Fermion Propagators Interpolating between the Instant and Front Forms of Relativistic Dynamics

Bailing Ma
PhD Student, Ji's group
NC State University

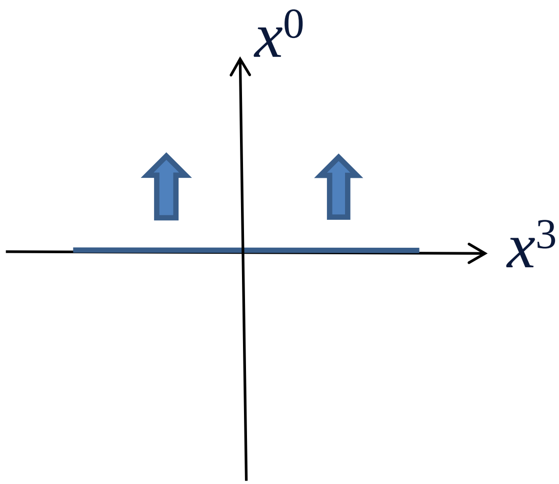
Outline

- Light Front Quantization and its properties
- Interpolation from equal-time quantization surface to the light-cone
- Fermion propagator interpolation and its fate when approaching the exact light front

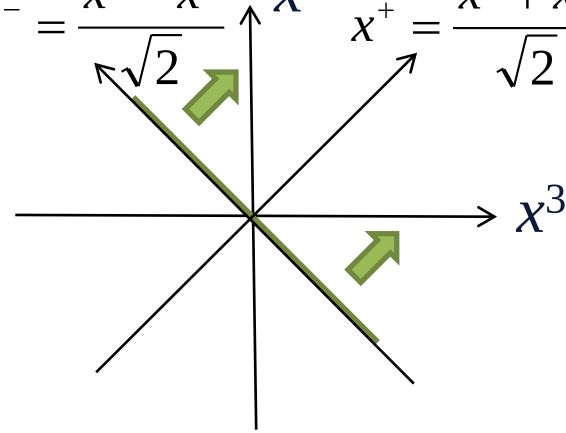
Light Front Quantization

Reference: P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

- Instant Form Dynamics (IFD)



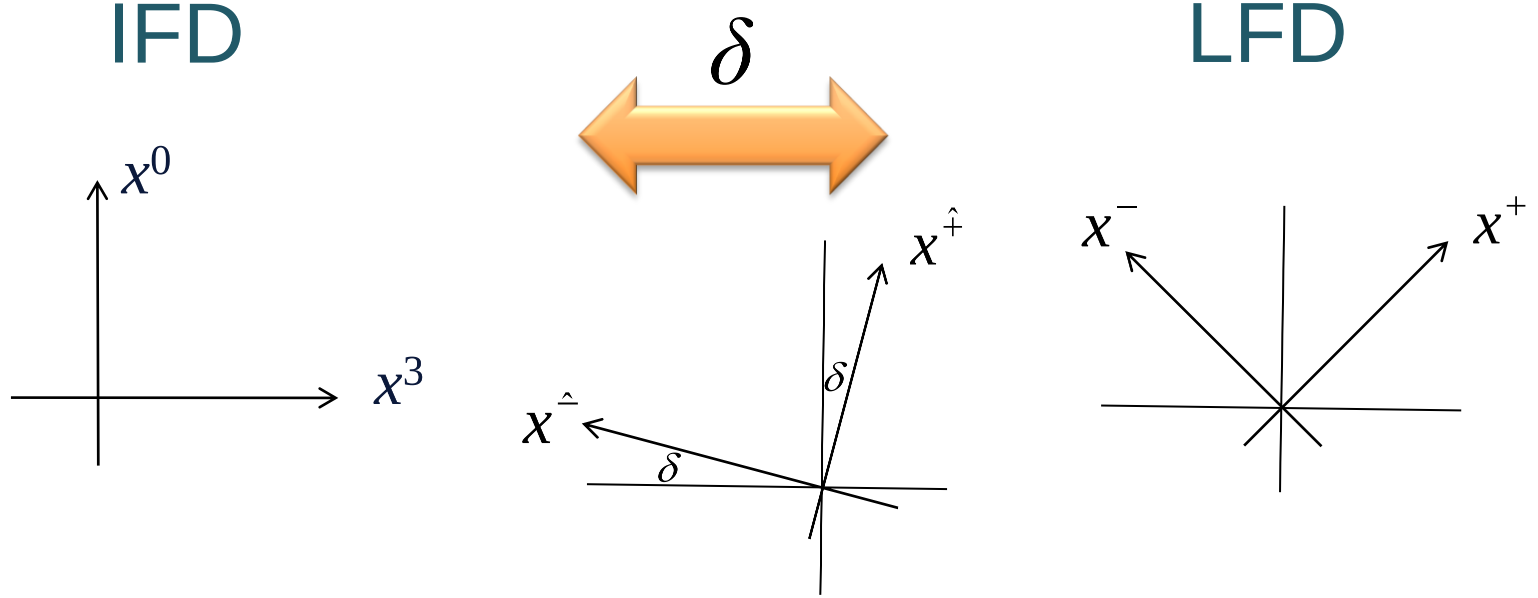
- Light Front Dynamics (LFD)



