Introduction to QCD

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Lecture three/four

The plan for my six lectures

- **The Goal:**

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- **The Plan (approximately):**

  From hadrons to partons, the quarks and gluons in QCD

  Fundamentals of QCD,

  Factorization, Evolution, and

  Elementary hard processes

  Four lectures

  Hadron structures and properties in QCD

  Parton distribution functions (PDFs),

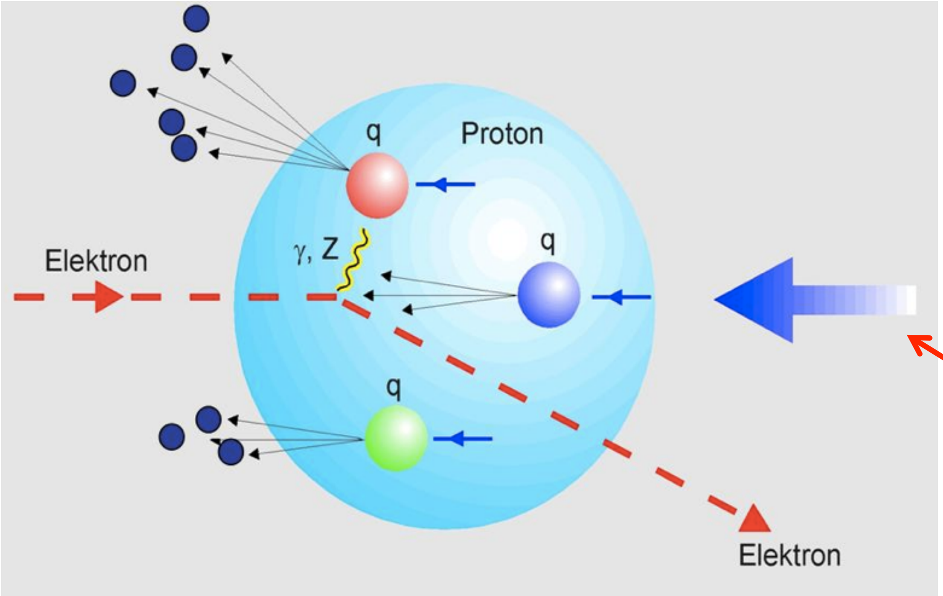
  Transverse momentum dependent PDFs (TMDs),

  Generalized PDFs (GPDs), and

  Multi-parton correlation functions

  Two lectures

Observables with ONE identified hadron



$$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})}$$

Identified initial-state hadron-proton!

Cross section is infrared divergent, and nonperturbative!

QCD factorization (approximation!)

Cross Section $=$ Infrared-Safe $\otimes$ Nonperturbative-distribution

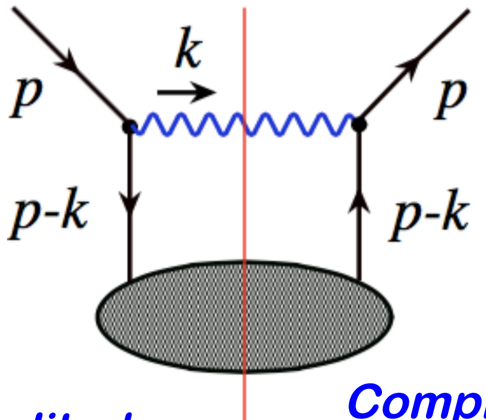
↑ **Measured** ↑ **Hard-probe** ↑ **Universal-hadron structure**

Pinch singularity and pinch surface

- "Square" of the diagram **with a *"unobserved gluon"*:**

"Cut-line" – final-state

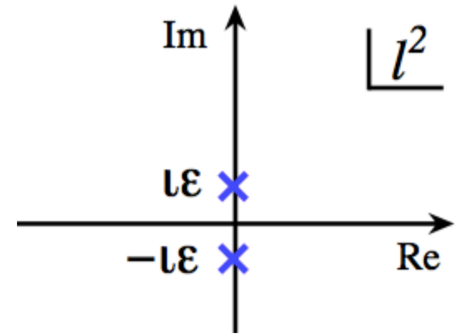
– in a "cut-diagram" notation



$$\propto \int \mathcal{T}(p-k,Q)\, \frac{1}{(p-k)^2+i\epsilon}\, \frac{1}{(p-k)^2-i\epsilon}\, d^4k\, \delta(k^2)_+$$

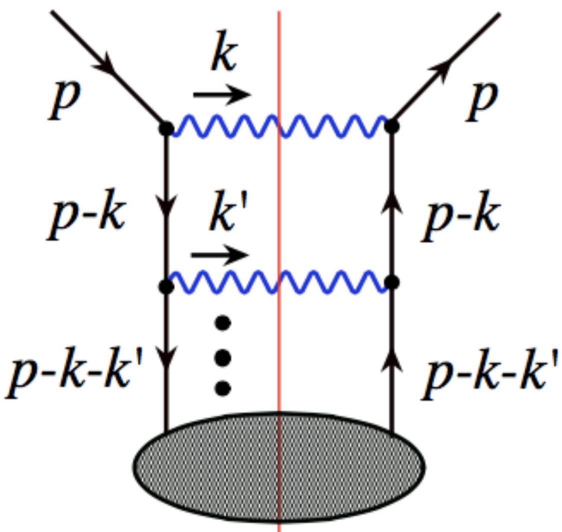
$$\propto \int \mathcal{T}(l,Q)\, \frac{1}{l^2+i\epsilon}\, \frac{1}{l^2-i\epsilon}\, dl^2$$

$$\Rightarrow \infty$$



Amplitude

Complex conjugate of the Amplitude



Pinch surfaces

Pinch singularities "perturbatively"

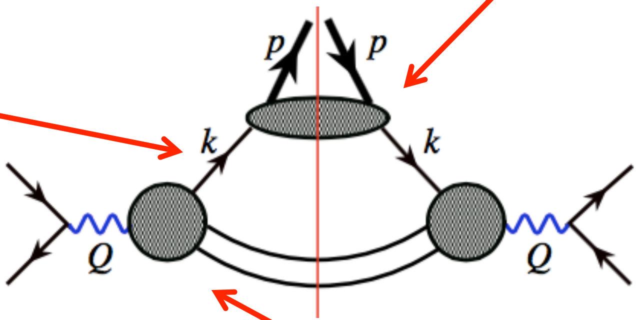
= "surfaces" in k, k', …

determined by $(p\text{-}k)^2=0$, $(p\text{-}k\text{-}k')^2=0$, …

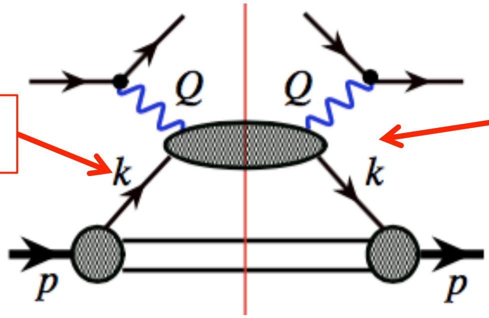
"perturbatively"

Hard collisions with identified hadron(s)

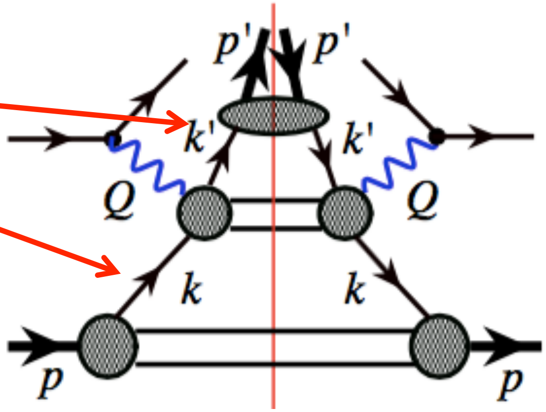
- ❑ Creation of an identified hadron:



- ❑ Identified initial hadron:



- ❑ Initial + created identified hadron(s):



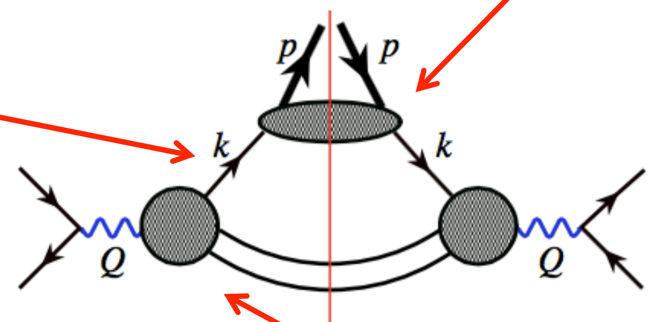
Cross section with identified hadron(s) is NOT perturbatively calculable

Hard collisions with identified hadron(s)

- Creation of an identified hadron:

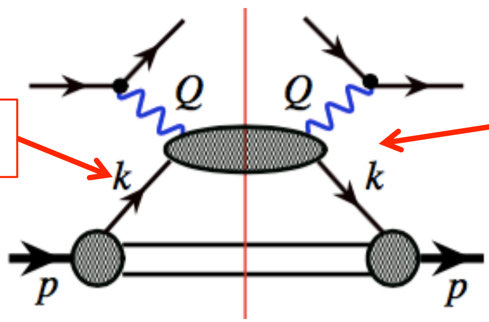
Non-perturbative!

Pinch in k^2



Perturbative!

- Identified initial hadron:



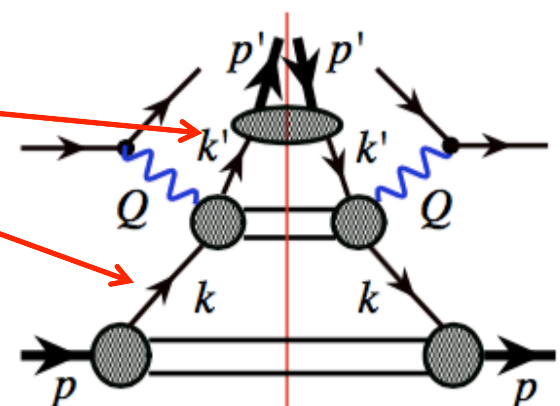
Pinch in k^2

Perturbative!

Non-perturbative!

- Initial + created identified hadron(s):

Pinch in both k^2 and k'^2



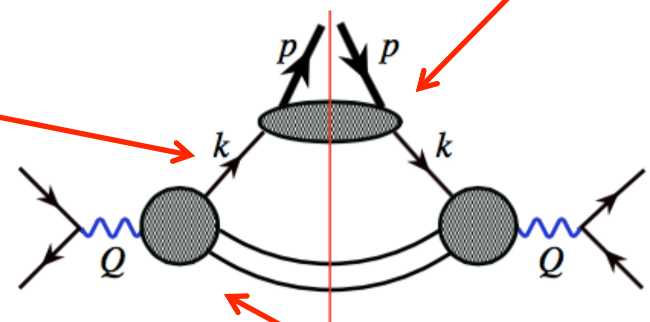
Dynamics at a HARD scale is linked by partons almost on Mass-Shell

Hard collisions with identified hadron(s)

- ❑ Creation of an identified hadron:

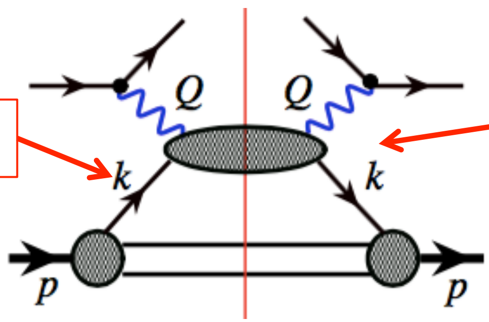
Non-perturbative!

