Accessing Quark Orbital Angular Momentum through GTMDs

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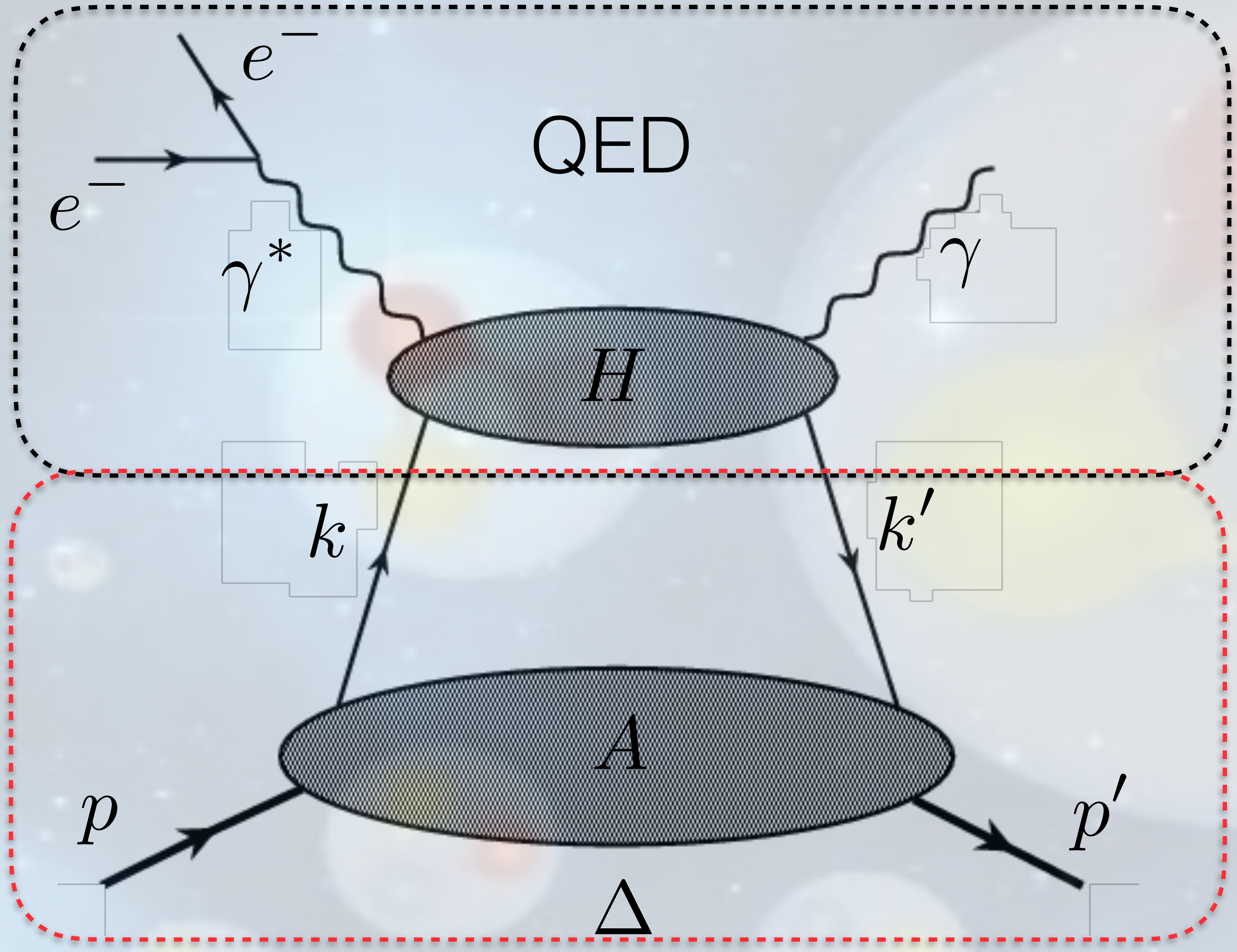
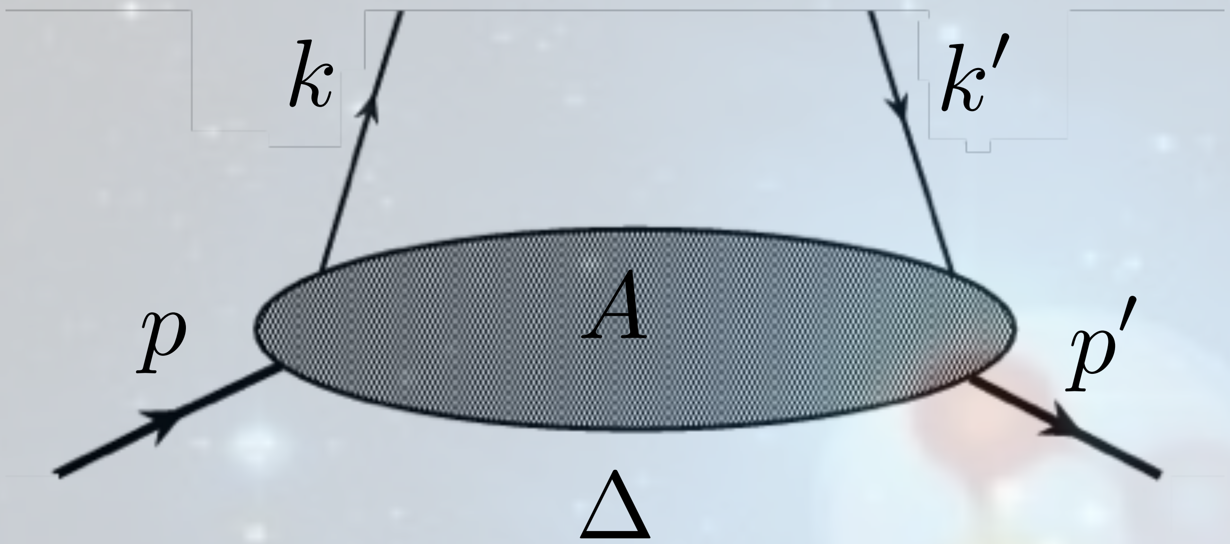
DVCS and Factorization (Lightning Quick Review)



