Determination of the Polarization Observables $C_x, C_z, \text{ and } P_y$ for the Quasi-Free Mechanism in

$$\gamma d \rightarrow K^+ \Lambda n$$

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Main Mechanisms of $\gamma d \rightarrow K^+ \vec{\Lambda} n$

- **Quasi-free (QF)**
  - $\gamma \rightarrow K^+$
  - $p \rightarrow \Lambda \rightarrow n$

- **Pion mediated scattering**
  - $\gamma \rightarrow N$
  - $N \rightarrow \pi \rightarrow K^+$
  - $N \rightarrow \Lambda \rightarrow n$

- **$\Lambda n$ rescattering**
  - $\gamma \rightarrow \Lambda \rightarrow K^+$
  - $p \rightarrow \Lambda \rightarrow \Lambda \rightarrow n$

- **$Kn$ rescattering**
  - $\gamma \rightarrow \Lambda \rightarrow K^+$
  - $p \rightarrow K^+ \rightarrow K^+ \rightarrow n$
Experimental Observables

Helicity-dependent polarized differential cross section for hyperon photoproduction off the nucleon.

$$\frac{d\sigma^{\pm}}{d\Omega} = \frac{d\sigma}{d\Omega}_{unpol} \left( 1 \pm \alpha P_{circ} C_x \cos \theta_x \pm \alpha P_{circ} C_z \cos \theta_z + \alpha P_y \cos \theta_y \right)$$

$\Lambda$ self-analyzing power: $\alpha = 0.642 \pm 0.013$

$$\hat{x} = \hat{y} \times \hat{z}$$

$$\hat{z} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|}$$

$$\hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|}$$
The g13a Experiment

- Circularly polarized photon beam
- $E_e = 2 \text{ GeV}; 2.65 \text{ GeV}$
- Electron beam polarization: up to 85%
- Photon beam polarization: [27%, 80%]
- Target: LD$_2$, unpolarized, 40 cm long, upstream of CLAS center

$$\gamma d \rightarrow K^+ \Lambda n$$

Detected in CLAS
Selection of Quasi-Free Events

\[ P_X (\overrightarrow{\gamma} d \rightarrow K^+ \overrightarrow{\Lambda} X) \]

Counts

Quasi-free mechanism
# of events: \(1.9 \times 10^6\)

Final-state interactions
# of events: \(3.1 \times 10^5\)
Background Subtraction

\[ MM = \sqrt{(\vec{p}_{\gamma} + \vec{p}_d - \vec{p}_{K^+} - \vec{p}_p - \vec{p}_{\pi^-})^2} \]

\[ M_n = 0.9396 \text{ GeV/c}^2 \]
Observable-Extraction Methods

- **One-dimensional fit:**

\[
Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int \int \frac{d\sigma^+}{d\Omega} d(\cos \theta_y) d(\cos \theta_{z/x}) - \int \int \frac{d\sigma^-}{d\Omega} d(\cos \theta_y) d(\cos \theta_{z/x})}{\int \int \frac{d\sigma^+}{d\Omega} d(\cos \theta_y) d(\cos \theta_{z/x}) + \int \int \frac{d\sigma^-}{d\Omega} d(\cos \theta_y) d(\cos \theta_{z/x})} = \alpha P_{\text{circ}} C_{x/z} \cos \theta_{x/z}
\]

- **Two-dimensional fit:**

\[
Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int \frac{d\sigma^+}{d\Omega} d(\cos \theta_y)) - \int \frac{d\sigma^-}{d\Omega} d(\cos \theta_y)}{\int \frac{d\sigma^+}{d\Omega} d(\cos \theta_y)) + \int \frac{d\sigma^-}{d\Omega} d(\cos \theta_y)} = \alpha P_{\text{circ}} C_{x} \cos \theta_{x} + \alpha P_{\text{circ}} C_{z} \cos \theta_{z}
\]

- **Maximum likelihood Method:**

\[
PDF = \left. \frac{d\sigma}{d\Omega} \right|_{\text{unpol}} (1 \pm \alpha P_{\text{circ}} C_{x} \cos \theta_{x} \pm \alpha P_{\text{circ}} C_{z} \cos \theta_{z} + \alpha P_{y} \cos \theta_{y})
\]
Results for $C_x$ and $C_z$ from Different Methods

$\cos\theta_K$: [0.35, 0.55]  

$C_x$:

1d fit  
2d fit  
Maximum likelihood

$E_\gamma$ (GeV)

$C_z$:

$E_\gamma$ (GeV)
Comparison With g1c Results

\( C_x \) and \( C_z \) from Robert K. Bradford

\( P_y \) from John W.C. McNabb

- Dataset: g1c
- Reaction: \( \vec{\gamma} p \rightarrow K^+ \bar{\Lambda} \)

\( \cos \theta_K: [0.35, 0.55] \)
Simulation Study to Understand Different Methods

A study was used to evaluate potential bias of the maximum likelihood method and the binned methods.

