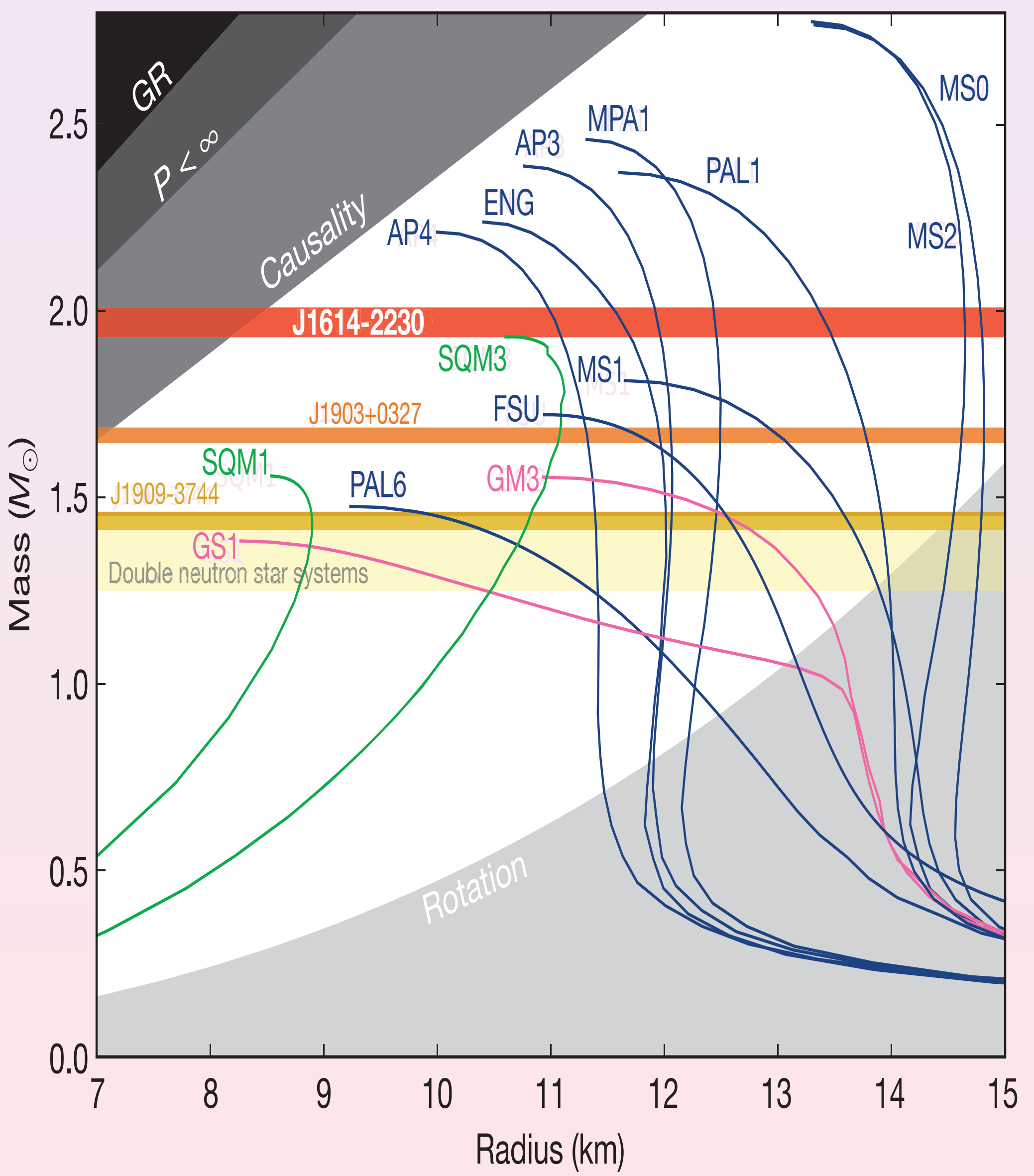




Neutron Stars: Unique Cosmic Laboratories

Neutron stars are the remnants of massive stellar explosions (CCSN)

- Bound by gravity — NOT by the strong force
- Satisfy the Tolman-Oppenheimer-Volkoff equation ($v_{esc}/c \sim 1/2$)
- Only Physics that the TOV equation is sensitive to: Equation of State
- EOS must span about 11 orders of magnitude in baryon density



$$\frac{dM}{dr} = 4\pi r^2 \mathcal{E}(r)$$

$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

Need an EOS: $P = P(\mathcal{E})$ relation

$\boxed{\text{Nuclear Physics Critical}}$

NEVER UNDERESTIMATE THE JOY PEOPLE DERIVE FROM HEARING SOMETHING THEY ALREADY KNOW. ... ENRICO FERMI

"SIMPLE MODELS OF NUCLEAR MATTER AND NEUTRON MATTER"

• THE NEED FOR AN EQUATION STATE

THE STRUCTURE, COMPOSITION, AND MANY OTHER PROPERTIES OF NEUTRON STARS ARE DESCRIBED BY THE "TOLMAN-OPPENHEIMER-VOLKOFF" (TOV) EQUATIONS:

(1) $$\frac{dP(r)}{dr} = -G\frac{\left[\mathcal{E}(r)+P(r)\right]\left[M(r)+4\pi r^3 P(r)\right]}{r^2\left[1-\frac{2GM(r)}{r}\right]}$$

(2) $$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r)$$

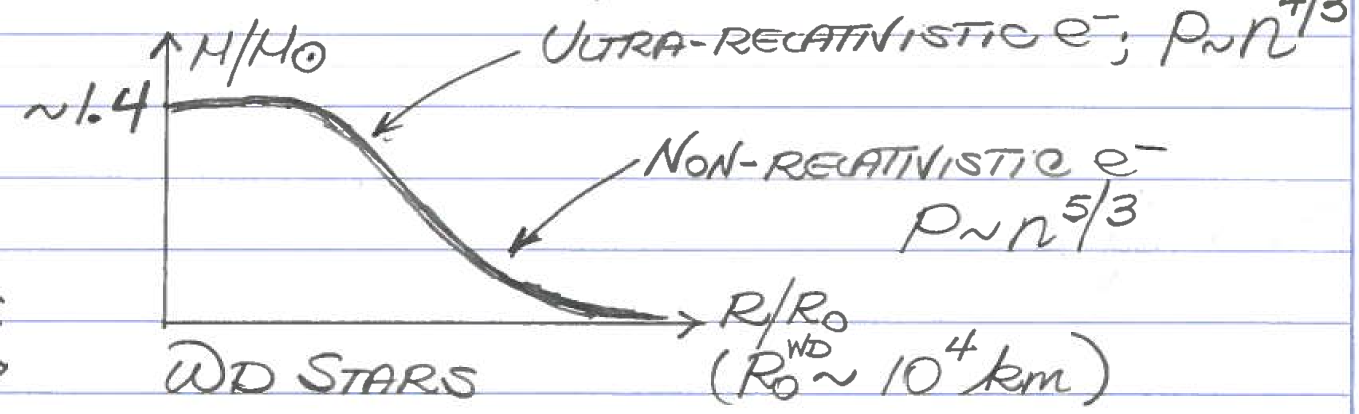
THE TOV EQUATIONS ARE THE GENERALIZATION OF NEWTONIAN GRAVITY (FOR SPHERICAL STARS IN HYDROSTATIC EQUILIBRIUM) TO THE DOMAIN OF GENERAL RELATIVITY. IN THE NEWTONIAN LIMIT— APPROPRIATE FOR WHITE-DWARF STARS — ONE OBTAINS:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\mathcal{E}(r)}{r^2}; \quad \frac{dM}{dr} = 4\pi r^2 \mathcal{E}(r)$$

