Heavy-quarkonium theory in the LHC era

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In collaboration with Mathias Butenschön and Zhiguo He
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Outline

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2. NLO NRQCD: General concept, singularities
3. Global fit: Unpolarized $J/\psi$ yield
4. Further tests: ATLAS, FTPS, ZEUS
5. Polarization: HERA, Tevatron, LHC
6. $\eta_c$ yield: LHC
7. Summary: NRQCD at the crossroads
Introduction: CEM, CSM, NRQCD factorization

Color evaporation model [Fritzsch 77; Halzen 77; Glück Owens Reya 78]

\[ \sigma_{J/\psi} \approx \frac{1}{9} \rho_{J/\psi} \int_{2m_c}^{2m_D} ds c\bar{c} \frac{d\sigma_{c\bar{c}}}{ds c\bar{c}} \]

- **1/9**: statistical probability that \(3\times\bar{3}\) \(c\bar{c}\) pair is asymptotically in color-single state
- **\(\rho_{J/\psi}\)**: fraction of charmonia that materialize as \(J/\psi\)
- Based local parton-hadron duality
- Assumes soft-gluon exchange with underlying event
- **\(2S+1 L^c_J\)** quantum numbers do not enter
- Useful qualitative picture, rather than rigorous theory

[Schuler Vogt 96; Vogt 99; Frawley Ullrich Vogt 08]

~ Talk by Vincent Cheung.
Color-singlet model vs. NRQCD factorization

Color-singlet model [Berger Jones 81; Baier Rückl 81]
- $c\bar{c}$ pair in physical color-singlet state, e.g. $c\bar{c}[^3S_1]$ for $J/\psi$.
- Nonperturbative information in $J/\psi$ wave function at origin.
- Leftover IR divergences for P-wave quarkonia $\sim$ inconsistent!
- Predicted cross section factor $10^1$–$10^2$ below Tevatron data.

NRQCD factorization [Bodwin Braaten Lepage 95]
- Rigorous effective field theory.
- Based on factorization of soft and hard scales
  (Scale hierarchy: $Mv^2 \lesssim \Lambda_{\text{QCD}} \ll Mv \ll M$).
- Theoretically consistent: no leftover singularities.
- Proof of factorization [Nayak Qiu Sterman 05; Nayak 15].
- Can explain unpolarized yield at Tevatron and elsewhere.
NRQCD factorization in a nutshell

**Factorization theorem** \( \sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O_{J/\psi}^n \rangle \)

- \( n \): every possible Fock state, including color-octet states.
- \( \sigma_{c\bar{c}[n]} \): production rate of \( c\bar{c}[n] \), calculated in perturbative QCD.
- \( \langle O_{J/\psi}^n \rangle \): long-distance matrix elements (LDMEs), nonperturbative, extracted from experiment, universal?

**Scaling rules** [Lepage Magnea\(^2\) Nakhle Hornbostel 92]
LDMEs scale with relative velocity \( v \) (\( v^2 \approx 0.2 \)).

<table>
<thead>
<tr>
<th>scaling</th>
<th>( v^3 ) (CS state)</th>
<th>( v^7 ) (CO states)</th>
<th>( v^{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( 3S_1^{[1]} )</td>
<td>( 1S_0^{[8]}, 3S_1^{[8]}, 3P_0^{[8]} )</td>
<td>( 0/1/2 )</td>
</tr>
</tbody>
</table>

- **Double expansion** in \( v \) and \( \alpha_s \).
- **Leading term** in \( v \) (\( n = 3S_1^{[1]} \)) corresponds to **color-singlet model**.
NLO NRQCD calculations

- Petrelli Cacciari Greco Maltoni Mangano 98:
  Photo- and hadroproduction (only $2 \rightarrow 1$ processes)

- Klasen BK Mihaila Steinhauser 05:
  Two-photon scattering (w/o resolved photons)

- Butenschön BK 09:
  Photoproduction (w/o resolved photons)

- Zhang Ma Wang Chao 10:
  $e^+ e^-$ annihilation

- Ma Wang Chao 10, Butenschön BK 10:
  Hadroproduction

- Butenschön BK 11:
  $\gamma p$ and $\gamma \gamma$ (resolved photons) $\sim$ global fit of CO LDMEs