- Factorization allows us to separate the hard scattering (H) from the non-perturbative term (A).

Kinematic Definitions



$$k = \bar{k} + \frac{\Delta}{2} \qquad p = P + \frac{\Delta}{2}$$

$$k' = \bar{k} - \frac{\Delta}{2} \qquad p' = P - \frac{\Delta}{2}$$

$$\bar{k} = \frac{k + k'}{2} \qquad P = \frac{p + p'}{2}$$

skewness

$$\xi = \frac{\Delta^+}{P^+} = 0$$

off-forwardness

$$\Longrightarrow \qquad t = -\Delta_T^2$$

How do we describe A?

Quark-Quark Correlation Function

$$\langle \Omega_f | T\phi(x)\phi(y) | \Omega_i \rangle$$

Amplitude for an excitation's propagation from x to y.

$$W^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big)\Gamma \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big)\psi\Big(\frac{1}{2}z\Big)|p,\Lambda\rangle$$

Describes the quark-quark "current" inside of the proton (the collinear term of the Feynman diagram).

$$F^{\Gamma}_{\lambda\lambda'}(P,x,\Delta,N) = \int dk^- d^2\vec{k_T} W^{\Gamma}_{\lambda\lambda'}(P,k,\Delta,N;\eta)$$

Integrating out the transverse components and implement the use of light cone coordinates.

$$F^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big)\Gamma \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big)\psi\Big(\frac{1}{2}z\Big)|p,\Lambda\rangle\Big|_{z^+=z_T=0}$$

 S. Meissner, A. Metz and M. Schlegel, JHEP 0908 (2009) 056.

Let's take a closer look …

$$F^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big) \Gamma \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big) \psi\Big(\frac{1}{2}z\Big) | p, \Lambda \rangle \Big|_{z^+ = z_T = 0}$$

$$\mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big) = P e^{-igt_a \int_{-\frac{1}{2}z}^{\frac{1}{2}z} dw A^+_a(w)}$$

