# An application of Talmi-Moshinsky transformations to a QCD oriented Hamiltonian

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in collaboration with

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## Justification

- Description of QCD at the non-perturbative regime.
- Lattice Gauge calculation, MIT Bag Model, Dyson-Schwinger
- The present model pretends to offer an alternative:
  - That requires less computational power.
  - That requires as less parameters as possible.
  - That offers a more intuitive physical interpretation.
  - That is realistic, ie, that contains the important ingredients of the theory.
  - Non-perturbative!.

## Characteristics of this model

How to approach to such a complex problem?

- One considers an effective QCD Hamiltonian in the Coulomb gauge.
- The fermionic fields are expanded in the harmonic oscillator basis in the coordinate space.
- Traslational invariance of the center of mass is recovered with the help of the Talmi-Moshinsky transformations.
- Many-body methods such as TDA and RPA are used to diagonalize the effective Hamiltonian.
- Flavor simmetry breaking is set by assuming different bare masses for different quarks.

## The election of a gauge

- Why is it a good idea to work in the Coulomb gauge?
  - Elimination of nondynamical degrees of freedom creates an instantaneous confining interaction.
  - This potential introduces the effect of gluons in an effective manner.
  - Retardation effects are minimized for heavy quarks, making this a natural framework for studying nonrelativistic bound states.
  - It is also relevant for light flavors once the constituent quarks are identified with the quasiparticle excitations, which saturate at $\sim$200 MeV as the bare mass is reduced.
  - The appearance of a quark-antiquark vacuum condensate is typically associated with the confinement potential.

[ A. Szczepaniak, E. Swanson, Phys. Rev. D, **55**, 1578, (1997) ]

[ S. L. Adler, A. C. Davis, Nucl. Phys. B **244**, 469 (1984) ] .

## The election of a basis for the fields

- Why is it a good idea to use the harmonic oscillator basis?
  - One expects the fields to be restricted to a finite volume.
  - Talmi-Moshinsky transformations guarantee translational invariance of the center of mass

$$\left[\Psi_{n_a l_a}(\mathbf{x}) \otimes \Psi_{n_b l_b}(\mathbf{y})\right]^L_M = \sum_{n_r l_r n_R l_R} \langle n_r l_r, n_R l_R; L | n_a l_a n_b l_b; L\rangle \left[\Psi_{n_r l_r}(\mathbf{r}) \otimes \Psi_{n_R l_R}(\mathbf{R})\right]^L_M$$

$$\mathbf{r} = \frac{1}{\sqrt{2}}(\mathbf{x} - \mathbf{y}) \quad , \qquad \mathbf{R} = \frac{1}{\sqrt{2}}(\mathbf{x} + \mathbf{y})$$

  - Only one integral is to be solved in this frame, with analytical results!

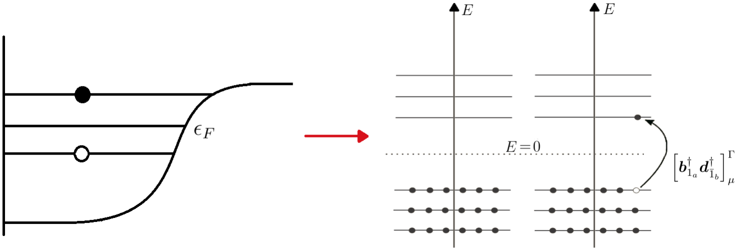
$$I(n_i, l_i, L) = \int R^2 dR \int r^2 dr R_{n_1 l_1}(r) R_{n_2 l_2}(r) \left(-\frac{\alpha}{r} + \beta r\right) R_{n_3 l_3}(R) R_{n_4 l_4}(R)$$

  - The integrals are long-range and short-range well-behaved
  - Plane waves are not the last word! (Jost-Schroer Theorem)

[ G.P. Kamuntavicius, R.K. Kalinauskas, Nuclear Physics A 695 (2001) 191-201. ]

## The election of a diagonalization method

- Why is it a good idea to use Many-body methods?
  - In analogy to the particle-hole excitations well known in nuclear physics, colorless quark-antiquark pairs are used to model mesons.
  - As said, Coulomb gauge gives rise to a vacuum with structure of a quark-antiquark condensate.



