HUGS 2016

Extraction of unpolarized TMDs from SIDIS and Drell-Yan processes

Filippo Delcarro

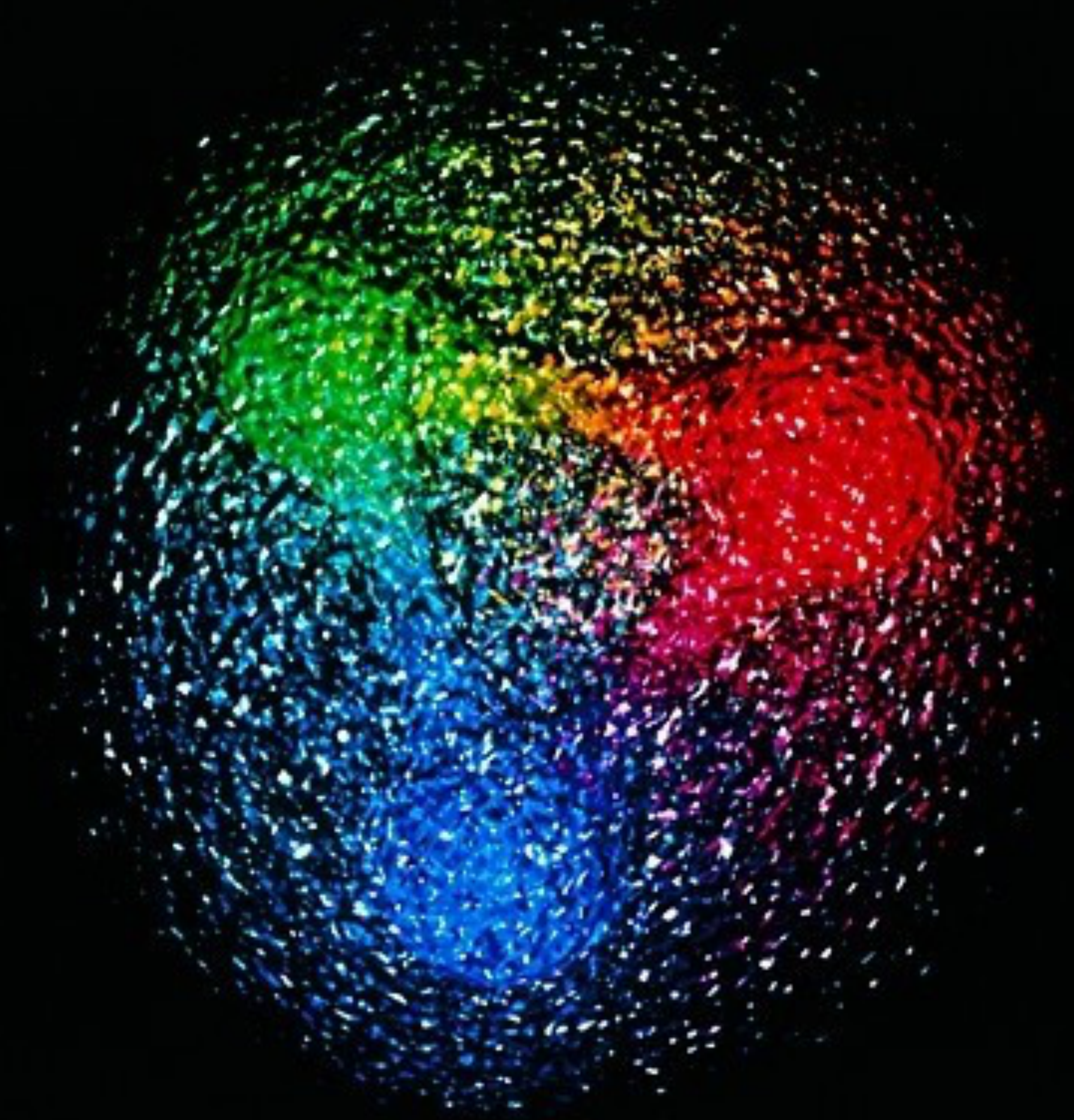
3DSPIN
MAPPING THE PROTON IN 3D

European Research Council
Established by the European Commission

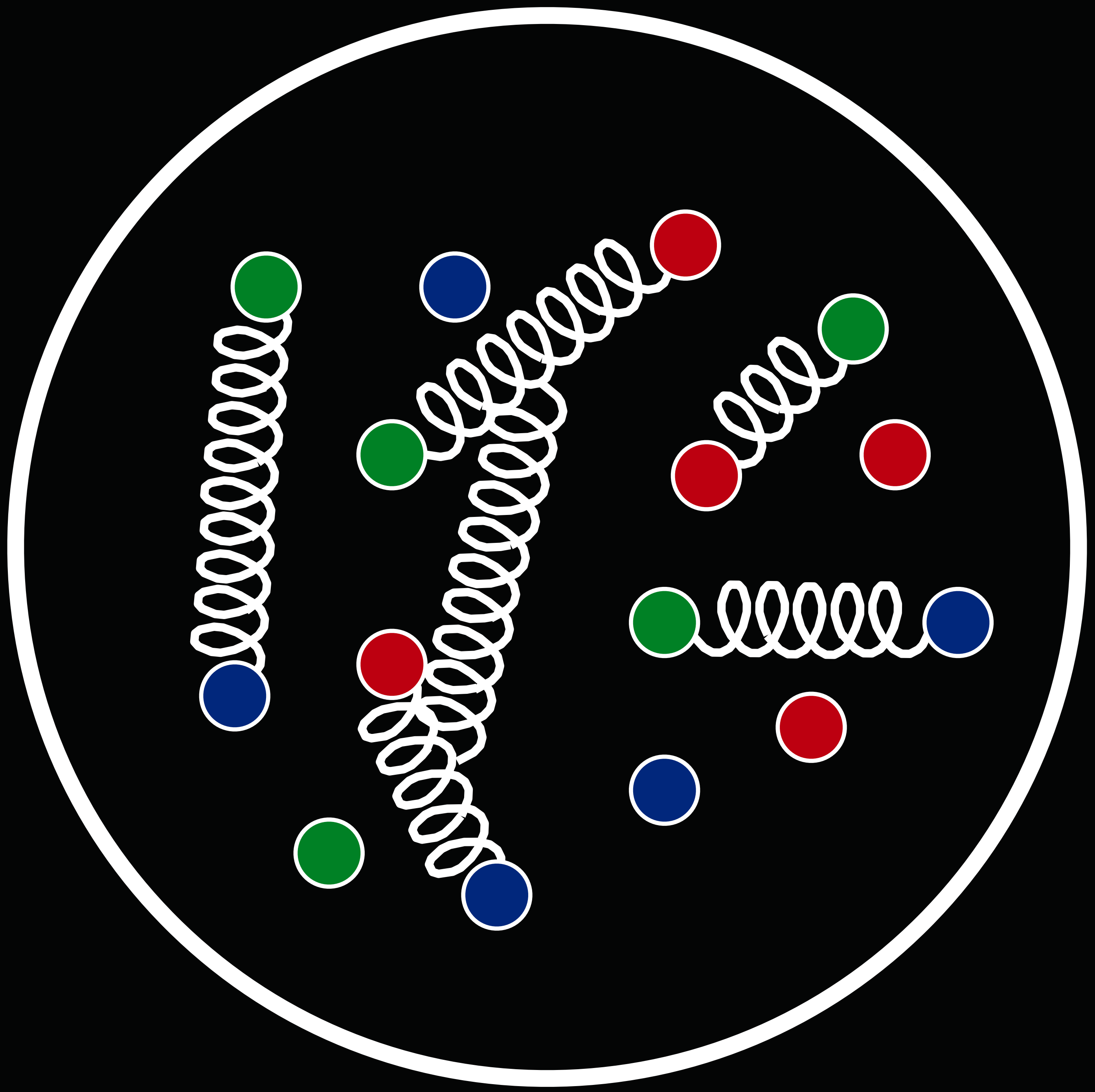
INFN

UNIVERSITÀ DI PAVIA

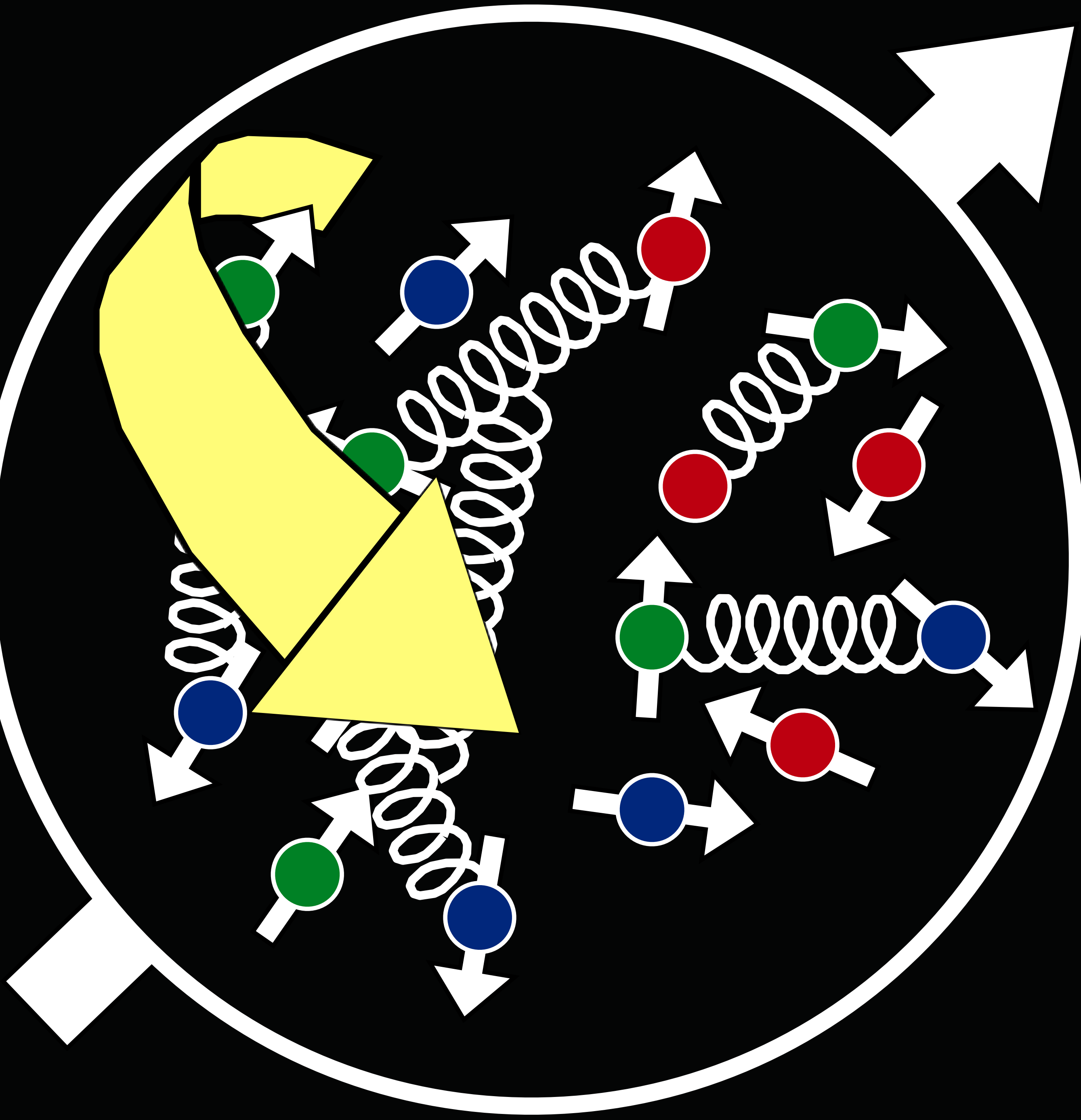
What is the structure of the nucleons?



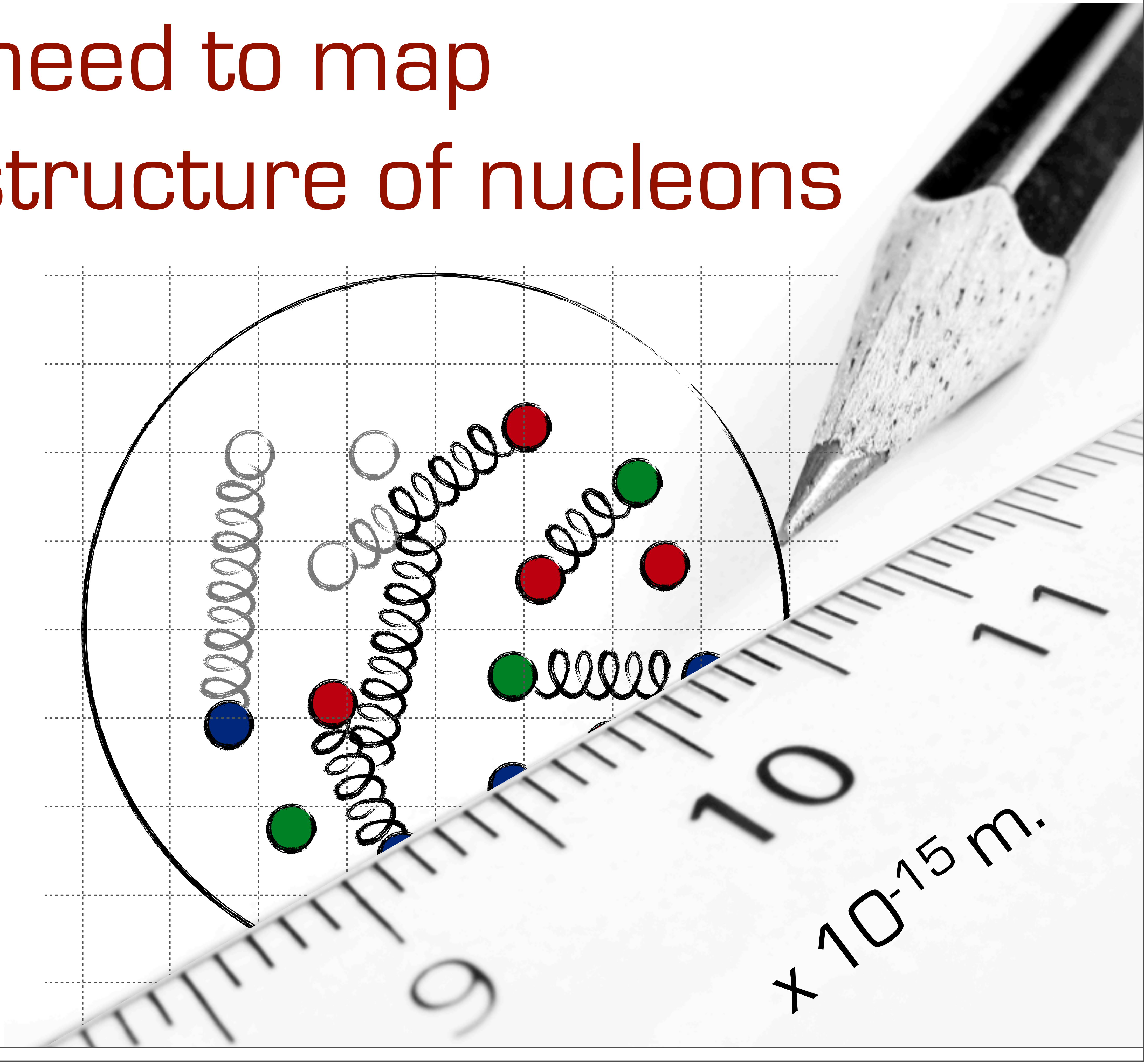
Is this structure explained by QCD?



Where does the spin of the nucleon come from?



We need to map the structure of nucleons



TMD distributions

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

quark pol.