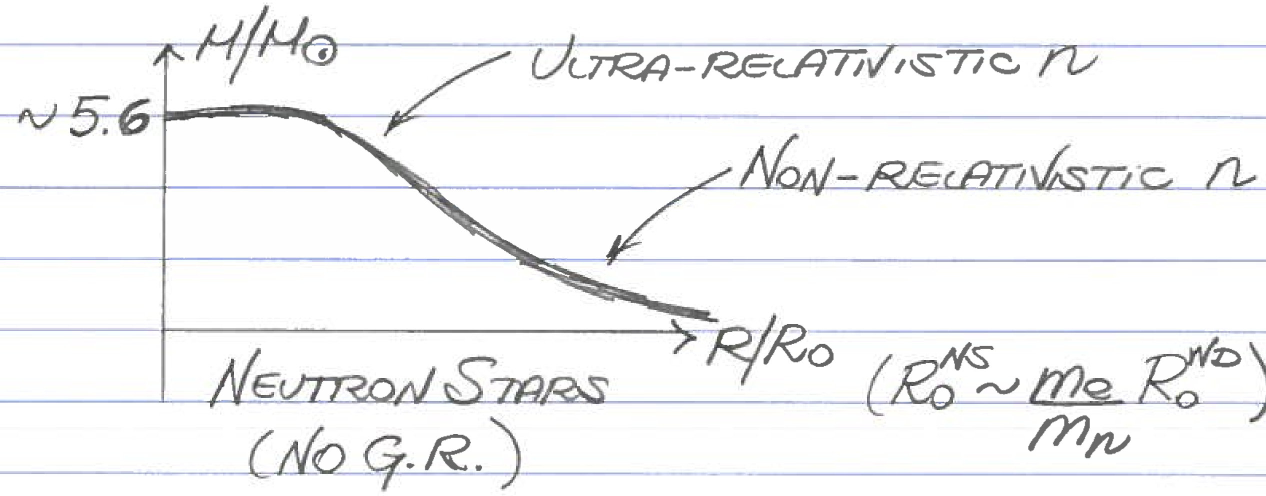
HERE $P(r)$, $M(r)$, $\mathcal{E}(r)$ ARE PRESSURE, MASS, AND ENERGY DENSITY PROFILES.

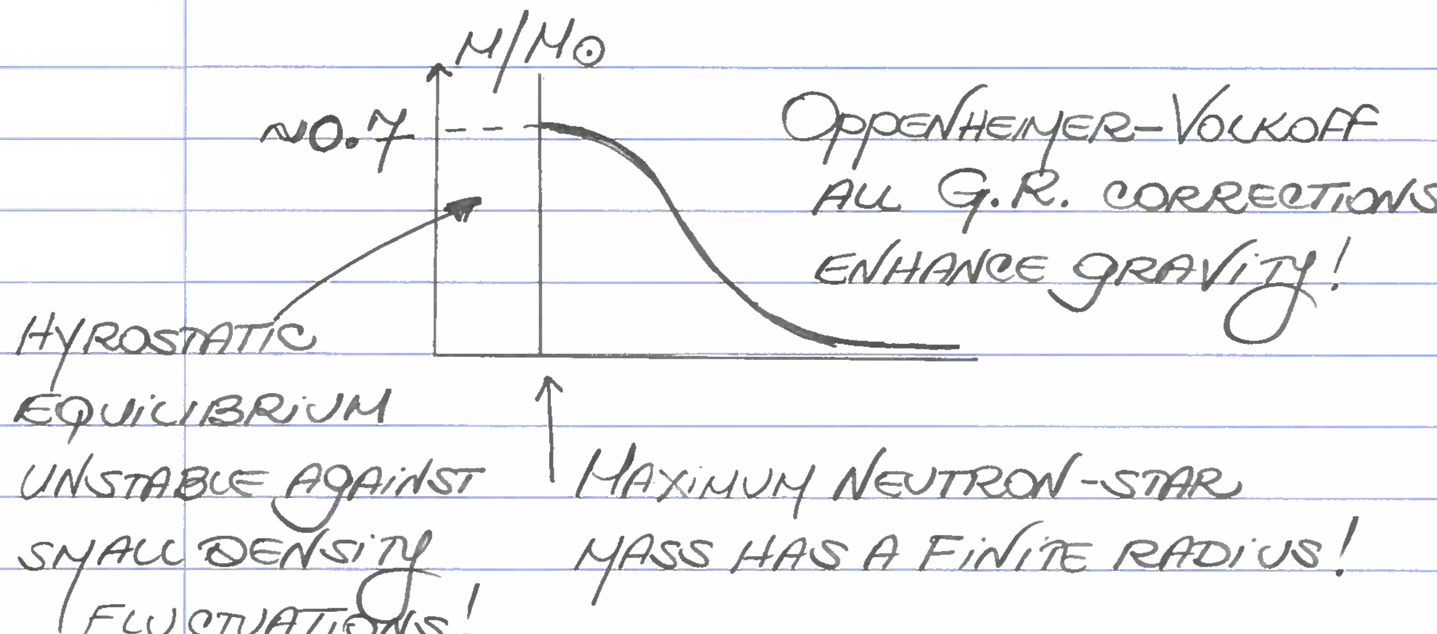
"Collapsed Stars"

There is something fundamentally different about the Chandrasekhar mass and the maximum neutron star mass.



$M_{ch} \sim \left(\frac{\hbar c}{G}\right)$[illegible]





In order to solve the TOV equations one must specify:

$$P(r=0) \equiv P_c \,;\quad M(r=0) = 0 \quad \ldots$$ and an

<u>equation of state</u> relating the pressure to the energy density, i.e., $P = P(\varepsilon)$

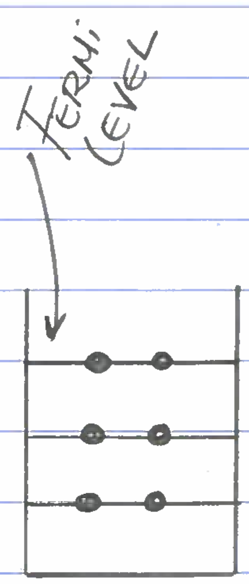
The critical role of nuclear physics:

- With an equation of state assuming a degenerate Fermi-gas of neutrons, Oppenheimer obtained a maximum neutron-star mass of only $0.72 M_\odot$. Recent observations have determined neutron-star masses of $\sim 2 M_\odot$; the extra pressure support must come from nuclear interactions.

