Pion electromagnetic form factor at high $Q^2$ from lattice QCD

Bipasha Chakraborty

[With Raul Briceno, Robert Edwards, Adithia Kusno, Kostas Orginos, David Richards, Frank Winter]

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Space like “$q$”:

\[ q^2 = (p_2 - p_1)^2 \leq 0 \]

\[ Q^2 = -q^2 \]

\[
\langle \pi^+ (p') | j^\mu | \pi^+ (p) \rangle = (p + p')^\mu F_\pi (Q^2)
\]

(in units of ‘e’)
Interplay between hard and soft scales

Need better understanding of the transition to the asymptotic region

Hard tail \((Q^2 \to \infty)\) from pQCD:

\[ F_\pi(Q^2) \to \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2} \]

G. P. Lepage, S.J. Brodsky, Phys. Lett. 87B(1979)359

Soft part \((Q^2 < 1 \text{ GeV}^2)\):
vector meson dominance with \(F_\pi(0) = 1\),
data fits well

G. Huber and D. Gaskell
Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?
Lattice recipe for meson correlators

- Expectation values of observables:
  \[ \int DUD\bar{\psi}\psi \exp(-\int L_{QCD} d^4x) \]
- 4-D space-time lattice
- Gauge configurations: gluons + sea quarks

Discretise:
\[ L_q \equiv \bar{\psi}(\gamma\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma \Delta + ma)\psi \]

- Inversion of Dirac matrix: propagator
- 2-point, 3-point correlation functions: extract meson properties
- Corrections for lattice artifacts
Two-point correlator construction

\[ C_{ij}(t) = \langle 0 | O_i(t) O_j^\dagger(0) | 0 \rangle \]

- Basis of operators

\[ \mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi \]

- Optimized operator for state \( |n> \)

\[ \Omega_n^\dagger = \sum_i w_i^{(n)} O_i^\dagger \]

in a variational sense by solving generalized eigenvalue problem-

\[ C(t) \nu^{(n)} = \lambda_n(t) C(t_0) \nu^{(n)} \]

- Diagonalize the correlation matrix – eigenvalues

\[ \lambda_n(t) = \exp[-En(t - t_0)] \]
Two-point correlator construction

Correlator Construction: smearing of quark fields - ‘distillation’ with

\[ \square xy(t) = \sum_{n=1}^{N_D} \xi^{(n)}_\bar{x}(t) \xi^{(n)\dagger}_\bar{y}(t) \]

Low lying hadron states

Meson creation operator:

\[ \mathcal{O}^\dagger(p) = \bar{\psi}_\bar{x} \square xy e^{-i \vec{p} \cdot \vec{y}} \Gamma yz \square z\bar{w} \psi \bar{w} \]

Paramambulators by inverting the Dirac matrix

Operator construction with momentum projection
Meson Spectrum

Tools well established for spectroscopy

Hadron Spectrum Collaboration

Form factor calculation

Need three-point correlator

\[ C_{f\mu i}(\Delta t, t) = \langle 0| O_f(\Delta t) j_\mu(t) O_i^\dagger(0) |0 \rangle \]

\[ Z_V < \pi^+(p_2)| J_\mu^\pi(0) |\pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2) \]

Clover discretised fermion action

\[ Z_V \] calculated using \( F_\pi(q^2 = 0) = 1 \)
Pion electromagnetic form factor: up to $Q^2 = 1 \text{ GeV}^2$

Amendolia et al.
JLAB expt.
JLAB lattice ongoing

Anisotropy

In agreement with recent lattice result from HPQCD (up to 0.25 GeV$^2$) Phys.Rev. D93 (2016)

$m_\pi = 750 \text{ MeV}$

$m_\pi = 450 \text{ MeV}$

$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$

$a_s = 0.12 \text{ fm}$, $\frac{a_s}{a_t} = 3.44$
Towards higher $Q^2$

More difficult on lattice for higher momenta

Signal-to-noise ratio:

2-point correlators:

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

3-point correlators:

$$\exp[-(E_\pi(pi) + E_\pi(pf) - 2m_\pi)t/2]$$

in the middle of the plateau

Minimize energies for a given $Q^2$ to get better signal
Towards higher $Q^2$

Dispersion relation:

$$E^2 = m^2 + p^2$$

$$\left( a_t E \right)^2 = \left( a_t m \right)^2 + \left( \frac{2\pi}{\xi (L/a_s)} \right)^2 |\vec{n}_{\vec{p}}|^2$$

$$\vec{n}_{\vec{p}} = [0, 0, 0], [0, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0, 2] \ldots$$

Achieve maximum $Q^2$ by using Breit frame:

$$\vec{P}_f = -\vec{P}_i$$
Outlook

Immediate goals:

- Pion form factor at $Q^2 \geq 6 \text{ GeV}^2$
- Extend to more ensembles with lighter pion masses, multiple volumes, multiple lattice spacings
- Take care of lattice artifacts

Long term goals:

- Hadron structure program – distribution amplitude, PFDs, Quasi PDFs
- Extend to nucleons & more – charges, moments, TMDs, GPDs ....