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Theory and Phenomenology of Generalized Parton Distributions

Nabil Chouika

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- Experimental access to GPDs
- Nucleon imaging
- Theoretical constraints on GPDs

2 Ab initio Model for GPDs

- From Dyson-Schwinger equations to GPDs
- Radon Transform

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Experimental acce	ess (example: DVCS		



Deeply Virtual Compton Scattering channel of photon electroproduction.

$$\Delta = P_2 - P_1 \ , \ t = \Delta^2 < 0$$

$$Q^2 = -q_1^2 > 0$$

$$P = rac{1}{2} \left(P_1 + P_2
ight) \; , \; \xi = -rac{\Delta^+}{2 \, P^+}$$



Compton Form Factors: (Belitsky et al., 2002)

E, Ê (x,ξ,t)

$$\mathcal{F}\left(\xi, t, Q^{2}\right) = \int_{-1}^{1} \mathrm{d}x \, C\left(x, \xi, \alpha_{S}\left(\mu_{F}\right), \frac{Q}{\mu_{F}}\right) F\left(x, \xi, t, \mu_{F}\right), \qquad (1)$$

where $F \in \{H, E, \tilde{H}, \tilde{E}, ...\}$ is a Generalized Parton Distribution.

 $N\left(P_{1}
ight)$

 $P = \frac{1}{2} (P_1 + P_2) , \ \xi = -\frac{\Delta^+}{2 P^+}$

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Nucleon imaging			

• Correlation of the longitudinal momentum and the transverse position of the partons inside the hadron.

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Nucleon imaging			

- Correlation of the longitudinal momentum and the transverse position of the partons inside the hadron.
- Probability density (Fourier transform of GPD): (Burkardt, 2000)

$$q\left(x,\vec{b_{\perp}}\right) = \int \frac{\mathrm{d}^{2}\vec{\Delta_{\perp}}}{\left(2\pi\right)^{2}} e^{-i\vec{b_{\perp}}\cdot\vec{\Delta_{\perp}}} H^{q}\left(x,0,-\vec{\Delta_{\perp}}^{2}\right) .$$
(2)

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Figure: Hadron tomography.

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Theoretical const	raints on GPDs		

• Support: $x, \xi \in [-1, 1]$.

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- Polynomiality:

$$\int_{-1}^{1} \mathrm{d}x \, x^m \, H(x,\xi,t) = \text{Polynomial in } \xi \,. \tag{3}$$

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From Lorentz invariance.

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- From Lorentz invariance.
- Positivity:

$$H^{q}(x,\xi,t) \leq \sqrt{q\left(rac{x-\xi}{1-\xi}
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Cauchy-Schwarz theorem in Hilbert space.

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(Maris and Roberts, 1997)

equations

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$$\downarrow$$
 \downarrow

Hadronic Fock space

$$\Psi\left(k^{+}, \vec{k_{\perp}}, P
ight) \ \propto \int \mathrm{d}k^{-} \operatorname{Tr}\left[\gamma^{+} \gamma_{5} \chi\left(k, P
ight)
ight]$$

Lightcone Wavefunction Ψ

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Lightcone Wavefunction Ψ

Overlap of LCWF



Generalized Parton Distribution $H(x, \xi, t)$

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Double Distributi	ion (DD)		

• Overlap representation: positivity fulfilled!

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- Overlap representation: **positivity** fulfilled!
- DGLAP region only: |x| > |ξ|. Need ERBL to complete polynomiality.

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Double Distributi	on (DD)		

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- Model fulfills Lorentz invariance:
 - ▶ Double distribution $f(\beta, \alpha)$ (1CDD): (Belitsky et al., 2001)

$$H(x,\xi) = x \int_{|\alpha|+|\beta| \leq 1} \mathrm{d}\beta \,\mathrm{d}\alpha \,f(\beta,\alpha) \,\delta(x-\beta-\alpha\xi) \;.$$

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• Exists and is unique (up to an ambiguity on $\beta = 0$ axis).

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$$H(x,\xi) = x \int_{|\alpha|+|\beta| \leq 1} \mathrm{d}\beta \,\mathrm{d}\alpha \, f(\beta,\alpha) \,\,\delta(x-\beta-\alpha\xi) \,\,.$$

- Exists and is unique (up to an ambiguity on $\beta = 0$ axis).
- Reconstruct GPD everywhere.

(Moutarde, 2015)

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Inverse Radon 7	Fransform		

Find $f(\beta, \alpha)$ on rhombus $\{|\alpha| + |\beta| \le 1\}$ such that

$$H(x,\xi)|_{\text{DGLAP}} = x \int d\beta \, d\alpha \, f(\beta,\alpha) \, \delta(x-\beta-\alpha\xi) \; .$$

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Inverse Radon Ti	ransform		

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 Inverse Radon Transform: (mildly) ill-posed problem!