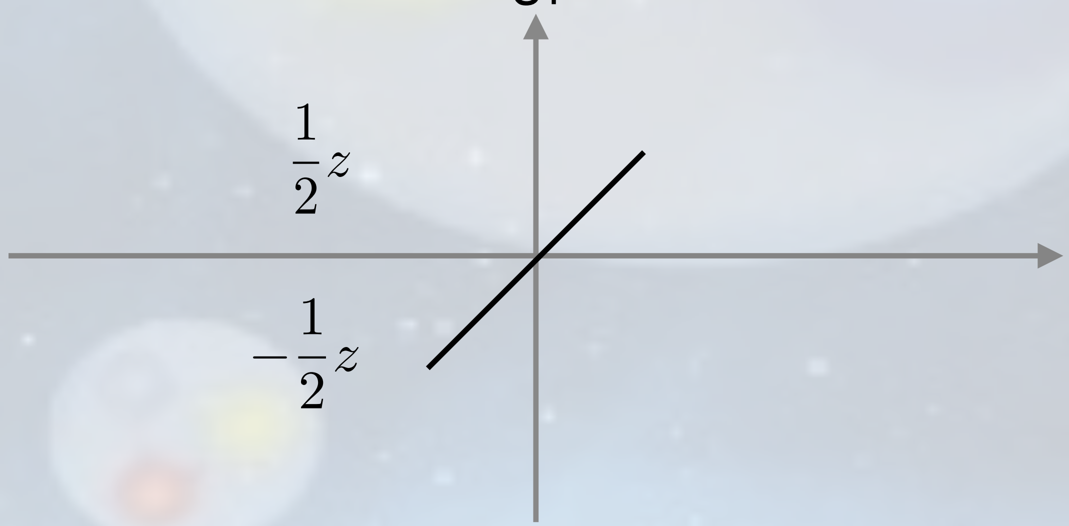
Connects the two spacetime points by all possible ordered paths of the gluon field

Definition of OAM produces different Wilson Lines.

Jaffe-Manohar

$$\mathcal{L}_{JM} = i\vec{r} \times \vec{\partial}$$

Ji



$$\mathcal{L}_{Ji} = i\vec{r} \times \vec{\mathcal{D}}$$

M. Burkardt, Phys. Rev. D 69, 057501 (2004)

Let’s take a closer look …

$$F^{\Gamma}_{\Lambda,\Lambda'} = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big)\Gamma \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big)\psi\Big(\frac{1}{2}z\Big)|p,\Lambda\rangle \Big|_{z^+=z_T=0}$$

Helicity Distribution

Γ is a function of the basis $\gamma^+, \gamma^+\gamma^5, i\sigma^{i+}$ we can determine the helicity distribution of the quarks through a specific projection operator defined as

$$\gamma^+(1 \pm \gamma^5)$$

This allows us to define a helicity amplitude of the quark current in the proton as

$$A_{\Lambda,\pm,\Lambda',\pm} = \int \frac{dz^-}{2\pi} e^{ik\cdot z^-} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big)\gamma^+(1\pm\gamma^5)\psi(\frac{1}{2}z\Big)|p,\Lambda\rangle$$

$$A_{\Lambda,\pm,\Lambda',\pm} = \int \frac{dz^-}{2\pi} e^{ik\cdot z^-} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big)\gamma^+(1\pm\gamma^5)\psi(\frac{1}{2}z\Big)|p,\Lambda\rangle$$

$$F^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2}(A_{\Lambda,+,\Lambda',+} + A_{\Lambda,-,\Lambda',-})$$

unpolarized quarks

$$F^{\gamma^+\gamma^5}_{\Lambda,\Lambda'} = \frac{1}{2}(A_{\Lambda,+,\Lambda',+} - A_{\Lambda,-,\Lambda',-})$$

polarized quarks

Orbital angular momentum contribution requires unpolarized quarks or else the spin contributes as well.

$$\mathcal{L} = A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}$$

Problem!!!

No GPD, PDF, or TMD at leading twist contributes to OAM by this definition

A. Courtoy, G. R. Goldstein, J. O. G. Hernandez, S. Liuti and A. Rajan, Phys. Lett. B 731, 141 (2014)

The Fix?

Add in partonic transverse momentum into the correlator (generalize)

$$F^{\gamma^+}_{\Lambda,\Lambda'} = \int \frac{dz^- d^2\vec{z}_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' | \bar{\psi}\Big(-\frac{1}{2}z\Big) \Gamma \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z | n\Big) \psi\Big(\frac{1}{2}z\Big) | p, \Lambda\rangle \Big|_{z^+=0}$$

What does this do for us?