• A one-component Free Fermi Gas; $T \equiv 0$.

$$N = \sum_{\vec{k}s} \theta(k_F - |\vec{k}|) = 2 \int V \frac{d^3k}{(2\pi)^3}\, \theta(k_F - |\vec{k}|)$$

$$= \frac{V}{\pi^2} \int_0^{k_F} k^2\, dk = V \frac{k_F^3}{3\pi^2}$$



Fermi Bucket

or

$$\boxed{\frac{N}{V} = n = \frac{k_F^3}{3\pi^2}}$$ Independent of the dispersion relation

For a fully relativistic dispersion relation

$$\epsilon(k) = \sqrt{k^2 + m^2} \longrightarrow \begin{cases} m + \frac{k^2}{2m}; & NR \\ k; & UR \end{cases}$$

$$E = \sum_{\vec{k}s} \epsilon(k)\,\theta(k_F - |\vec{k}|) = \frac{V}{\pi^2}\int_0^{k_F} k^2 \epsilon(k)\,dk$$

Nice "Trick" Define,

$$k = m\sinh t = \frac{m}{2}(e^t - e^{-t}) = \frac{m}{2}\left(x - \frac{1}{x}\right)$$

$$\epsilon = m\cosh t = \frac{m}{2}(e^t + e^{-t}) = \frac{m}{2}\left(x + \frac{1}{x}\right)$$

Then,

$$\frac{E}{V} = \frac{1}{\pi^2}\left(\frac{m}{2}\right)^4 \int_1^{x_F} \frac{(x^4-1)^2}{x^5}\,dx \; ; \; x_F \equiv \frac{k_F + \epsilon_F}{m}$$

or

$$\boxed{\frac{E}{N} = \frac{3}{8}\left[\frac{\epsilon_F}{k_F^2}(k_F^2 + \epsilon_F^2) - \frac{m^4}{k_F^3}\ln\left(\frac{k_F + \epsilon_F}{m}\right)\right]}$$

Energy per particle of a relativistic Fermi gas.

– Non-relativistic limit: $k_F \ll m$

$$\left(\frac{E}{N}\right)_{NR} = m\left[1 + \frac{3k_F^2}{10m^2} - \frac{3}{56}\frac{k_F^4}{m^4} + \ldots\right]$$

$$= m + \frac{3}{5}\epsilon_F + \ldots$$

– Ultra-relativistic limit: $k_F \gg m$

$$\left(\frac{E}{N}\right)_{UR} = \frac{3}{4}k_F\left[1 + \frac{m^2}{k_F^2} + \ldots\right] \simeq \frac{3}{4}k_F$$

FOR A SYSTEM AT $T=0$ THE PRESSURE MAY BE WRITTEN AS

$$P = -\left(\frac{\partial E}{\partial V}\right)_N = -\frac{\partial}{\partial V}\left[V\mathcal{E}\right] = -\mathcal{E} - V\left(\frac{\partial \mathcal{E}}{\partial V}\right)_N$$

OR

$$\boxed{P = n\frac{\partial \mathcal{E}}{\partial n} - \mathcal{E}}$$

EXACT EXPRESSION AT $T=0$.
$\mathcal{E} \equiv E/V$

IN THE CASE OF A FREE FERMI GAS

$$P = \frac{1}{3}k_F\left(\frac{\partial \mathcal{E}}{\partial k_F}\right) - \mathcal{E} = \frac{k_F^3 E_F}{3\pi^2} - \mathcal{E}$$

OR

$$\boxed{P = \frac{1}{24\pi^2}\left[2k_F^3 E_F - 3m^2 k_F E_F + 3m^4 \ln\left(\frac{k_F + E_F}{m}\right)\right]}$$

AGAIN, TAKE LIMITS:

– NON-RELATIVISTIC LIMIT: $k_F \ll m$

$$P_{NR} = \frac{k_F^5}{15m\pi^2} = \left(\frac{k_F^3}{3\pi^2}\right)\left(\frac{k_F^2}{5m}\right) = \frac{2}{5}nE_F$$

COMPARE WITH $\mathcal{E}_{NR} = \frac{3}{5}nE_F = \frac{3}{2}P_{NR}$

– ULTRA-RELATIVISTIC LIMIT: $k_F \gg m$

$$P_{UR} = \frac{k_F^4}{12\pi^2} = \left(\frac{k_F^3}{3\pi^2}\right)\left(\frac{k_F}{4}\right) = \frac{1}{4}nk_F$$

COMPARE WITH $\mathcal{E}_{UR} = \frac{3}{4}nk_F = 3P_{UR}$

THIS IS THE ORIGIN OF THE CHANDRASEKHAR MASS LIMIT FOR WHITE-DWARF STARS!

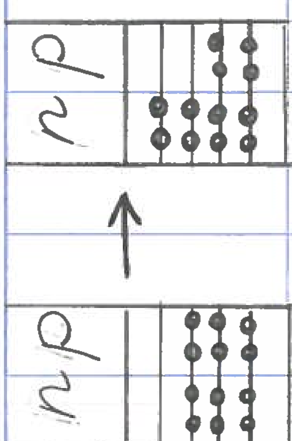
Liquid-Drop Formula

$$\frac{B(N,Z)}{A} = a_v - \ldots - a_a \frac{(N-Z)^2}{A^2}$$

Symmetry Energy

• A two component Free Fermi Gas: "The Nuclear Symmetry Energy"

We have just computed the energy and pressure of a relativistic Free Fermi Gas. We are now interested in computing the energy of a two-component system; for example protons and neutrons. Given that neutrons can change into protons and protons into neutrons, it is best to compute the energy of the system as a function of the total density and isospin asymmetry. That is,



$$\rho = \rho_n + \rho_p \quad \text{and} \quad \alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$

Hence,

$$\rho_n = \frac{(k_F^{(n)})^3}{3\pi^2} = \left(\frac{1+\alpha}{2}\right)\rho \equiv \left(\frac{1+\alpha}{2}\right)\cdot\left(\frac{2k_F^3}{3\pi^2}\right)$$

$$\rho_p = \frac{(k_F^{(p)})^3}{3\pi^2} = \left(\frac{1-\alpha}{2}\right)\rho \equiv \left(\frac{1-\alpha}{2}\right)\left(\frac{2k_F^3}{3\pi^2}\right)$$

With these definitions we have

$$k_F^{(n)} = (1+\alpha)^{1/3} k_F; \quad k_F^{(p)} = (1-\alpha)^{1/3} k_F$$

Now the total energy per nucleon is given by

$$\frac{E}{A}(\rho,\alpha) = \left(\frac{N}{A}\right)\frac{E}{N}(k_F^{(n)}) + \left(\frac{Z}{A}\right)\frac{E}{Z}(k_F^{(p)})$$

$$\boxed{\frac{E}{A}(\rho,\alpha) = \left(\frac{1+\alpha}{2}\right)\frac{E}{N}(k_F^{(n)}) + \left(\frac{1-\alpha}{2}\right)\frac{E}{Z}(k_F^{(p)})}$$