- Rational energy-momentum relation ➡ simpler vacuum structure
  - Light front energy: $k^- = k_+ = \frac{k_0 + k_3}{\sqrt{2}} = \frac{k^0 - k^3}{\sqrt{2}}$
  - Light front momentum: $k^+ = k_- = \frac{k^0 + k^3}{\sqrt{2}}, \ \vec{\mathbf{k}}_\perp$

  ➡ $k^+ k^- = \frac{(k^0)^2 - (k^3)^2}{2} = \frac{(\vec{\mathbf{k}}_\perp)^2 + m^2}{2}$ ➡ $k^+ \geq 0$
- Boost invariance: boost operation becomes kinematic, however transverse rotation becomes a dynamical problem

Interpolation Method

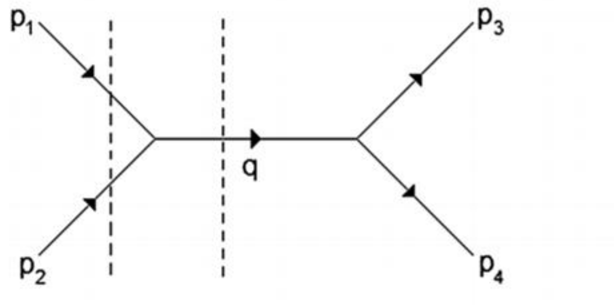


$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos\delta & 0 & 0 & \sin\delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\delta & 0 & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

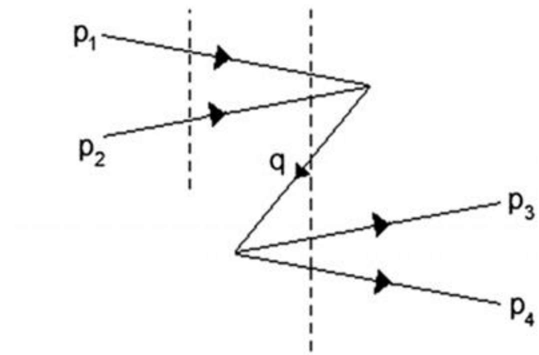
- Relate IFD and LFD, show the whole landscape in between
- Avoid singularity at $k^+=0$
- Gain insight into Light Front itself

Interpolation of Scalar Field Theory

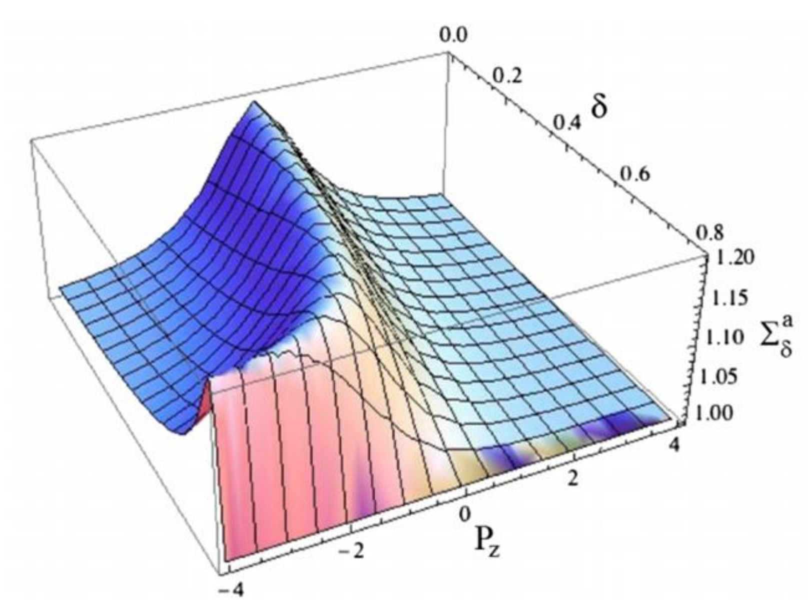
Reference: C.-R. Ji and A. T. Suzuki, Phys. Rev. D 87, 34 (2013).

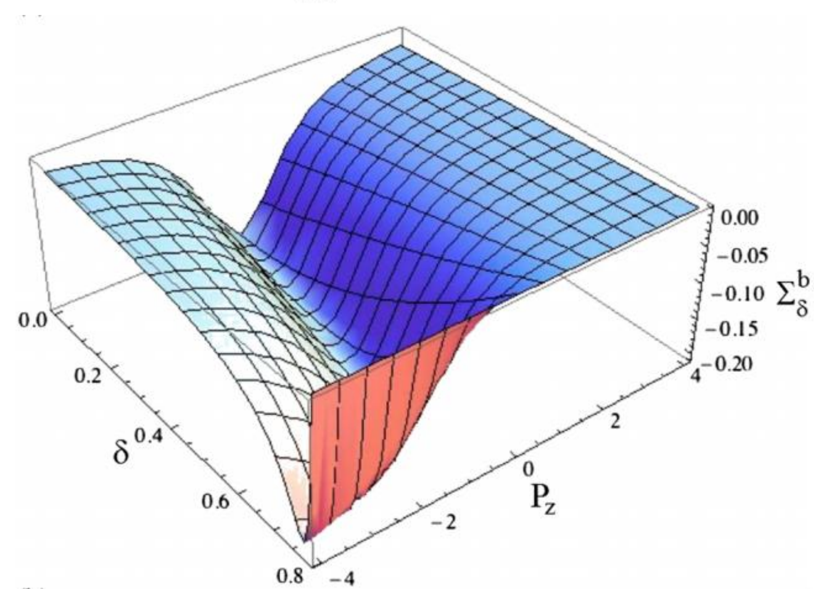


(a)