We can parameterize our correlator in terms of GTMDs (Generalized Transverse Momentum Distribution functions)

$$W^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{u}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}^+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}^+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}\right]u(p,\Lambda)$$

We wish to describe OAM, so which GTMD will describe this and can we reduce it?

$$\frac{1}{2\bar{p}^+}\bar{u}(p',\Lambda')\gamma^+ u(p,\Lambda) = \frac{\sqrt{1-\xi}}{1-\xi/2}\delta_{\Lambda,\Lambda'}$$

$$\frac{\sqrt{1-\xi}}{2\bar{p}^+}\bar{u}(p',\Lambda')\frac{i\sigma^{+\mu}}{2M}\Delta_\mu u(p,\Lambda) = \frac{-\xi^2/4}{1-\xi/2} + \frac{-\Lambda\Delta^1 - i\Delta^2}{2M}\delta_{\Lambda\Lambda'}$$

$$\xi = 0$$

using these relations we can put the correlator in terms of proton non-flip and proton flip GTMD contributions.

$$= \Big(F_{11} + i\Lambda\frac{\vec{k_T}\times\vec{\Delta_T}}{M^2}F_{14}\Big)\delta_{\Lambda\Lambda'} + \Big(\frac{\Lambda\Delta^1 + i\Delta^2}{2M}(2F_{13} - F_{11}) + \frac{\Lambda k^1 + ik^2}{M}F_{12}\Big)\Delta_{\Lambda\Lambda'}$$

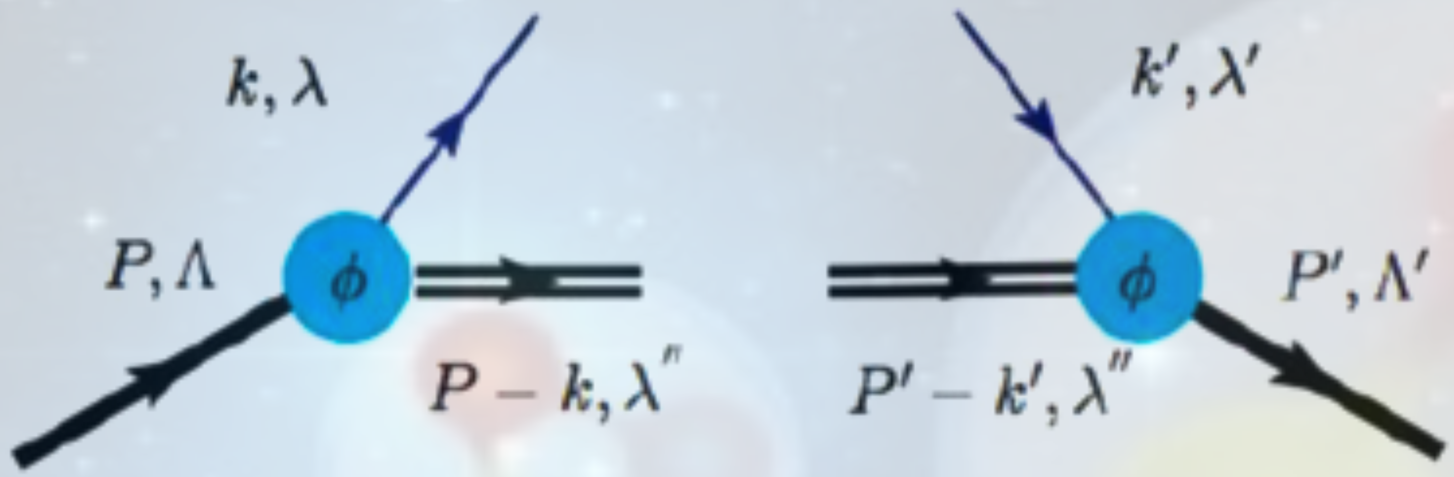
To describe OAM, we look at the proton helicity non-flip case, and looking at the form of the OAM definition

$$\mathcal{L} = A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}$$

Opposite hadronic helicities subtract and by definition of the correlator, we see that only the GTMD F_{14} survives.