THE NUCLEAR SYMMETRY ENERGY: LIMITS

- NON-RELATIVISTIC LIMIT

$$\frac{E}{A}(\rho,\alpha)=\left(\frac{1+\alpha}{2}\right)\left[m+\frac{3}{5}\epsilon_F^{(n)}\right]+\left(\frac{1-\alpha}{2}\right)\left[m+\frac{3}{5}\epsilon_F^{(p)}\right]$$

$$=\left(\frac{1+\alpha}{2}\right)\left[m+\frac{3}{10}(1+\alpha)^{2/3}\frac{k_F^2}{m}\right]$$

$$+\left(\frac{1-\alpha}{2}\right)\left[m+\frac{3}{10}(1-\alpha)^{2/3}\frac{k_F^2}{m}\right]$$

$$=m+\left(\frac{3k_F^2}{10m}\right)\left[\frac{(1+\alpha)^{5/3}+(1-\alpha)^{5/3}}{2}\right]$$

$$f_{NR}(\alpha)\equiv\frac{1}{2}\left[(1+\alpha)^{5/3}+(1-\alpha)^{5/3}\right]$$

$$=1+\frac{5}{9}\alpha^2+\frac{5}{243}\alpha^4+\ldots$$

HENCE

$$\frac{E}{A}(\rho,\alpha)=\left(m+\frac{3k_F^2}{10m}\right)+\left(\frac{k_F^2}{6m}\right)\alpha^2+\ldots$$

SYMMETRIC NUCLEAR MATTER SYMMETRY ENERGY

- RELATIVISTIC LIMIT

$$\frac{E}{A}(\rho,\alpha)=\left(\frac{1+\alpha}{2}\right)\left[\frac{3}{4}(1+\alpha)^{1/3}k_F+\frac{3}{4}(1-\alpha)^{1/3}k_F\right]$$

$$=\frac{3}{4}k_F\left[\frac{(1+\alpha)^{4/3}+(1-\alpha)^{4/3}}{2}\right]$$

$$f_{UR}(\alpha)\equiv\frac{1}{2}\left[(1+\alpha)^{4/3}+(1-\alpha)^{4/3}\right]=1+\frac{2}{9}\alpha^2+\frac{5}{243}\alpha^4+\ldots$$

HENCE

$$\frac{E}{A}(\rho,\alpha)=\frac{3}{4}k_F+\left(\frac{k_F}{6}\right)\alpha^2+\ldots$$

SYMMETRIC N.M. SYMMETRY ENERGY

GIVEN THAT $|\alpha| \leq 1$ IT IS OFTEN CONVENIENT TO EXPAND THE ENERGY PER NUCLEON IN POWERS OF α. SINCE NO DISTINCTION HAS BEEN MADE BETWEEN NEUTRONS AND PROTONS, ONLY EVEN POWERS OF α APPEAR IN THE EXPANSION. THAT IS,

$$\mathcal{E}(\rho,\alpha) = \mathcal{E}_{SNM}(\rho) + \alpha^2 S(\rho) + \alpha^4 S_4(\rho) + \ldots$$

WHERE $\mathcal{E}_{SNM}(\rho)$ WAS ALREADY COMPUTED AND

$$\boxed{\begin{aligned} S(\rho) &= \frac{k_F^2}{6\epsilon_F} \; ; \; \text{\underline{IS THE SYMMETRY ENERGY}} \\ S_4(\rho) &= \frac{k_F^2}{648\epsilon_F^5}\left[3k_F^4 + 3k_F^2\epsilon_F^2 + 4\epsilon_F^4\right] \end{aligned}}$$

THE SYMMETRY ENERGY IS THE "PENALTY" IMPOSED ON THE SYSTEM FOR TURNING PROTONS INTO NEUTRONS; THIS IS A PURE QUANTUM MECHANICAL EFFECT ("PAULI EXCLUSION PRINCIPLE"). IN FACT, TO A VERY GOOD APPROXIMATION THE SYMMETRY ENERGY REPRESENTS THE ENERGY COST OF TURNING ALL PROTONS INTO NEUTRONS. THAT IS,

$$S(\rho) \equiv \frac{1}{2}\left(\frac{\partial^2 \mathcal{E}(\rho,\alpha)}{\partial \alpha^2}\right)_{\alpha=0} \simeq \underbrace{\mathcal{E}_{PNM}(\rho)}_{\alpha=1} - \underbrace{\mathcal{E}_{SNM}(\rho)}_{\alpha=0}$$

THE DENSITY DEPENDENCE OF THE NUCLEAR SYMMETRY ENERGY IS POORLY KNOWN AND CRITICAL TO MANY INTERESTING PHENOMENA IN NUCLEAR AND NEUTRON-STAR STRUCTURE.

Nuclear Symmetry Energy: Relativistic Free Fermi Gas

restart :

$\varepsilon F := \sqrt{kF^2 + m^2}$:

$E := \frac{3}{8} \cdot \left(\frac{\varepsilon F}{kF^2} \cdot \left(kF^2 + \varepsilon F^2\right) - \frac{m^4}{kF^3} \cdot \ln\left(\frac{(kF + \varepsilon F)}{m} \right) \right) - m$: # *Energy per particle*

$kp := (1-\alpha)^{\frac{1}{3}} \cdot kF : kn := (1+\alpha)^{\frac{1}{3}} \cdot kF$:

$Ep := \frac{(1-\alpha)}{2} \cdot subs(kF = kp, E)$: # *proton energy per nucleon*

$En := \frac{(1+\alpha)}{2} \cdot subs(kF = kn, E)$: # *neutron energy per nucleon*

$E0 := Ep + En$: # *total energy per nucleon*

$Epnm := subs(\alpha = 1, En)$: # *pure nuclear matter*

$Esnm := simplify(coeftayl(E0, \alpha = 0, 0))$: # *symmetric nuclear matter*

$Symm := simplify(coeftayl(E0, \alpha = 0, 2))$: # *symmetry energy*

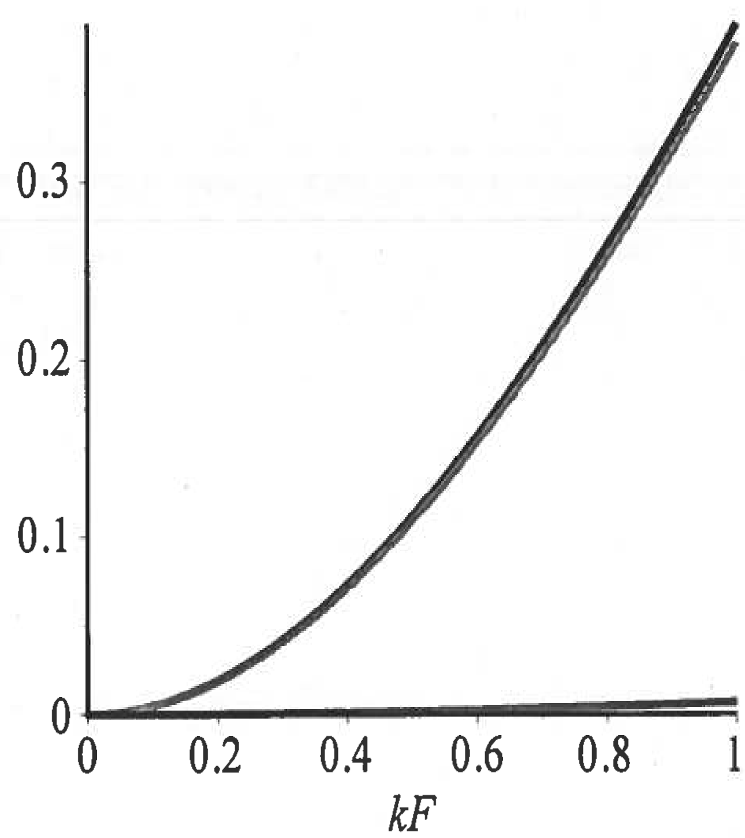
$S4 := simplify(coeftayl(E0, \alpha = 0, 4))$: # *correction to symmetry energy*