U	L	T
D_1		$H_1^\perp$

“Amsterdam Notation”

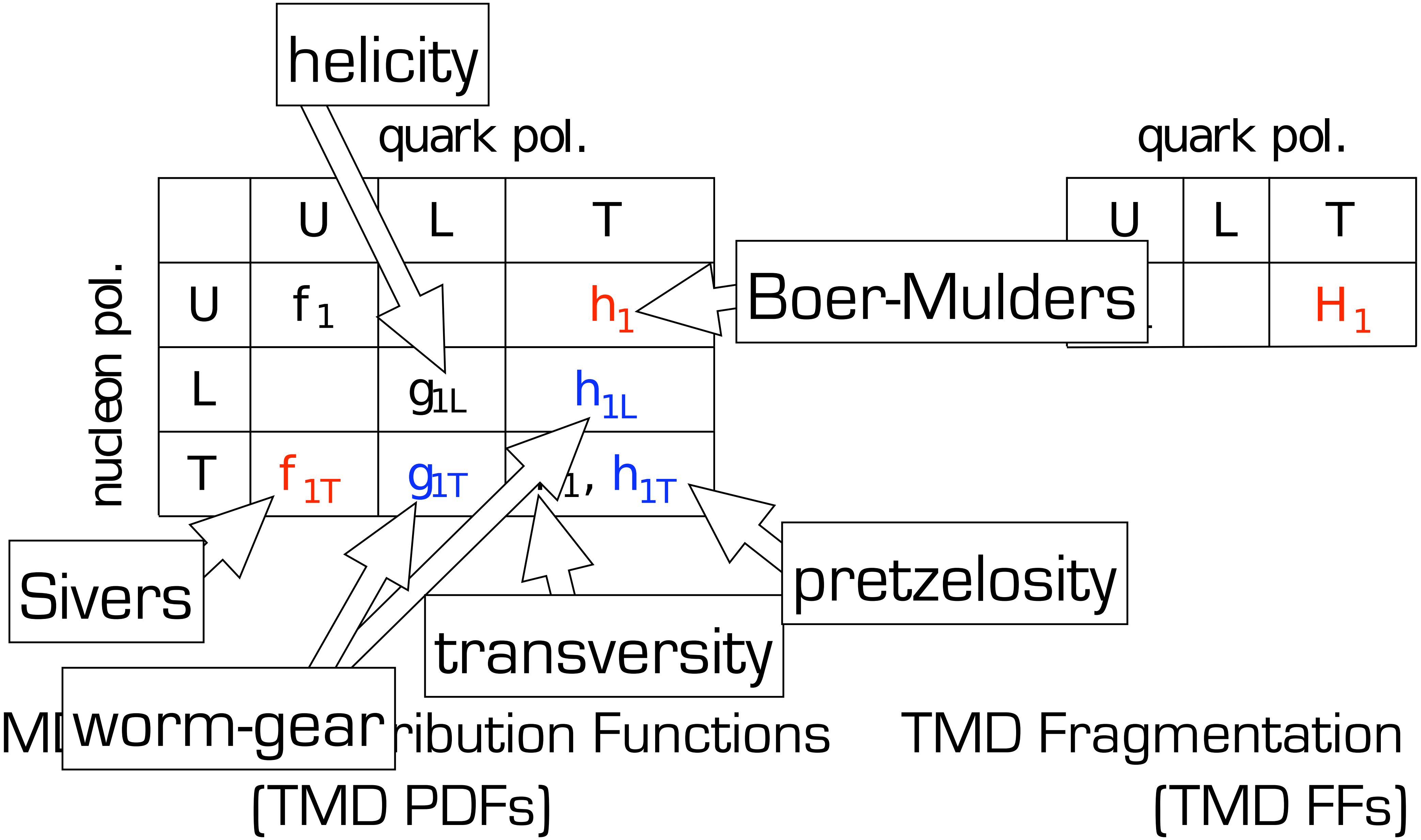
TMD Parton Distribution Functions
(TMD PDFs)

TMD Fragmentation Functions
(TMD FFs)

TMDs in black survive transverse-momentum integration
TMDs in red are T-odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

TMD distributions



helicity

Boer-Mulders

Sivers

worm-gear

transversity

pretzelosity

quark pol. / nucleon pol.

	U	L	T
U	f_1		h_1
L		g_{1L}	h_{1L}
T	f_{1T}	g_{1T}	$_1$, h_{1T}

TMI worm-gear ribution Functions
(TMD PDFs)

quark pol.

U	L	T
		H_1

TMD Fragmentation Functions
(TMD FFs)

TMD distributions

	U
U	f_1

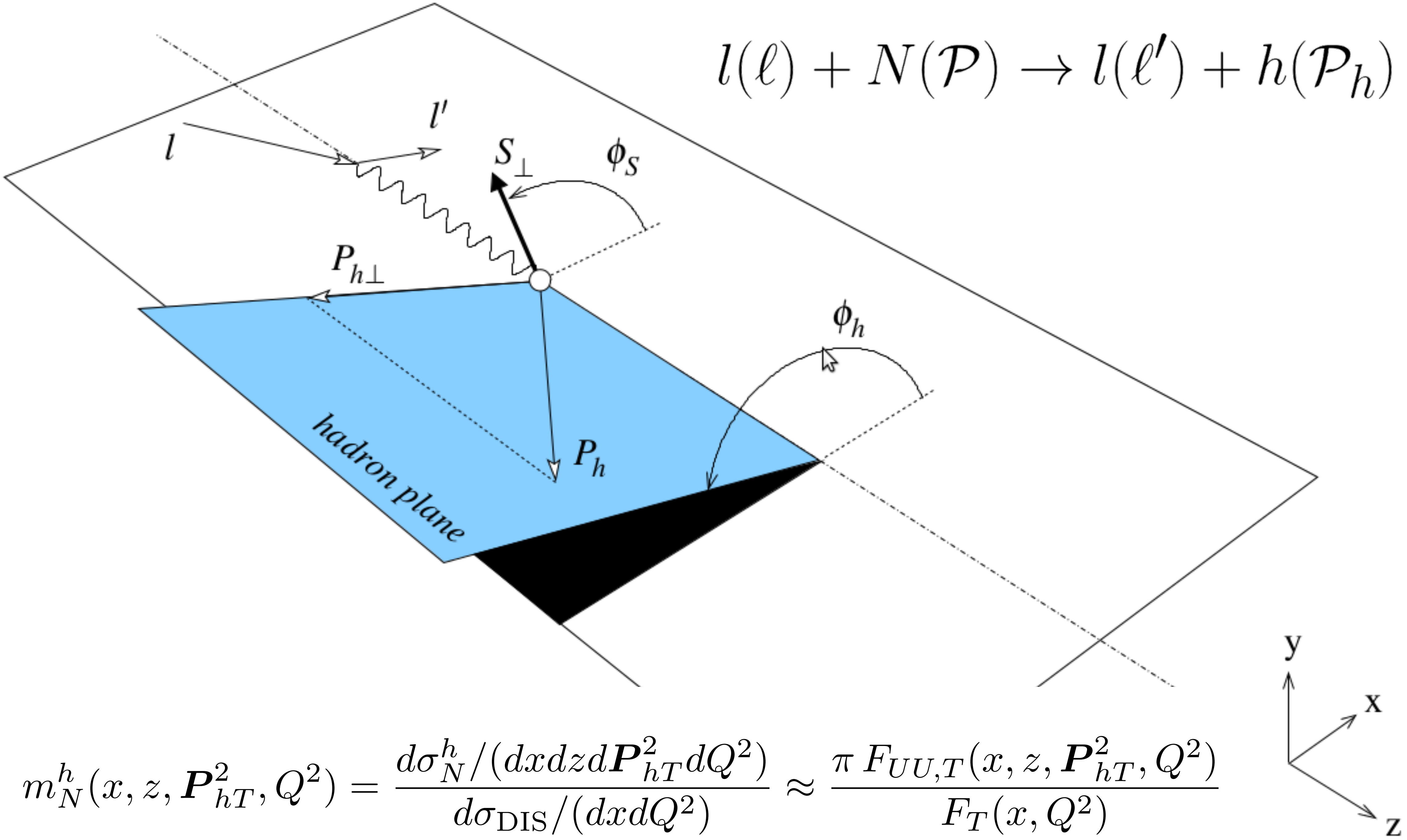
U
D_1

TODAY: only “unpolarized”

TMD Parton Distribution Functions
(TMD PDFs)

TMD Fragmentation Functions
(TMD FFs)

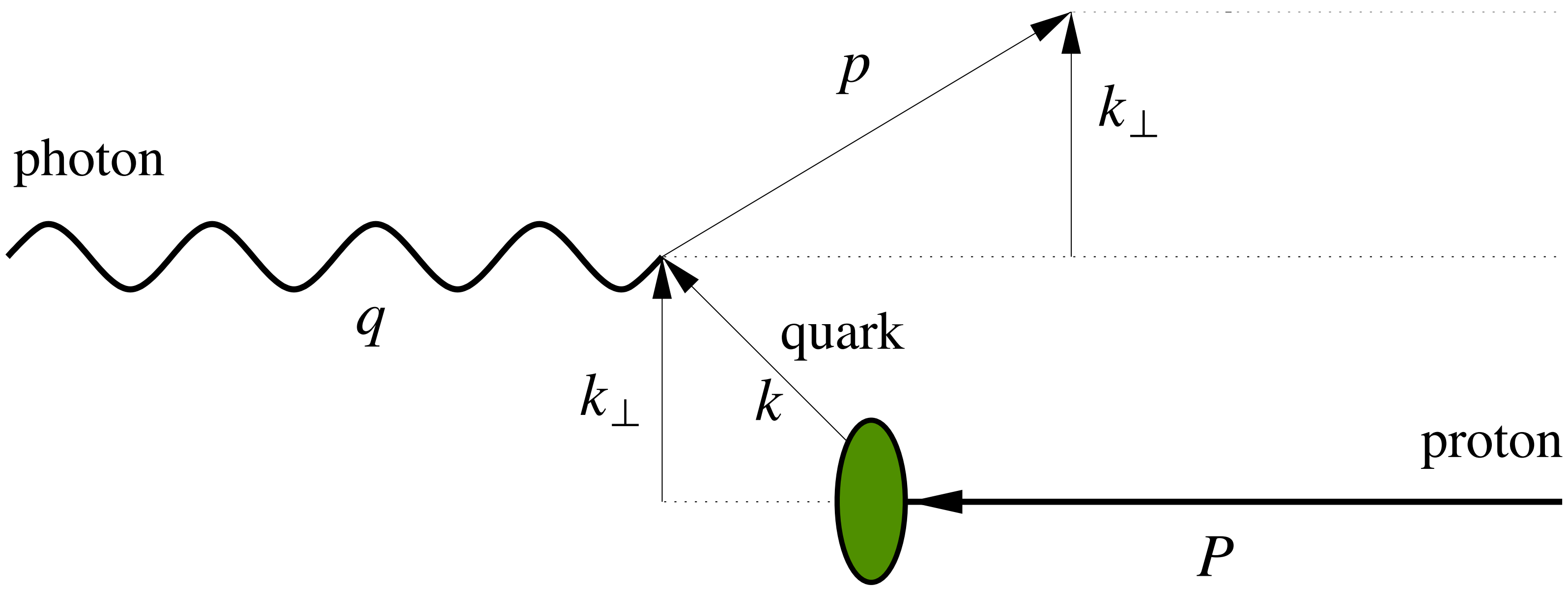
Semi-inclusive DIS



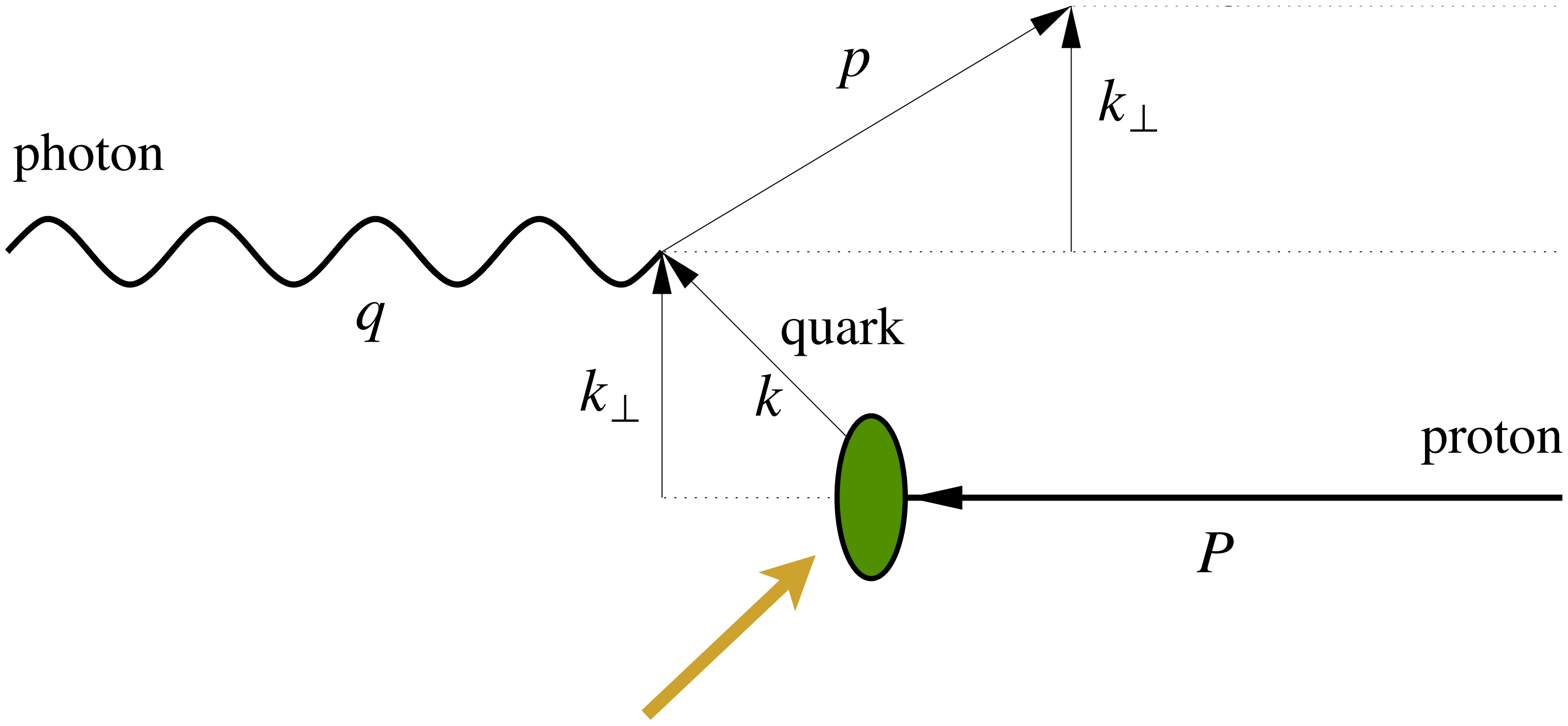
$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

$$m_N^h(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \frac{d\sigma_N^h/(dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2)}{d\sigma_{\mathrm{DIS}}/(dx\,dQ^2)} \approx \frac{\pi\, F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)}{F_T(x, Q^2)}$$