S. Meissner, A. Metz and M. Schlegel, JHEP 0908 (2009) 056.

Diquark Model



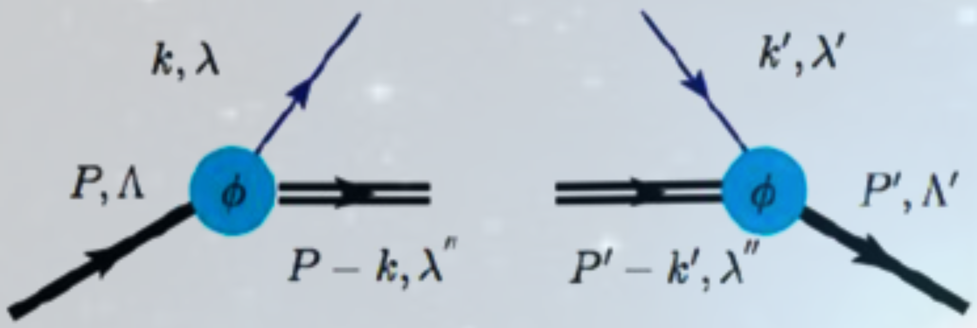
The proton dissociates into a quark and recoiling mass system with quantum numbers of a diquark

$$\Gamma = g_s \frac{k^2 - m^2}{(k^2 - M_\Lambda^2)^2}$$

Vertex coupling of proton-quark-diquark.
We can describe a vector or scalar diquark through variation of the mass parameters.

G. R. Goldstein, J.O.G. Hernandez, S. Liuti, Phys. Rev. D 84, 034007 (2011)

Diquark Model Calculation of F_{14}



$$A_{\Lambda\lambda,\Lambda'\lambda'} = \phi^*_{\Lambda'\lambda'}(k', P')\phi_{\Lambda\lambda}(k, P)$$

$$\phi_{\Lambda\lambda}(k, P) = \Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2 - m^2}$$

$$\phi^*_{\Lambda'\lambda'}(k', P') = \Gamma(k)\frac{\bar{U}(P',\Lambda')u(k',\lambda')}{k'^2 - m^2}$$

Relevant helicity amplitudes needed for OAM calculation

$$A_{++,++} = \phi^*_{++}(k', P')\phi_{++}(k, P)$$
$$A_{+-,+-} = \phi^*_{+-}(k', P')\phi_{+-}(k, P)$$
$$A_{-+,-+} = \phi^*_{-+}(k', P')\phi_{-+}(k, P)$$
$$A_{--,--} = \phi^*_{--}(k', P')\phi_{--}(k, P)$$

$$\bar{u}(k,\lambda)U(P,\Lambda) = \sqrt{k^+p^+}\Big(\frac{M}{p^+} + \frac{m}{k^+}\Big)\delta_{\Lambda,\lambda} + \Lambda\sqrt{k^+p^+}\Big(\frac{p_\Lambda}{p^+} - \frac{k^*_\lambda}{k^+}\Big)\delta_{\Lambda,-\lambda}$$

$$k_\lambda = \bar{k}_\lambda - \Delta_\lambda/2 \quad \Longrightarrow \quad k_\lambda = \bar{k}^1 + i\lambda\bar{k}^2 - (\Delta^1 + i\lambda\Delta^2)/2$$

$$p_\Lambda = \bar{p}_\Lambda - \Delta_\Lambda/2$$

$$\bar{p}_\Lambda = 0 \quad \Longrightarrow \quad p_\Lambda = -(\Delta^1 + i\Lambda\Delta^2)/2$$

$$\because\ \xi = 0,\ p^+ = \bar{p}^+ \wedge \text{ and } k^+ = \bar{k}^+ \qquad \bar{k}^+ = x\bar{p}^+$$

$$\bar{u}(k,\lambda)U(p,\Lambda) = \sqrt{x}\Big[\Big(M + \frac{m}{x}\Big)\delta_{\Lambda,\lambda} - \Lambda\Big(\frac{\Delta_\Lambda}{2} + \frac{\bar{k}^*_\lambda - \Delta^*_\lambda/2}{x}\Big)\delta_{\Lambda,-\lambda}\Big]$$

$$\bar{k}^*_\lambda = \bar{k}^1 - i\lambda\bar{k}^2 \qquad \overset{\delta_{\Lambda,-\lambda}}{\Longrightarrow} \qquad \bar{k}^*_\lambda = \bar{k}_\Lambda$$

$$\Delta^*_\lambda = \Delta^1 - i\lambda\Delta^2/2 \qquad \Delta^*_\lambda = \Delta_\Lambda$$