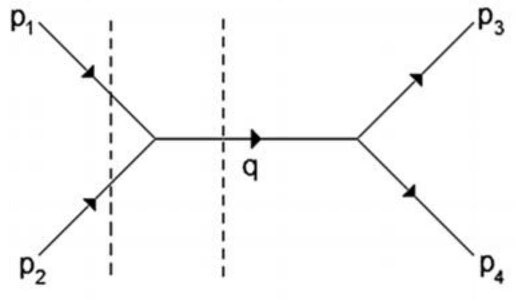
(b)



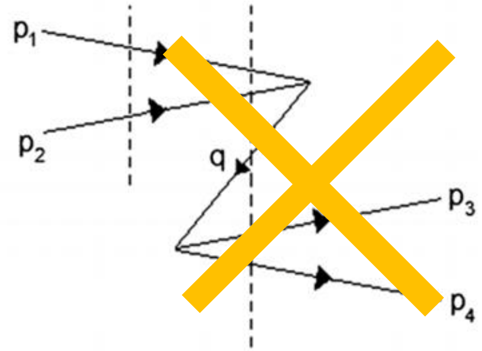


Interpolation of Scalar Field Theory

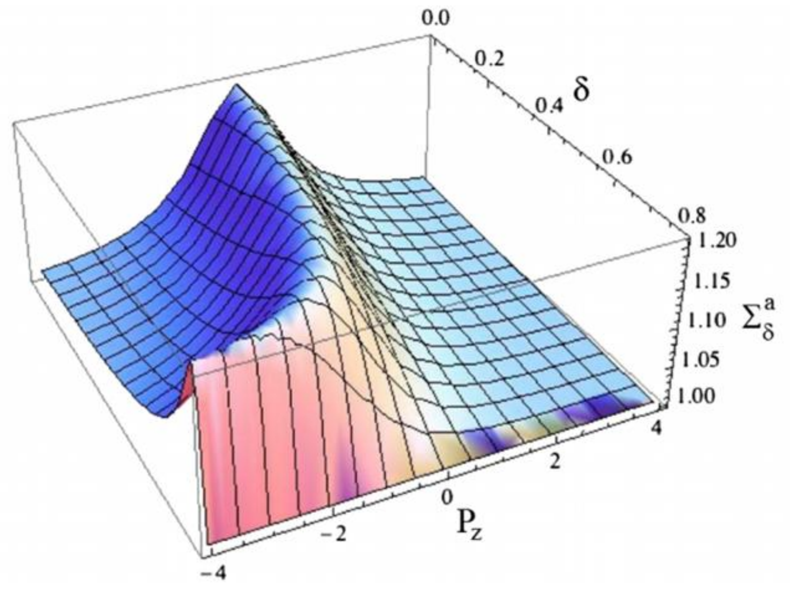
Reference: C.-R. Ji and A. T. Suzuki, Phys. Rev. D 87, 34 (2013).

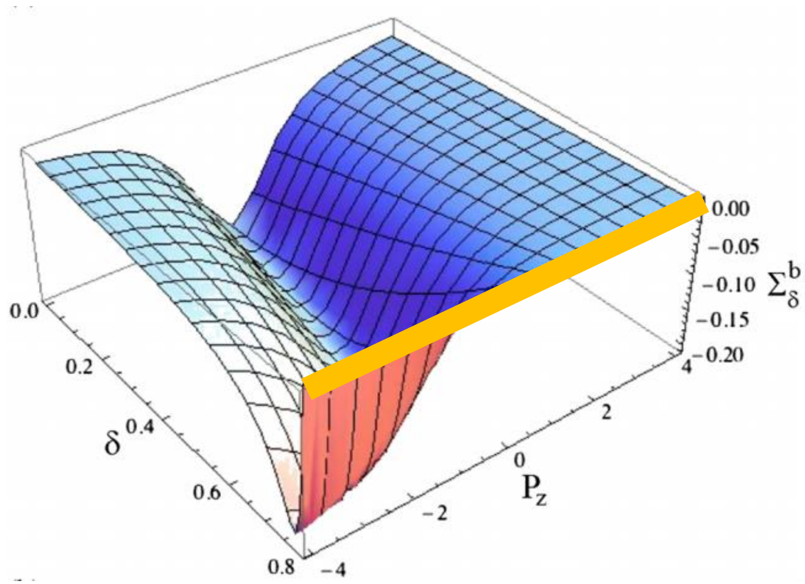


(a)



(b)