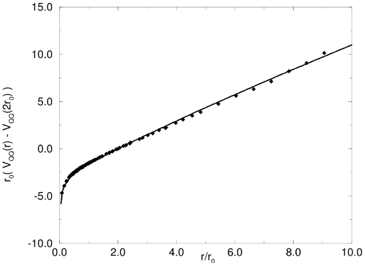
## Flavor symmetry breaking

- Introduction of different masses for different quarks.
  $m_{\frac{1}{3}\frac{1}{2}} = 0.008$ GeV, $\quad m_{-\frac{2}{3}0} = 0.092$ GeV.
- $\epsilon^{\Gamma}_{ab} \sim m_{Y_a T_a} + m_{Y_b T_b}$.
- Partially decouple the $SU(3)$ flavor symmetry into an $SU(2) \times U(1)$ algebra. By doing so, the interaction can still be coupled to an isospin $SU(2)$ subalgebra.

## The full Hamiltonian

A Cornell potential simulates the quark-antiquark interaction due to gluons.

$$\langle a, \mathbf{r}| \frac{1}{\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}}(-\boldsymbol{\nabla}^2)\frac{1}{\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}}|a'\mathbf{r}'\rangle \longrightarrow V(|\mathbf{r}-\mathbf{r}'|) = \frac{-\alpha}{|\mathbf{r}-\mathbf{r}'|} + \beta|\mathbf{r}-\mathbf{r}'|$$




$$H \approx \underbrace{\int \boldsymbol{\psi}^\dagger(\mathbf{r})[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}]\boldsymbol{\psi}(\mathbf{r})d\mathbf{r}}_{\mathbf{H}_{K_q}} + \underbrace{\int \boldsymbol{\psi}^\dagger(\mathbf{r})[\beta m]\boldsymbol{\psi}(\mathbf{r})d\mathbf{r}}_{\mathbf{H}_{m_q}} + \underbrace{\frac{1}{2}g^2\delta_{a'a}\int \rho_a^{(q)}(\mathbf{r})V(|\mathbf{r}-\mathbf{r}'|)\rho^{(q)a}(\mathbf{r}')d\mathbf{r}d\mathbf{r}'}_{\mathbf{V}},$$

## Mass and kinetic terms for quark sector

The quark fields are expanded in the harmonic oscillator basis

$$\psi^{\dagger}_{1,2}(\mathbf{r},\sigma,c,f) \quad = \quad \sum_{Nlm\sigma cf} \mathbf{b}^{\dagger}_{\pm\frac{1}{2},Nlm,\sigma cf} R^{*}_{Nl}(r) Y^{*}_{lm}(\hat{r}) \chi^{\dagger}_{\sigma} \ ,$$

The kinetic term is not diagonal in this basis! So a unitary transformation is introduced to diagonalize the free Hamiltonian

$$\begin{pmatrix} \mathbf{b}^{\dagger}_{\frac{1}{2}Nljmcf} \\ \mathbf{b}^{\dagger}_{-\frac{1}{2}Nljmcf} \end{pmatrix} = \sum_k \begin{pmatrix} \gamma^{j}_{Nl,k} & -\beta^{j}_{Nl,k} \\ \beta^{j}_{Nl,k} & \gamma^{j}_{Nl,k} \end{pmatrix} \begin{pmatrix} \mathbf{b}^{\dagger}_{kjmcf} \\ \mathbf{d}_{kjmfc} \end{pmatrix} ,$$

with this, the mass and kinetic terms turn to be

$$\mathbf{K} \quad = \quad \mathbf{H}_{K_q} + \mathbf{H}_{m_q} = \sum_{kj} \tilde{\epsilon}_{kj} \left[ \left( \mathbf{b}^{\dagger}_{kj} \cdot \mathbf{b}^{kj} - \mathbf{d}_{kj} \cdot \mathbf{d}^{\dagger\, kj} \right) \right] \ .$$

[ A. Amor-Quiroz, P. O. Hess, O. Civitarese, T. Yépez-Martínez, J. Phys.: Conf. Ser. **639** 011001, (2015). ]

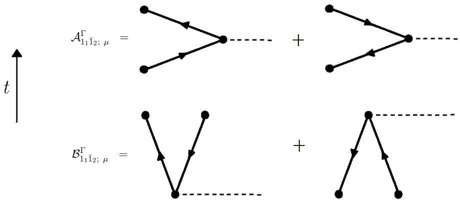
## The effective quark interaction

By defining the short-hand notation

$$\begin{aligned}\mathcal{A}^{\Gamma}_{1_1\bar{1}_2;\ \mu} &= \left[\mathbf{b}^{\dagger}_{1_1}\mathbf{b}_{\bar{1}_2}\right]^{\Gamma}_{\mu} - \left[\mathbf{d}_{1_1}\mathbf{d}^{\dagger}_{\bar{1}_2}\right]^{\Gamma}_{\mu} \\ \mathcal{B}^{\Gamma}_{1_1\bar{1}_2;\ \mu} &= \left[\mathbf{b}^{\dagger}_{1_1}\mathbf{d}^{\dagger}_{\bar{1}_2}\right]^{\Gamma}_{\mu} + \left[\mathbf{d}_{1_1}\mathbf{b}_{\bar{1}_2}\right]^{\Gamma}_{\mu},\end{aligned}$$