Pinch in k^2



Perturbative!

- ❑ Identified initial hadron:



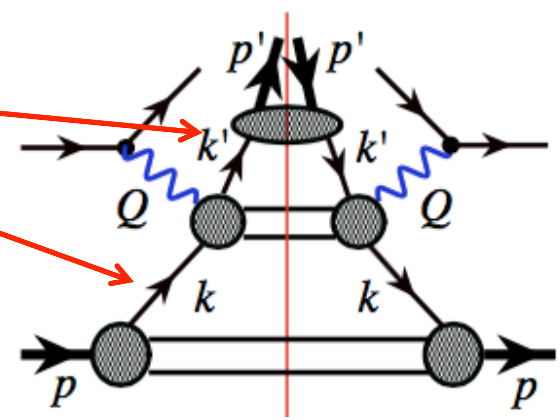
Pinch in k^2

Perturbative!

Non-perturbative!

- ❑ Initial + created identified hadron(s):

Pinch in both k^2 and k'^2

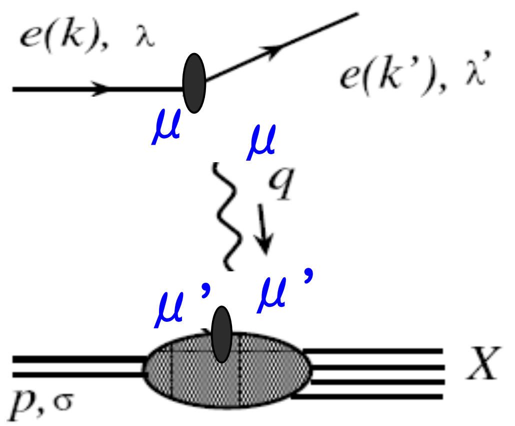


Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!

Inclusive lepton-hadron DIS – one hadron

- **Scattering amplitude:**

$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$
$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$
$$* \left\langle X\right|eJ^{em}_{\mu'}(0)\left|p,\sigma\right\rangle$$



- **Cross section:**

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_{X}\sum_{\lambda,\lambda',\sigma}\left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2\left[\prod_{i=1}^{X}\frac{d^3l_i}{(2\pi)^3 2E_i}\right]\frac{d^3k'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X}l_i+k'-p-k\right)$$

$$\Rightarrow \quad E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

- **Leptonic tensor:**

– **known from QED**

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu}+k^{\nu}k'^{\mu}-k\cdot k' g^{\mu\nu}\right)$$

DIS structure functions

- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\ \langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

- **Symmetries:**

  - Parity invariance (EM current) → $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
  - Time-reversal invariance → $W_{\mu\nu} = W_{\mu\nu}^*$ real
  - Current conservation → $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2 = -q^2 \qquad x_B = \frac{Q^2}{2p\cdot q}$$

- **Structure functions – infrared sensitive:**

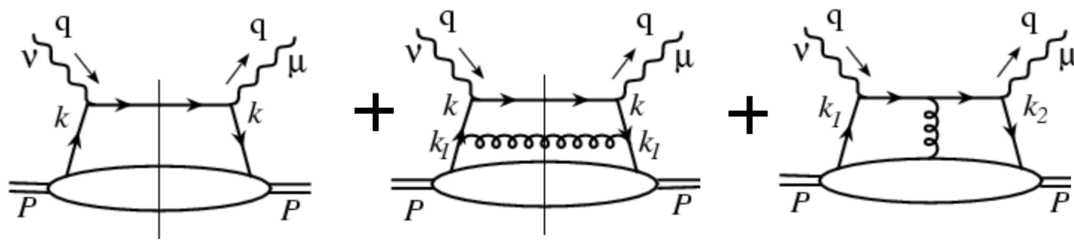
$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

Long-lived parton states

- **Feynman diagram representation of the hadronic tensor:**

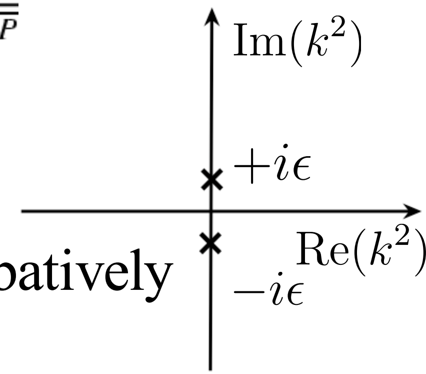
$W^{\mu\nu} \propto$



$+ \ldots$

- **Perturbative pinched poles:**

$$\int d^4k \; \mathrm{H}(Q,k) \left(\frac{1}{k^2 + i\varepsilon} \right) \left(\frac{1}{k^2 - i\varepsilon} \right) \mathrm{T}(k, \frac{1}{r_0}) \;\Rightarrow\; \infty \;\; \text{perturbatively}$$



- **Perturbative factorization:**

$$k^\mu = x p^\mu + \frac{k^2 + k_T^2}{2xp \cdot n} n^\mu + k_T^\mu$$

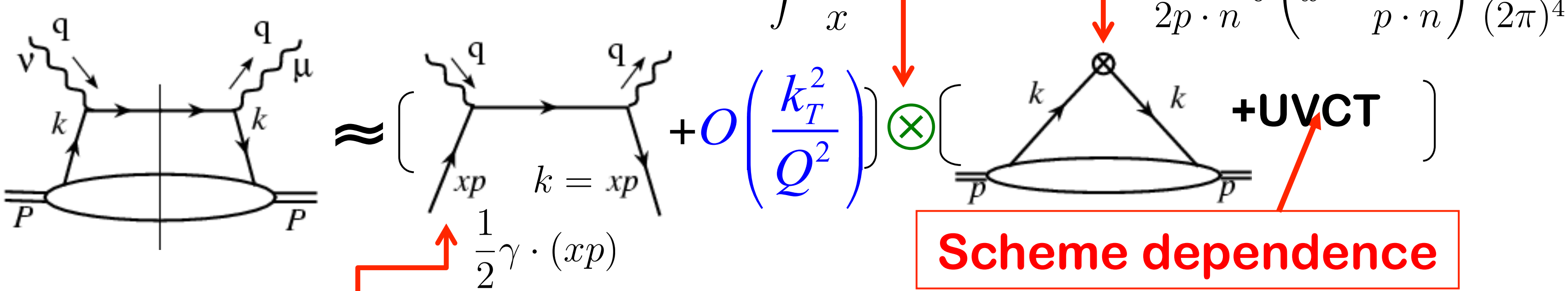
Nonperturbative matrix element

$$\int \frac{dx}{x} d^2k_T \; \mathrm{H}(Q, k^2 = 0) \int dk^2 \left(\frac{1}{k^2 + i\varepsilon} \right) \left(\frac{1}{k^2 - i\varepsilon} \right) \mathrm{T}(k, \frac{1}{r_0}) + \mathcal{O}\left(\frac{\langle k^2 \rangle}{Q^2} \right)$$

Short-distance

Collinear factorization – further approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T, \sqrt{k^2}$
  - **– Lowest order:**



$$\approx \left[\ldots \right] + O\left(\frac{k_T^2}{Q^2}\right) \otimes \left[\ldots + \text{UVCT} \right]$$

$$\int \frac{dx}{x} \qquad \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x - \frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4} \qquad \frac{1}{2}\gamma\cdot(xp)$$

Same as an elastic x-section

Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \longrightarrow \infty$, while x_B fixed

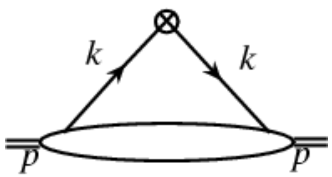
Feynman's parton model and Bjorken scaling

$$F_2(x_B, Q^2) = x_B \sum_f e_f^2\, \phi_f(x_B) = 2x_B F_1(x_B, Q^2)$$

Spin-½ parton!

- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle / Q^2\right)$

Parton distribution functions (PDFs)



❑ **PDFs as matrix elements of two parton fields:**

– **combine the amplitude & its complex-conjugate**

$$\phi_{q/h}(x,\mu^2)=\int\frac{p^+dy^-}{2\pi}\,e^{ixp^+y^-}\langle h(p)|\bar{\psi}_q(0)\,\frac{\gamma^+}{2p^+}\,\psi_q(y^-)|h(p)\rangle\mathcal{Z}_{\mathcal{O}}(\mu^2)$$

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

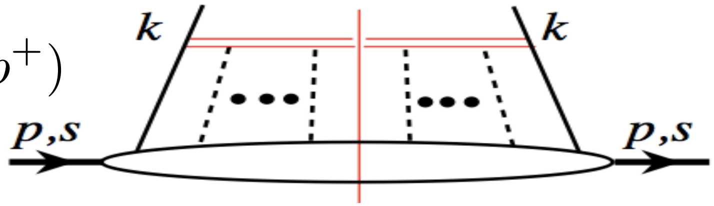
But, it is NOT gauge invariant! $\psi(x)\to e^{i\alpha_a(x)t_a}\psi(x)\quad\bar{\psi}(x)\to\bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– **need a gauge link:**

$$\phi_{q/h}(x,\mu^2)=\int\frac{dy^-}{2\pi}\,e^{ixp^+y^-}\langle h(p)|\bar{\psi}_q(0)\left[\mathcal{P}e^{-ig\int_0^{y^-}d\eta^-A^+(\eta^-)}\right]\frac{\gamma^+}{2p^+}\,\psi_q(y^-)|h(p)\rangle\mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– **corresponding diagram in momentum space:**

$$\int\frac{d^4k}{(2\pi)^4}\,\delta(x-k^+/p^+)$$



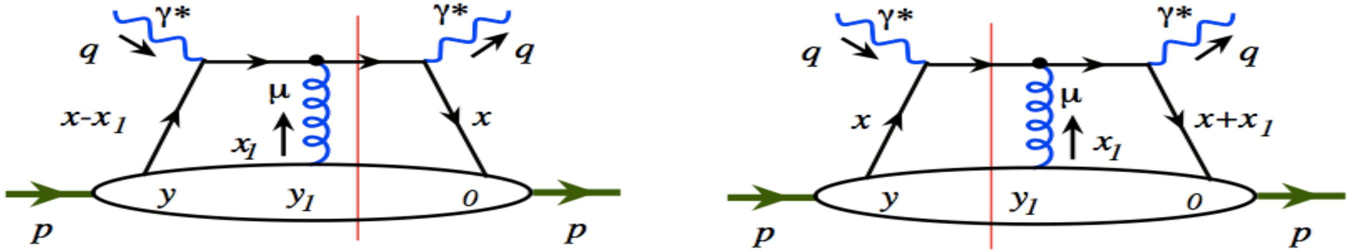
$+\,\mathrm{UVCT}(\mu^2)$

μ **-dependence**

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

❑ **Longitudinal gluon:**



❑ **Left diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, e^{ix_1 p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

❑ **Right diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+ y_1^-}\, n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, e^{ix_1 p^+ y_1^-}\right]\mathcal{M} \;= ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

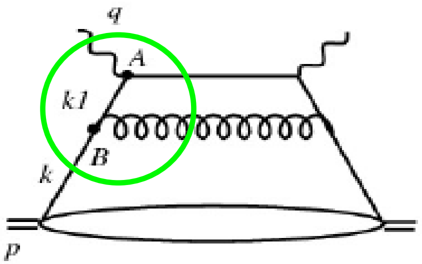
❑ **Total contribution:**

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\rm LO}$$

O(g)-term of the gauge link!