Interpolation of QED

Previous works:

Reference: C.-R. Ji, Z. Li, and A. T. Suzuki, Phys. Rev. D 91, 065020 (2015), Z. Li, M. An and C.-R Ji, Phys. Rev. D 92, 105014 (2015)

- Gauge Fields
  - Polarization Vectors
  - Gauge Condition
  - Photon Propagator
- Helicity Spinors

This work:

- Fermion Propagators

$$\Sigma_a = \frac{1}{2Q^{\hat{+}}} \frac{\not{Q}_a + m}{q_{\hat{+}} - Q_{a\hat{+}}} \qquad \Sigma_b = \frac{1}{2Q^{\hat{+}}} \frac{-\not{Q}_b + m}{-q_{\hat{+}} - Q_{b\hat{+}}}$$

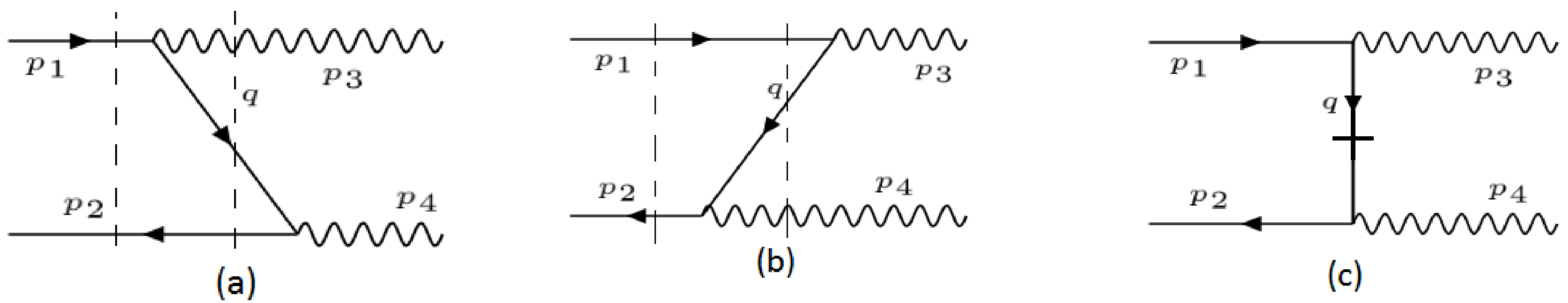
Light Front Limit of Fermion Propagator

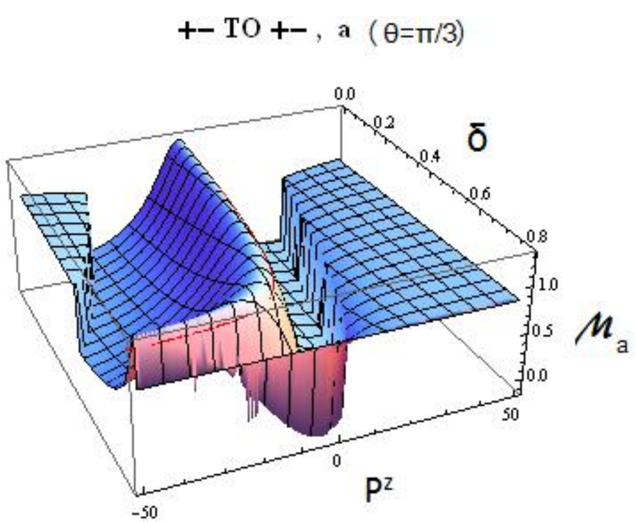
- When taking the limit δ→π/4, the propagator corresponding to positive energy change to the LF on-shell propagator.
- The negative energy (anti particle) propagator change to LF instantaneous propagator.

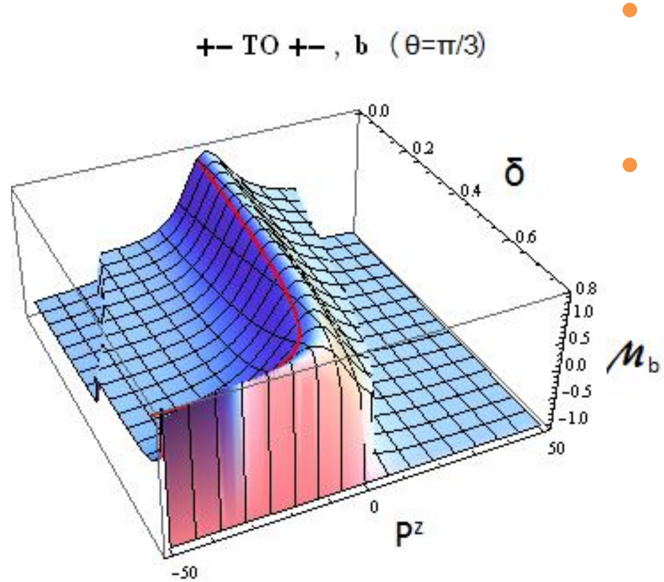
$$\begin{aligned}\Sigma_{a,\delta\to\frac{\pi}{4}} &= \lim_{\mathbb{C}\to 0}\left(\frac{1}{2Q^{\hat{+}}}\frac{\not{Q}_a + m}{q_{\hat{+}} - Q_{a\hat{+}}}\right)\\ &= \frac{1}{2q_-}\frac{\not{Q}_a + m}{q^- - Q_a^-}\\ &= \frac{\not{q}_{on} + m}{2q^+\left(q^- - q_{on}^-\right)}\\ &= \frac{\not{q}_{on} + m}{q^2 - m^2}\end{aligned}$$