Semi-inclusive DIS

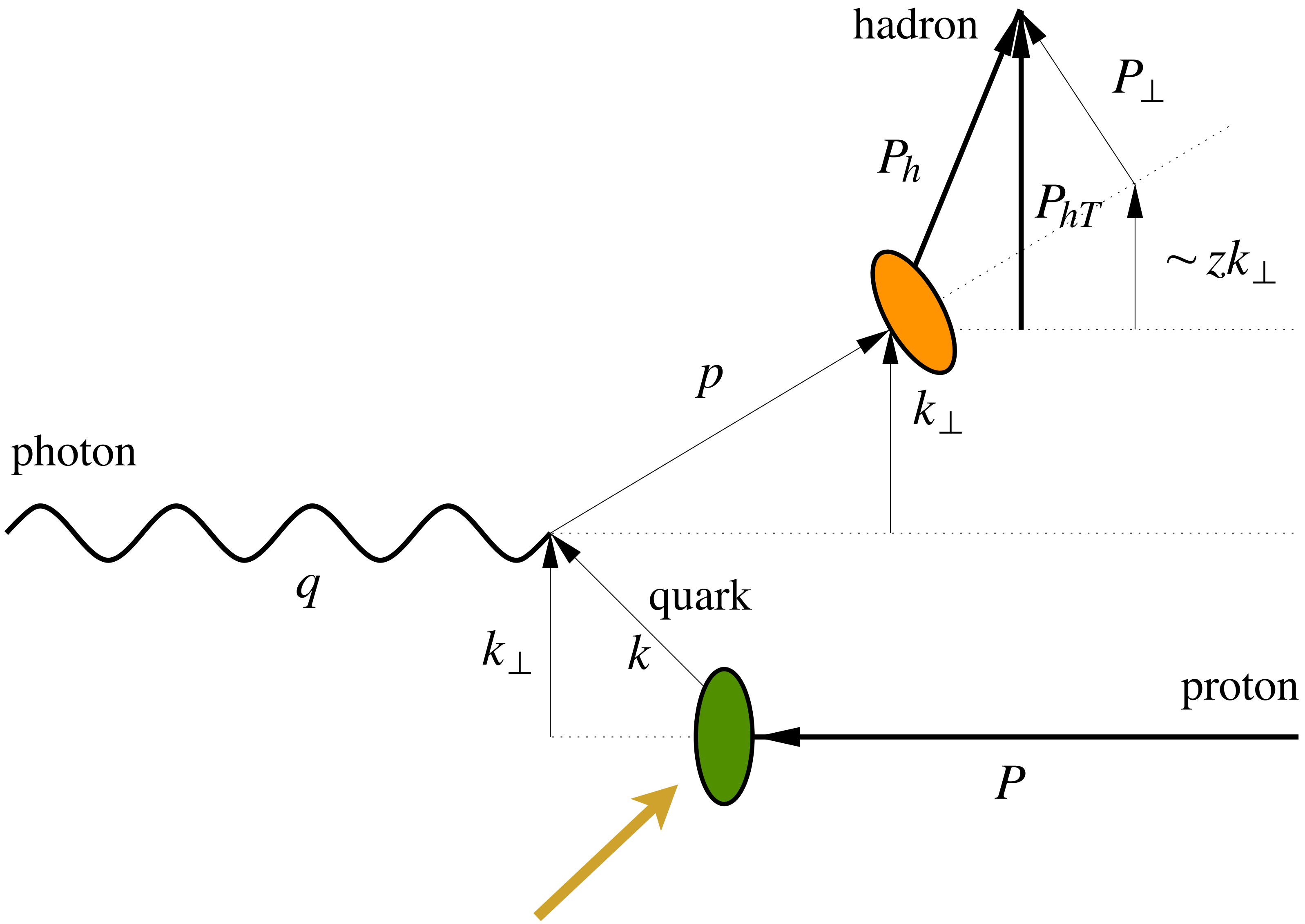


Semi-inclusive DIS



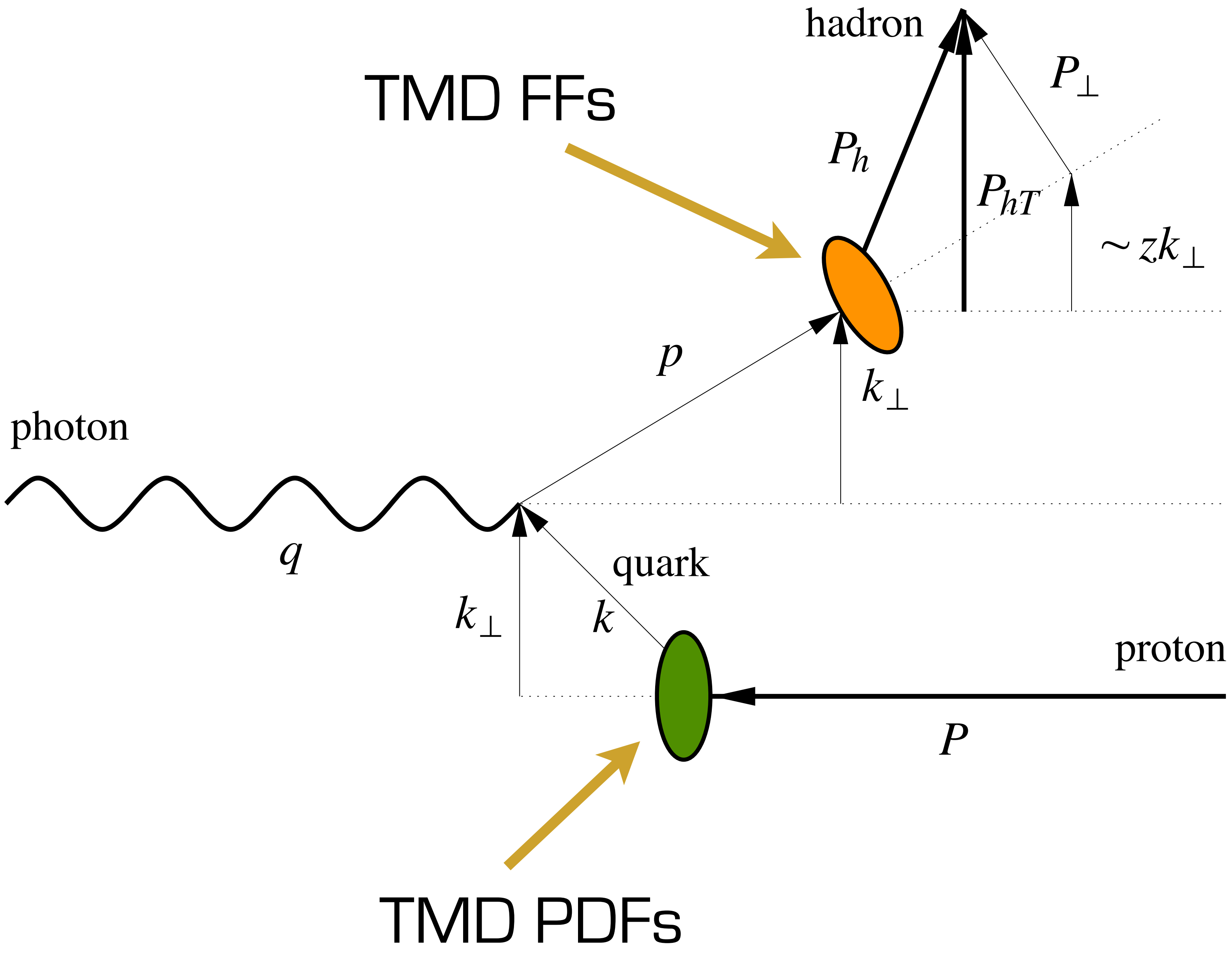
TMD PDFs

Semi-inclusive DIS



TMD PDFs

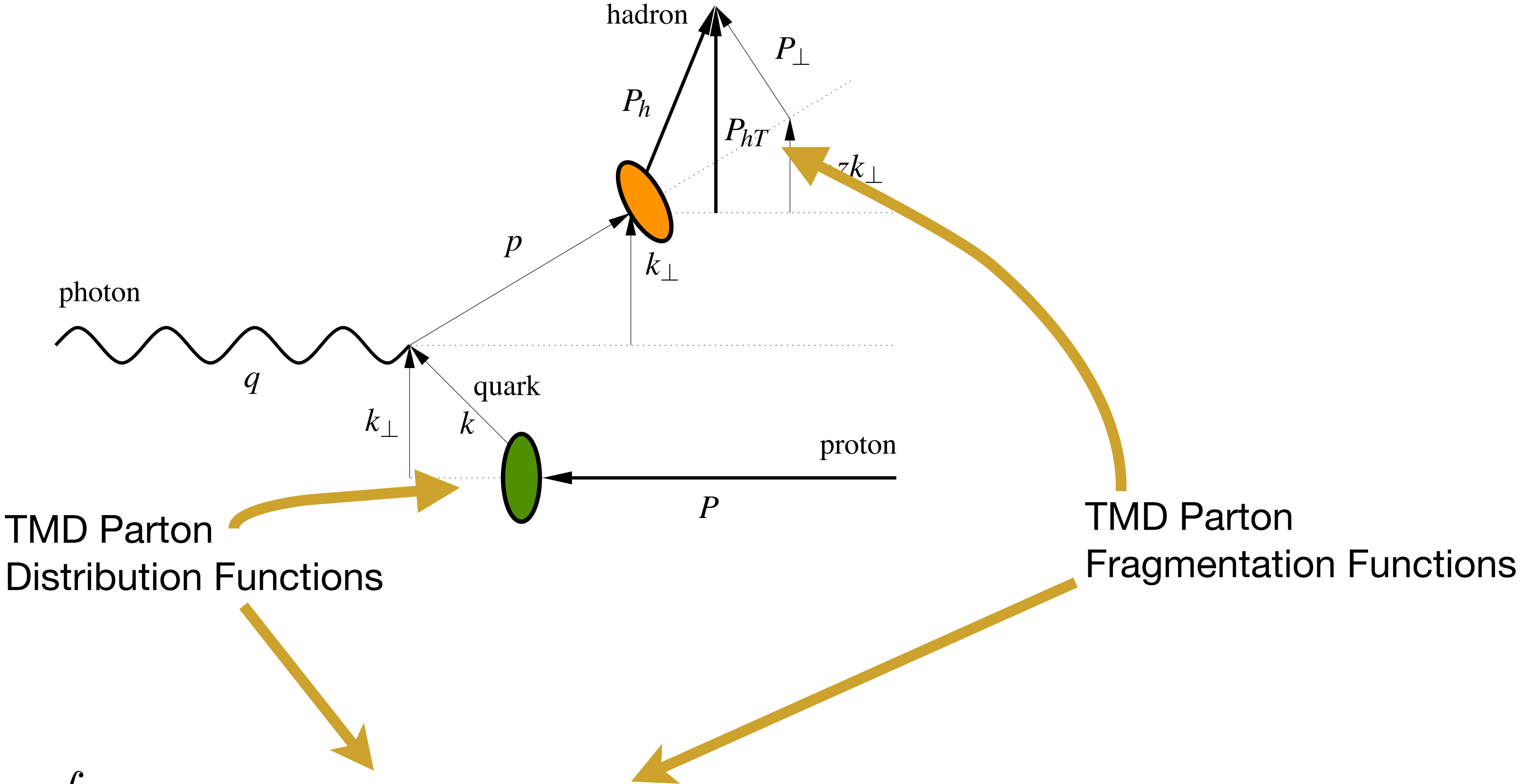
Semi-inclusive DIS



TMD FFs

TMD PDFs

Structure functions and TMDs



TMD Parton
Distribution Functions

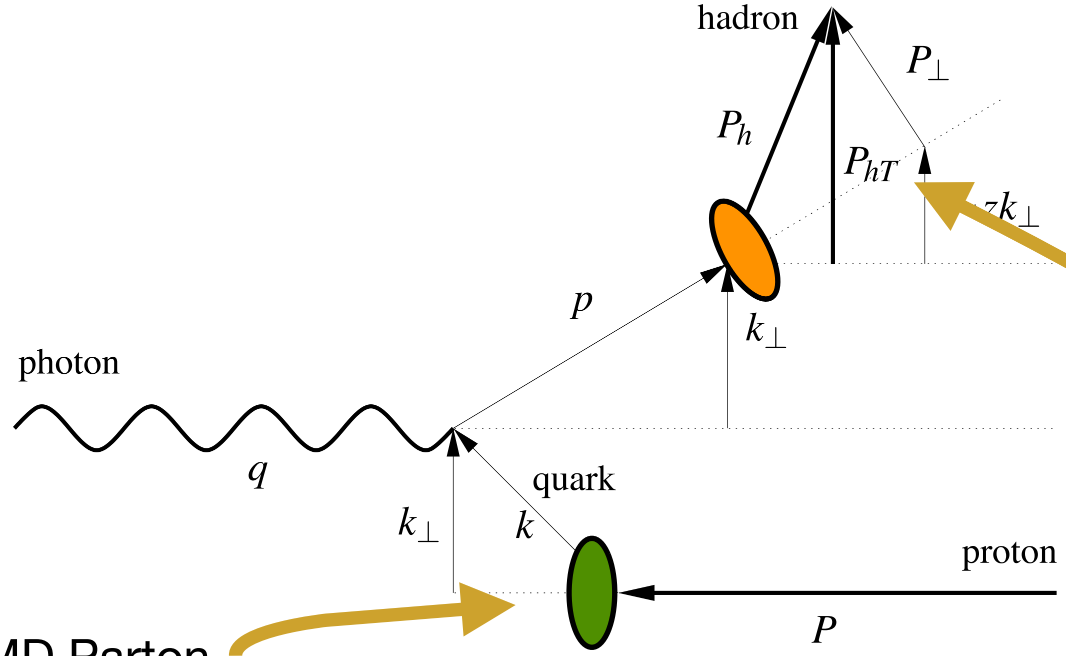
TMD Parton
Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)=\sum_a\int d\boldsymbol{k}_\perp\, d\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2)\,\delta(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp)+\mathcal{O}(M^2/Q^2)$$

“Parton model” or “Phase 1”

e.g., Pavia 2014, Torino 2014

Structure functions and TMDs



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)=\sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2)\int d\boldsymbol{k}_\perp\, d\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp)$$
$$+\,Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2)+\mathcal{O}(M^2/Q^2)$$

With QCD corrections or "Phase 2"

e.g., DEMS 2014 for D-Y

TMD evolution

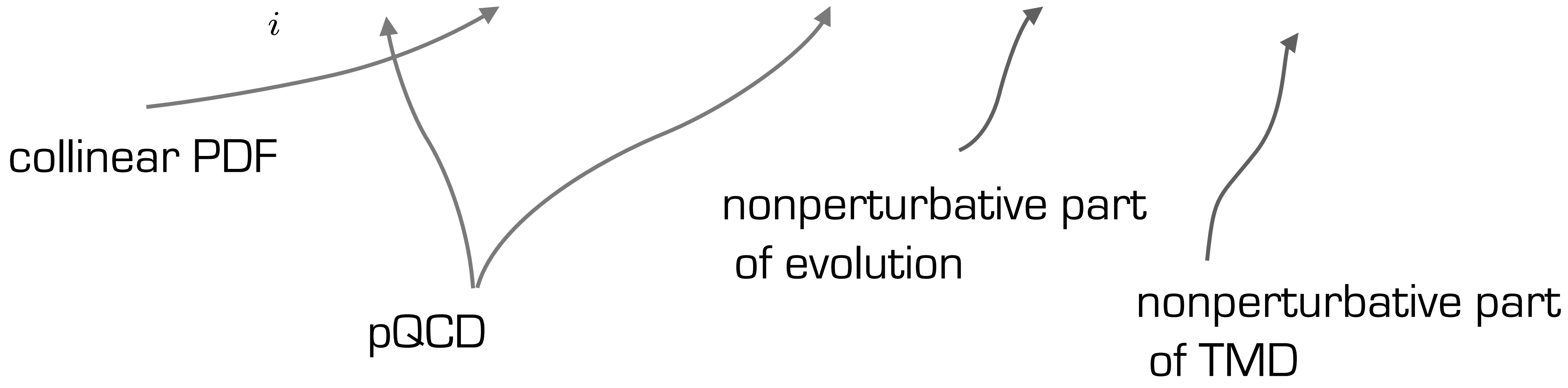
$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_\perp e^{-i b_\perp \cdot k_\perp} \widetilde{f}_1^a(x, b_\perp; \mu^2)$$

see, e.g., Rogers, Aybat, PRD 83 (11)
Collins, "Foundations of Perturbative QCD" (11)
Collins, Soper, Sterman, NPB250 (85)

TMD evolution

$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi}\int d^2 b_\perp e^{-ib_\perp \cdot k_\perp} \widetilde{f}_1^a(x, b_\perp; \mu^2)$$