QCD high order corrections

- NLO partonic diagram to structure functions:



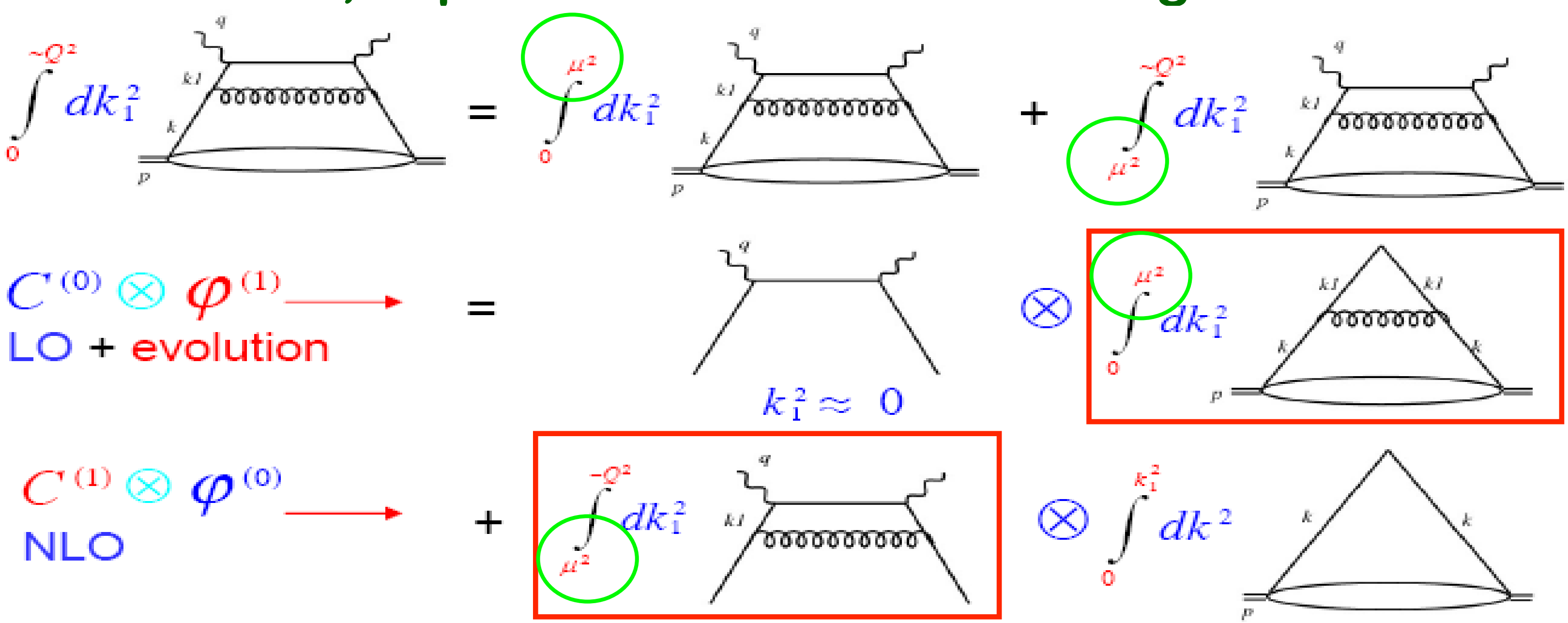
$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by

$$\begin{cases} k_1^2 \sim 0 \\ t_{AB} \rightarrow \infty \end{cases}$$

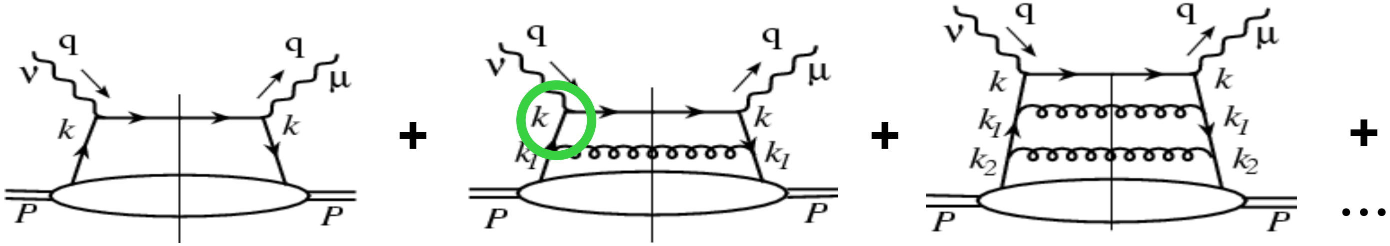
Diagram has both long- and short-distance physics

- Factorization, separation of short- from long-distance:

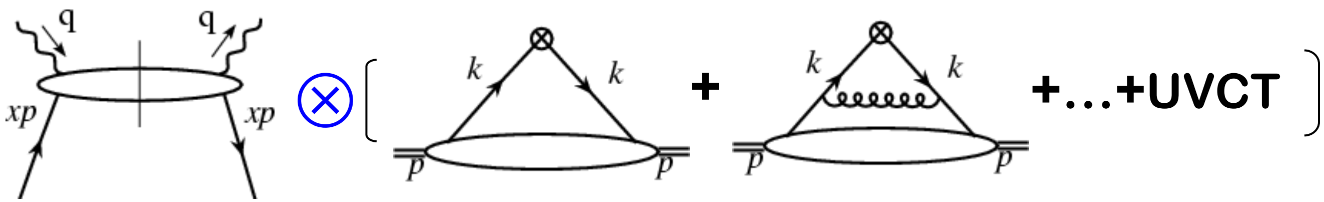


QCD leading power factorization

- QCD corrections: pinch singularities in $\int d^4 k_i$



- Logarithmic contributions into parton distributions:



$$F_2(x_B,Q^2) = \sum_f C_f\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_f\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- Factorization scale: μ_F^2

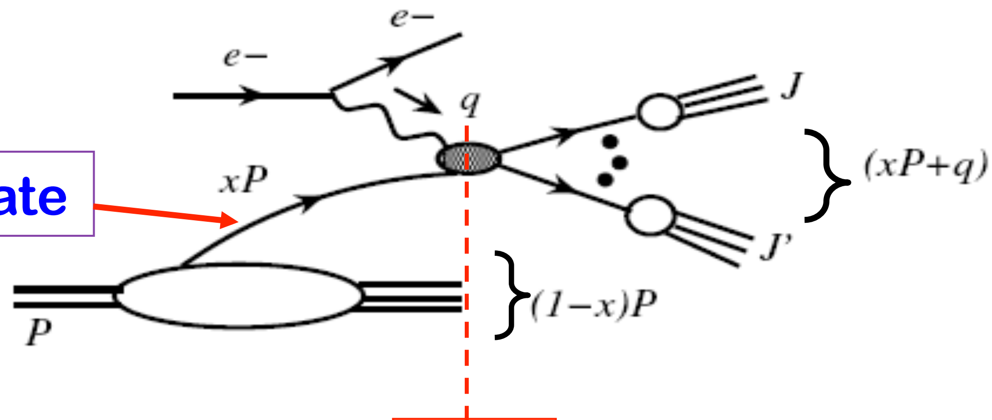
→ To separate the collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution

Picture of factorization for DIS

- **Time evolution:**

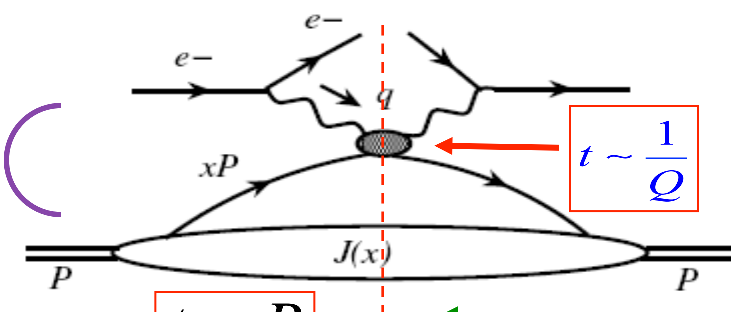
Long-lived parton state



Time: "Past" Now "Future"

- **Unitarity – summing over all hard jets:**

$$\sigma_{\rm tot}^{\rm DIS} \propto \text{Im}\Big(\quad \Big)$$



$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

How to calculate the perturbative parts?

❑ Use DIS structure function F_2 as an example:

$$F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

✧ Apply the factorized formula to parton states: $h \rightarrow q$

Feynman diagrams →

$$F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$

← Feynman diagrams

✧ Express both SFs and PDFs in terms of powers of α_s:

0th order:

$$F_{2q}^{(0)}(x_B,Q^2) = C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

⇒

$$C_q^{(0)}(x) = F_{2q}^{(0)}(x) \qquad \varphi_{q/q}^{(0)}\left(x\right) = \delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F_{2q}^{(1)}(x_B,Q^2) = C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right) + C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

⇒

$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

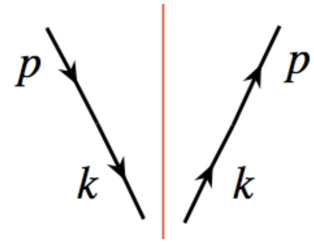
PDFs of a parton

- **Change the state without changing the operator:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\,\psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\text{parton}(p)\rangle$ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

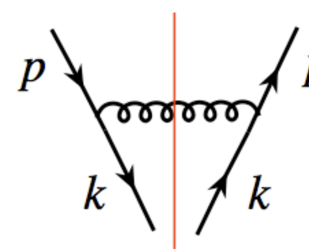
- **Lowest order quark distribution:**



  - ✧ **From the operator definition:**

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

- **Leading order in α_s quark distribution:**



$+\ldots$

  - ✧ **Expand to $(g_s)^2$ – logarithmic divergent:**

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

Partonic cross sections

❑ Projection operators for SFs:

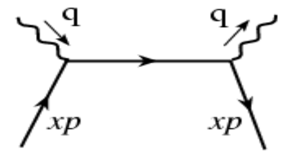
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x, Q^2\right)$$

$$F_1(x, Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

$$F_2(x, Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x, Q^2)$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left[\frac{1}{4\pi} \right.$$



$$\left.\right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2} \gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x \delta(1-x)$$

NLO coefficient function – complete example

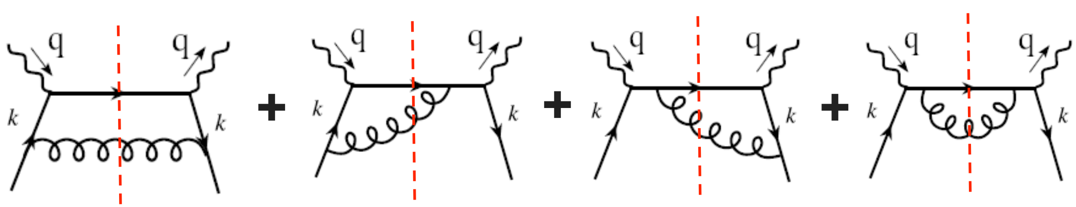
$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

$$\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}$$

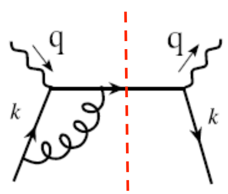
- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$

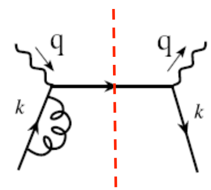


} **Real**

Virtual { +



+ c.c. (+



+ c.c. + UV CT)

- **Calculation:** $-g^{\mu\nu}W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

❑ **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

❑ **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

❑ **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ln(1-z) f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\Bigg\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x) \ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)$$

$$+ C_F \left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$

$$\left. + 3 - x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right]\Bigg\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

❑ One loop contribution to F_2 of a quark:

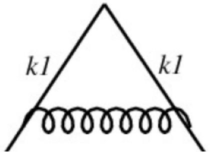
$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2}\right)\right.$$

$$\left. +C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ **MS scheme:** $\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$