- Butenschön BK 11:
  Polarization in photoproduction

- Butenschön BK 12, Chao Ma K. Wang Y.-J. Zhang 12, Gong, Wan, J.-X. Wang, H.-F. Zhang 12, Shao, Ma, K. Wang, Chao 14:
  Polarization in hadroproduction
Sample diagrams for $J/\psi$ photoproduction in NRQCD

(a) \[ \gamma \rightarrow c \bar{c} \]

(b) \[ \gamma \rightarrow c g c\]

(c) \[ \gamma \rightarrow c c g\]

(d) \[ \gamma \rightarrow q g q\]

(e) \[ \gamma \rightarrow q g q\]

(f) \[ \gamma \rightarrow q g q\]
Amplitudes for $c\bar{c}[n]$ production by projector application:

\[
A_{c\bar{c}[1S_0^8]} = \text{Tr} \left[ C_8 \Pi_0 A_{c\bar{c}} \right] |_{q=0}
\]
\[
A_{c\bar{c}[3S_1^{1/8}]} = \epsilon_\alpha \text{Tr} \left[ C_{1/8} \Pi^\alpha A_{c\bar{c}} \right] |_{q=0}
\]
\[
A_{c\bar{c}[3P_J^{8}]} = \epsilon_{\alpha\beta} \frac{d}{dq_\beta} \text{Tr} \left[ C_8 \Pi^\alpha A_{c\bar{c}} \right] |_{q=0}
\]

- $A_{c\bar{c}}$: amputated pQCD amplitude for open $c\bar{c}$ production.
- $q$: relative momentum between $c$ and $\bar{c}$.
- $C_{1/8}$: color projectors
- $\Pi_{0/1}$: spin projectors
- $\epsilon$: polarization vectors and tensors
Main Difference to Previous Calculations

Virtual corrections: Two different approaches:

- First loop integration, then projectors: (Previous publications)
  - Loop integrals **Coulomb divergent**.
- First projectors, then loop integration: (Our method)
  + No Coulomb singularities.
  + One scale less in loop integration.
  - Loop integrals not standard form.

Where do Coulomb divergences come from?

- Projectors: Relative momentum \( q \to 0 \).
- Scalar diagrams with gluon between external \( c \) and \( \bar{c} \), e.g.:

\[
I(q) \equiv \lim_{q \to 0} \frac{P/2+q}{c} = \frac{A}{q^2} + \frac{B}{\epsilon} + C \\
\text{But: } I(0) = \frac{B}{\epsilon} + C
\]

\[\implies \text{No Coulomb singularities in dimensional regularization!} \]
Cancellation of divergences

**UV divergences:** Cancellation within virtual corrections:
- Loop integrals
- Charm mass renormalization
- Strong coupling constant renormalization
- Wave function renormalization of external particles

**IR divergences:** Cancellation between:
- Virtual corrections (loop integrals + wave function renormal.)
- Soft and collinear parts of real corrections
- Universal part absorbed into proton and photon PDFs
- Radiative corrections to long distance matrix elements
Overview of IR singularity structure
Structure of Soft Singularities

Soft limits of the real corrections:

\[ P/2 + q \rightarrow P/2 - q \rightarrow k_4 \rightarrow 0 \]  
\[ k_4 \text{ soft} \]

\[ A_{\text{soft}}(q) = A_{\text{Born}}(q) \cdot E(q) \]

S and P states: Soft #1 + Soft #2 + Soft #3 terms:

\[ A_{\text{soft},s} = A_{\text{soft}}(0) = A_{\text{Born},s} \cdot E(0) \]
\[ A_{\text{soft},p} = A'_{\text{soft}}(0) = A_{\text{Born},p} \cdot E(0) + A_{\text{Born},s} \cdot E'(0) \]

\[ |A_{\text{soft},s}|^2 = |A_{\text{Born},s}|^2 \cdot E(0)^2 \]
\[ |A_{\text{soft},p}|^2 = |A_{\text{Born},p}|^2 \cdot E(0)^2 + 2 \text{ Re } A_{\text{Born},s}^* A_{\text{Born},p} \cdot E(0) E'(0) \]
\[ + |A_{\text{Born},s}|^2 \cdot E'(0)^2 \]
Radiative Corrections to Long Distance MEs

In NRQCD: Long distance MEs = $c\bar{c}$ scattering amplitudes:

$$\langle O^{J/\psi}[n] \rangle = \begin{pmatrix} c \\ \bar{c} \end{pmatrix} \begin{pmatrix} c \\ \bar{c} \end{pmatrix} \begin{pmatrix} O[n] \end{pmatrix}$$