$$\begin{aligned}\Sigma_{b,\delta\to\frac{\pi}{4}} &= \lim_{\mathbb{C}\to 0}\left(\frac{1}{2Q^{\hat{+}}}\frac{\not{Q}_b - m}{q_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + Q^{\hat{+}}}{\mathbb{C}}}\right)\\ &= \lim_{\mathbb{C}\to 0}\left(\frac{1}{2Q^{\hat{+}}}\frac{\mathbb{C}\left(\gamma^{\hat{+}}\frac{\mathbb{S}q_{\hat{-}} + Q^{\hat{+}}}{\mathbb{C}} - \gamma^{\hat{-}}q_{\hat{-}} - \gamma^{\perp}.q_{\perp} - m\right)}{\mathbb{C}q_{\hat{+}} + \mathbb{S}q_{\hat{-}} + Q^{\hat{+}}}\right)\\ &= \frac{\gamma^+\left(q_- + Q^+\right)}{2q^+\left(q_- + Q^+\right)}\\ &= \frac{\gamma^+}{2q^+}\end{aligned}$$

Example Application: The Annihilation of Electron- positron Pair into Two Photons



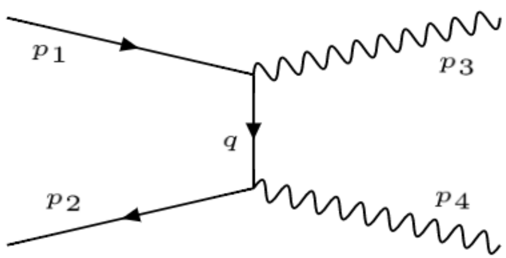




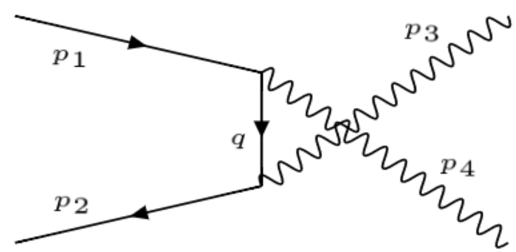
- Diagram (c) only exists in LFD
- Only one of (a) and (b) is allowed in LFD and the other one changes to instantaneous interaction in LFD