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$



collinear PDF

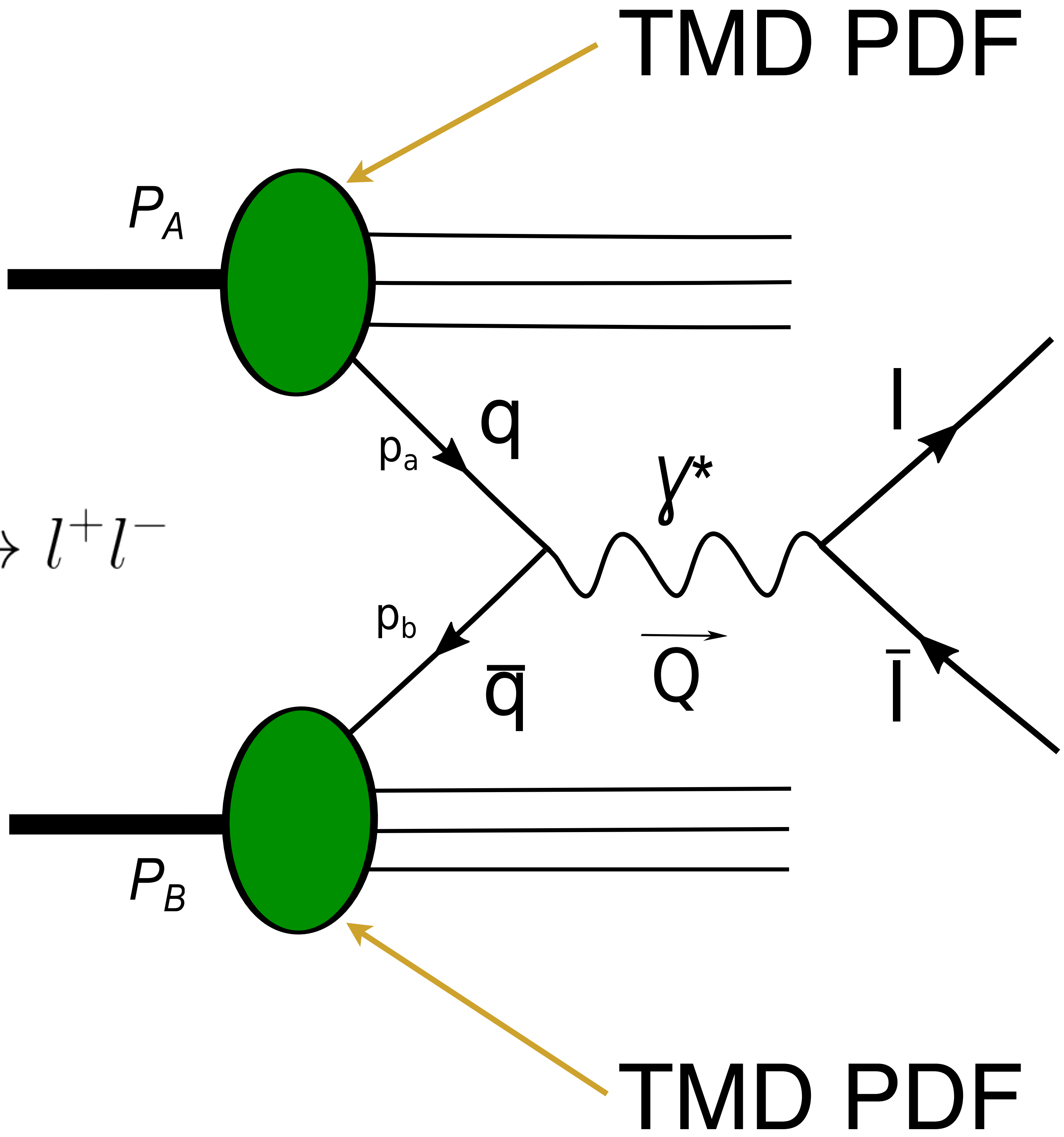
pQCD

nonperturbative part
of evolution

nonperturbative part
of TMD

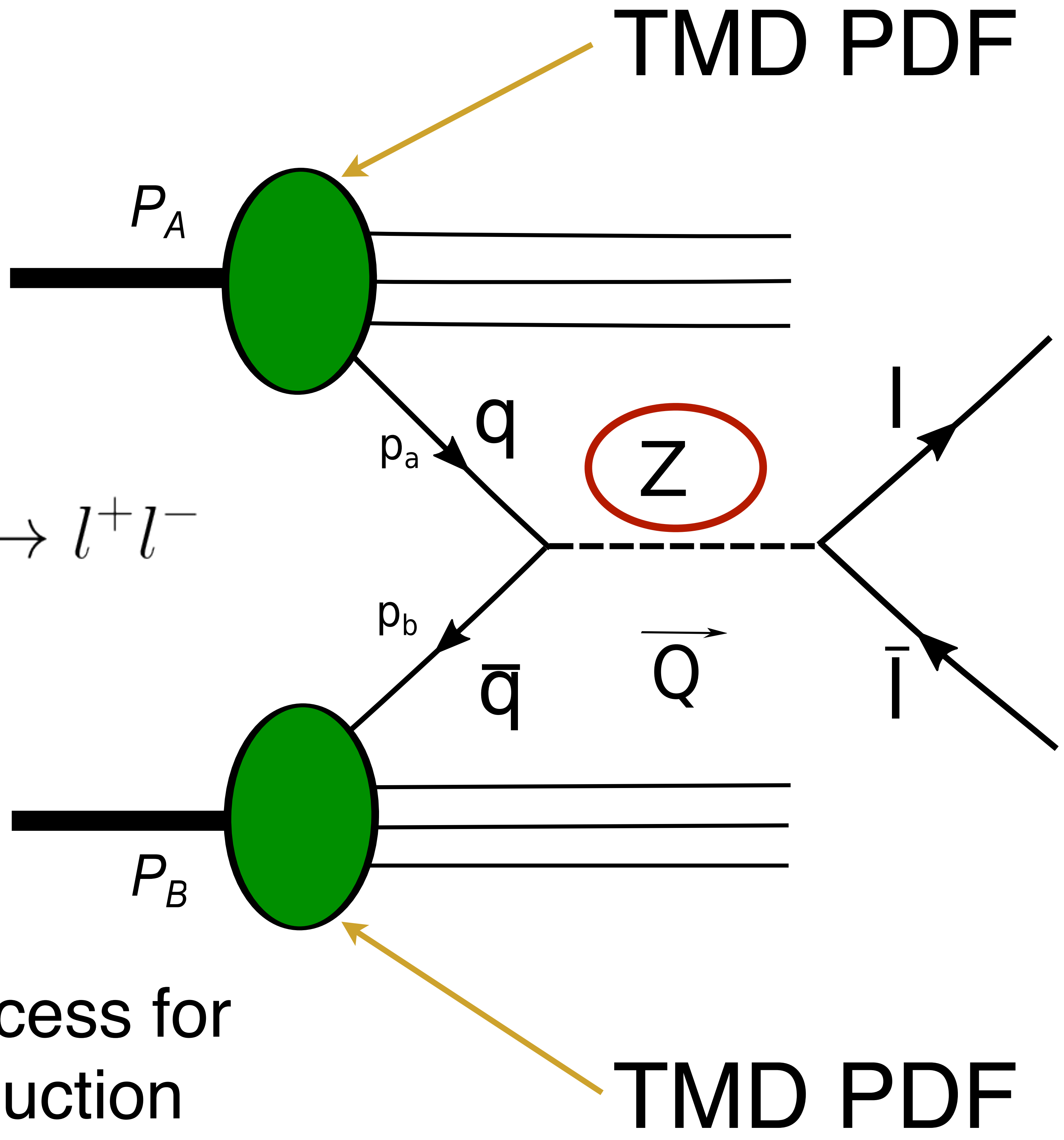
see, e.g., Rogers, Aybat, PRD 83 (11)
Collins, "Foundations of Perturbative QCD" (11)
Collins, Soper, Sterman, NPB250 (85)

Drell-Yan processes



$$A + B \to \gamma^* \to l^+ l^-$$

Drell-Yan processes

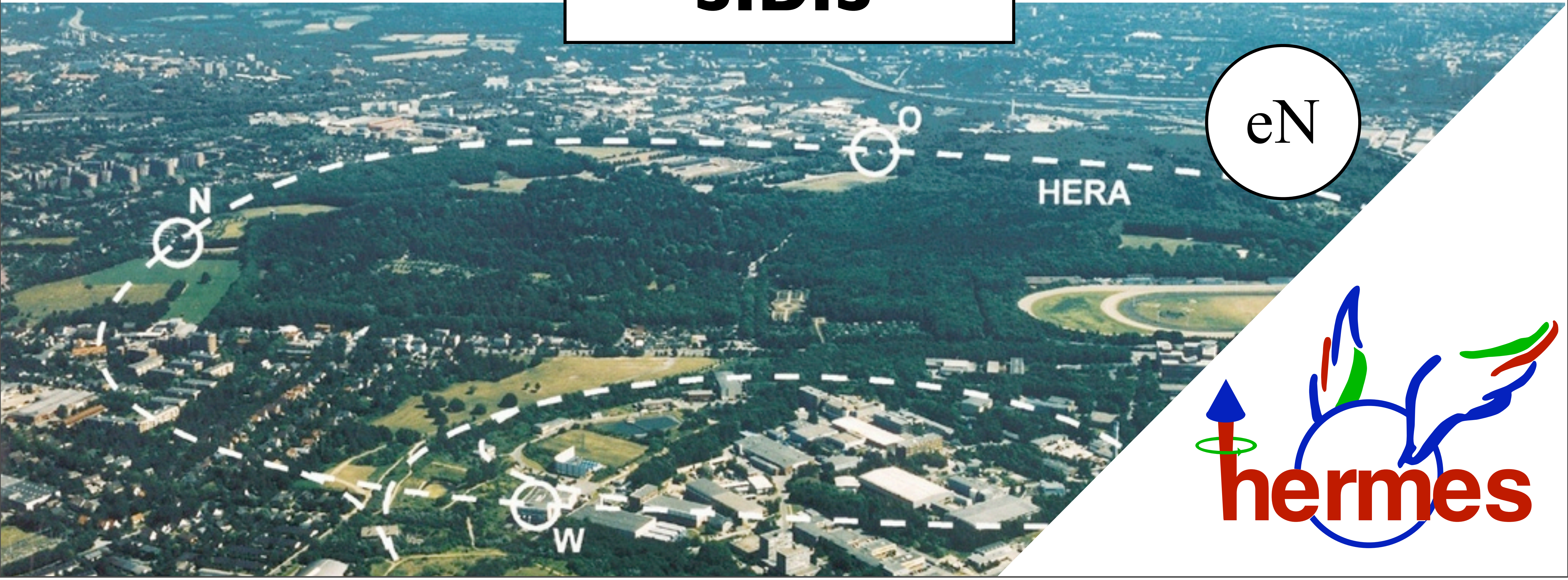


$$A + B \to Z \to l^+ l^-$$

Analogous process for Z boson production

COMPASS

SIDIS



hermes

DRELL-YAN

Z

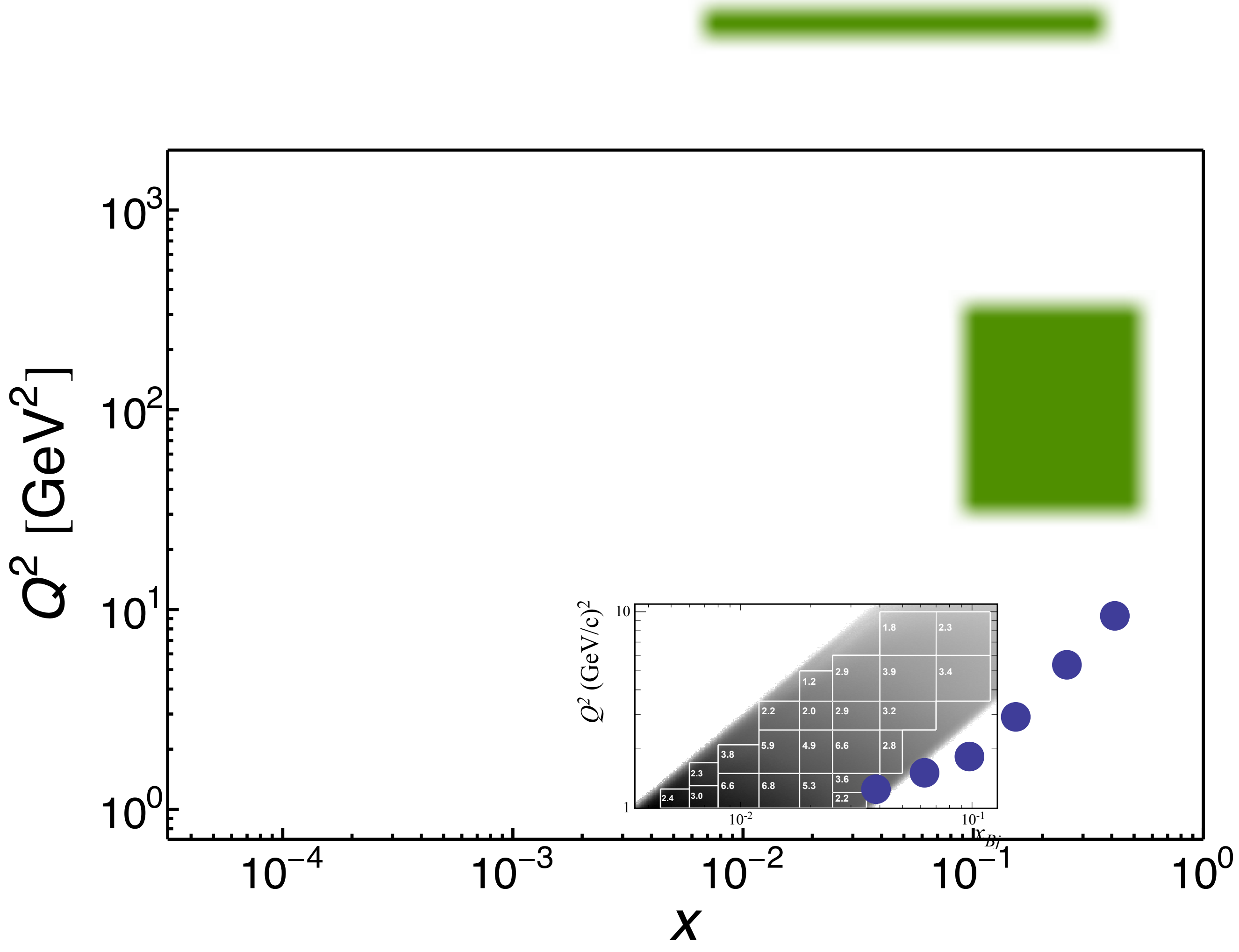
$p\bar{p}$

pN

Υ^*

E288
E605

Available data



Z production

Drell-Yan@

Fermilab

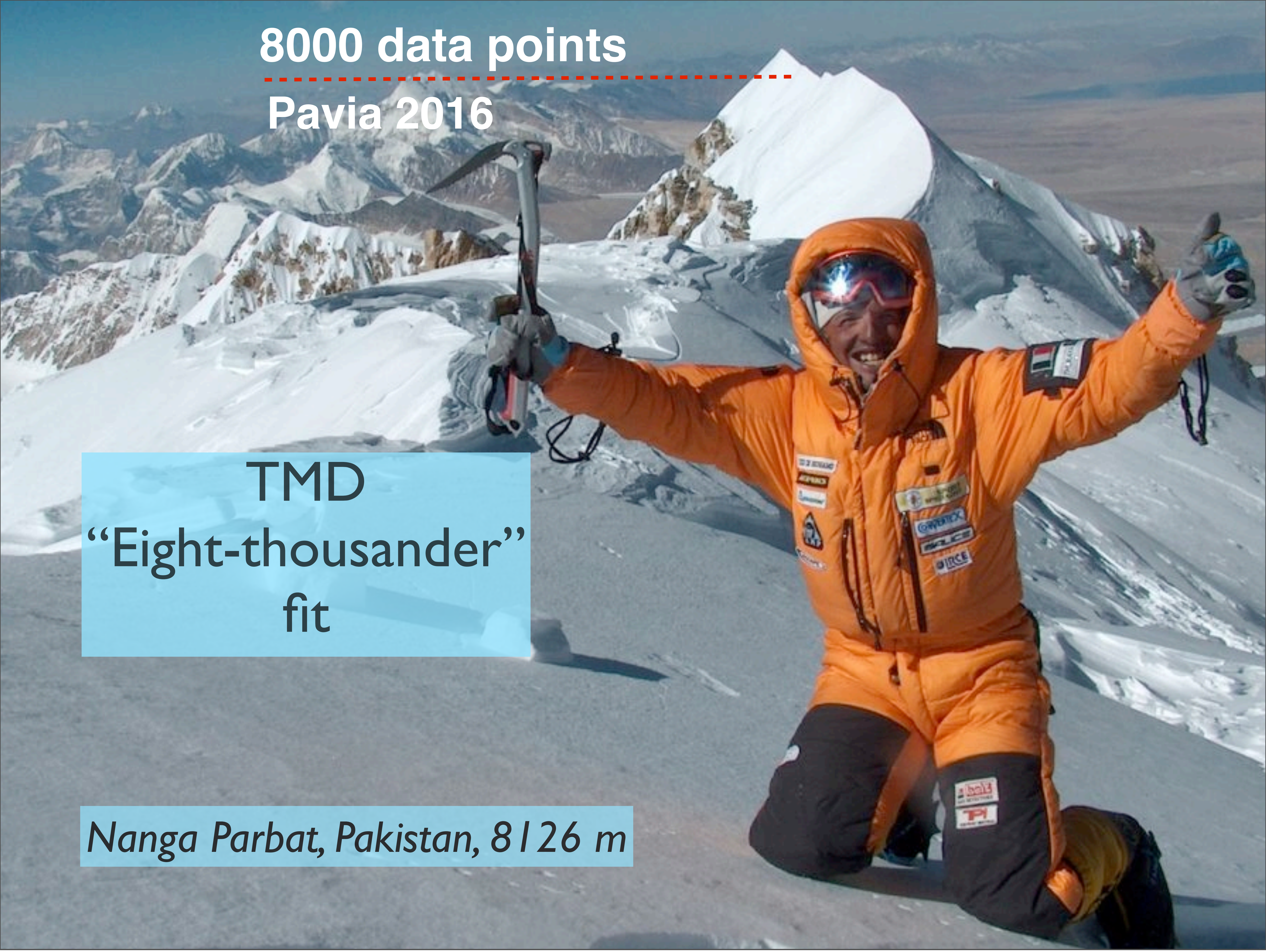
hermes

COMPASS

future data of EIC?
small-x, high Q2

Published and *soon* available fits

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	NLL	✗	✗	✓	✓	98
Pavia 2013 (+Amsterdam,Bilbao) arXiv:1309.3507	No evo	✓	✗	✗	✗	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✓	✓	500 (?)
Pavia 2016	NLL	✓	✓	✓	✓	8156