A. Rajan, S. Liuti, arXiv:1602.00160

$$\bar{u}(k,\lambda)U(p,\Lambda) = \sqrt{x}\Big[\Big(M + \frac{m}{x}\Big)\delta_{\Lambda,\lambda} - \Lambda\Big(\frac{\Delta_\Lambda}{2} + \frac{\bar{k}^*_\lambda - \Delta^*_\lambda/2}{x}\Big)\delta_{\Lambda,-\lambda}\Big]$$

$$\bar{k}^*_\lambda = \bar{k}_\Lambda \quad \Downarrow \quad \Delta^*_\lambda = \Delta_\Lambda$$

$$\bar{u}(k,\lambda)U(p,\Lambda) = \sqrt{x}\Big(M + \frac{m}{x}\Big)\delta_{\Lambda,\lambda} - \Lambda\sqrt{x}\Big(\frac{\Delta_\Lambda}{2}\Big(1 - \frac{1}{x}\Big) + \frac{\bar{k}_\Lambda}{x}\Big)\delta_{\Lambda,-\lambda}$$

Now using our definition of $\phi_{\Lambda,\lambda}$,we can construct the helicity amplitudes we need.

$$\phi_{++}(k,P) = \frac{\sqrt{x}}{k^2 - m^2}\left(M + \frac{m}{x}\right)\Gamma(k)$$

$$\phi_{+-}(k,P) = -\frac{\sqrt{x}}{k^2 - m^2}\left[\frac{(\Delta^1 + i\Delta^2)}{2}\Big(1 - \frac{1}{x}\Big) + \frac{\bar{k}^1 + i\bar{k}^2}{x}\right]\Gamma(k)$$

$$\phi{-+}(k,P) = \frac{\sqrt{x}}{k^2 - m^2}\left[\frac{(\Delta^1 - i\Delta^2)}{2}\Big(1 - \frac{1}{x}\Big) + \frac{\bar{k}^1 - i\bar{k}^2}{x}\right]\Gamma(k)$$

$$\phi_{--}(k,P) = \frac{\sqrt{x}}{k^2 - m^2}\left(M + \frac{m}{x}\right)\Gamma(k)$$

Similar calculation for the k',P'

$$\Delta_\perp \to -\Delta_\perp$$

$$\phi_{++}(k',P') = \frac{\sqrt{x}}{k'^2 - m^2}\left(M + \frac{m}{x}\right)\Gamma(k')$$

$$\phi_{+-}(k',P') = \frac{\sqrt{x}}{k'^2 - m^2}\left[\frac{(\Delta^1 + i\Delta^2)}{2}\left(1 - \frac{1}{x}\right) - \frac{\bar{k}^1 + i\bar{k}^2}{x}\right]\Gamma(k')$$

$$\phi{-+}(k',P') = \frac{\sqrt{x}}{k'^2 - m^2}\left[\frac{-(\Delta^1 - i\Delta^2)}{2}\left(1 - \frac{1}{x}\right) + \frac{\bar{k}^1 - i\bar{k}^2}{x}\right]\Gamma(k')$$

$$\phi_{--}(k',P') = \frac{\sqrt{x}}{k'^2 - m^2}\left(M + \frac{m}{x}\right)\Gamma(k')$$

Now we can calculate some helicity amplitudes

$$A_{\Lambda\lambda,\Lambda'\lambda'} = \phi^*_{\Lambda'\lambda'}(k',P')\phi_{\Lambda\lambda}(k,P)$$

$$
\begin{aligned}
A_{++,++} &= \phi^*_{++}(k',P')\phi_{++}(k,P) \\
&= \frac{\sqrt{x}}{k'^2-m^2}\left(M+\frac{m}{x}\right)\Gamma(k')\frac{\sqrt{x}}{k^2-m^2}\left(M+\frac{m}{x}\right)\Gamma(k) \\
&= \frac{x}{(k'^2-m^2)(k^2-m^2)}\left(M+\frac{m}{x}\right)^2\Gamma(k')\Gamma(k) \\
&= A_{--,--}
\end{aligned}
$$