Scattering Angle Dependence of the Annihilation Amplitudes:Total Probability

Scattering Angle Dependence of the Annihilation Amplitudes:Total Probability



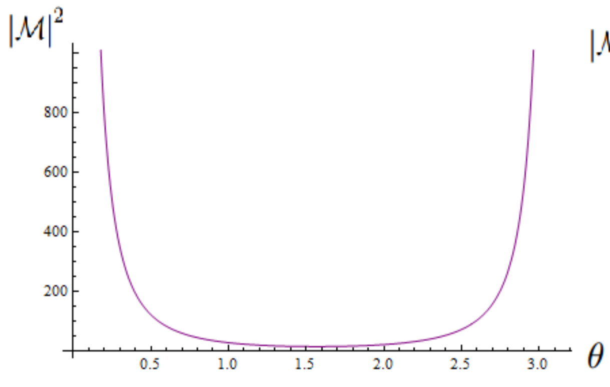
Direct Diagram

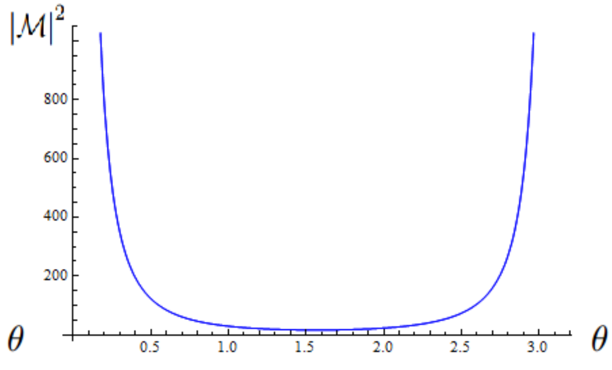


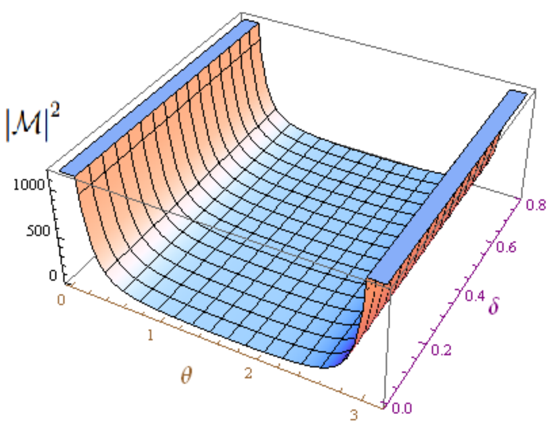
Exchanged Diagram

Scattering Angle Dependence of the Annihilation Amplitudes:Total Probability

Taking electron mass zero







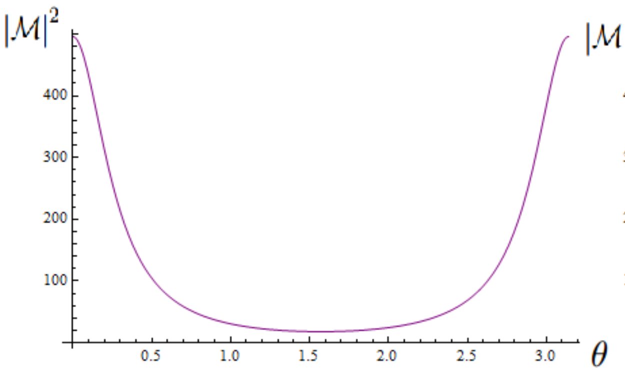
—— Textbook calculation

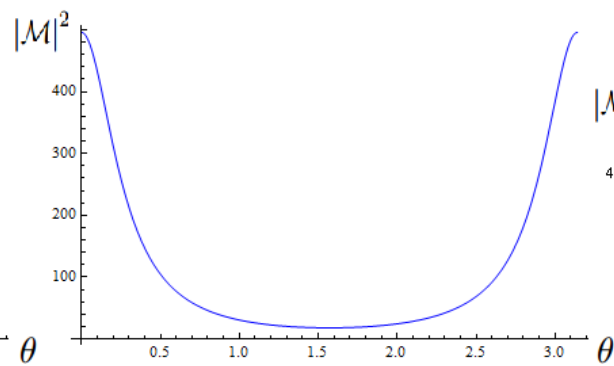
—— Our interpolation method

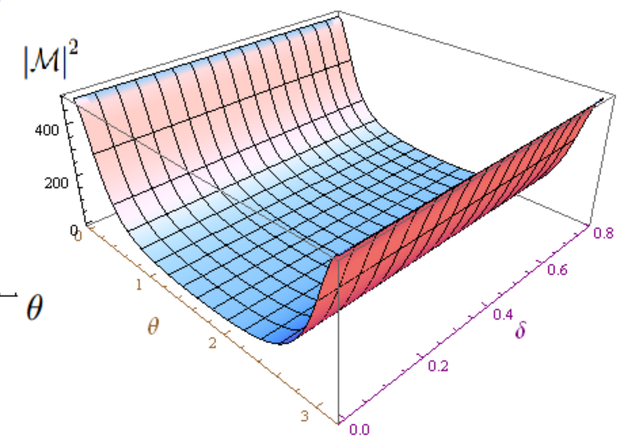
$$|\mathcal{M}|^2 = 2e^4\left(\frac{u}{t}+\frac{t}{u}\right)$$

Scattering Angle Dependence of the Annihilation Amplitudes:Total Probability

Including electron mass







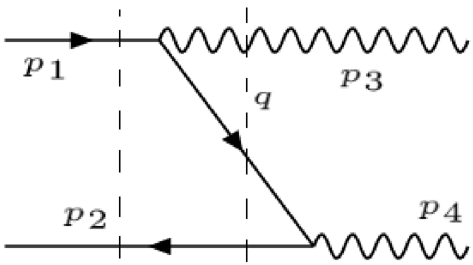
— Textbook calculation

— Our interpolation method

$$|\mathcal{M}|^2 = 2e^4\left[\frac{u_m}{t_m} + \frac{t_m}{u_m} + 2m^2\left(\frac{s_m}{t_m u_m} - \frac{1}{t_m} - \frac{1}{u_m}\right) - 4m^4\left(\frac{1}{{t_m}^2} + \frac{1}{{u_m}^2}\right)\right]$$

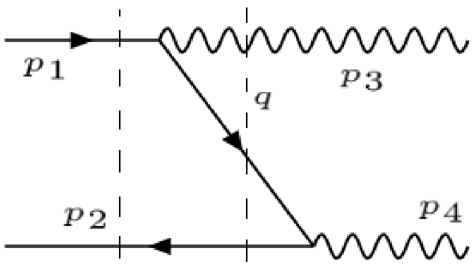
where $t_m \equiv t - m^2$, $u_m \equiv u - m^2$, and $s_m \equiv s - 4m^2$.