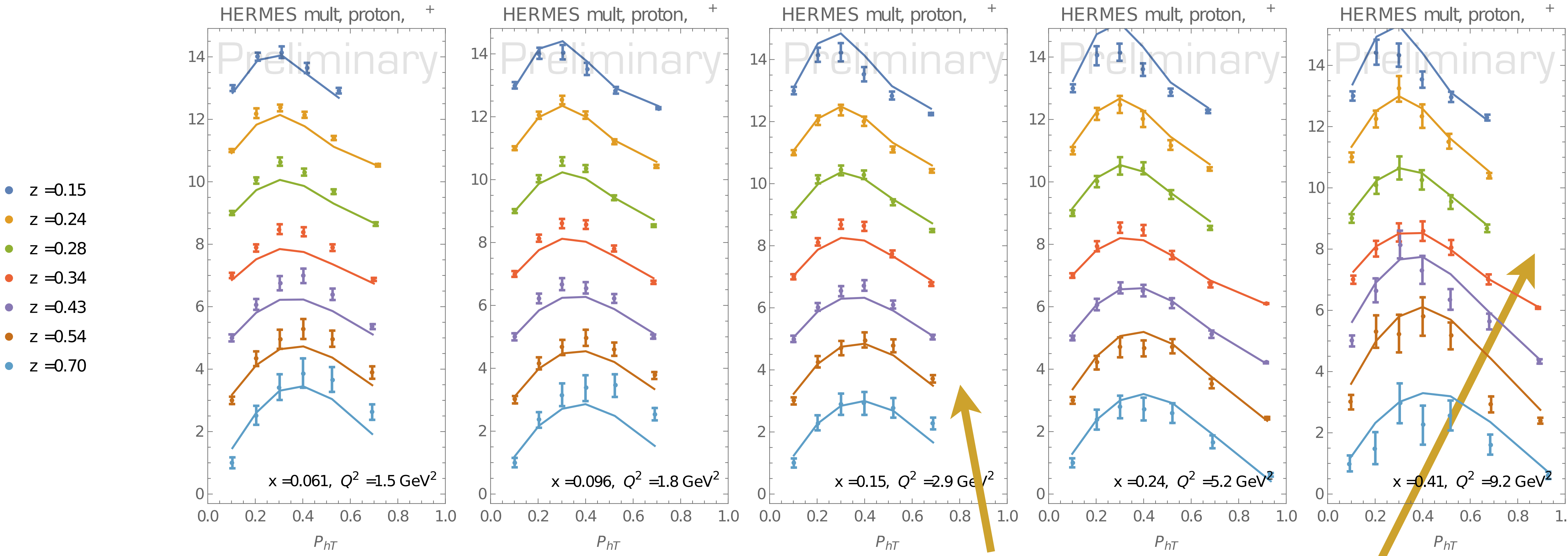
8000 data points

Pavia 2016

TMD
“Eight-thousander”
fit

Nanga Parbat, Pakistan, 8126 m

HERMES (some selected bins)



$\chi^2/\text{dof} = 4.20$ for proton π^+

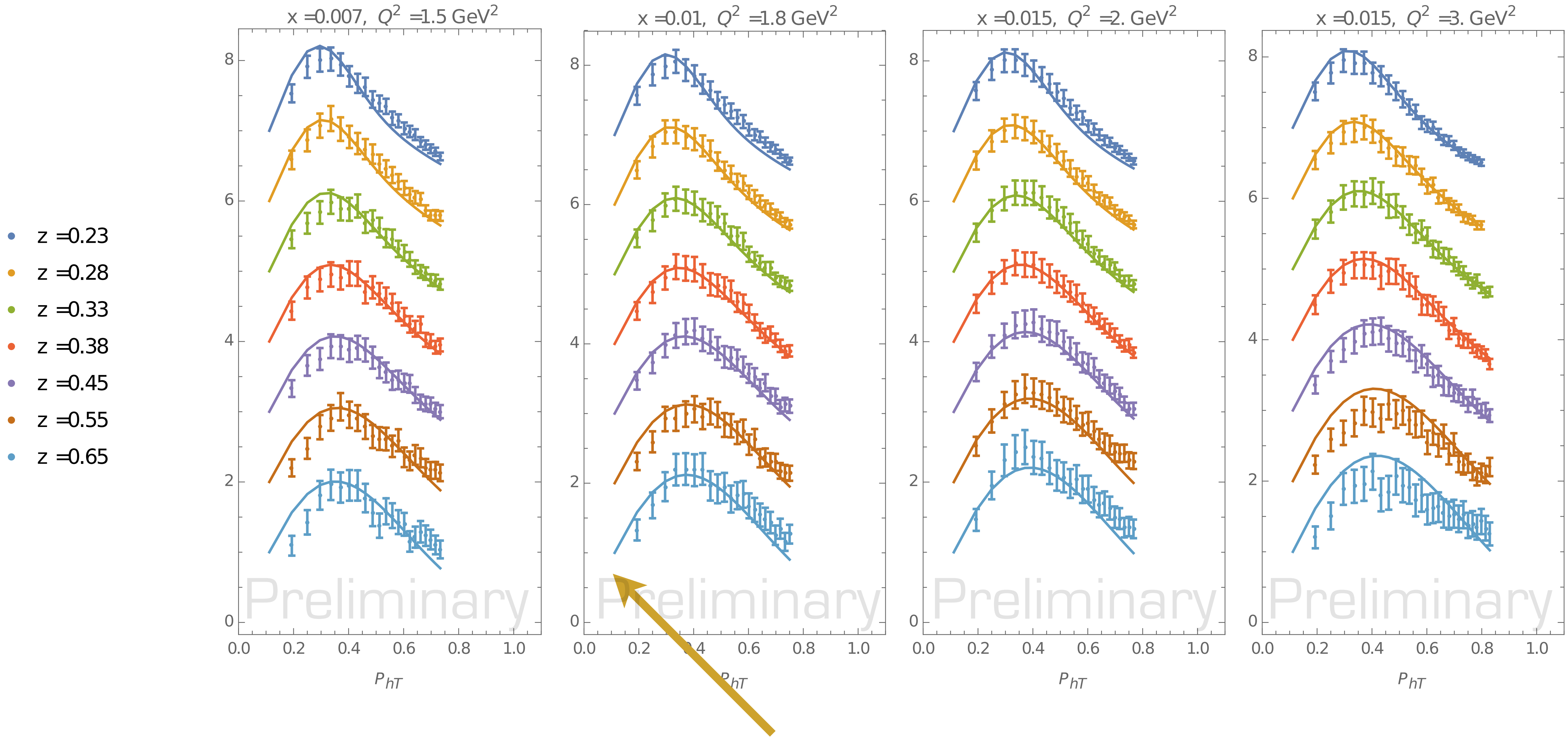
cut on $P_{hT} < 0.2\ Q + 0.5$

(other 7 channels are better)

stronger cut on P_{hT} at low z

However, normalizing the theory curves to the first bin, without changing the parameters of the fit, $\chi^2/\text{dof} = 1.94$

Compass (some selected bins)

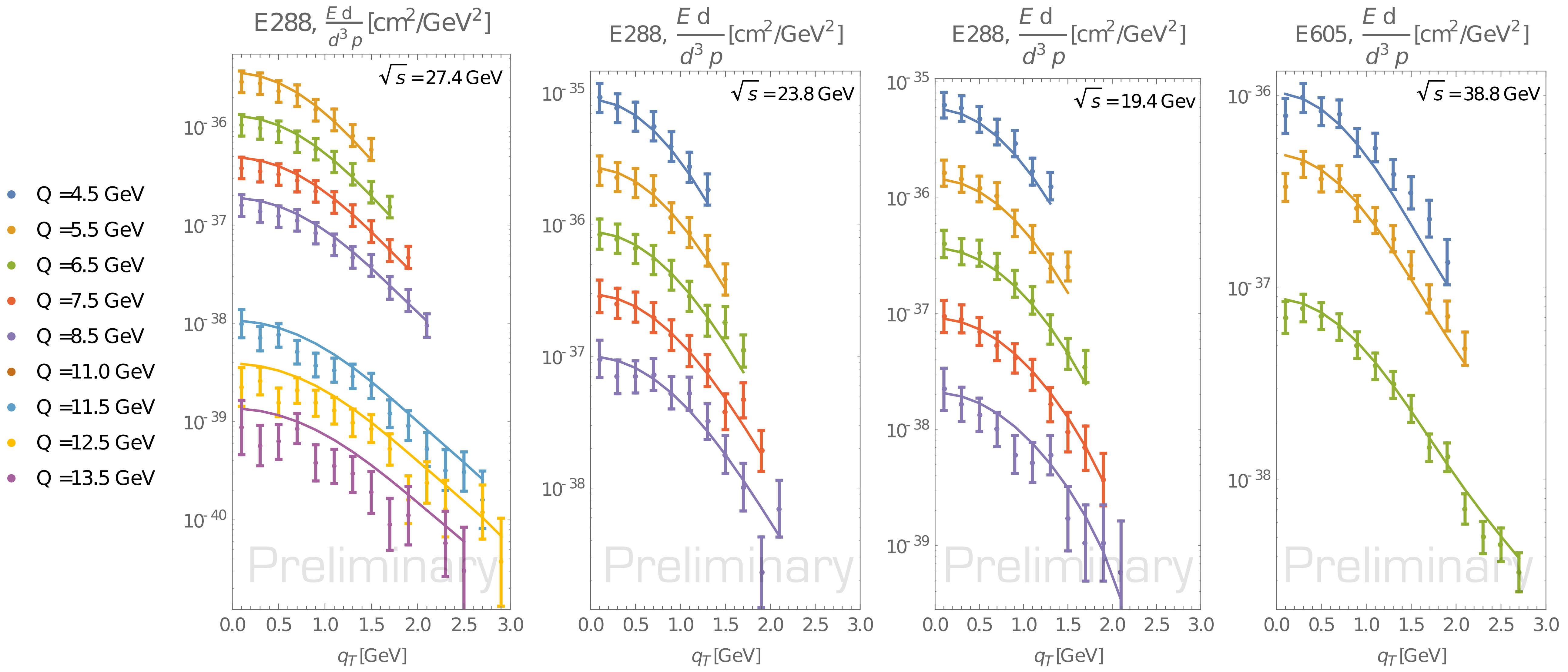


First points are not fitted, but used as normalization to avoid problems related to data normalization