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- Inverse Radon Transform: (mildly) ill-posed problem!
- Limited angle inverse transform $(|\xi| < 1)$: severely ill-posed!



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Inverse Radon Tra	ansform		

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$$H(x,\xi)|_{\mathrm{DGLAP}} = x \int \mathrm{d}\beta \,\mathrm{d}\alpha \, f(\beta,\alpha) \,\,\delta(x-\beta-\alpha\xi) \;.$$

- Inverse Radon Transform: (mildly) ill-posed problem!
- Limited angle inverse transform $(|\xi| < 1)$: severely ill-posed!
- Access only to a limited region (DGLAP: |x| > |ξ|): things are probably worse!



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Inverse Radon T	ransform		

Find $f(\beta, \alpha)$ on rhombus $\{|\alpha| + |\beta| \le 1\}$ such that

$$H(x,\xi)|_{\mathrm{DGLAP}} = x \int \mathrm{d}\beta \,\mathrm{d}\alpha \, f(\beta,\alpha) \,\,\delta\left(x-\beta-\alpha\xi\right) \,.$$

Test with a constant DD:



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Inverse Radon T	ransform		

Find $f(\beta, \alpha)$ on rhombus $\{|\alpha| + |\beta| \le 1\}$ such that

$$\left|H(x,\xi)\right|_{\mathrm{DGLAP}} = x \int \mathrm{d}\beta \,\mathrm{d}\alpha \,f\left(\beta,\alpha\right) \,\delta\left(x-\beta-\alpha\xi\right) \,.$$

Test with a constant DD on half rhombus:



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(Berthou et al., 2016)

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Computing GPDs

	gpdExample()		computeOneGPD.xml
1	// Lots of includes	1	xml version="1.0" encoding="UTF-8" standalone="ves" ?
- 2	#include <src partons.h=""></src>	2	<scenario date="" description="Example.:computation.of.one.GPD</td></tr><tr><td>3</td><td>3</td><td></td><td>model.(GK11),without,evolution" id="01"></scenario>
4	1	3	Select type of computation
Ę	5 // Retrieve GPD service	4	<task method="computeGPDModel" service="GPDService"></task>
6	GPDService* pGPDService = ServiceObjectRegistry::getGPDService();	5	Specify kinematics
7	7 // Load GPD module with the BaseModuleFactory	6	<gpdkinematic></gpdkinematic>
8	3 GPDModule* pGK11Model = ModuleObjectFactory::newGPDModule(7	<pre><pre>cparam name="x" value="0.1" /></pre></pre>
	GK11Model::classId);	8	<pre><param name="xi" value="0.00050025"/></pre>
9	// Create a GPDKinematic(x, xi, t, MuF, MuR)	9	<pre><param name="t" value="-0.3"/></pre>
10	GPDKinematic gpdKinematic(0.1, xBToXi(0.001), -0.3, 8., 8.);	10	<pre><pre>param name="MuF2" value="8" /></pre></pre>
11	1 // Compute data and store results	11	<pre><pre>/> </pre>/> </pre>
12	2 GPDResult gpdResult = pGPDService->	12	
	CDDT	13	Choose GPD model and set parameters
	GPDType::ALL);	14	<gpdmodule></gpdmodule>
12	ctd::cout << gpdPacult toString() << std::opdf;	15	<pre><pre>cparam name="id" value="GK11Model" /></pre></pre>
14	stallout << gpartesut.tostring() << stallendi,	16	
16	delete pGK11Model:	17	
17	pGK11Model = 0;	18	

(Berthou et al., 2016)

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gpdExample()	computeOneGPD.xml
1 // Lots of includes	1 xml version="1.0" encoding="UTF-8" standalone="yes" ?
2 #include <src partons.h=""></src>	2 <scenario date="" description="Example_:_computation_of_one_GPD</p></td></tr><tr><td>3</td><td>_model_(GK11)_without_evolution" id="01"></scenario>
4	3 Select type of computation
5 // Retrieve GPD service	4 <task method="computeGPDModel" service="GPDService"></task>
6 GPDService* pGPDService = ServiceObjectRegistry::getGPDService();	5 Specify kinematics
7 // Load GPD module with the baseModuleFactory CDDM-shule* = CK11M-shule - Mashule ObjectEasternum CDDM-shule (6 <gpdkinematic></gpdkinematic>
8 GEDModule: pGKTIModel = ModuleObjectFactory.:newGEDModule(CK11Medelveleetel);	7 <pre><pre>param name="x" value="0.1" /></pre></pre>
<pre>GK11Wodel.classid); // Create a CPDKinematic(v_vi_t_MuE_MuP)</pre>	<pre>8 <param name="xi" value="0.00050025"/></pre>
9 // Create a Gr Dikinematic(X, Xi, t, Mult, Mult) 10 CPDKinematic and Kinematic(0.1, vBToXi(0.001) = 0.3, 8, 8);	9 <param name="t" value="-0.3"/>
11 // Compute data and store results	10 <param name="MuF2" value="8"/>
12 GPDResult and Result = nGPDService->	11 <pre><pre>param name="MuR2" value="8" /></pre></pre>
computeGPDModelRestrictedByGPDType(gpdKinematic_pGK11Model	12
GPDType: ALL):	13 <=> Choose GPD model and set parameters>
13 // Print results	14 <gpdmodule></gpdmodule>
14 std::cout << gpdResult.toString() << std::endl:	15 <pre><pre>cparam name="id" value="GK11Model" /></pre></pre>
15	16
16 delete pGK11Model;	17
17 pGK11Model = 0;	18
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• Keep it simple.