Symm, *S4*;

$$\frac{1}{6} \frac{kF^2}{\sqrt{kF^2 + m^2}}, \frac{1}{648} \frac{\left(10\, kF^4 + 11\, kF^2\, m^2 + 4\, m^4\right) kF^2}{\left(kF^2 + m^2\right)^{5/2}} \qquad (1)$$

$m := 1$:

plot([*Epnm*, *Esnm* + *Symm*, *S4*], *kF* = 0 ..1, *color* = [*black*, *red*, *blue*], *thickness* = 2);



- A THREE COMPONENT FREE FERMI GAS: "CHEMICAL (OR BETA) EQUILIBRIUM"

IN NEUTRON STARS TIME IS ON YOUR SIDE; THAT IS THE SYSTEM EVOLVES INTO ITS ABSOLUTE GROUND STATE FOR EVERY GIVEN DENSITY. THAT IS, YOU DON'T GET TO SELECT THE NEUTRON-PROTON ASYMMETRY.

BECAUSE THE LONG-RANGE NATURE OF THE COULOMB FORCE, BESIDES CHEMICAL EQUILIBRIUM ONE MUST ALSO ENFORCE CHARGE NEUTRALITY; THAT IS, THE NUMBER OF ELECTRONS MUST EQUAL THE NUMBER OF PROTONS.

THAT IS, α IS OBTAINED BY MINIMIZING THE TOTAL ENERGY PER BARYON (INCLUDING) ELECTRONS. THAT IS,

$$\frac{E}{A}(\rho,\alpha) = \left(\frac{1+\alpha}{2}\right)\frac{E}{N}(k_F^{(n)}, m) + \left(\frac{1-\alpha}{2}\right)\left[\frac{E}{Z}(k_F^{(p)}, m) + \frac{E}{Z}(k_F^{(p)}, m_e)\right]$$

NOW, CHEMICAL EQUILIBRIUM, i.e. $\frac{\partial E/A}{\partial \alpha} = 0$ YIELDS THE EQUALITY OF THE CHEMICAL POTENTIALS. THAT IS,

$$\mu_n = \mu_p + \mu_e$$

OR

$$\sqrt{k_F^{(n)2} + m^2} = \sqrt{(k_F^{(p)})^2 + m^2} + \sqrt{(k_F^{(p)})^2 + m_e^2}$$

THIS YIELDS α AS A FUNCTION OF DENSITY... AND OF THE INDIVIDUAL MASSES.

IN GENERAL, THIS TRASCENDENTAL EQUATION MUST BE SOLVED NUMERICALLY. HOWEVER, IN THE (LIKELY UNREALISTIC) CASE THAT $k_F^{(n)} \gg m$ AND $k_F^{(p)} \gg m \gg m_e$ ONE OBTAINS:

$$k_n = 2k_p \quad \left(\text{WITH } k_n \equiv k_F^{(n)} \text{ AND } k_p \equiv k_F^{(p)}\right)$$

AND

$$\rho_n = 8\rho_p \rightarrow \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = \frac{7}{9} \quad \left(\text{OR } Y_p = \frac{1}{9}\right)$$

A FAIRLY REALISTIC CASE IS $k_p \gg m_e$ (ELECTRONS ARE ULTRARELATIVISTIC)

THEN,

$$\mu_n = \mu_p + k_p \rightarrow k_n^2 = 2k_p^2 + 2k_p \mu_p$$

OR

$$k_n^4 - 4k_n^2 k_p^2 - 4k_p^2 m^2 = 0$$

OR

$$\boxed{(1+\alpha)^{4/3} x_F^2 - 4(1-\alpha^2)^{2/3} x_F^2 - 4(1-\alpha)^{2/3} = 0}$$

WHERE $x_F \equiv k_F/m$. WE "SOLVE" THIS TRASCENDENTAL EQUATION BY WRITING $x_F = x_F(\alpha)$ RATHER THAN $\alpha = \alpha(x_F)$. THAT IS,

$$\boxed{x_F^2 = \frac{4(1-\alpha)^{2/3}}{(1+\alpha)^{4/3} - 4(1-\alpha^2)^{2/3}}}$$

CLEARLY, ONE REQUIRES THAT $(1+\alpha)^{4/3} - 4(1-\alpha^2)^{2/3} \geq 0$. THIS IMPLIES THAT

$$9\alpha^2 + 8\alpha - 7 = (3\alpha+3)(3\alpha - 7/3) \geq 0$$

OR IN THIS LIMIT OF ULTRARELATIVISTIC ELECTRONS:

$$\boxed{\alpha \geq 7/9 \leftrightarrow Y_p \leq 1/9}$$

NEUTRON-STAR MATTER IS NEUTRON RICH.

Simple Models of Infinite Nuclear Matter

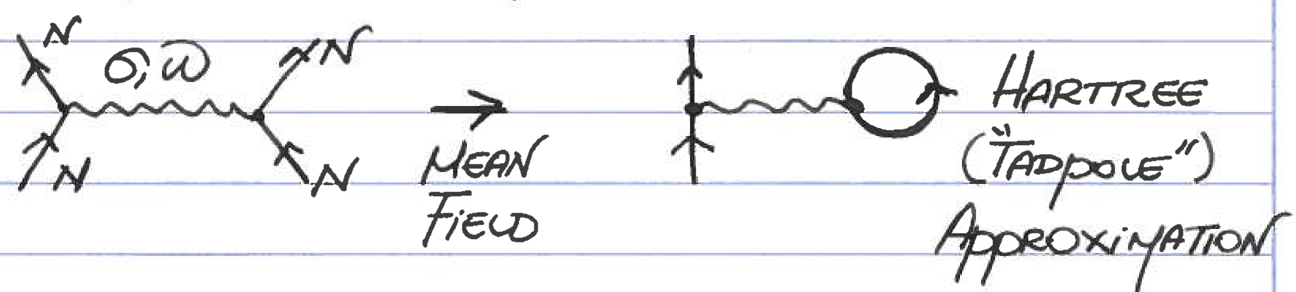
- The Walecka Model (J. D. Walecka, Ann. Phys. 83, 491 (1974))

Nucleons interact by the exchange of two massive mesons

Scalar-Isoscalar ϕ $\quad (J^\pi = 0^+; T=0)$

Vector-Isoscalar V^μ $\quad (J^\pi = 1^-; T=0)$

$$\mathcal{L}_{int} = \bar{\psi}\left(g_s\phi - g_v V_\mu \gamma^\mu\right)\psi$$



Assuming translationally invariant uniform nuclear matter, the self-consistent set of mean-field equation reads:

$$m_s^2\phi = g_s\rho_s \leftarrow \text{Lorentz-scalar nucleon density}$$

$$m_v^2 V_0 = g_v\rho_0 \leftarrow \text{Lorentz-vector nucleon density}$$

Nucleons are still Fermi gases — but no longer free. Dispersion Relation:

$$\epsilon(k) = \sqrt{k^2+M^2} \rightarrow \boxed{\epsilon(k) = \sqrt{k^2+M^{*2}} + g_v V_0}$$

where $M^* = M - g_s\phi$

Note that both the nucleon effective mass M^* and energy shift depend on the density.