✧ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$

✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x,Q^2/\mu^2)\big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x)\ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Evolution

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ **Evolution (differential-integral) equation for PDFs**

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\rm QCD}^2\right)$

Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

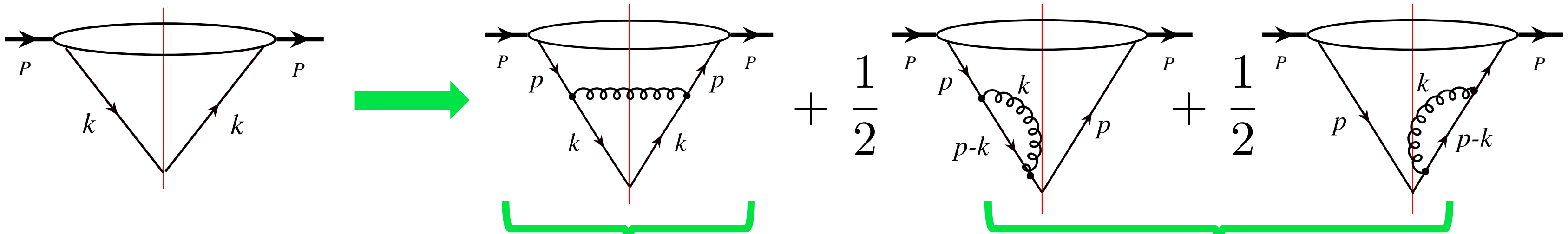
→ **DGLAP evolution equation:**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

Calculation of evolution kernels

- Evolution kernels are process independent
  - ✧ Parton distribution functions are universal
  - ✧ Could be derived in many different ways

- Extract from calculating parton PDFs' scale dependence



Collins, Qiu, 1989

$$Q^2 \frac{d}{dQ^2} q_i(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dx_1}{x_1} q_i(x_1,Q^2) \gamma_{qq}\left(\frac{x}{x_1}\right) - \frac{\alpha_s}{2\pi} q_i(x,Q^2) \int_0^1 dz\, \gamma_{qq}(z)$$

Change "Gain" "Loss"

  - ✧ Same is true for gluon evolution, and mixing flavor terms

- One can also extract the kernels from the CO divergence of partonic cross sections

Global QCD analyses – Testing QCD

- Factorization for observables with identified hadrons:
  - ✧ Factorized cross sections (DIS):

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - ✧ DGLAP Evolution:

$$\frac{\partial f(x,\mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

  - ✧ Adding more observables:
    Factorized cross section with multiple-hadrons (next lecture)

Testing QCD: Universal PDFs for all cross sections?

- Input for QCD Global analysis/fitting:
  - ✧ World data with "Q" > 2 GeV
  - ✧ PDFs at an input scale: $\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$

Input scale ~ GeV

Fitting paramters

Global QCD analysis – Testing QCD

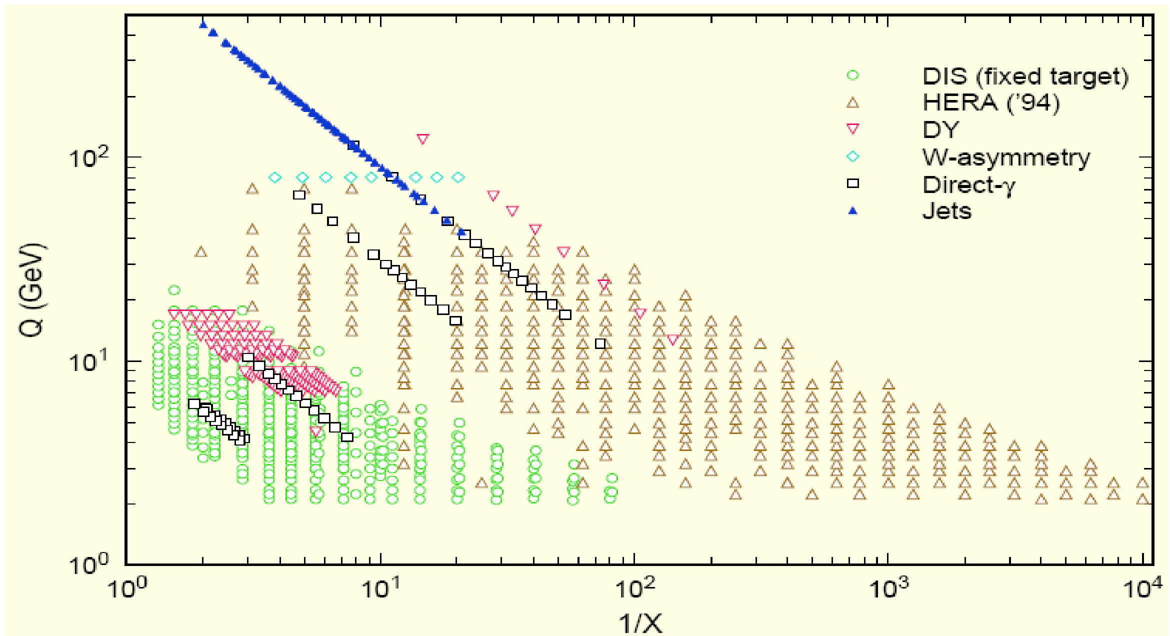
Input DPFs at Q_0

$\varphi_{f/h}\left(x, \{a_j\}\right)$

Vary $\{a_j\}$

Minimize *Chi²*

DGLAP



$\varphi_{f/h}(x)$ **at $Q>Q_0$**

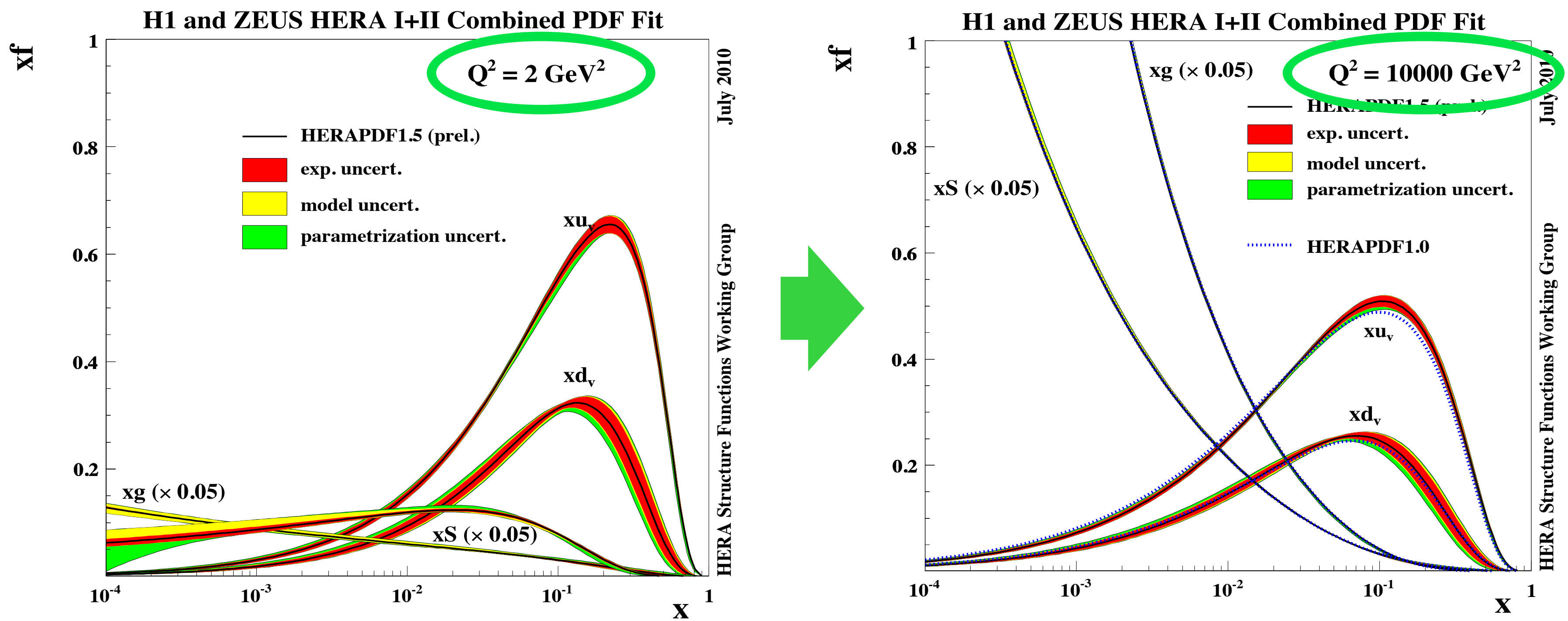
QCD calculation

Comparison with Data at various x and Q

Procedure: Iterate to find the best set of $\{a_j\}$ for the input DPFs

PDFs from DIS

❑ Q2-dependence is a prediction of pQCD calculation:



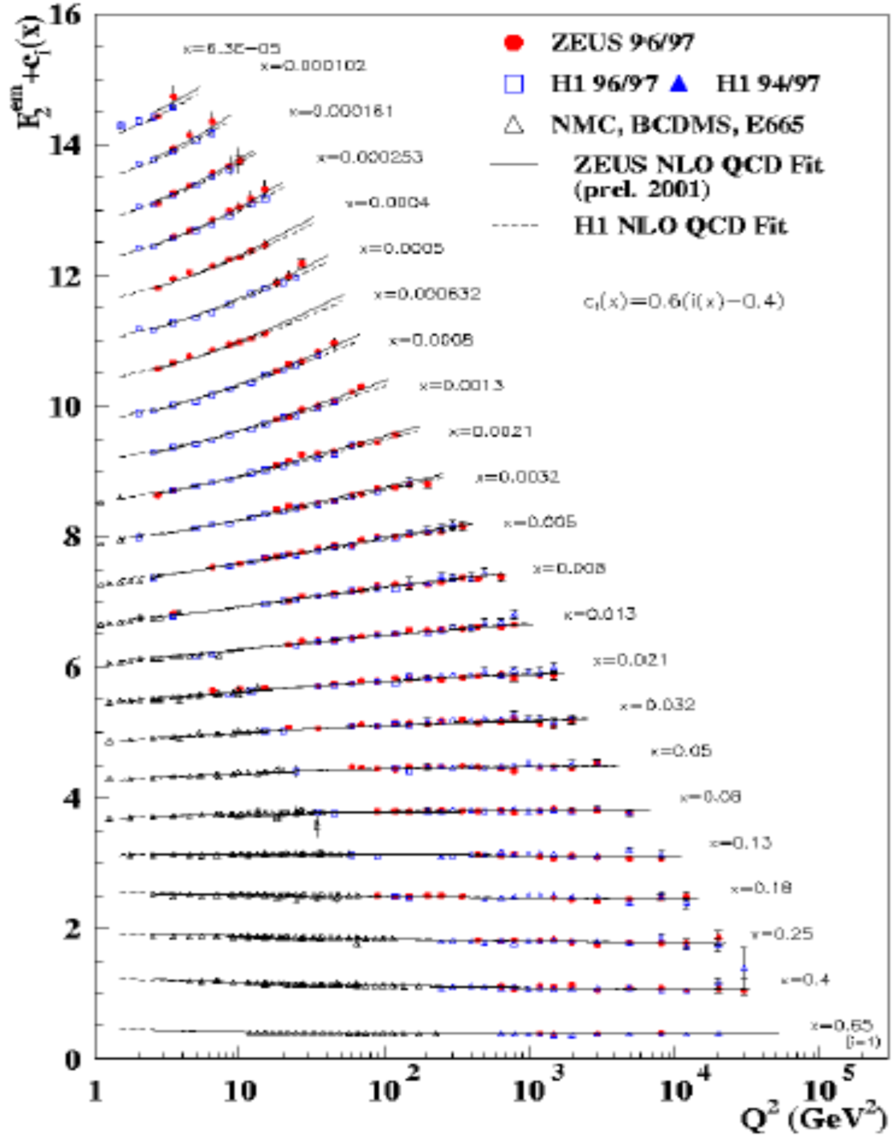
❑ Physics interpretation of PDFs:

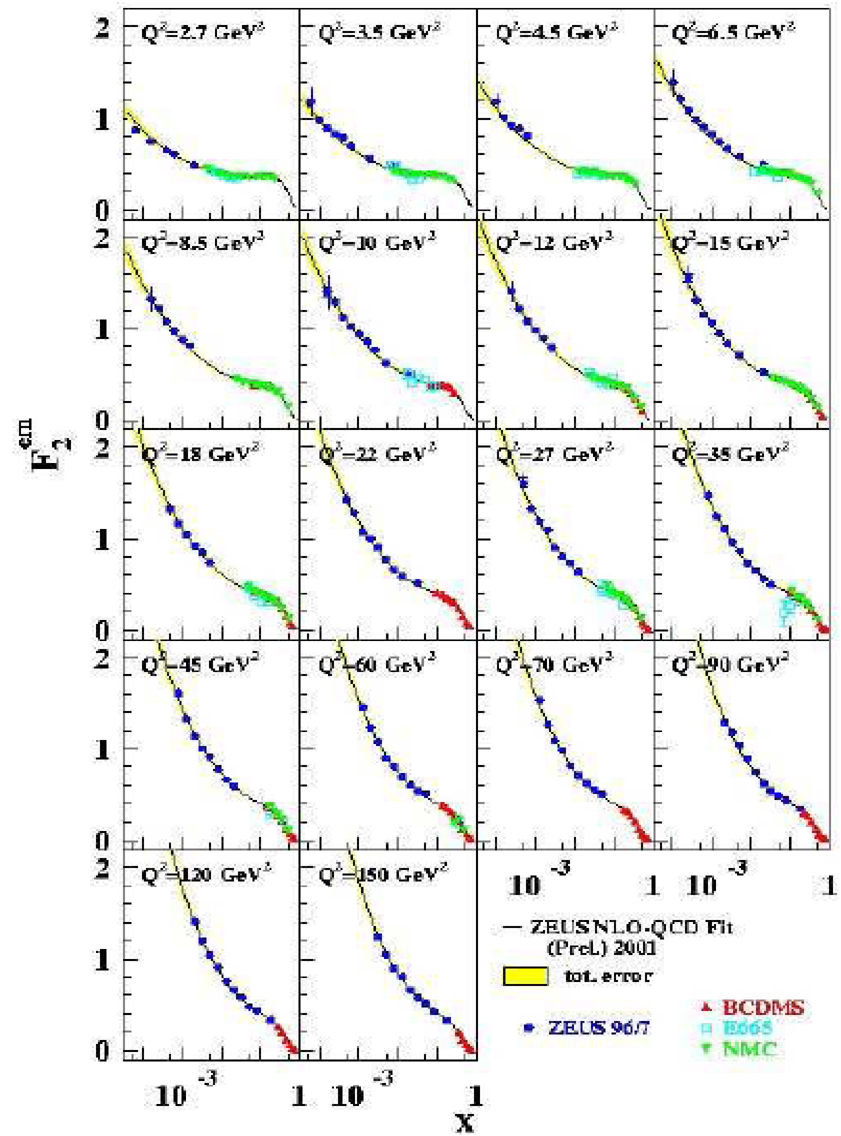
$f(x, Q^2)$: Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q²"

✧ **Number of partons:** $\int_0^1 dx\, u_v(x, Q^2) = 2, \quad \int_0^1 dx\, d_v(x, Q^2) = 1$

✧ **Momentum fraction:** $\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x, Q^2) \quad \Longrightarrow \quad \sum_f \langle x(Q^2)\rangle = 1$

Scaling and scaling violation

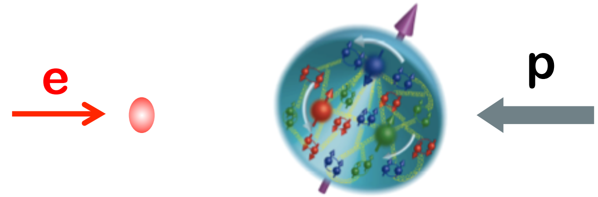


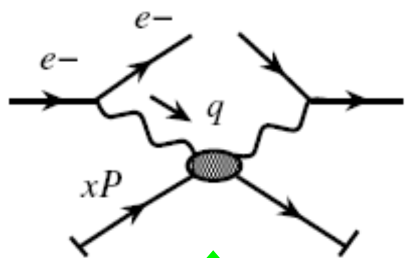
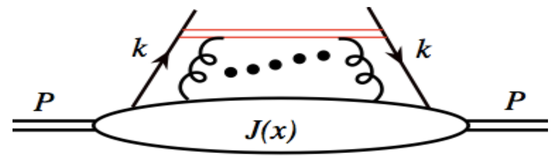


Q^2-dependence is a prediction of pQCD calculation

From one hadron to two hadrons

- One hadron:



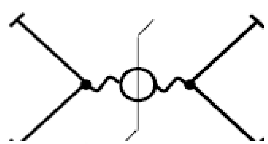
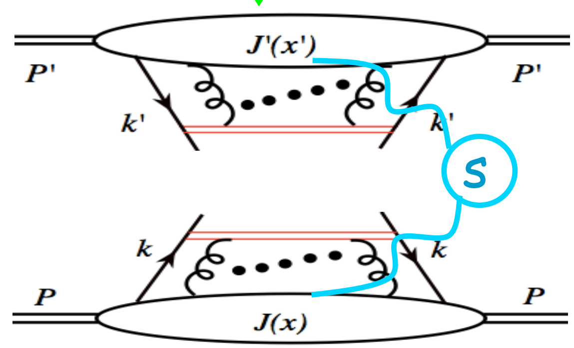
$\sigma_{\text{tot}}^{\text{DIS}} \sim$  $\otimes$  $+\, O\left(\frac{1}{QR}\right)$

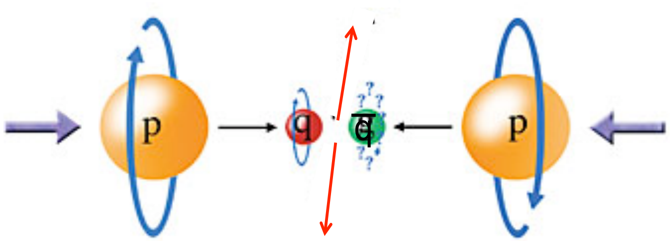
Hard-part
Probe

Parton-distribution
Structure

Power corrections
Approximation

- Two hadrons:

$\sigma_{\text{tot}}^{\text{DY}} \sim$  $\otimes$  $+\, O\left(\frac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

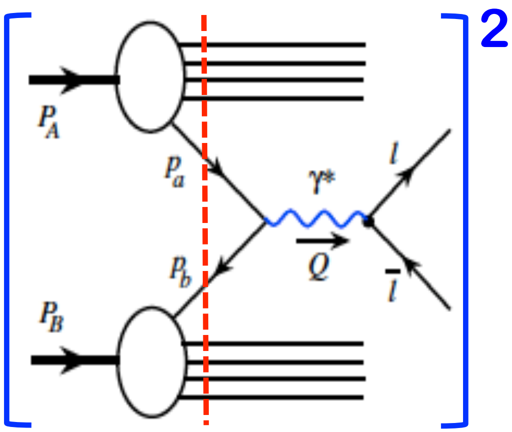
❑ Drell-Yan mechanism:

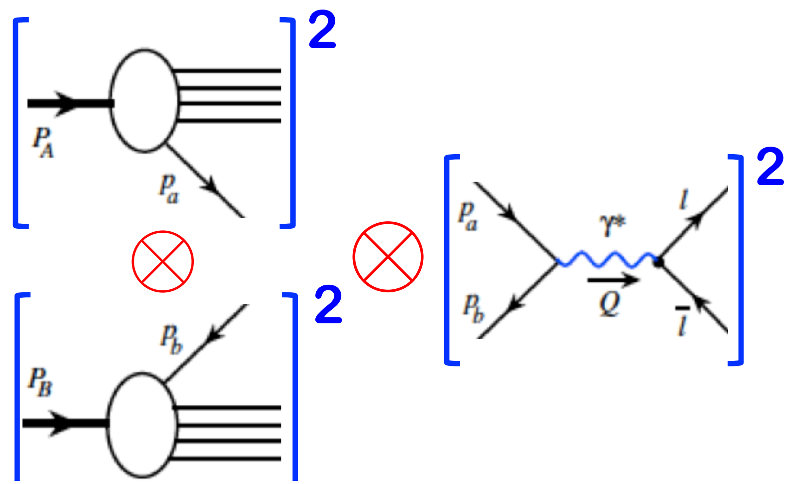
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X$$ **with** $$q^2 \equiv Q^2 \gg \Lambda_{\mathrm{QCD}}^2 \sim 1/\mathrm{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general, a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

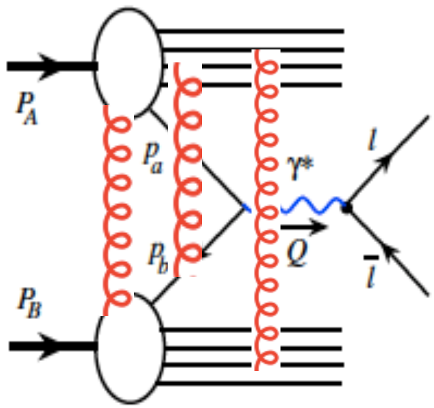
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}} e^{y} \qquad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

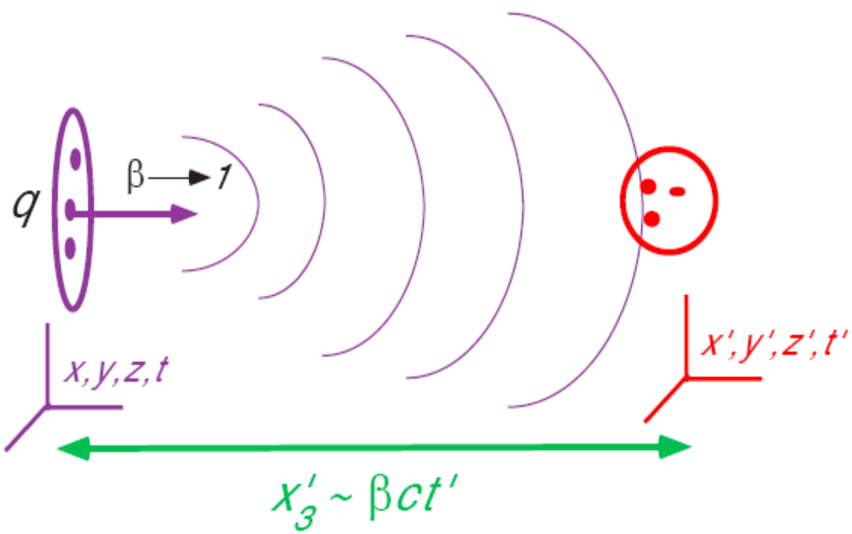
❑ Beyond the lowest order:



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$ $\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