the final structure of an arbitrary local potential turns to be

$$\mathbf{V} = \sum_{L_0}\sum_{\{k,j\}}\left(E^{L_0}_{\{k,j\}}\left[\mathcal{A}_{12}\mathcal{A}_{34}\right]^0_0 + F^{L_0}_{\{k,j\}}\left[\mathcal{A}_{12}\mathcal{B}_{34}\right]^0_0 + F'^{L_0}_{\{k,j\}}\left[\mathcal{B}_{12}\mathcal{A}_{34}\right]^0_0 + G^{L_0}_{\{k,j\}}\left[\mathcal{B}_{12}\mathcal{B}_{34}\right]^0_0\right),$$




$$E, F, F', G \sim \int R^2 dR \int r^2 dr R_{n_1 l_1}(r) R_{n_2 l_2}(r)\left(-\frac{\alpha}{r} + \beta r\right) R_{n_3 l_3}(R) R_{n_4 l_4}(R)$$

## Exploring the use of Many-body methods

The Hamiltonian to diagonalize is then given by

$$
\begin{aligned}
H = \sum_{kj} 3\hat{j} C_{kj} \left( \left[ \boldsymbol{b}^\dagger_{kj} \boldsymbol{b}_{\bar{k}j} \right]^0_0 + \left[ \boldsymbol{d}_{kj} \boldsymbol{d}^\dagger_{\bar{k}j} \right]^0_0 \right) \\
+ \sum_{L_0} \sum_{\{k,j\}} \left( E^{L_0}_{\{k,j\}} \left[ \mathcal{A}_{12}\mathcal{A}_{34} \right]^0_0 + F^{L_0}_{\{k,j\}} \cancel{\left[ \mathcal{A}_{12}\mathcal{B}_{34} \right]^0_0} + F'^{L_0}_{\{k,j\}} \cancel{\left[ \mathcal{B}_{12}\mathcal{A}_{34} \right]^0_0} + G^{L_0}_{\{k,j\}} \left[ \mathcal{B}_{12}\mathcal{B}_{34} \right]^0_0 \right)
\end{aligned}
$$

To apply the Many-body methods, the mesonic states are build up as quark-antiquark pairs coupled to a singlet in color (physical states); and by defining some transformation laws

$$
\begin{aligned}
\gamma^\dagger_{1_a \bar{1}_b;\Gamma\mu} &\equiv \left[ \mathbf{b}^\dagger_{1_a} \mathbf{d}^\dagger_{\bar{1}_b} \right]^\Gamma_\mu \,, \\
\gamma^{\bar{1}_b 1_a;\, \Gamma\mu} &\equiv \left( \gamma^\dagger_{1_a \bar{1}_b;\Gamma\mu} \right)^\dagger \,, \\
\gamma^{\dagger 1_b \bar{1}_a;\, \bar{\Gamma}\bar{\mu}} &\equiv (-1)^{\phi_\mu} \gamma^\dagger_{1_a \bar{1}_b;\Gamma\mu} \,,
\end{aligned}
$$

## Tamm-Dankoff Method (TDA)

The TDA phonon is defined as a combination of quark-antiquark pairs $\gamma^\dagger$

$$\Gamma^\dagger_{\Gamma\mu;\alpha} \equiv \sum_{ab} X^{\alpha}_{ab;\Gamma} \gamma^\dagger_{ab;\Gamma\mu} \ .$$

The method consists in mapping a non-diagonal Hamiltonian into an effective harmonic oscillator Hamiltonian in TDA pairs basis.

$$H \rightarrow \sum_{\Gamma\mu} \sum_{\alpha} \hbar\Omega^{\Gamma}_{\alpha} \Gamma^\dagger_{\Gamma\mu;\alpha} \Gamma^{\Gamma\mu;\alpha} \ .$$

The *forward matrix* is defined as

$$\mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \equiv \left[ \gamma^{b'a';\Gamma'\mu'}, \left[ H, \gamma^{\dagger\Gamma}_{ab;\mu} \right] \right] = \left( \epsilon^{\Gamma}_{ab} \delta_{a'a} \delta_{b'b} + V^{\Gamma}_{(a'b')(ab)} \right) \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} \ .$$

By doing so, one gets the following eigenvalues equation

$$\sum_{ab} \mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} X^{\alpha}_{ab;\Gamma} = \hbar\Omega^{\Gamma}_{\alpha} X^{\alpha}_{a'b';\Gamma} \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} \ ,$$

But the TDA vacuum is the same as the free-theory vacuum!