"NUCLEAR SATURATION"

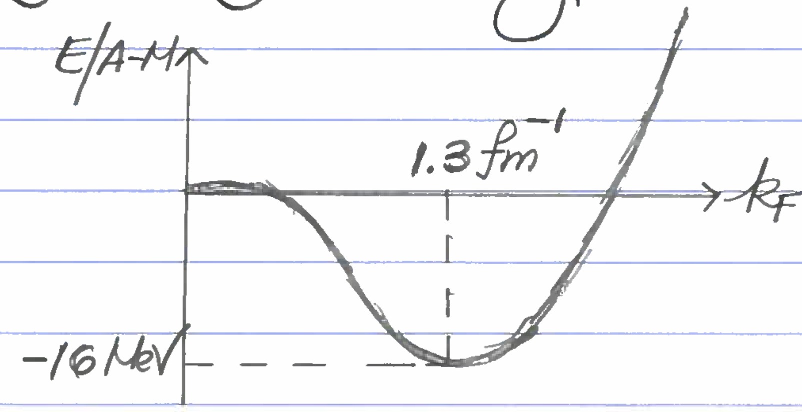
THE EXISTENCE OF AN EQUILIBRIUM DENSITY — NEAR THAT DISPLAYED BY HEAVY NUCLEI AT THE CENTER ($\sim 0.15\,fm^{-3}$) — IS A HALLMARK OF THE NUCLEAR DYNAMICS. IN THE WALECKA MODEL SATURATION EMERGES EXCLUSIVELY FROM THE LORENTZ CHARACTER OF THE MESON FIELDS. RECALL THAT THE SCALAR AND VECTOR FIELDS SATISFY:

$$\Phi = \frac{g_s^2}{m_s^2}\rho_s = \frac{g_s^2}{m_s^2}\sum_{\vec{k}s} \frac{M^*}{\sqrt{k^2+M^{*2}}}\,\Theta(k_F-|\vec{k}|)$$

$$W_0 = \frac{g_v^2}{m_v^2}\rho_0 = \frac{g_v^2}{m_v^2}\sum_{\vec{k}s} 1\cdot\Theta(k_F-|\vec{k}|)$$

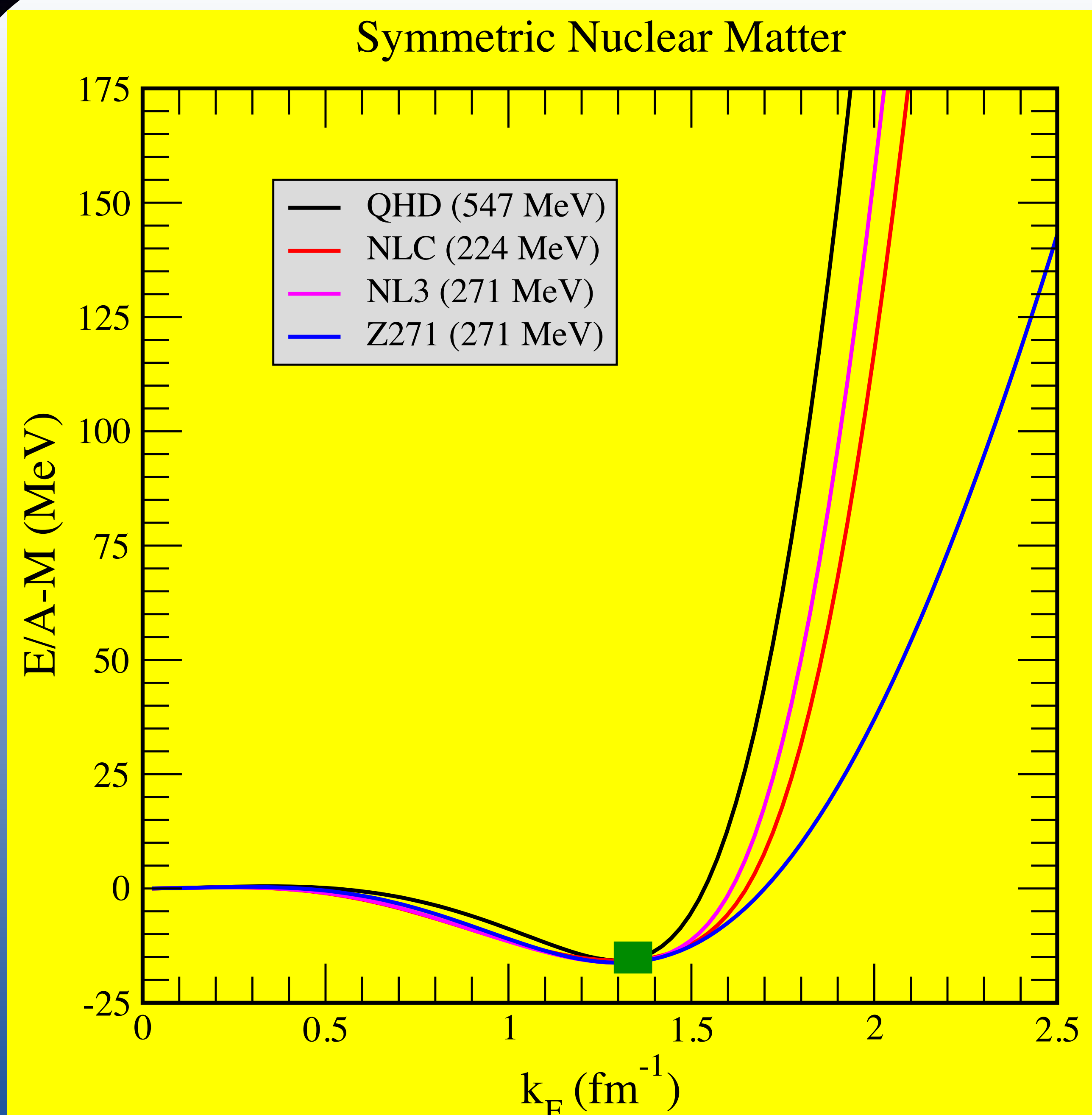
SO IF $\dfrac{g_s^2}{m_s^2} > \dfrac{g_v^2}{m_v^2}$ THEN AT VERY LOW DENSITIES THE ATTRACTION "WINS".