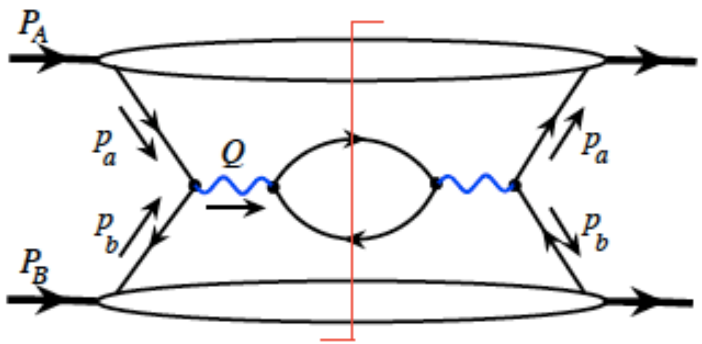
Drell-Yan process in QCD – factorization

Collins, Soper, Sterman, 1988

❑ Factorization – approximation:

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \rightarrow \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

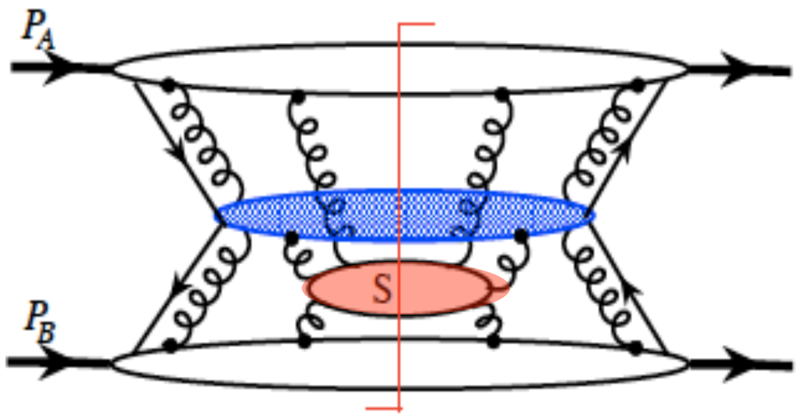
✧ Infrared safe of partonic parts:

Cancelation of IR behavior
Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD – factorization

- ❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

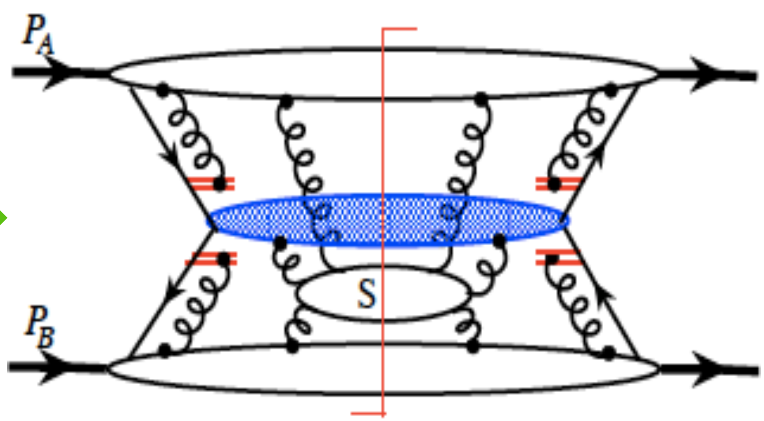
Collinear:

- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

- ❑ **Collinear gluons:**

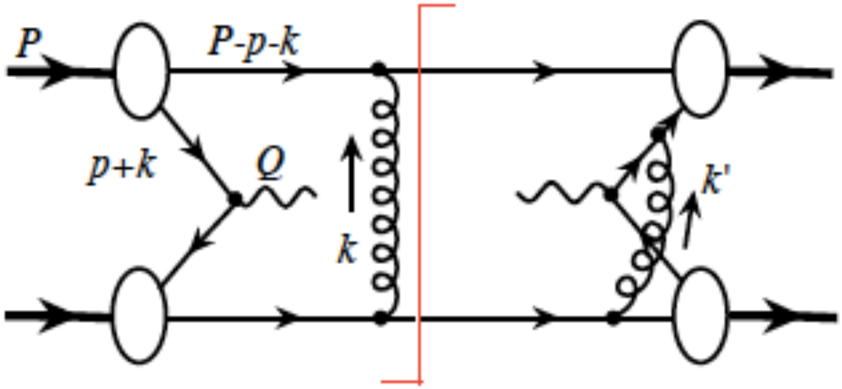
  - ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
  - ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

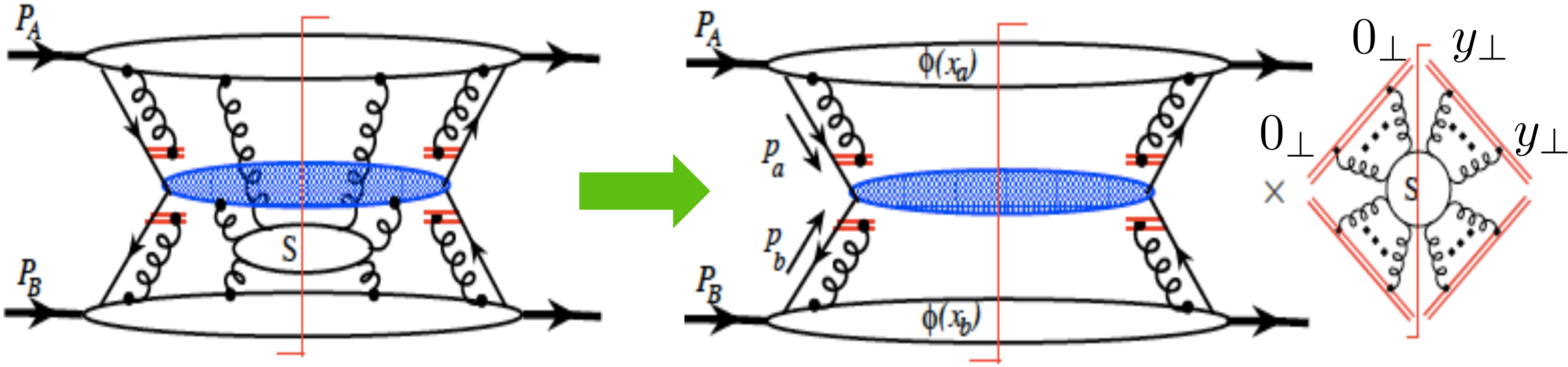
✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^{\pm}$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a,\mu) \int dx_b\, f_{b/B}(x_b,\mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

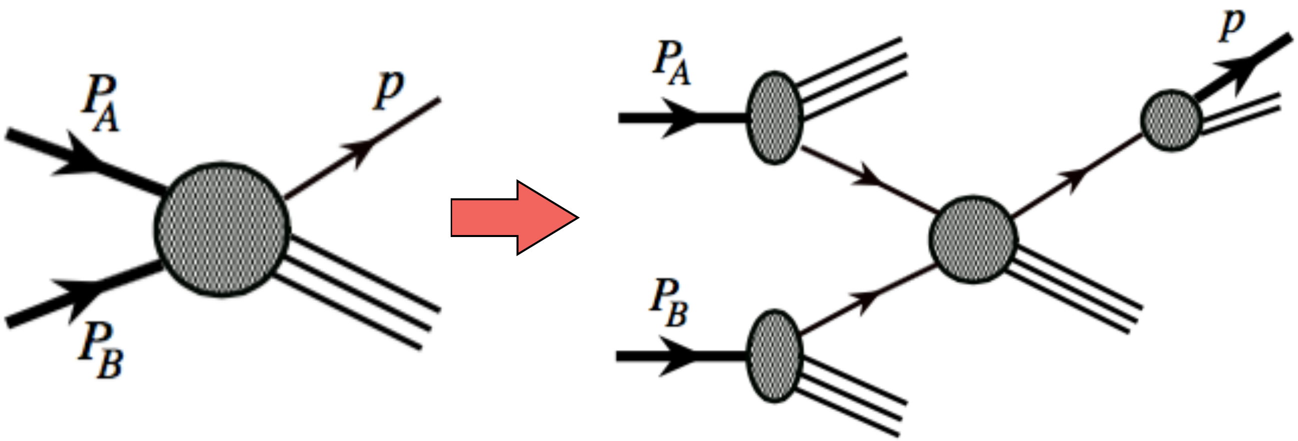
The factorization arguments are independent of the spin states of the colliding hadrons

➡ same formula with polarized PDFs for γ^*,W/Z, H^0...

Factorization for more than two hadrons

- **Factorization for high p_T single hadron:**

Nayak, Qiu, Sterman, 2006



$\gamma, W/Z, \ell(s), \text{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P$_T^2$)

$p_T \gg m \gtrsim \Lambda_{\rm QCD}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

✧ **Fragmentation function:** $D_{c\to C}(z, \mu_F^2)$

✧ **Choice of the scales:** $\mu_{\rm Fac}^2 \approx \mu_{\rm ren}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Global QCD analyses – Testing QCD

- **Factorization for observables with identified hadrons:**
  - **One-hadron (DIS):**

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

  - **Two-hadrons (DY, Jets, W/Z, …) :**

$$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

  - **DGLAP Evolution:**

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

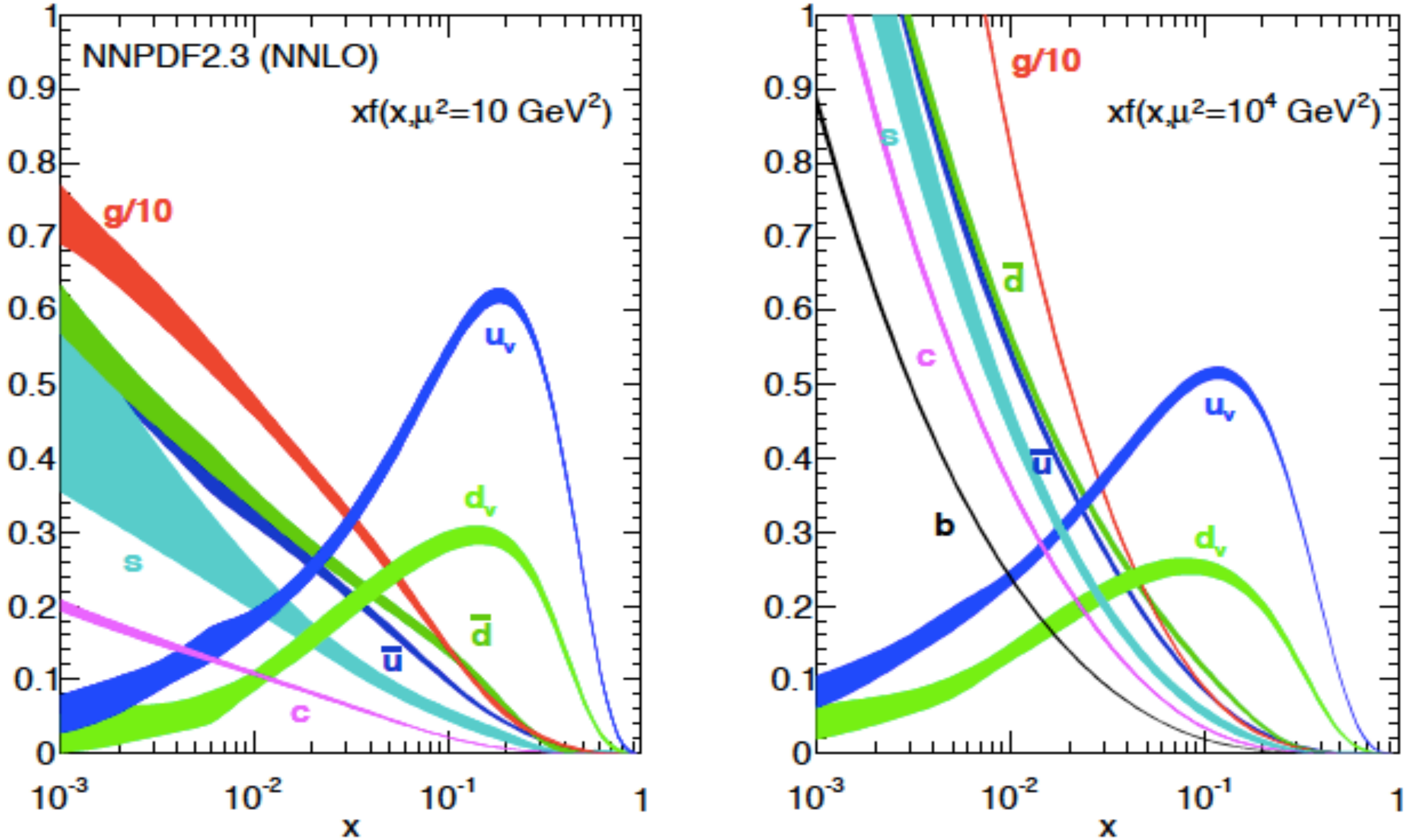
- **Input for QCD Global analysis/fitting:**
  - **World data with "Q" > 2 GeV**
  - **PDFs at an input scale:** $\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$