(Berthou et al., 2016)

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4		3	Select type of computation
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11	CDDDarith and Store results	11	<pre><param name="MuR2" value="8"/></pre>
12	GPDResult gpdResult = pGPDService->	12	
	CDDTures(ALL):	13	Choose GPD model and set parameters
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- Keep it simple.
- Do no reinvent the wheel.

(Berthou et al., 2016)

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_	gpdExample()		computeOneGPD.xml
1 // 2 # 4 // 6 G 7 // 8 G 9 // 10 G 11 // 12 G G 13 // 14 st		1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 -	<pre>computeOneGPD.xml c?xml version="1.0" encoding="UTF-8" standalone="yes" ?> scenario id="01" date="" description="%zample_:i_computation_of_one_GPD model_(GK11),u:thout,evolution"> <!-- Select type of computation--> <task method="computeGPDModel" service="GPDService"> <!-- Select type of computation--> <task method="computeGPDModel" service="GPDService"> <!-- Select type of computation--> <gpdkinematics> <gpdkinematics <param name="%xl" value="0.00050025"/> <param 0.00050025"="" name="%xl" value=""/> <param 0.3"="" name="%xl" value=""/> <gpdkinematic> <!-- Choose GPD model and set parameters--> <gpdmodule> <!-- Choose GPD model and set parameters--> <!-- Comparamem=""1" value="GK11Model" /--> <!-- Comparameme=""1" value="GK11Model" /--></gpdmodule></gpdkinematic></gpdkinematics </gpdkinematics></task></task></pre>
15 16 d	elete pGK11Model;	16 17	
I/ p	GKTIMODEI = 0;		

- Keep it simple.
- Do no reinvent the wheel.
 - Modularity.

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(Berthou et al., 2016)
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From GPDs to observables





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• Dyson-Schwinger model for GPDs in progress:

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- Dyson-Schwinger model for GPDs in progress:
 - Inverse Radon Transform to reconstruct the GPD in the entire physical domain.

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- Dyson-Schwinger model for GPDs in progress:
 - Inverse Radon Transform to reconstruct the GPD in the entire physical domain.
 - Move from pion to **proton**.

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Form factors for the pion



(Mezrag, 2015)

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Quark Dyson-Schwinger equation (Gap Equation)

• Quark Propagator:

$$S(p)^{-1} = A(p^2) (i\gamma \cdot p + M(p^2)) , \qquad (5)$$

with $M(p^2)$ the dynamical mass of the quark.

• Quark Gap equation (represented in Page 8):

$$S(p)^{-1} = Z_{2}(i\gamma \cdot p + m_{b})$$

$$+ Z_{1} \int^{\Lambda} \frac{d^{4}q}{(2\pi)^{4}} g^{2} D_{\mu\nu}(p-q) \frac{\lambda^{a}}{2} \gamma_{\mu} S(q) \frac{\lambda^{a}}{2} \Gamma_{\nu}(q,p).$$
(7)

• Truncation for the Vertex. Example (Rainbow-Ladder truncation):

$$\Gamma^{RL}_{\mu}(q,p) = \gamma_{\mu} \,. \tag{8}$$

 Must choose a (phenomenological) model for the Gluon Propagator in the infra-red (ultra-violet domain reproduces perturbative QCD). Example of effective interaction (Maris-Tandy):

$$\mathcal{G}_{IR}^{MT}\left(k^{2}\right) = \frac{4\pi^{2}}{\omega^{6}} D k^{2} e^{-\frac{k^{2}}{\omega^{2}}}.$$
(9)

Dynamical mass for the quark



Figure: Dynamical mass M function of $\sqrt{p^2}$ (in GeV). Numerical resolution of the Quark Dyson-Schwinger equation on the real axis. In blue (solid line), the positive Nambu solution displays dynamical mass generation: large constituent-quark mass at small momenta, small current-quark mass at large momenta. In red (dashed), solution with no mass generated; in green (dots), negative Nambu solution.