- 6000 different experiments, with $10^6$ events in each experiment, were generated according to the differential polarized cross section with realistic values of $C_x$, $C_z$, and $P_y$ for $\gamma p \rightarrow K^+ \Lambda$.
- Generated data were processed through GSIM and gpp.
- After raw data were skimmed, the observables were extracted using the maximum likelihood method and the binned methods.

![Graphs showing differences between estimated and true values of $C_x$ and $C_z$.]
Simulation Study to Compare Different Methods

- Mean of Obs - Obs\text{ext}

Graph

- Maximum Likelihood
- 1d binned method
- 2d binned method

Legend

- C_x for Constant acceptance
- C_x for GSIM
- C_z for Constant acceptance
- C_z for GSIM
- P_y for Constant acceptance
- P_y for GSIM
Examples of 1D Fit

\[ E_\gamma: [1.5875, 1.6875] \text{ GeV and } \cos \theta_K: [0.35, 0.55] \]

\[ C_x = \frac{slope}{\alpha P_{\text{circ}}} \]

\[ C_z = \frac{slope}{\alpha P_{\text{circ}}} \]
Why is the Bias Small for $C_z$ from 1D Fit?

In the spherical coordinate system:

$$\begin{align*}
\cos \theta_x &= \sin \theta \cos \phi \\
\cos \theta_y &= \sin \theta \sin \phi \\
\cos \theta_z &= \cos \theta
\end{align*}$$

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$\theta_x$, $\theta_y$, and $\theta_z$ are not independent.

Event yield: $Y^\pm(\theta, \phi) = N_\gamma^\pm N_T \sigma^\pm(\theta, \phi) A(\theta, \phi)$

Integral over $\phi$: $Y^\pm(\theta) = c(A(\theta) \pm \alpha P_{circ} C_x \sin \theta A_x(\theta) \pm \alpha P_{circ} C_z \cos \theta A(\theta) + \alpha P_y \sin \theta A_y(\theta))$

$$A(\theta) = \int_0^{2\pi} A(\theta, \phi) \, d\phi; \quad A_x(\theta) = \int_0^{2\pi} A(\theta, \phi) \cos \phi \, d\phi; \quad A_y(\theta) = \int_0^{2\pi} A(\theta, \phi) \sin \phi \, d\phi$$

$$A_x(\theta) = \int_0^{2\pi} A(\theta, \phi) \cos \phi \, d\phi < \int_0^{2\pi} A(\theta, \phi) |\cos \phi| \, d\phi < |\cos \phi|_{\text{max}} \int_0^{2\pi} A(\theta, \phi) \, d\phi = A(\theta)$$

Asymmetry: $Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\alpha P_{circ} C_x \sin \theta A_x(\theta) + \alpha P_{circ} C_z \cos \theta A(\theta)}{A(\theta) + \alpha P_y \sin \theta A_y(\theta)}$

Generally, $|C_x| << |C_z|, |P_y| < |C_z|$

Therefore, $Asym \approx \alpha P_{circ} C_z \cos \theta_z$
Why is the Bias Large for $C_x$ from 1D Fit?

Spherical coordinate system for the convenience of $C_x$ analysis:

\[
\begin{align*}
\cos \theta_x &= \cos \theta \\
\cos \theta_y &= \sin \theta \cos \phi \\
\cos \theta_z &= \sin \theta \sin \phi
\end{align*}
\]

Asymmetry: 
\[
Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\alpha P_{\text{circ}} C_x \cos \theta A(\theta) + \alpha P_{\text{circ}} C_z \sin \theta A_z(\theta)}{A(\theta) + \alpha P_y \sin \theta A_y(\theta)}
\]

In general, $C_x$ is small relative to $C_z$ and $P_y$, so $C_z$ and $P_y$ terms do not cancel. Therefore, the asymmetry for $C_x$ is not a linear function of $\cos \theta_x$.