Input scale ~ GeV

Fitting paramters

PDFs of a spin-averaged proton

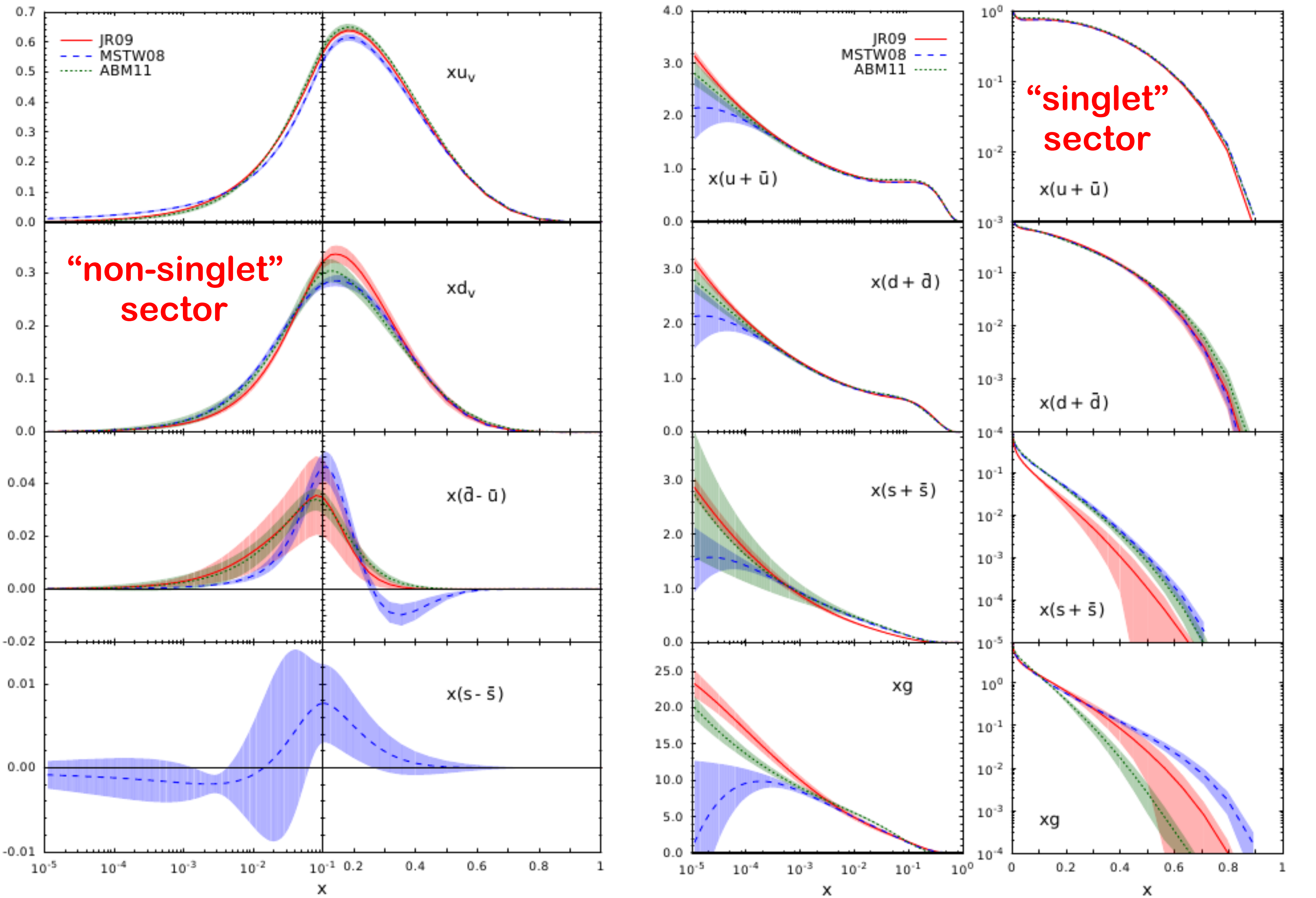
- **Modern sets of PDFs @NNLO with uncertainties:**



K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)

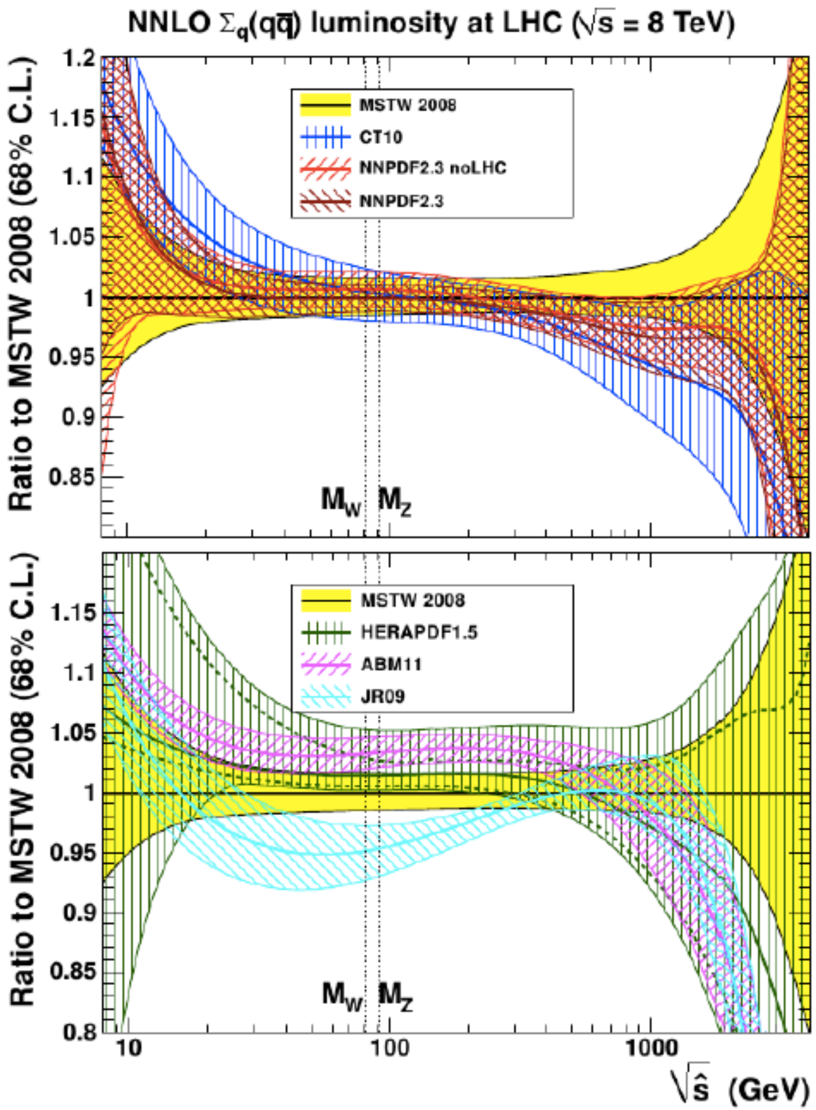
Consistently fit almost all data with Q > 2GeV

Uncertainties of PDFs

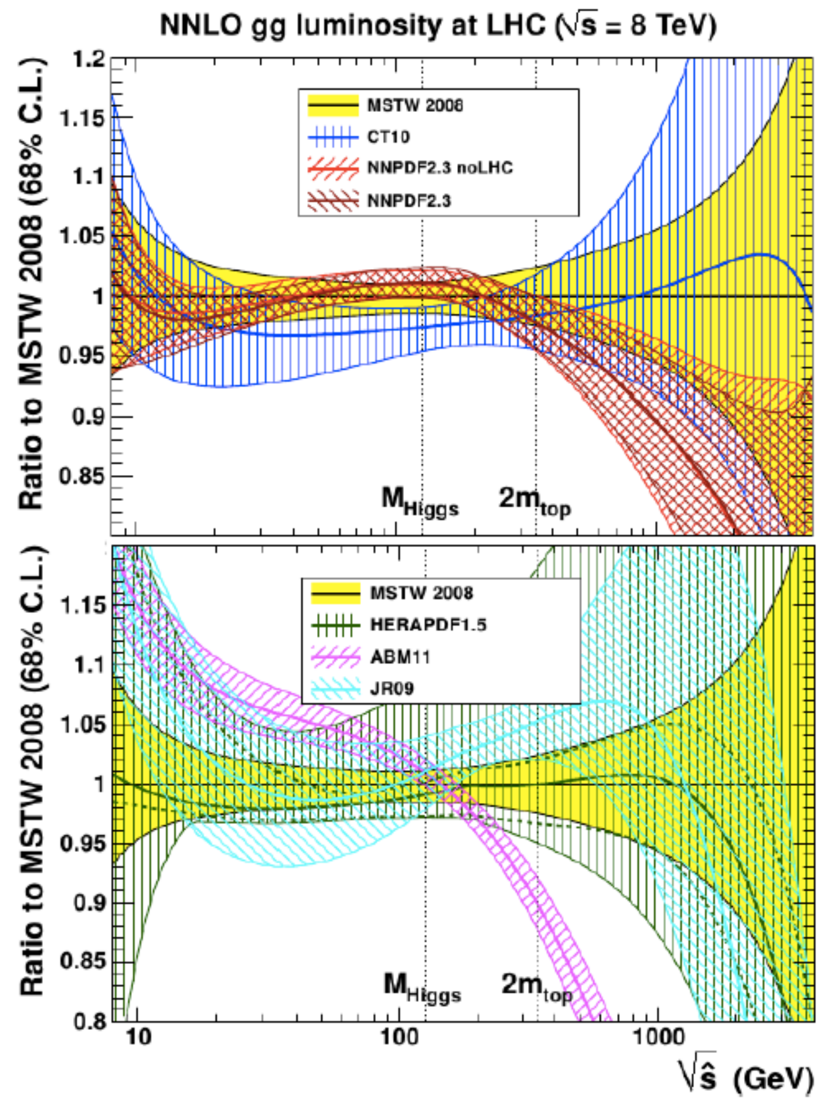


Partonic luminosities

q - qbar



g - g



PDFs at large x

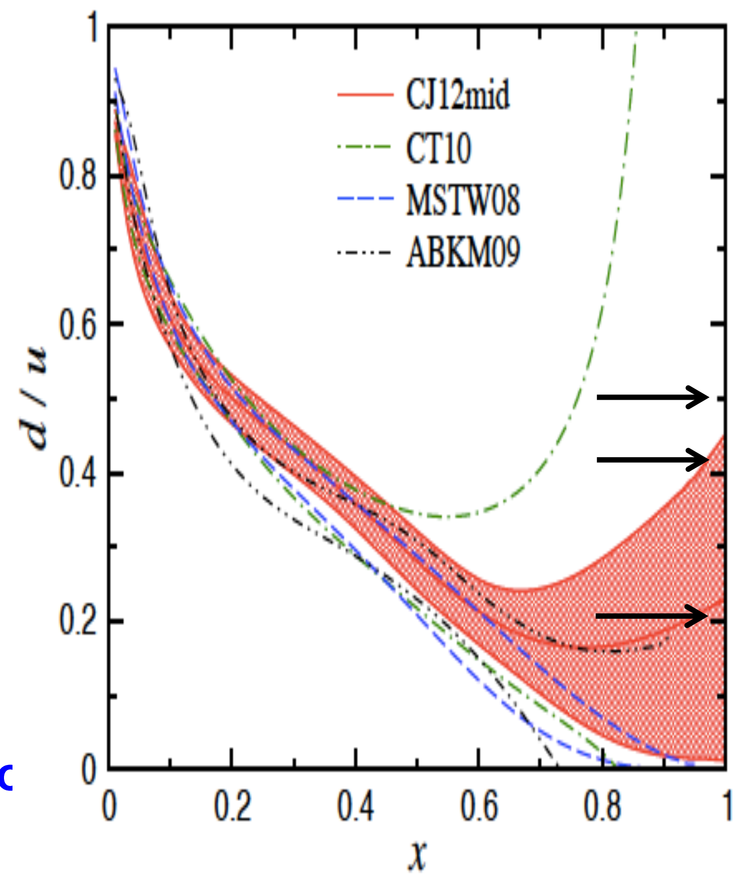
- Testing ground for hadron structure at x → 1:

✧ $d/u \to 1/2$ — SU(6) Spin-flavor symmetry

✧ $d/u \to 0$ — Scalar diquark dominance

✧ $d/u \to 1/5$ — pQCD power counting

✧ $$d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$$ — Local quark-hadrc duality



PDFs at large x

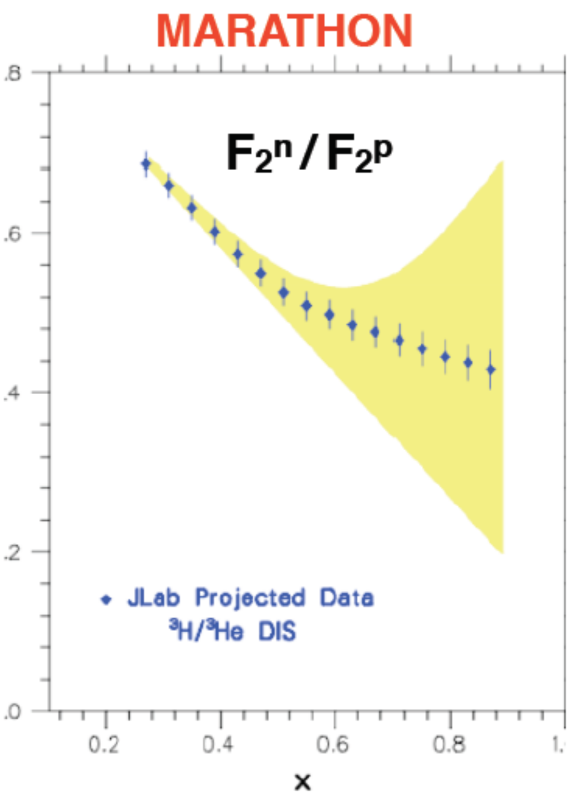
- Testing ground for hadron structure at **x → 1**:

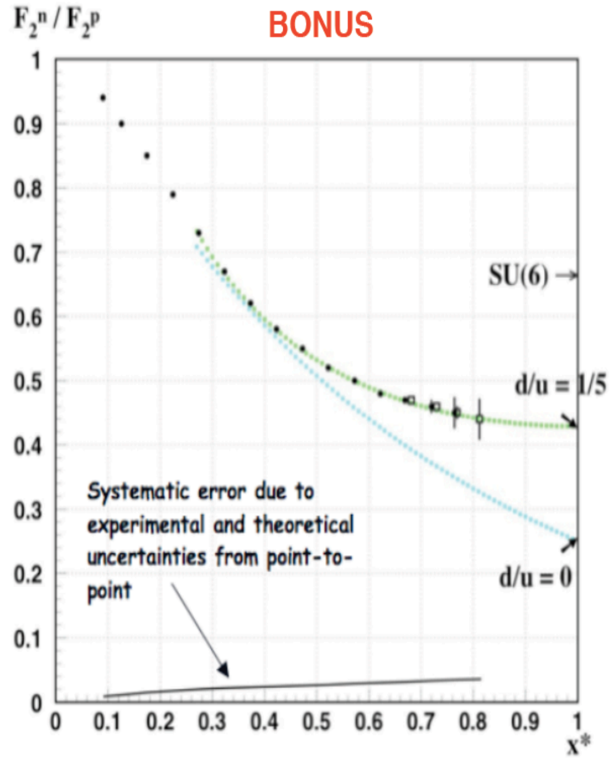
✧ $d/u \to 1/2$	SU(6) Spin-flavor symmetry	✧ $\Delta u/u \to 2/3$ $\Delta d/d \to -1/3$
✧ $d/u \to 0$	Scalar diquark dominance	✧ $\Delta u/u \to 1$ $\Delta d/d \to -1/3$
✧ $d/u \to 1/5$	pQCD power counting	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$
✧ $d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$	Local quark-hadron duality	✧ $\Delta u/u \to 1$ $\Delta d/d \to 1$

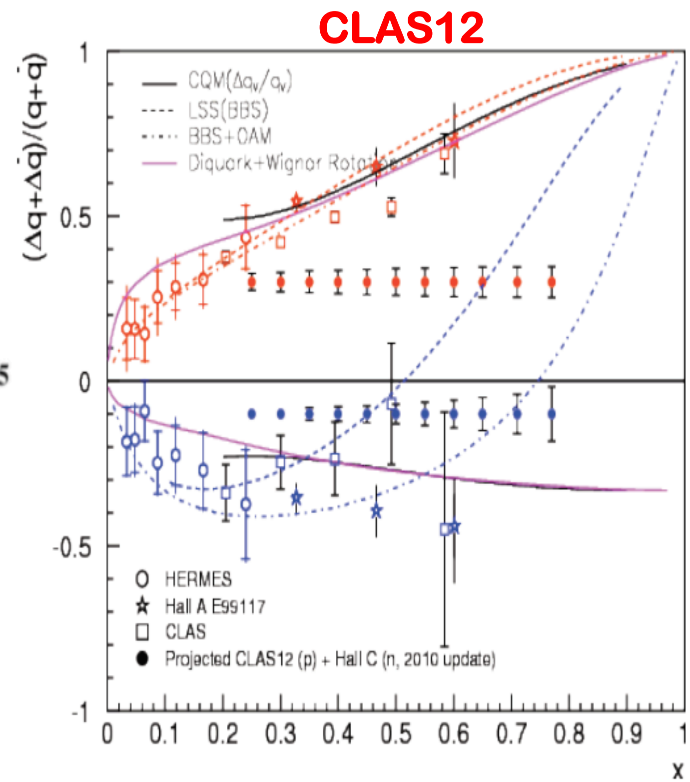
What data try to say?

Future large-x experiments – JLab12

❑ NSAC milestone HP14 (2018):







Plus many more JLab experiments:

E12-06-110 (Hall C on ^{3}He), E12-06-122 (Hall A on ^{3}He),

E12-06-109 (CLAS on NH_3, ND_3), …

and Fermilab E906, …

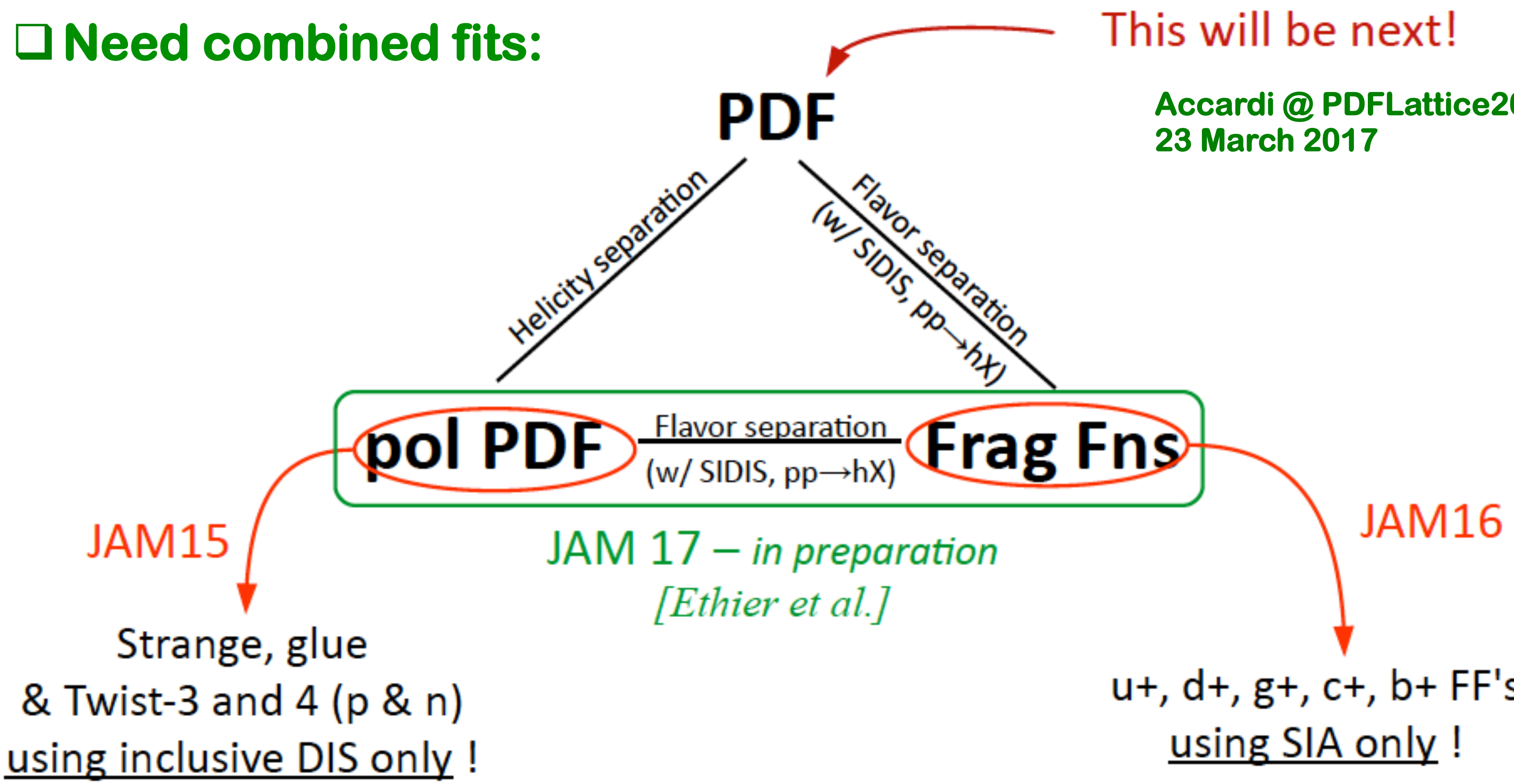
Can lattice QCD help?

Backup slides

JLab Theory Effort

- **Need combined fits:**

Accardi @ PDFLattice2017
23 March 2017



Iterative Monte Carlo: the JAM approach

Sato, Ethier, Melnitchouk, Kuhn, Accardi, Hirai, Kumano

PRD93 (2016) 074005 and PRD94 (2016) 114004

JLab Theory Effort

Accardi @ PDFLattice2017
23 March 2017

IMC method in action