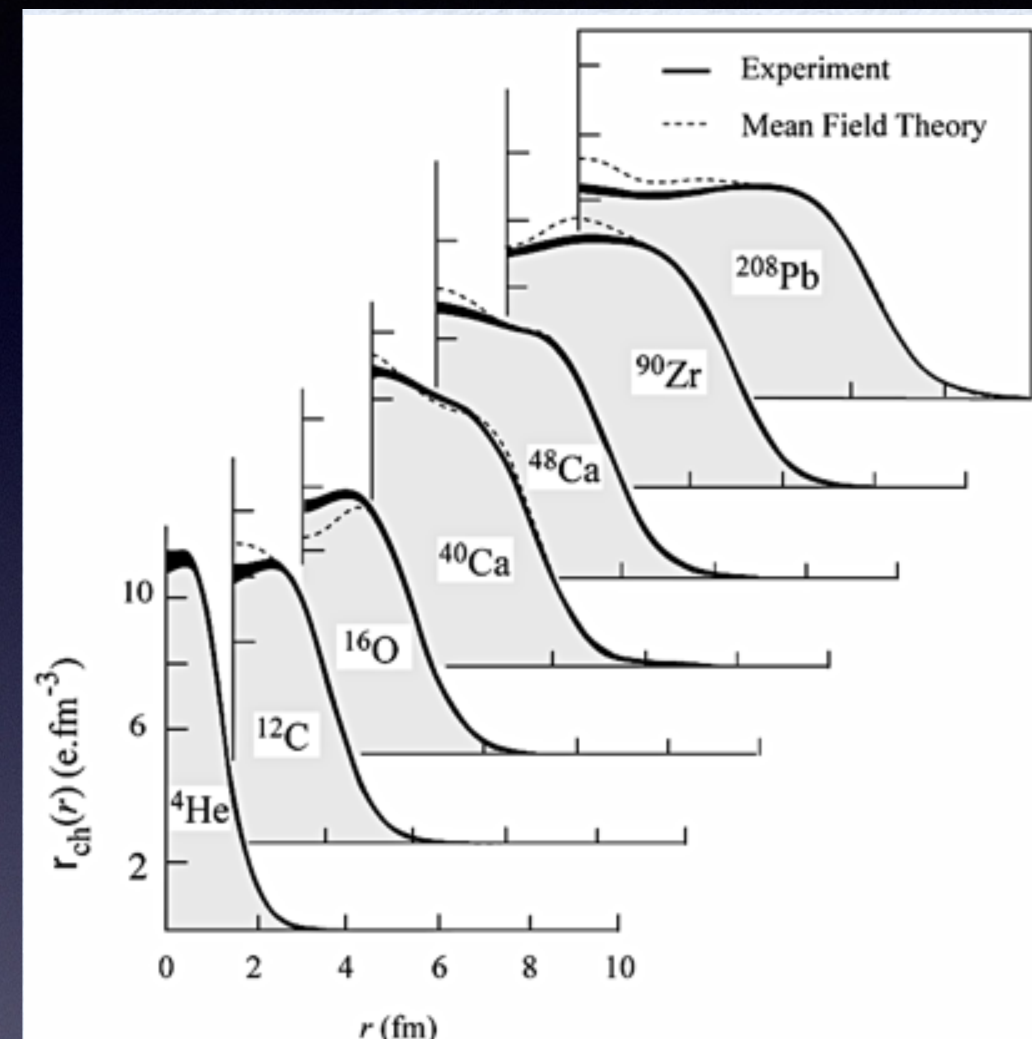
HOWEVER, AS THE DENSITY INCREASES, THE CONTRIBUTION TO THE SCALAR DENSITY FROM NUCLEONS WITH LARGE MOMENTA IS DILUTED BY THE M^*/E_k^* FACTOR. SO AT HIGH ENOUGH DENSITY THE VECTOR REPULSION WINS!



NUCLEAR-MATTER SATURATION IN THE WALECKA MODEL: A SIMPLE 2-PARAMETER MODEL CONSISTENT WITH SATURATION AND THAT ALLOWS FOR A COVARIANT AND CAUSAL EXTRAPOLATION.

Nuclear Saturation
A Hallmark of the Nuclear Dynamics





Uniform interior is a clear manifestation of nuclear saturation, namely the existence of an equilibrium density

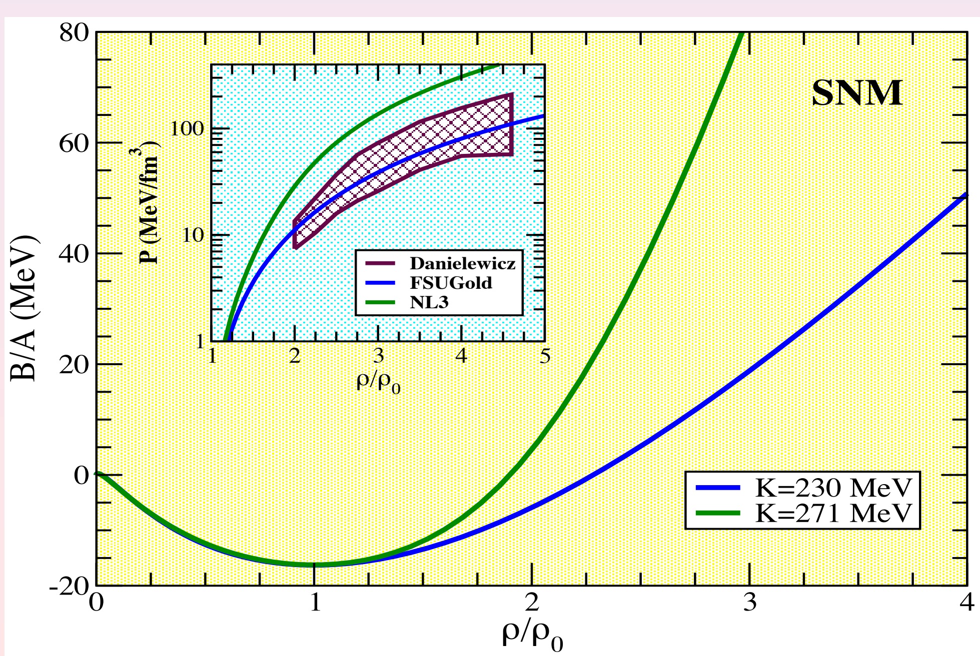
Relativistic Density Functional Theory: From Finite Nuclei to Neutron Stars