- The effect of acceptance cannot be ignored in 1D fit, especially for $C_x$.
- The situation with $P_y$ is somewhat in-between $C_x$ and $C_z$ if it’s extracted by 1D fit.
- 2D fitting can reduce the effect of the acceptance to some extent.
Effect of Missing Momentum Cut

$\cos \theta_K: [0.15, 0.35]$

The missing momentum was cut at points 0.2, 0.1 and 0.05 GeV/c.
$C_x$ (g13 Results)
C_z (g13 Results)

\[\begin{align*}
  &E: [0.9, 1.2434] \text{ GeV} \\
  &E: [1.2434, 1.3284] \text{ GeV} \\
  &E: [1.3284, 1.3896] \text{ GeV} \\
  &E: [1.3896, 1.4474] \text{ GeV} \\
  &E: [1.4474, 1.4984] \text{ GeV} \\
  &E: [1.4984, 1.546] \text{ GeV} \\
  &E: [1.546, 1.5936] \text{ GeV} \\
  &E: [1.5936, 1.6446] \text{ GeV} \\
  &E: [1.6446, 1.699] \text{ GeV} \\
  &E: [1.699, 1.7534] \text{ GeV} \\
  &E: [1.7534, 1.8078] \text{ GeV} \\
  &E: [1.8078, 1.9608] \text{ GeV} \\
  &E: [1.9608, 2.107] \text{ GeV} \\
  &E: [2.107, 2.2974] \text{ GeV} \\
  &E: [2.2974, 2.6] \text{ GeV}
\end{align*}\]
$P_y$ (g13 Results)
\[ R = \sqrt{C_x^2 + C_z^2 + P_y^2} \]
Summary and Outlook

• Comprehensive results for $C_x$, $C_z$, and $P_y$ in the kinematic bins of $E_y$ (0.9 – 2.6 GeV) and $\cos\theta_K$ (-1, 1) have been obtained for $K^+\Lambda$ photoproduction off the bound proton.

• Fermi-motion does not seem to significantly influence the polarization observables $C_x$, $C_z$, and $P_y$.

• Systematic uncertainties to be estimated.

• Currently the $C_x$ discrepancy g1c vs g13 is not understood.

• To analyze free proton data from g13.

• To publish results eventually.
Backup Slides
Why Study $K\Lambda$ Channel?

• The study of nucleon resonance excitation plays an important role in building a comprehensive picture of the strong interaction.

• The theoretical work on quark models in the intermediate energy range predicts a rich resonance spectrum.

• Besides have been observed in $\pi N \rightarrow \pi N$ scattering experiments, those “missing” resonances may couple strongly to other channels, such as $K\Lambda$ and $K\Sigma$ channels.
Interests of QF Study in $\vec{\gamma} d \rightarrow K^+ \vec{\Lambda} n$

• Many experimental results for $\vec{\gamma} p \rightarrow K^+ \vec{\Lambda}$ have been published, from total cross section to polarization observables.

• This study is to extract polarization observables for the quasi-free mechanism in $\vec{\gamma} d \rightarrow K^+ \vec{\Lambda} n$, where the target proton is not free, but bounded in deuterium.

• The comparison of results between reactions with free and bounded proton target can be used to test how great influence of the fermi-motion in the deuterium system.
Wien Angle

- The electron moves in a straight line but the spin precesses around the magnetic field. The rotated angle is called Wien angle which determines the degree of beam polarization.
Photon Polarization

The electron polarization for some special runs were measured by the Möller polarimeter.

The polarization of the photon beam was calculated using the Maximon and Olson relation

\[ P_{cir} = \frac{E \gamma (E + \frac{1}{3}E') P_e}{E^2 + E'^2 - \frac{2}{3}EE'} \]

![Graph showing electron beam polarization and photon polarization calculations](image-url)
Reaction Yields

Particle Identification

\[ \Delta \beta = \beta_{\text{meas}} - \beta_{\text{calc}} = \beta_{\text{meas}} - \frac{p^2}{m^2 + p^2} \]

Photon Selection

\[ \Delta t = t_\nu - t_\gamma = (t_{SC} - \frac{d_{SC}}{c \beta_{\text{calc}}}) - (t_{TAGR} + \frac{z + 20 \text{ (cm)}}{c}) \]

Z-Vertex Cut

\[ M_{p\pi^-} = \sqrt{\left(\vec{p}_p + \vec{p}_{\pi^-}\right)^2} \]

Invariant-Mass Cut

\[ M_\Lambda = 1.1157 \text{ GeV}/c^2 \]
Method 2 of Observable-Extraction: Weight for Each Event

Weight = (All – Backgrounds) / All
Method 2 of Observable-Extraction: The Maximum Likelihood Method

Non-normalized Probability Density Function (PDF) defined from the polarized differential cross section:

$$PDF = \frac{d\sigma}{d\Omega}_{unpol} (1 \pm \alpha P_{circ} C_x \cos \theta_x \pm \alpha P_{circ} C_z \cos \theta_z + \alpha P_y \cos \theta_y)$$

Total likelihood is the product of the likelihoods for all individual events:

$$logL = b + \sum_{i=1}^{n^+} \log[(1 + \alpha P_{circ}^i C_x \cos \theta_x^i + \alpha P_{circ}^i C_z \cos \theta_z^i + \alpha P_y \cos \theta_y^i)w^i]$$

$$\sum_{j=1}^{n^-} \log[(1 - \alpha P_{circ}^j C_x \cos \theta_x^j - \alpha P_{circ}^j C_z \cos \theta_z^j + \alpha P_y \cos \theta_y^j)w^j]$$

Next side will introduce how to set weight $w^i$ and $w^j$ for each event.
Why the Maximum Likelihood Method Can Ignore Acceptance?