[ P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980) ]

## Random Phase Approximation (RPA)

The RPA phonon is defined as

$$\Gamma^{\dagger}_{\Gamma\mu;\alpha} \equiv \sum_{ab} \left[ X^{\alpha}_{ab;\Gamma} \gamma^{\dagger}_{ab;\Gamma\mu} - Y^{\alpha}_{ab;\Gamma} \gamma_{ab;\Gamma\mu} \right] .$$

This time the vacuum does change: it is correlated!

By defining the *Backward Matrix* $\mathbb{B}$ as

$$\mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \equiv - \left[ \gamma^{\dagger b'a';\Gamma'\mu'}, \left[ H, \gamma^{\dagger}_{ab;\Gamma\mu} \right] \right] = \frac{1}{2} \left( W^{\Gamma}_{(a'b')(ab)} + W^{\Gamma}_{(ab)(a'b')} \right) \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} ,$$

One obtains an eigenvalues equation of the form

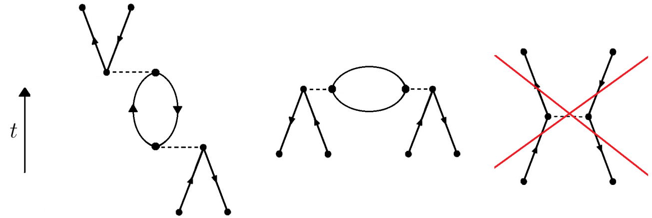
$$\sum_{ab} \begin{pmatrix} \mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} & \mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \\ -\mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} & -\mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \end{pmatrix} \begin{pmatrix} X^{\alpha}_{ab;\Gamma} \\ Y^{\alpha}_{ab;\Gamma} \end{pmatrix} = \hbar\Omega^{\Gamma}_{\alpha} \begin{pmatrix} X^{\alpha}_{a'b';\Gamma} \\ Y^{\alpha}_{a'b';\Gamma} \end{pmatrix} \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} ,$$

[ P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980) ]

## Random Phase Approximation (RPA)

This means that diagonalizing the above matrix equation allows to find the energy spectrum for the RPA phonon and the coefficients $X$ and $Y$ from the linear combination.

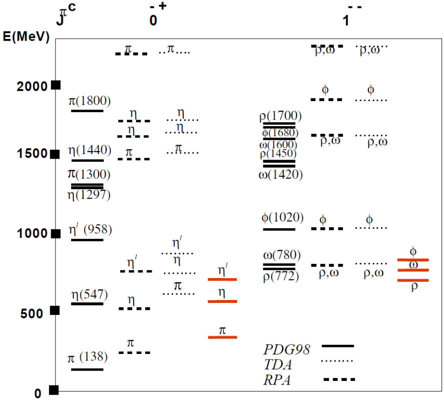
These coefficients may also allow us to construct the effective propagators and obtain transition amplitudes




[ P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980) ]

## Results

• It is not reasonable to compare with the physical spectrum. Instead, we compare to other many-body calculations.




| $N_{max} = 12$ | Spin | |
|---|---|---|
| ETDA[GeV] | J=0 | J=1 |
| E1 | 0.397 | 0.699 |
| E2 | 0.568 | 0.771 |
| E3 | 0.693 | 0.819 |

[ Phys Rev Lett, **84**, 6, Llanes-Estrada, Cotanch (2000) ]

## Conclusions

Actual status: Not ready yet!

- Recover traslational invariance.
- Recover good parity for the phonons.

Advantages:

- A simple yet semi-realistic model.
- The harmonic oscillator basis is confining and allows analytic integration.
- Rapid convergence of the solutions respect to the number of quanta considered.
- Does not require too many computational power.
- The coordinate space does not contain any divergences.
- The structure of a correlated vacuum simulates the virtual pairs creation.

## Summary

- Take many-body methods such as TDA or RPA.
- Mix with Talmi-Moshinsky transformations.
- Gently apply to an effective QCD-inspired Hamiltonian.
- A more *spicy* model is obtained if you add different *flavors* for different quarks!
- Cook in the coordinate space during the 4 years of your PhD along with your hopes and dreams.
- Serve it (hopefully in January 2017) expecting that it does taste like QCD!

Thank you!


This is Proton.
Proton has so many problems
But Proton still stays positive
Proton is Good
Be Like Proton