$$
\begin{aligned}
A_{+-,+-} &= \phi^*_{+-}(k',P')\phi_{+-}(k,P) \\
&= \frac{\sqrt{x}}{k'^2-m^2}\left[\frac{(\Delta^1-i\Delta^2)}{2}\left(1-\frac{1}{x}\right)-\frac{\bar{k}^1-i\bar{k}^2}{x}\right]\Gamma(k') \\
&\times \frac{-\sqrt{x}}{k^2-m^2}\left[\frac{(\Delta^1+i\Delta^2)}{2}\left(1-\frac{1}{x}\right)+\frac{\bar{k}^1+i\bar{k}^2}{x}\right]\Gamma(k) \\
&= -\frac{x}{(k'^2-m^2)(k^2-m^2)}\left[\frac{(\Delta^1-i\Delta^2)}{2}\left(1-\frac{1}{x}\right)-\frac{\bar{k}^1-i\bar{k}^2}{x}\right] \\
&\times \left[\frac{(\Delta^1+i\Delta^2)}{2}\left(1-\frac{1}{x}\right)+\frac{\bar{k}^1+i\bar{k}^2}{x}\right]\Gamma(k')\Gamma(k) \\
&= A^*_{-+,-+}
\end{aligned}
$$

Quark OAM

$$\mathcal{L} = A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}$$

$$A_{++,++} = A_{--,--}$$
$$A_{+-,+-} = A^*_{-+,-+}$$

Using these relations we can see that the real parts completely cancel from this equation and only the imaginary part survives of the mixed helicity amplitudes.

$$A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--} = \boxed{2i\,\mathrm{Im}(A_{+-,+-})}$$

$$\begin{aligned}
2i\,\mathrm{Im}(A_{+-,+-}) &= 2i\frac{-x}{(k'^2-m^2)(k^2-m^2)}\Gamma(k')\Gamma(k)\,\mathrm{Im}\left[\frac{(\Delta^1 - i\Delta^2)}{2}\left(1-\frac{1}{x}\right) - \frac{\bar{k}^1 - i\bar{k}^2}{x}\right] \\
&\times\left[\frac{(\Delta^1 + i\Delta^2)}{2}\left(1-\frac{1}{x}\right) + \frac{\bar{k}^1 + i\bar{k}^2}{x}\right] \\
&= 2i\frac{-x}{(k'^2-m^2)(k^2-m^2)}\Gamma(k')\Gamma(k)\left[\left(-\frac{\Delta^2}{2}\left(1-\frac{1}{x}\right)+\frac{\bar{k}^2}{x}\right)\left(\frac{\Delta^1}{2}\left(1-\frac{1}{x}\right)+\frac{\bar{k}^1}{x}\right)\right. \\
&\left.+\left(\frac{\Delta^1}{2}\left(1-\frac{1}{x}\right)-\frac{\bar{k}^1}{x}\right)\left(\frac{\Delta^2}{2}\left(1-\frac{1}{x}\right)+\frac{\bar{k}^2}{x}\right)\right] \\
&= \frac{2i\Gamma(k)\Gamma(k')}{(k'^2-m^2)(k^2-m^2)}\left(1-\frac{1}{x}\right)(\Delta^2\bar{k}^1 - \Delta^1\bar{k}^2)
\end{aligned}$$

$$= \Big(F_{11} \boxed{+ i\Lambda \frac{\vec{k_T} \times \vec{\Delta_T}}{M^2} F_{14}} \Big) \delta_{\Lambda\Lambda'} + \Big(\frac{\Lambda\Delta^1 + i\Delta^2}{2M} (2F_{13} - F_{11}) + \frac{\Lambda k^1 + ik^2}{M} F_{12} \Big) \Delta_{\Lambda\Lambda'}$$

$$F_{14} = \frac{-iM^2\mathcal{L}}{2(\bar{k}^1\Delta^2 - \bar{k}^2\Delta^1)}$$

plugging in our definition for $\mathcal{L}$ that we found before we find an expression for the GTMD

$$F_{14} = \frac{M^2\Gamma(k)\Gamma(k')}{(k'^2 - m^2)(k^2 - m^2)} \Big(1 - \frac{1}{x} \Big)$$

This GTMD describes the quark orbital angular momentum in the diquark model!

Further Work and Study

- Understand what to look for in experiment, how to measure this GTMD, and how to relate it to twist three GPDs
- Fock Expansion higher order terms
- Flavor Composition (S=1 vector diquark v. S=0 scalar diquark)
- Inclusion of final state interactions between the free quark and the spectator diquark
- Quark-gluon interactions
- Look at the evolution of these equations

Special Thanks!

HUGS coordinators, Simonetta Liuti, Abha Rajan