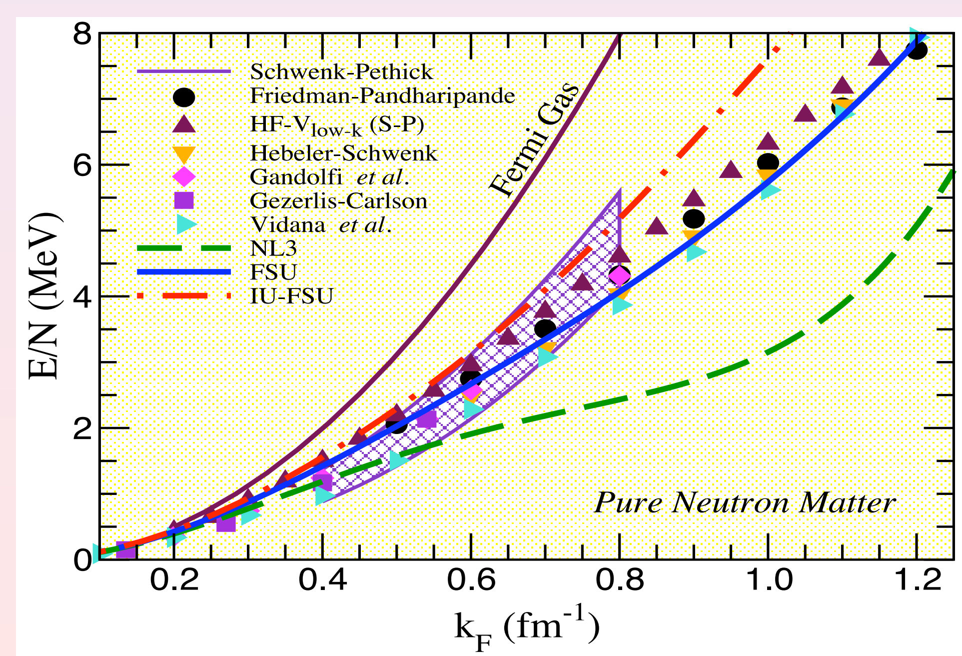
Relativistic Density Functional: The Effective Lagrangian Density

$$\mathscr{L}_{\rm int} = g_{\rm s}\bar{\psi}\psi\phi - g_{\rm v}\bar{\psi}\gamma^{\mu}\psi V_{\mu} - \frac{g_{\rho}}{2}\bar{\psi}\gamma^{\mu}\boldsymbol{\tau}\cdot\mathbf{b}_{\mu}\psi - e\bar{\psi}\gamma^{\mu}\tau_{p}\psi A_{\mu}$$
$$-\frac{\kappa}{3!}(g_{\rm s}\phi)^{3} - \frac{\lambda}{4!}(g_{\rm s}\phi)^{4} + \Lambda_{\rm v}(g_{\rm v}^{2}V^{\mu}V_{\mu})(g_{\rho}^{2}b^{\mu}b_{\mu}) + \frac{\zeta}{4!}g_{\rm v}^{4}(V_{\mu}V^{\mu})^{2}$$

The Encoding:

- $g_{\rm s}$ and $g_{\rm v}$: saturation properties (ρ_0 , ε_0 $\longrightarrow$ masses, charge radii)
- g_{ρ}: symmetry energy ($J \equiv a_4$ $\longrightarrow$ masses, charge radii)
- κ and λ: nuclear compressibility (K_0 $\longrightarrow$ ISGMR)
- $\Lambda_{\rm v}$: slope symmetry energy (L $\longrightarrow$ neutron skins, neutron-star radii)
- ζ: high-density component of EOS (limiting neutron-star mass)





IDEALLY, ONE WOULD LIKE TO DETERMINE THE MODEL PARAMETERS FROM FIRST PRINCIPLES. IN PRACTICE, AND IN THE SPIRIT OF EFFECTIVE FIELD THEORIES, ONE FITS TO DATA. INDEED, IF

$$C_s^2 \equiv \frac{g_s^2}{m_s^2} M^2 = 357.4; \quad C_v \equiv \frac{g_v^2}{m_v^2} M^2 = 273.8$$

SYMMETRIC NUCLEAR MATTER SATURATES AT AN EQUILIBRIUM DENSITY OF $k_F^0 = 1.30\,\text{fm}^{-1}$ (OR $\rho_0 \simeq 0.15\,\text{fm}^{-3}$) AND A BINDING ENERGY PER NUCLEON OF $\varepsilon_0 = 15.75$ MeV. HOWEVER, EVERY OTHER PROPERTY OF THE EOS BECOMES A PREDICTION; e.g., THE INCOMPRESSIBILITY (OR CURVATURE) AT SATURATION $K_0 \simeq 550$ MeV (MORE ON THIS LATER).

HOW ABOUT THE SYMMETRY ENERGY IN THE WALECKA MODEL? GIVEN THAT BOTH MESONS ARE ISOSCALAR, i.e., THEY COUPLE IDENTICALLY TO NEUTRONS AND PROTONS, THE SIMPLE RESULT IN PAGE 6 REMAINS VALID. THAT IS,

$$S(\rho) = \frac{k_F^2}{6E_F^*} = \frac{k_F^2}{6(k_F^2 + M^{*2})^{1/2}}$$

AT NUCLEAR MATTER SATURATION DENSITY

$$k_F^0 = 1.30\,\text{fm}^{-1} \text{ AND } \frac{M^*}{M} = 0.541 \text{ WE OBTAIN}$$

$$J \equiv S(\rho_0) = 19.27 \text{ MeV}.$$

VERY SMALL TO REPRODUCE PHENOMENOLOGY OF NUCLEI WITH A SIGNIFICANT NEUTRON EXCESS.

WHERE ARE WE TODAY?

"RELATIVISTIC DENSITY FUNCTIONAL THEORY"

ENORMOUS PROGRESS SINCE THE EARLY DAYS...

- $$\mathcal{L}_{int} = \bar{\psi}\left[g_s\phi - \left(g_v V_\mu + \frac{g_\rho}{2}\vec{\tau}\cdot\vec{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi - \frac{\kappa}{3!}\Phi^3 - \frac{\lambda}{4!}\Phi^4 + \frac{\zeta}{4!}(W_\mu W^\mu)^2 + \Lambda_v(B^\mu B_\mu)(W^\nu W_\nu)$$

- PARAMETERS OF THE MODEL CONSTRAINED BY PHYSICAL OBSERVABLES ONLY, SUCH AS NUCLEAR MASSES, CHARGE RADII, NUCLEAR EXCITATIONS, AND MAXIMUM NEUTRON-STAR MASSES. NO LONGER RELIANCE ON BULK PARAMETERS OF INFINITE NUCLEAR MATTER; THESE BECOME PREDICTIONS OF THE MODEL.

- ALL PREDICTIONS ACCOMPANIED BY THEORETICAL ERRORS AND CORRELATION COEFFICIENTS; TOWARDS A NEW ERA OF "UNCERTAINTY QUANTIFICATION."

- ONE-TO-ONE CORRESPONDENCE BETWEEN MODEL PARAMETERS AND FUNDAMENTAL PROPERTIES OF THE EQUATION OF STATE

$$E_{SNM}(\rho) = \epsilon_0 + \frac{1}{2}K_0 x^2 + \ldots \qquad x \equiv (\rho - \rho_0)/3\rho_0$$

$$S(\rho) = J + Lx + \ldots$$

$$(\rho_0, \epsilon_0, K_0, M_0^*, J, L) \leftrightarrow (g_s, g_v, g_\rho, \kappa, \lambda, \Lambda_v)$$