Compass deuteron h$^+$

$\chi^2/\text{dof} = 1.49$

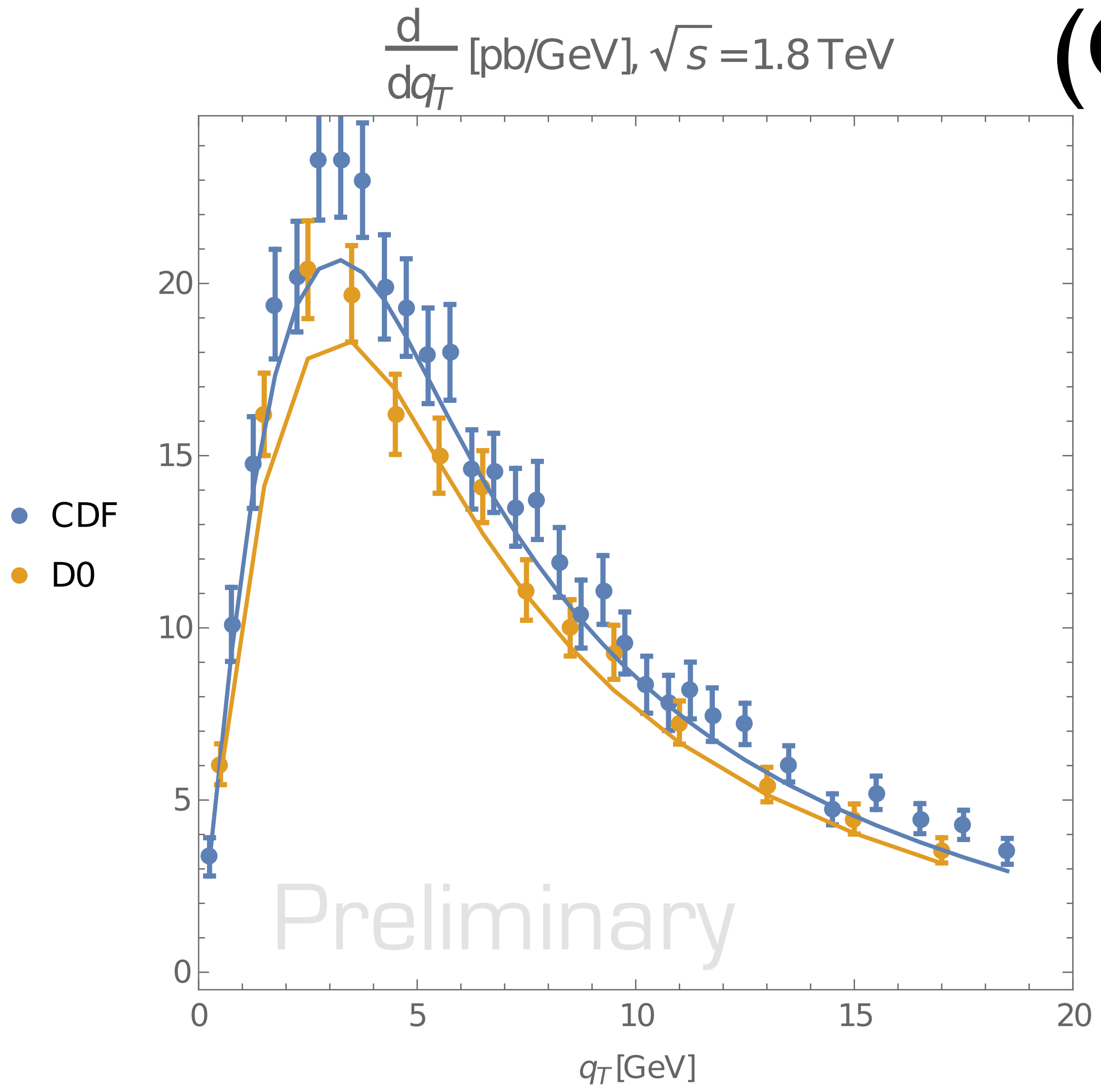
Drell-Yan data

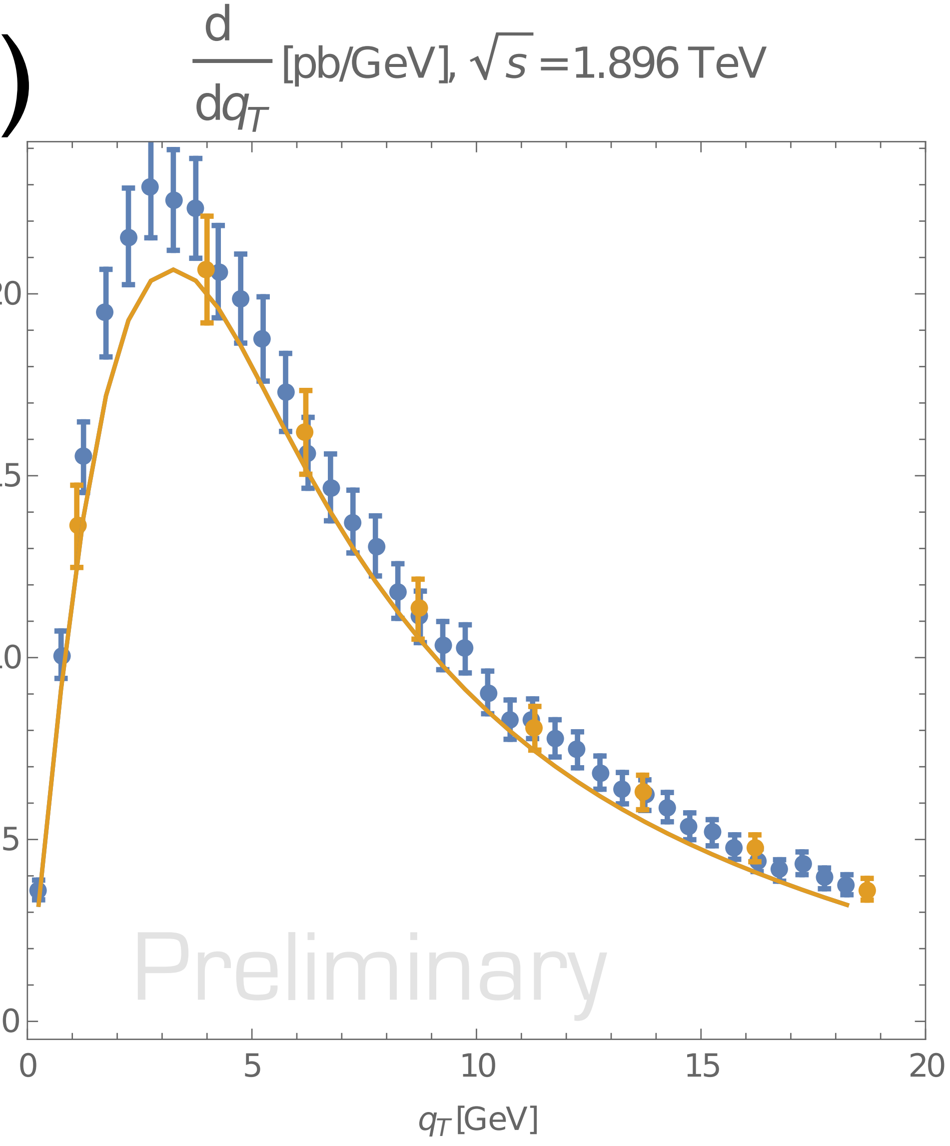


χ^2/dof = 1.57 χ^2/dof = 0.48 χ^2/dof = 0.42 χ^2/dof = 0.97

Z Boson production data

$(Q \simeq M_Z)$





$\chi^2/\mathrm{dof} = 1.36$

$\chi^2/\mathrm{dof} = 1.11$

$\chi^2/\mathrm{dof} = 2.00$

$\chi^2/\mathrm{dof} = 1.73$

Conclusions

Conclusions

- We showed how useful is to fit the TMD FFs and PDFs to different different processes to test their universality.

Conclusions

- We showed how useful is to fit the TMD FFs and PDFs to different different processes to test their universality.

- We demonstrated for the first time that it is possible to fit simultaneously SIDIS, DY, and Z boson data

Conclusions

- We showed how useful is to fit the TMD FFs and PDFs to different different processes to test their universality.
- We demonstrated for the first time that it is possible to fit simultaneously SIDIS, DY, and Z boson data
- We extracted unpolarized TMDs using several thousand data points.

Conclusions

- We showed how useful is to fit the TMD FFs and PDFs to different different processes to test their universality.
- We demonstrated for the first time that it is possible to fit simultaneously SIDIS, DY, and Z boson data
- We extracted unpolarized TMDs using several thousand data points.
- We are working on uncertainty studies and Y terms still to be implemented.

BACKUP

μ and b$_*$ prescriptions

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

μ and b$_*$ prescriptions

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, \overset{\text{Choice}}{\overbrace{b_*}}; \overset{\text{Choice}}{\overbrace{\mu_b}}) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

μ and b_* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x,b_T;\mu^2)=\sum_i(\tilde{C}_{a/i}\otimes f_1^i)(x,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu)}e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\hat{f}^a_{\mathrm{NP}}(x,b_T)$$

$$\mu_b=2e^{-\gamma_E}/b_* \qquad b_*\equiv\frac{b_T}{\sqrt{1+b_T^2/b_{\mathrm{max}}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b=2e^{-\gamma_E}/b_* \qquad b_*\equiv b_{\mathrm{max}}\left(1-e^{-\frac{b_T^4}{b_{\mathrm{max}}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b=Q_0+q_T \qquad b_*=b_T$$

DEMS 2014

μ and b$_*$ prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv \dfrac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$ Collins, Soper, Sterman, NPB250 (85)

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv b_{\max}\left(1 - e^{-\frac{b_T^4}{b_{\max}^4}}\right)^{1/4}$ Bacchetta, Echevarria, Mulders, Radici, Signori arXiv:1508.00402

$\mu_b = Q_0 + q_T$ $\quad b_* = b_T$ DEMS 2014

Complex-b prescription Laenen, Sterman, Vogelsang, PRL 84 (00)

Nonperturbative ingredients 1

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

Nonperturbative ingredients 1

$$\widetilde{f}_1^a(x,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes f_1^i\big)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \overset{\textbf{Choice}}{\hat{f}^a_{\mathrm{NP}}}(x,b_T)$$

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes f_1^i\big)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{f}^a_{\mathrm{NP}}(x,b_T)$$

$e^{-\frac{b_T^2}{\langle b_T^2\rangle}}$	almost everybody
$e^{-\frac{b_T^2}{\left\langle b_T^2(x)\right\rangle_a}}$	Pavia 2013, KN 2006
$e^{-\lambda_1 b_T}\left(1+\lambda_2 b_T^2\right)$	DEMS 2014

Nonperturbative ingredients 2

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

Nonperturbative ingredients 2

Choice

$$\widetilde{f}_1^a(x,b_T;\mu^2)=\sum_i\big(\tilde{C}_{a/i}\otimes f_1^i\big)(x,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu)}e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\hat{f}_{\mathrm{NP}}^a(x,b_T)$$

Nonperturbative ingredients 2

Choice

$$\widetilde{f}_1^a(x,b_T;\mu^2) = \sum_i \left(\tilde{C}_{a/i}\otimes f_1^i\right)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{f}^a_{\mathrm{NP}}(x,b_T)$$

$$-g_2\frac{b_T^2}{2}$$

Collins, Soper, Sterman, NPB250 (85)

$$-2\,g_2\ln\left(1+\frac{b_T^2}{4}\right)$$

Aidala, Field, Gamberg, Rogers
arXiv:1401.2654

$$-g_0(b_{\max})\left(1-\exp\left[-\frac{C_F\alpha_s(\mu_{b_*})b_T^2}{\pi g_0(b_{\max})b_{\max}^2}\right]\right)$$

Collins, Rogers
arXiv:1412.3820

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \rightarrow \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

see talks by Collins, Boglione, (Rogers?)

Low-b$_\mathrm{T}$ modifications

$$\log\left(Q^2 b_T^2\right) \rightarrow \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

see talks by Collins, Boglione, (Rogers?)

Data selection

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < 0.2\,Q + 0.5\ \mathrm{GeV}$ $\qquad$ $P_{hT} < 0.8\ \mathrm{GeV}\ (\mathrm{if}\ z < 0.3)$

Data selection

$Q^2 > 1.4 \text{ GeV}^2$

$0.2 < z < 0.7$

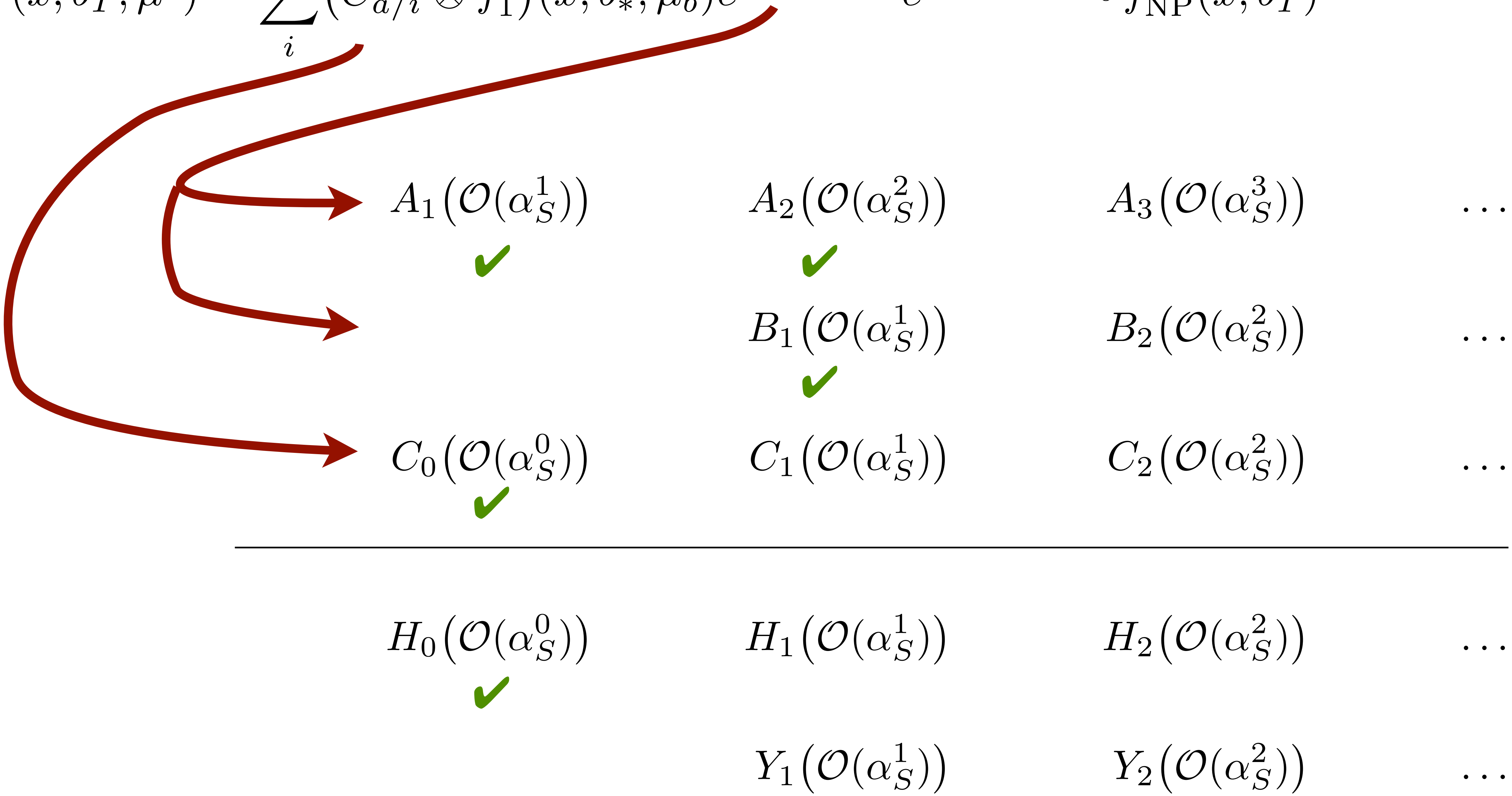
$P_{hT}, q_T < 0.2\, Q + 0.5 \text{ GeV}$ $\qquad$ $P_{hT} < 0.8 \text{ GeV } (\text{if } z < 0.3)$

Total number of data points: 8156

Total $\chi^2/\text{dof} = 1.45$

Pavia 2016 perturbative ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T) \ln \frac{\mu}{\mu_0}}\, \hat{f}^a_{\mathrm{NP}}(x, b_T)$$



$A_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$A_2\big(\mathcal{O}(\alpha_S^2)\big)$ ✓	$A_3\big(\mathcal{O}(\alpha_S^3)\big)$	$\ldots$
	$B_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$B_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$
$C_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$C_1\big(\mathcal{O}(\alpha_S^1)\big)$	$C_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$

$H_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$H_1\big(\mathcal{O}(\alpha_S^1)\big)$	$H_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$
	$Y_1\big(\mathcal{O}(\alpha_S^1)\big)$	$Y_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\rm max} \left(\frac{1 - e^{-b_T^4/b_{\rm max}^4}}{1 - e^{-b_T^4/b_{\rm min}^4}} \right)^{1/4} \qquad b_{\rm max} = 2e^{-\gamma_E}$$

$$b_{\rm min} = \frac{2e^{-\gamma_E}}{Q}$$

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad \bar{b}_* \equiv b_{\rm max}\left(\frac{1-e^{-b_T^4/b_{\rm max}^4}}{1-e^{-b_T^4/b_{\rm min}^4}}\right)^{1/4} \qquad b_{\rm max} = 2e^{-\gamma_E}$$

$$b_{\rm min} = \frac{2e^{-\gamma_E}}{Q}$$

$$g_K = -g_2\frac{b_T^2}{2} \qquad \mu_0 = 1\,\mathrm{GeV}$$

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_* \equiv b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

$$g_K = -g_2 \frac{b_T^2}{2}$$

$$\mu_0 = 1\,\mathrm{GeV}$$

$g_2 = 0.14\ \mathrm{GeV}^2$ from fit results

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_* \equiv b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

$$g_K = -g_2 \frac{b_T^2}{2}$$

$$\mu_0 = 1\,\mathrm{GeV}$$

$g_2 = 0.14\ \mathrm{GeV}^2$ from fit results

$$\hat{f}_{\mathrm{NP}}^a = e^{-\frac{b_T^2}{\langle b_T^2(x) \rangle_a}}$$

Pavia 2016 other ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}^a_{\mathrm{NP}}(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_* \equiv b_{\mathrm{max}} \left(\frac{1 - e^{-b_T^4/b_{\mathrm{max}}^4}}{1 - e^{-b_T^4/b_{\mathrm{min}}^4}} \right)^{1/4}$$

$$b_{\mathrm{max}} = 2e^{-\gamma_E}$$

$$b_{\mathrm{min}} = \frac{2e^{-\gamma_E}}{Q}$$

$$g_K = -g_2 \frac{b_T^2}{2}$$

$$\mu_0 = 1\,\mathrm{GeV}$$

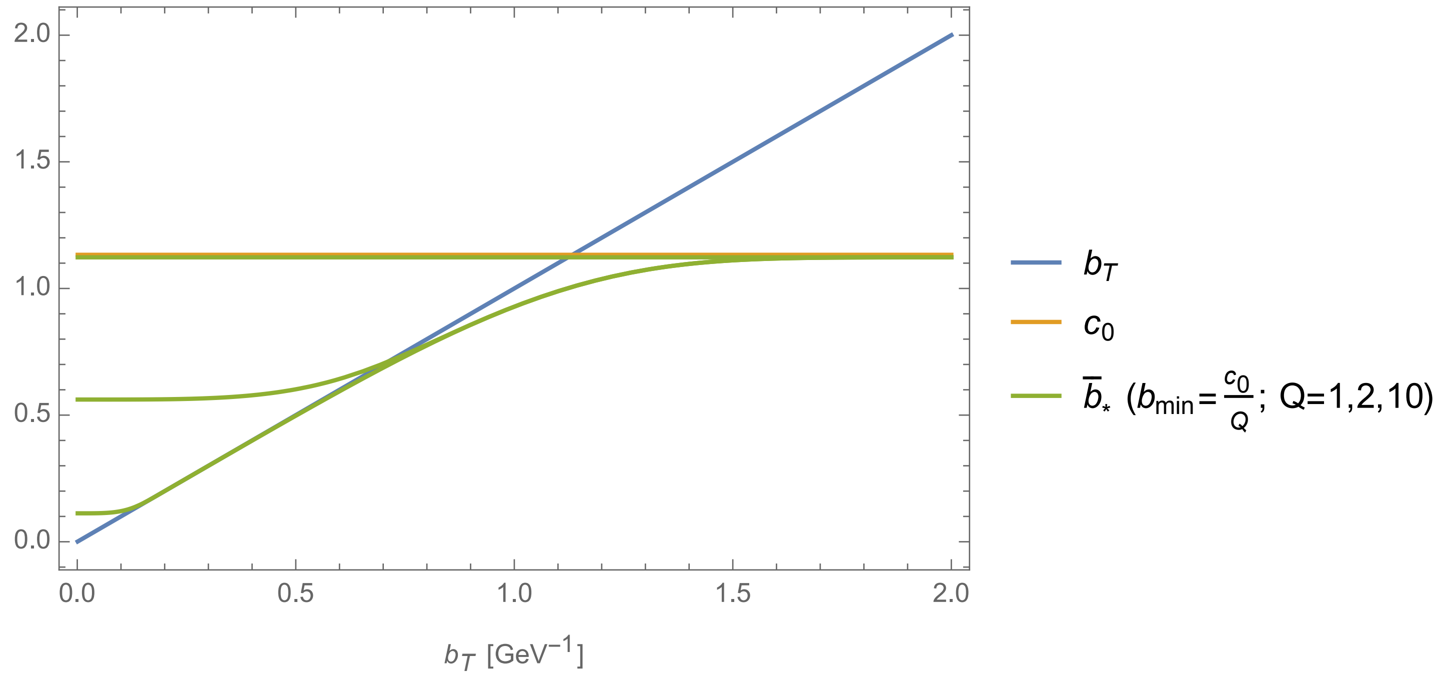
$g_2 = 0.14\ \mathrm{GeV}^2$ from fit results