Scattering Angle Dependence of the Annihilation Amplitudes: Chirality



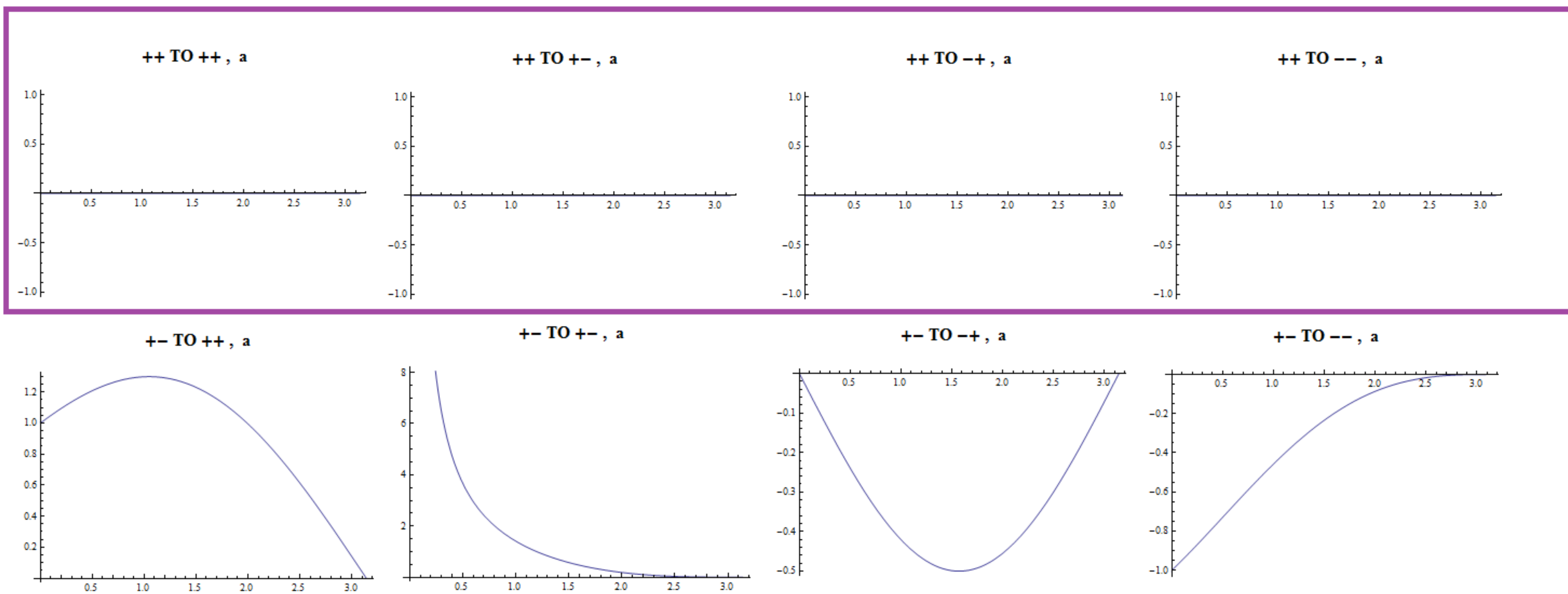
(a)

Scattering Angle Dependence of the Annihilation Amplitudes: Chirality

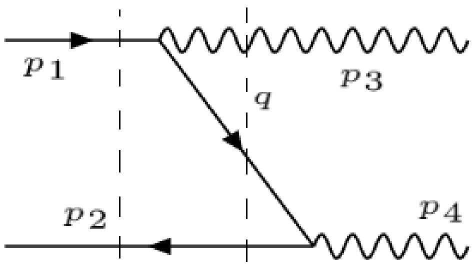


(a)

When m_e=0, chirality is conserved.

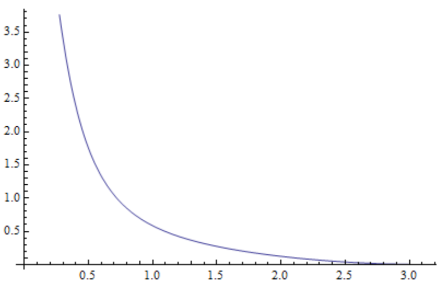


Scattering Angle Dependence of the Annihilation Amplitudes: Chirality

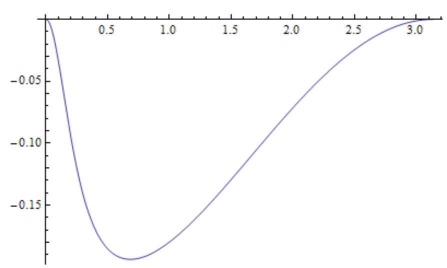


(a)

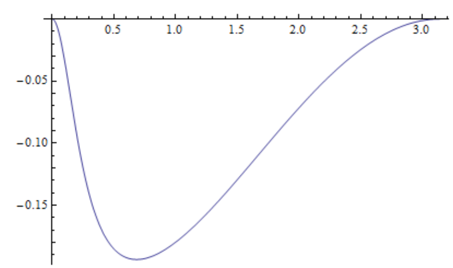
When $m_e \neq 0$, no such property.