$O[n] = 4$-fermion operators

$(n = 3S_1, 1S_0, 3S_1, 3P_0/1/2, \ldots)$

Corrections to $\langle O^{J/\psi}[3S_1^{1/8}] \rangle$ with NRQCD Feynman rules:

$$\begin{pmatrix} c \\ \bar{c} \end{pmatrix} \begin{pmatrix} c \\ \bar{c} \end{pmatrix} \begin{pmatrix} 3S_1 \end{pmatrix} + \text{similar diagrams} \propto \frac{4\alpha_s}{3\pi m_c^2} \left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) \begin{pmatrix} c \\ \bar{c} \end{pmatrix} \begin{pmatrix} 3P_0 + 3P_1 + 3P_2 \end{pmatrix}$$

- UV singularity cancelled by renormalization of 4-fermion operat.
- IR singularity cancels soft #3 terms of $P$ states!
# Global fit at NLO in NRQCD

<table>
<thead>
<tr>
<th>Fit</th>
<th>CO LDMEs to all available world data on $J/\psi$ inclusive production:</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>$\sqrt{s}$</td>
</tr>
<tr>
<td>$pp$</td>
<td>200 GeV</td>
</tr>
<tr>
<td>$\bar{p}p$</td>
<td>1.8 TeV</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>1.96 TeV</td>
</tr>
<tr>
<td>$pp$</td>
<td>7 TeV</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma p$</td>
<td>319 GeV</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>197 GeV</td>
</tr>
<tr>
<td>$e^+e^-$</td>
<td>10.6 GeV</td>
</tr>
</tbody>
</table>

Fit values for CO LDMEs:

- $10^{-2} \text{ GeV}^{3+2L}$ feed-down included
- $10^{-2} \text{ GeV}^{3+2L}$ feed-down subtracted

- $\langle O^{[1S_0]} \rangle = 4.97 \pm 0.44 \rightarrow 3.04 \pm 0.35$
- $\langle O^{[3S_1]} \rangle = 0.224 \pm 0.059 \rightarrow 0.168 \pm 0.046$
- $\langle O^{[3P_0]} \rangle = -1.61 \pm 0.20 \rightarrow -0.908 \pm 0.161$
- $\chi^2 / \text{d.o.f.} = 857/194 = 4.42 \rightarrow 725/194 = 3.74$

Note: CO LDMEs $\propto \sqrt{s} \times \langle O^{[3S_1]} \rangle \rightarrow$ NRQCD velocity scaling rules $\checkmark$
Comparison with world data

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Comparison with RHIC and Tevatron

- **RHIC PHENIX**
  - PHENIX data
  - CS, LO
  - CS, NLO
  - CS+CO, LO
  - CS+CO, NLO

- **Tevatron II CDF**
  - CDF data: Run 2
  - CS, LO
  - CS, NLO
  - CS+CO, LO
  - CS+CO, NLO

- **Decomposition of NLO NRQCD**
  - CDF data
  - $3S_J^{[1]}$, NLO
  - $1S_J^{[1]}$, NLO
  - $3S_J^{[8]}$, NLO
  - $3P_J^{[8]}$, NLO
  - $3P_J^{[8]}$, NLO
  - Total, NLO

- **Data** well described by CS+CO at NLO.
- **CS** orders of magnitudes below data.
- Sizeable NLO corrections, especially in the $3P_J^{[8]}$ channels.
Comparison with ATLAS and CMS at LHC
Comparison with ALICE and LHCb at LHC

Comparison with ALICE and LHCb at LHC
Comparison with ZEUS at HERA I

- **$W = \gamma p$** CM energy.
- **$z$** = fraction of $\gamma$ energy going to $J/\psi$ in $p$ rest frame.
- Compensation of $^{1}\Sigma_0^{[8]}$ vs. $^{3}P_j^{[8]} \rightsquigarrow$ regular $z \rightarrow 1$ behavior.
- Data well described by CS+CO at NLO.
- CS factor of 3–5 below the data.
Comparison with H1 at HERA I and II

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Comparison with DELPHI at LEP II

- Agreement with NRQCD at NLO worse than in 2002 at LO.
- Just 16 DELPHI events with $p_T > 1$ GeV.
- No results from ALEPH, L3, OPAL.
- Data exhausted by single-resolved contribution.