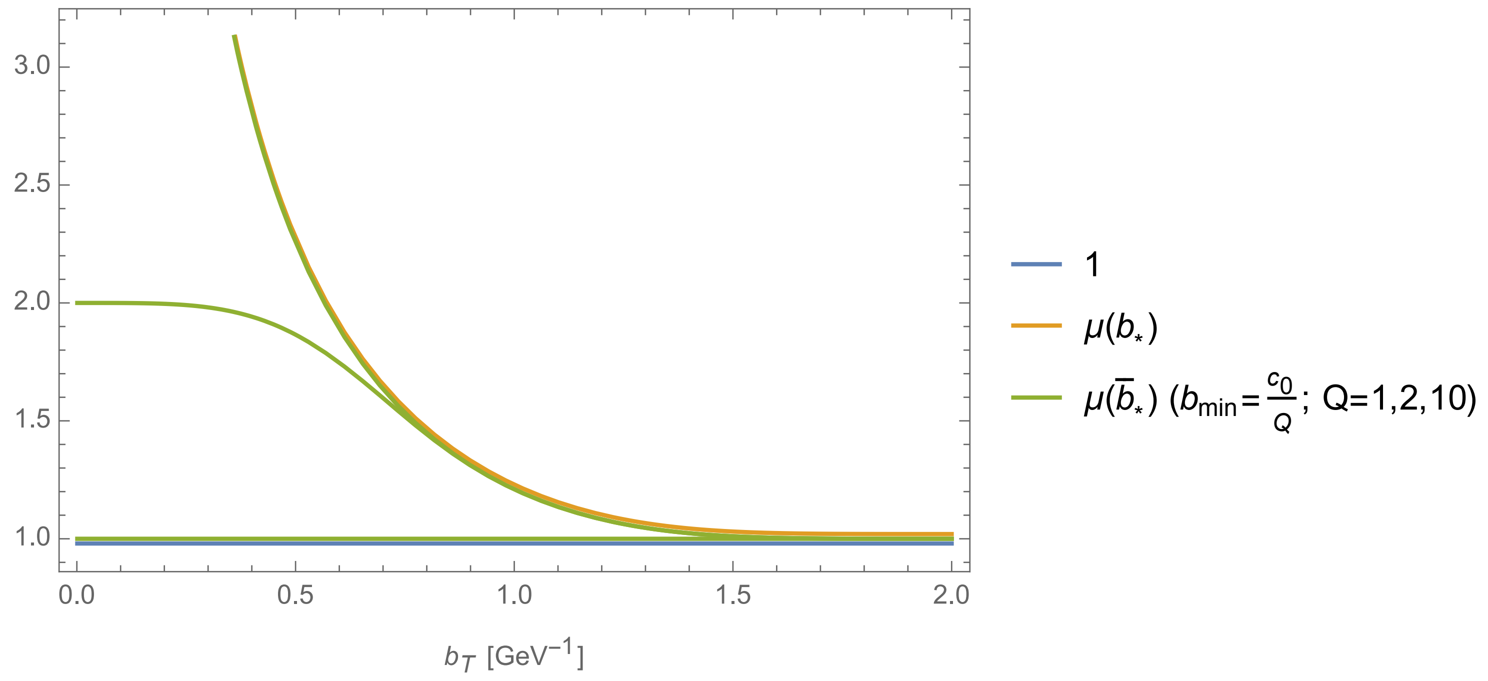
$$\hat{f}^a_{\mathrm{NP}} = e^{-\frac{b_T^2}{\langle b_T^2(x)\rangle_a}}$$

For fragmentation functions

$$\hat{f}^a_{\mathrm{NP}} = \mathrm{F.T.\ of} \left(e^{-\frac{P_\perp^2}{\langle P_\perp^2(z)\rangle_a}} + \lambda' P_\perp^2 e^{-\frac{P_\perp^2}{\langle P_\perp^2(z)\rangle'_a}} + \lambda'' P_\perp^4 e^{-\frac{P_\perp^2}{\langle P_\perp^2(z)\rangle''_a}} \right)$$

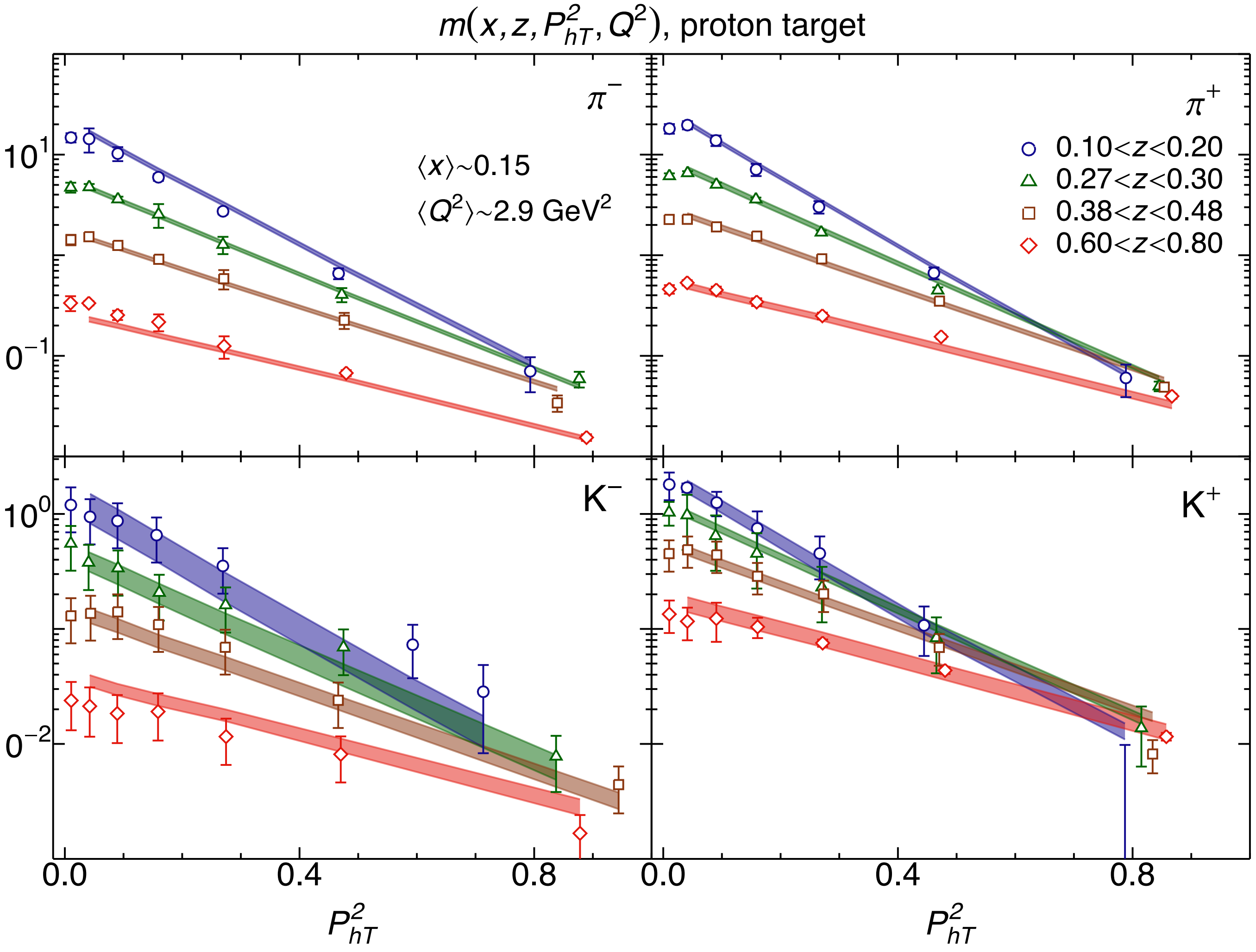
Effects of b_* prescription





Pavia 2013 (no TMD evo)

Global χ^2/dof = 1.63±0.12



1.80±0.27

2.64±0.21

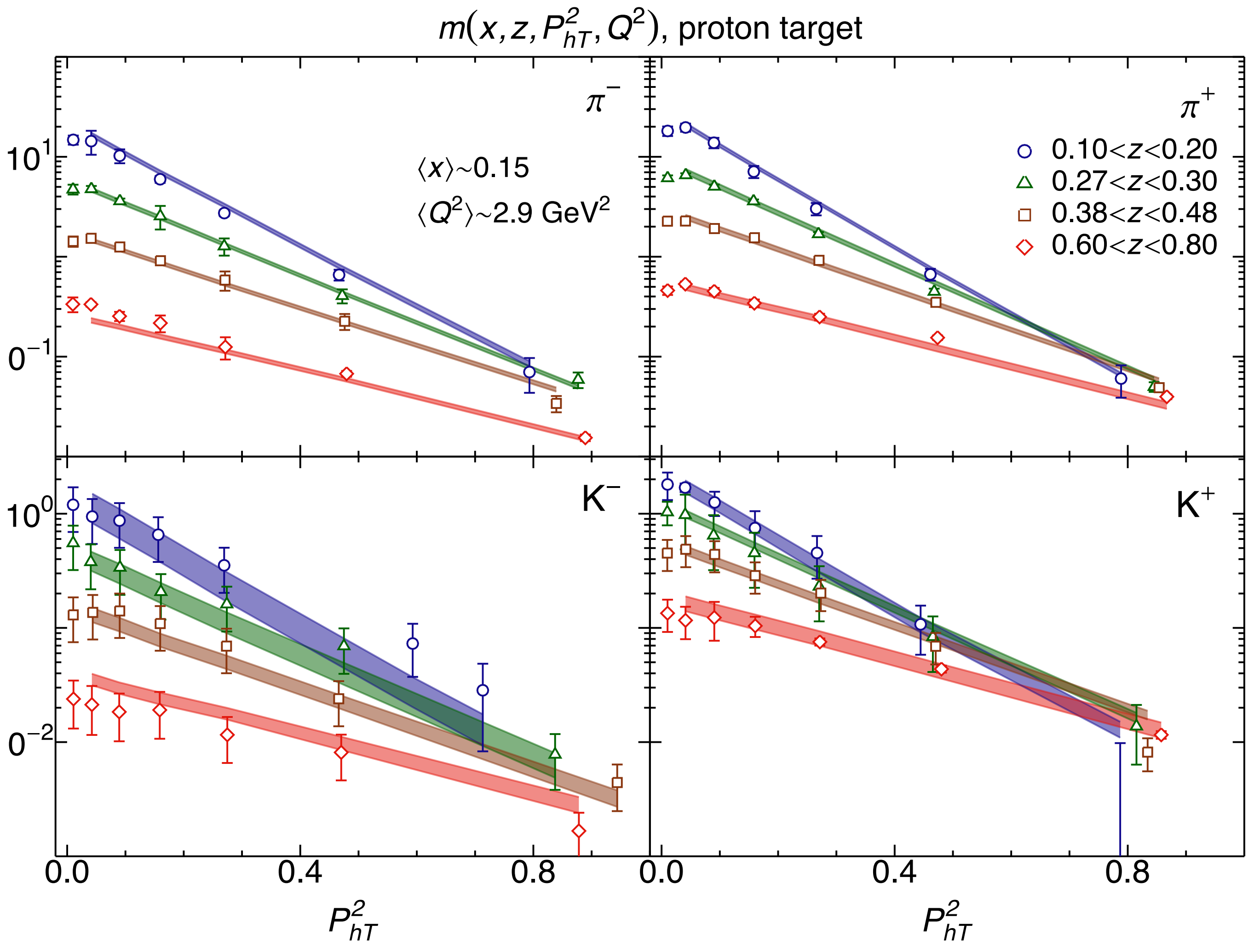
0.78±0.15

0.46±0.07

Pavia 2013 (no TMD evo)

Global χ^2/dof = 1.63±0.12

Without flavor dep.: global χ^2/dof = 1.72±0.11



1.80±0.27

1.83±0.25

2.64±0.21

2.89±0.23

0.78±0.15

0.87±0.16

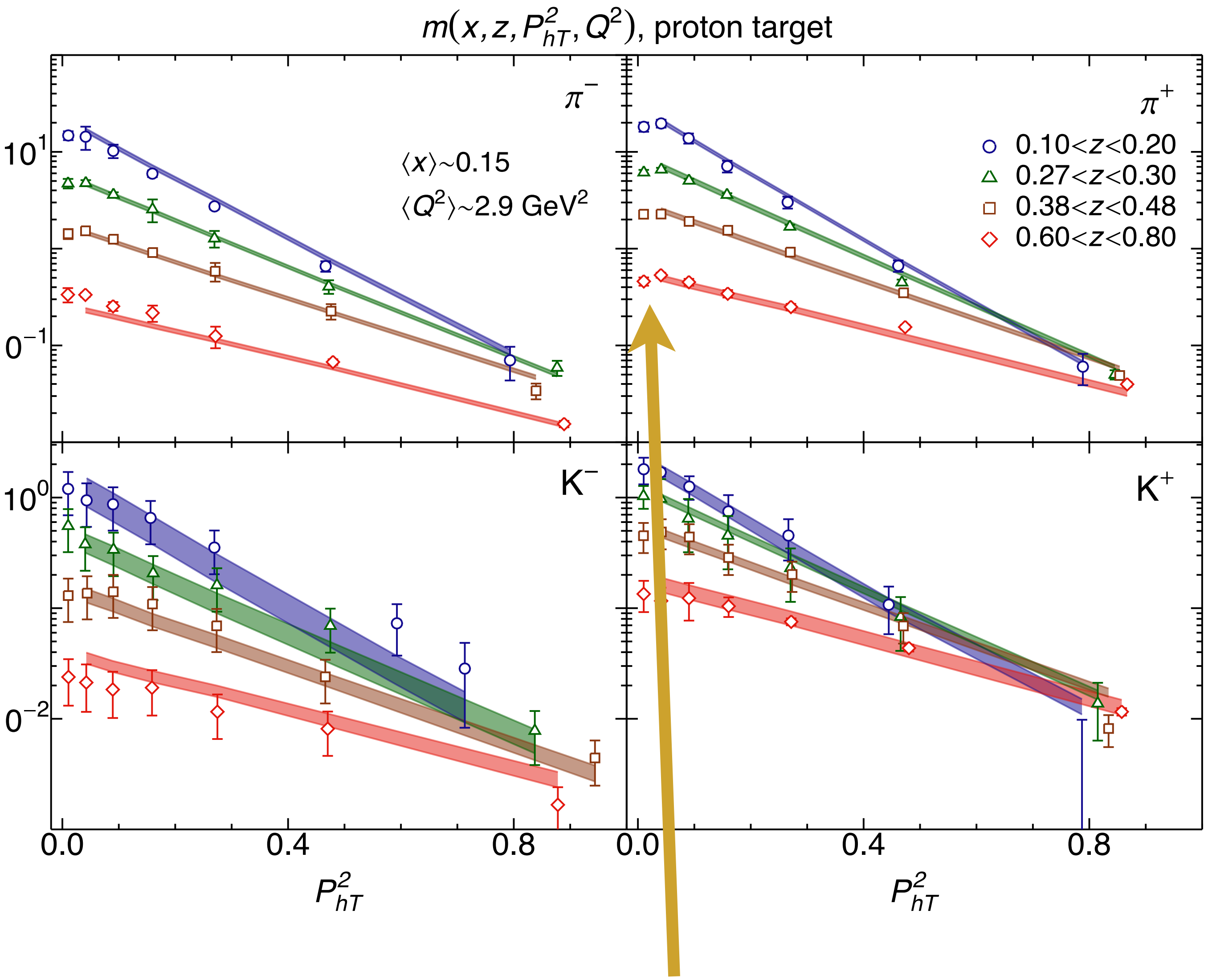
0.46±0.07

0.43±0.07

Pavia 2013 (no TMD evo)