Non-normalized PDF with consideration of acceptance:

\[ PDF = A(\theta, \phi)\sigma^\pm(\theta, \phi; C_x, C_z, P_y)w \]

Total likelihood:

\[ \log L(C_x, C_z, P_y) = \sum_{i=1}^{n^+} \log A(\theta_i, \phi_i) + \sum_{j=1}^{n^-} \log A(\theta_j, \phi_j) \]

\[ + \sum_{i=1}^{n^+} \log[\sigma^+(\theta_i, \phi_i; C_x, C_z, P_y)w_i] + \sum_{j=1}^{n^-} \log[\sigma^-(\theta_j, \phi_j; C_x, C_z, P_y)w_j] \]

Equation array to obtain \( C_x, C_z, \) and \( P_y \):

\[
\begin{align*}
\frac{\partial \log L(C_x, C_z, P_y)}{\partial C_x} &= \frac{\partial}{\partial C_x} \left( \sum_{i=1}^{n^+} \log[\sigma^+(\theta_i, \phi_i; C_x, C_z, P_y)w_i] \right) + \frac{\partial}{\partial C_x} \left( \sum_{j=1}^{n^-} \log[\sigma^-(\theta_j, \phi_j; C_x, C_z, P_y)w_j] \right) = 0 \\
\frac{\partial \log L(C_x, C_z, P_y)}{\partial C_z} &= \frac{\partial}{\partial C_z} \left( \sum_{i=1}^{n^+} \log[\sigma^+(\theta_i, \phi_i; C_x, C_z, P_y)w_i] \right) + \frac{\partial}{\partial C_z} \left( \sum_{j=1}^{n^-} \log[\sigma^-(\theta_j, \phi_j; C_x, C_z, P_y)w_j] \right) = 0 \\
\frac{\partial \log L(C_x, C_z, P_y)}{\partial P_y} &= \frac{\partial}{\partial P_y} \left( \sum_{i=1}^{n^+} \log[\sigma^+(\theta_i, \phi_i; C_x, C_z, P_y)w_i] \right) + \frac{\partial}{\partial P_y} \left( \sum_{j=1}^{n^-} \log[\sigma^-(\theta_j, \phi_j; C_x, C_z, P_y)w_j] \right) = 0 
\end{align*}
\]

Acceptance is cancelled because it’s independent of polarization observables.
Comparison of 2-Track and 3-Track Topology

2-Track

$C_x$ for 1d

3-Track

$C_x$ for 1d
**Axis Conventions**

**Axis Convention 1:**
\[
\begin{align*}
\hat{z} &= \hat{p}_\gamma \\
\hat{y} &= \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\
\hat{x} &= \hat{y} \times \hat{z}
\end{align*}
\]

**Axis Convention 2:**
\[
\begin{align*}
\hat{z} &= \hat{p}_\gamma \\
\hat{y} &= \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\
\hat{x} &= \hat{y} \times \hat{z}
\end{align*}
\]

**Axis Convention 3:**
\[
\begin{align*}
\hat{z} &= \hat{p}_\gamma \\
\hat{y} &= \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\
\hat{x} &= \hat{y} \times \hat{z}
\end{align*}
\]

**Axis Convention 4:**
\[
\begin{align*}
\hat{z} &= \frac{\hat{p}_\Lambda + \hat{p}_K}{|\hat{p}_\Lambda + \hat{p}_K|} \\
\hat{y} &= \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\
\hat{x} &= \hat{y} \times \hat{z}
\end{align*}
\]
\( C_x \)

\[ E_x: [1.4474, 1.4984] \text{ GeV} \]

\[ E_x: [1.4984, 1.546] \text{ GeV} \]

\[ E_x: [1.6446, 1.699] \text{ GeV} \]

\[ E_x: [1.699, 1.7534] \text{ GeV} \]
$C_z$
$C_y$

$E_\gamma$: [1.5875, 1.6875] GeV and $\cos\theta_K$: [0.35, 0.55]

Real data

Simulated data