[Klasen BK Mihaila Steinhauser 02]
Comparison with Belle at KEKB

- At NLO, both CSM and NRQCD agree with data.
- # of charged tracks > 4, missing events not corrected for.
  \sim Belle point likely higher.
Comparison with ATLAS (after fit) [NPB850(2011)387]

- Resummation of large logs $\ln(p_T^2/M^2)$ necessary at large $p_T$.
- New formalism to include non-leading powers in $p_T^2/M^2$ [Kang Qiu Sterman 2012].
Comparison with Fermilab Tagged-Photon Spectrometer data (excluded from fit) \[\text{[PRL52(1984)795]}\]

- **Inelastic scattering of 105 GeV photons on hydrogen target.**
- **Data remarkably well described** by CS+CO at NLO.
Comparison with ZEUS (after fit) [JHEP1302(2013)071]

- Notorious NRQCD overshoot at large $z$ overcome.
Decay angular distribution:
\[
\frac{d\Gamma(J/\psi \rightarrow l^+ l^-)}{d\cos\theta \, d\phi} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos(2\phi) + \lambda_\theta\phi \sin(2\theta)\cos\phi
\]

Polarization observables in spin density matrix formalism:
\[
\lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\phi = \frac{d\sigma_{1,-1} - d\sigma_{-1,1}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\theta\phi = \sqrt{2}\text{Re} \frac{d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}
\]
\[
\lambda = 0, \pm 1, -1: \text{unpolarized, transversely and longitudinally polarized.}
\]
Comparison with H1 and ZEUS

- H1 data
  - Helicity frame
    - CS, LO
    - CS, NLO
    - CS+CO, LO
    - CS+CO, NLO
  - Collins-Soper frame
    - CS, LO
    - CS, NLO
    - CS+CO, LO
    - CS+CO, NLO

- ZEUS data (till z=1)
  - Target frame
    - CS, LO
    - CS, NLO
    - CS+CO, LO
    - CS+CO, NLO

No z cut on ZEUS data $\sim$ diffractive production included.
- Perturbative stability in NRQCD higher than in CSM.
- $J/\psi$ preferably unpolarized at large $p_T$. 
Comparison with CDF and ALICE

- CDF $J/\psi$ polarization anomaly persists at NLO.
- 4/8 ALICE [PRL108 (2012) 082001] points agree w/ NLO NRQCD within errors, others $< 2\sigma$ away.

- ALICE and LHCb data mutually agree.
- NLO NRQCD predictions systematically disagree w/ data.
Comparison with CMS data on prompt $J/\psi$ and $\psi'$ polarization [PLB727(2013)381]

- NLO NRQCD predictions systematically disagree w/ data on $\lambda_\theta$. 

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Comparison with Gong et al. and Chao et al.

**e^+e^- yield**

- BK, MB
  - PRL108(2012)172002

- Gong et al.
  - PRL110(2013)042002

- Chao et al.
  - PRL108(2012)242004

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**γp yield**

- BK, MB
  - PRL108(2012)242004

- Gong et al.
  - PRL110(2013)042002

- Chao et al.
  - PRL108(2012)242004

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**p+p/pp yield**

- BK, MB
  - PRL108(2012)172002

- Gong et al.
  - PRL110(2013)042002

- Chao et al.
  - PRL108(2012)242004

---

**CDF polariz.**

- BK, MB
  - PRL108(2012)172002

- Gong et al.
  - PRL110(2013)042002

- Chao et al.
  - PRL108(2012)242004

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Heavy-quarkonium theory in the LHC era

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LHCb data on $\eta_c$ yield [EPJC75(2015)311]

M. Butenschoen, Z. He, BK, PRL114(2015)092004
NRQCD factorization provides rigorous framework for production and decay of heavy quarkonia; predicts:
- existence of CO states;
- universality of LDMEs.
NLO NRQCD nicely describes world data on unpolarized $J/\psi$ yield.
NLO CSM greatly undershoots data, except for $e^+e^-$ annihilation.
$\gamma\gamma$ scattering not conclusive yet.
Hadroproduction data alone cannot reliably fix all 3 CO LDMEs.
NLO NRQCD predictions for polarized $J/\psi$ hadroproduction based on global analysis of $J/\psi$ yield agrees with ALICE (low $p_T$), but strongly disagrees with CDF, CMS, and LHCb.
NLO NRQCD predictions for $\eta_c$ yield based on heavy-quark spin symmetry greatly overshoot LHCb data.
NRQCD factorization remains among the hottest topics of QCD @ LHC.