Global $\chi^2/\text{dof} = 1.63 \pm 0.12$

Without flavor dep.: global $\chi^2/\text{dof} = 1.72 \pm 0.11$



1.80±0.27

1.83±0.25

2.64±0.21

2.89±0.23

0.78±0.15

0.87±0.16

0.46±0.07

0.43±0.07

first P_{hT} excluded from fit

Pavia 2013 (no TMD evo)

Global χ^2/dof = 1.63±0.12

Without flavor dep.: global χ^2/dof = 1.72±0.11

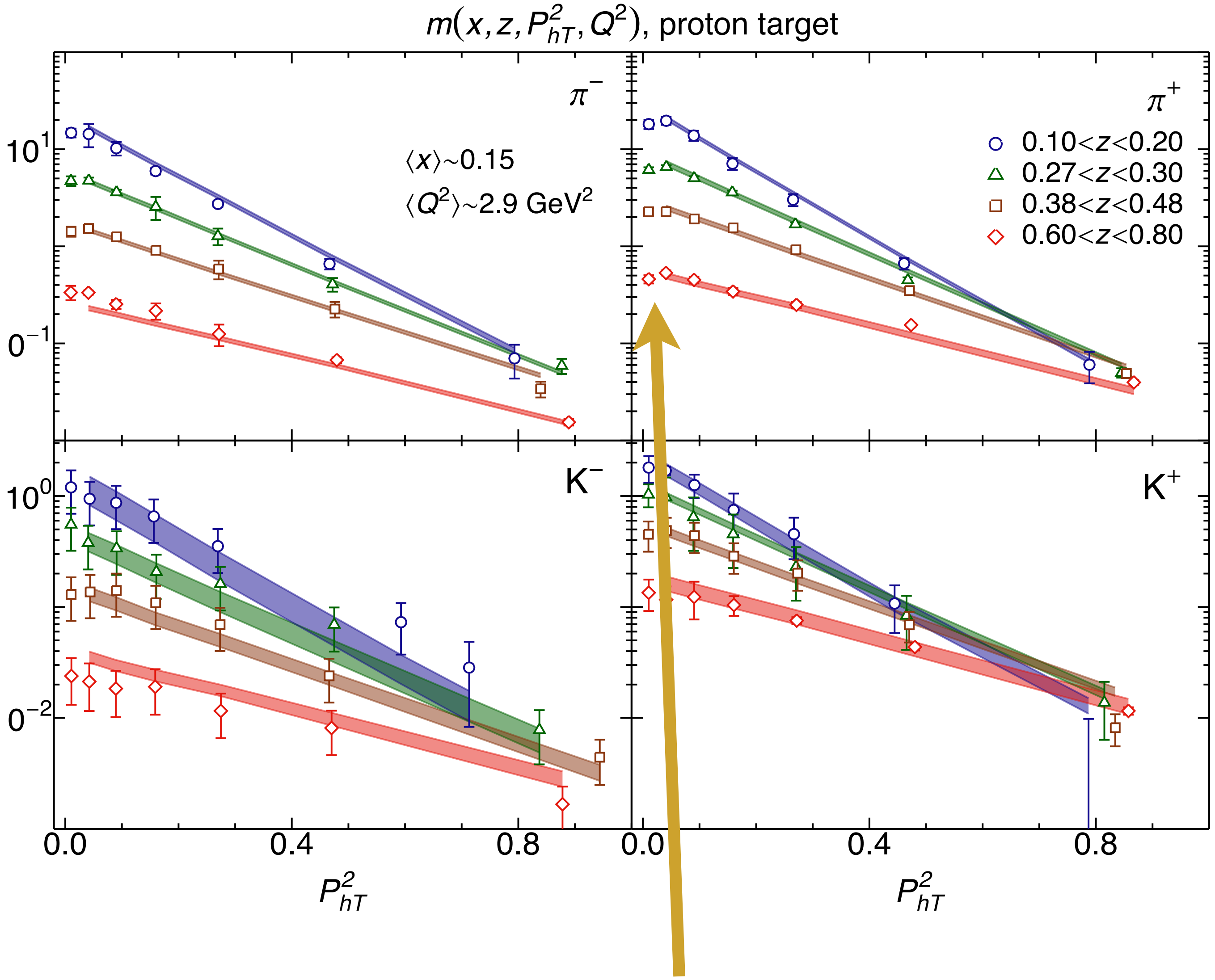
not so low χ^2

1.80±0.27

1.83±0.25

0.78±0.15

0.87±0.16



2.64±0.21

2.89±0.23

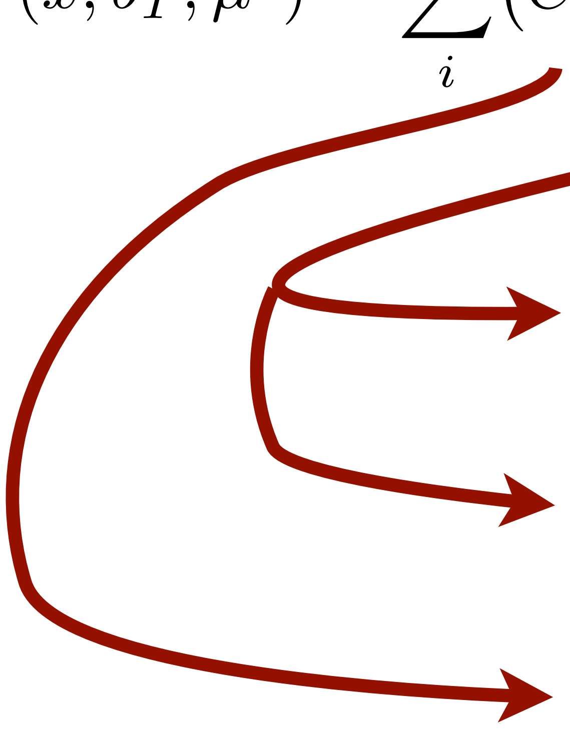
0.46±0.07

0.43±0.07

first P_{hT} excluded from fit

KN 2006 perturbative ingredients

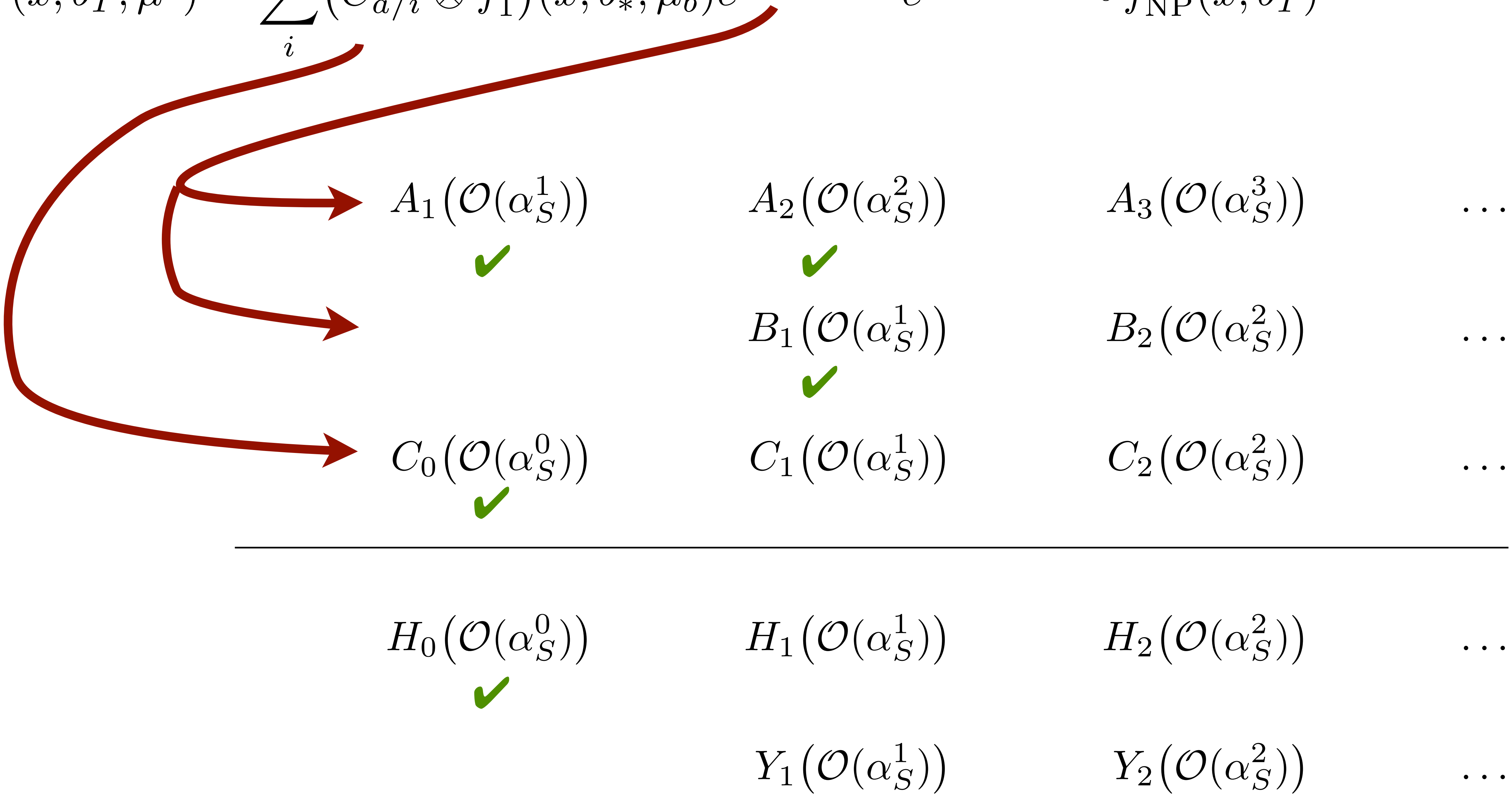
$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$



$A_1(\mathcal{O}(\alpha_S^1))$ ✓	$A_2(\mathcal{O}(\alpha_S^2))$ ✓	$A_3(\mathcal{O}(\alpha_S^3))$	$\ldots$
	$B_1(\mathcal{O}(\alpha_S^1))$ ✓	$B_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$C_0(\mathcal{O}(\alpha_S^0))$ ✓	$C_1(\mathcal{O}(\alpha_S^1))$ ✓	$C_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$H_0(\mathcal{O}(\alpha_S^0))$ ✓	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\ldots$

DEMS 2014 NLL

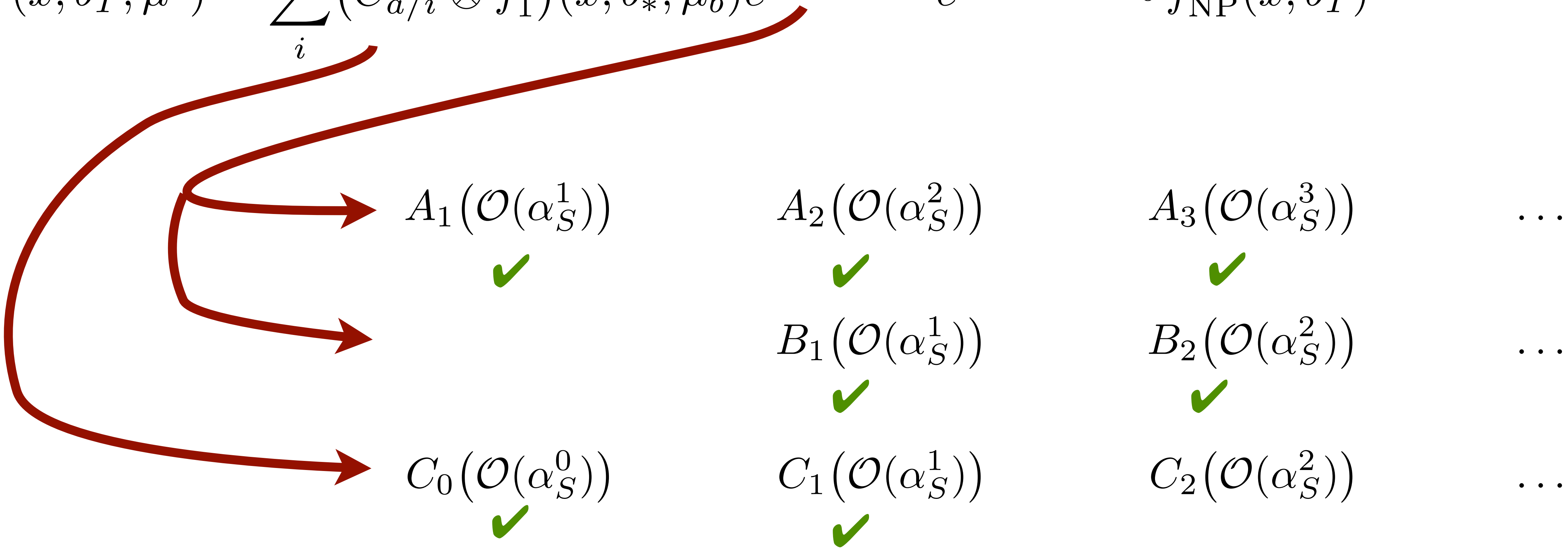
$$\widetilde{f}_1^{a}(x,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes f_1^{i}\big)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{f}^{a}_{\rm NP}(x,b_T)$$



$A_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$A_2\big(\mathcal{O}(\alpha_S^2)\big)$ ✓	$A_3\big(\mathcal{O}(\alpha_S^3)\big)$	$\dots$
	$B_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$B_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\dots$
$C_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$C_1\big(\mathcal{O}(\alpha_S^1)\big)$	$C_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\dots$
$H_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$H_1\big(\mathcal{O}(\alpha_S^1)\big)$	$H_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\dots$
	$Y_1\big(\mathcal{O}(\alpha_S^1)\big)$	$Y_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\dots$

DEMS 2014 NNLL

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$



$A_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$A_2\big(\mathcal{O}(\alpha_S^2)\big)$ ✓	$A_3\big(\mathcal{O}(\alpha_S^3)\big)$ ✓	$\ldots$
	$B_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$B_2\big(\mathcal{O}(\alpha_S^2)\big)$ ✓	$\ldots$
$C_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$C_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$C_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$

$H_0\big(\mathcal{O}(\alpha_S^0)\big)$ ✓	$H_1\big(\mathcal{O}(\alpha_S^1)\big)$ ✓	$H_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$
	$Y_1\big(\mathcal{O}(\alpha_S^1)\big)$	$Y_2\big(\mathcal{O}(\alpha_S^2)\big)$	$\ldots$