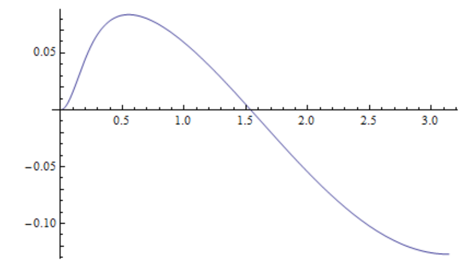
++ TO +− , a



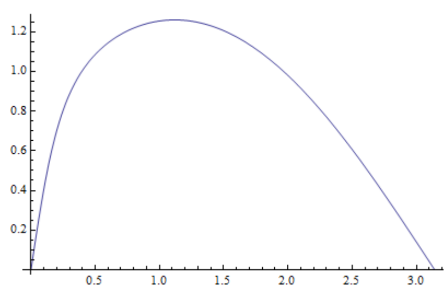
++ TO −+ , a



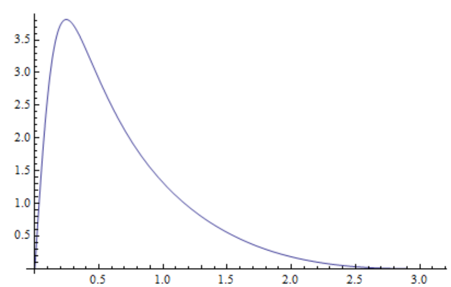
++ TO −− , a



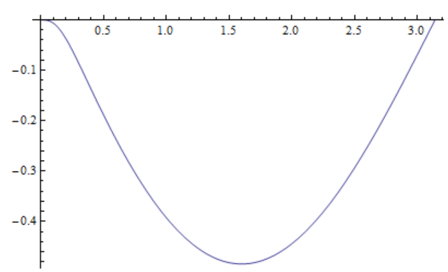
+− TO ++ , a



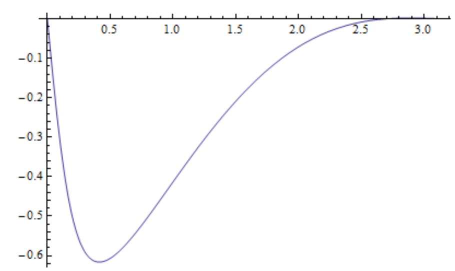
+− TO +− , a



+− TO −+ , a



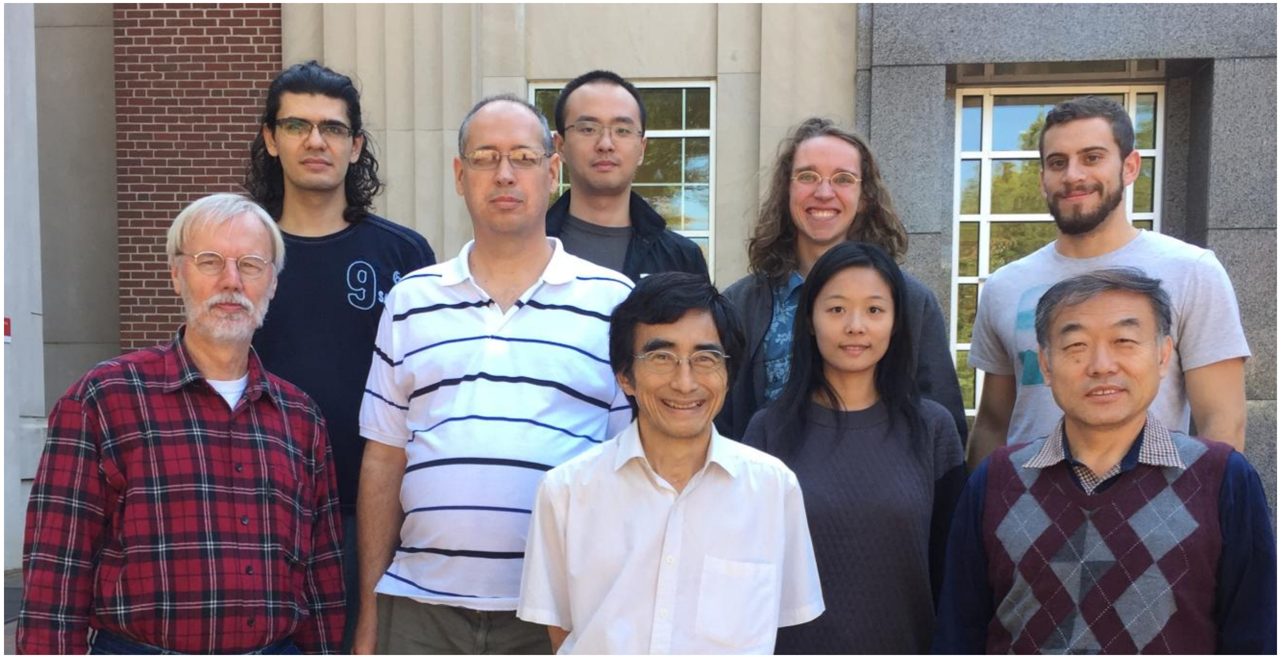
+− TO −− , a



Summary

- Light Front Quantization has distinguished vacuum structure and boost invariance property.
- Interpolation can connect IFD with LFD and clarify confusion about LFD.
- The interpolating time-ordered fermion propagators respectively change to the Light Front on-shell and instantaneous propagator when the interpolation angle approaches π/4.

Group Information



- From left:

Murat An, Ziyue Li, Benjamin Hamm, Colton Bradley,

Bernard Bakker, Ronaldo Thibes, Alfredo Suzuki, Bailing Ma, Chueng Ji.

Thank You!

Bailing Ma
PhD Student, Ji's group
NC State University