Transverse Momentum Dependent Fragmenting Jet Functions with Applications to Quarkonium Production

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Fragmenting Jet Functions (FJFs)

NRQCD and Quarkonium Production

Heavy Quarkonium FJFs

TMD-dependent FJFs and Heavy Quarkonium

Recent Data on Quarkonia in Jets (LHCb)
Fragmenting Jet Functions

jets with identified hadrons

Jet Energy: \( E \)
\[
p_H^+ = z p_{jet}^+
\]

cross sections determined by **fragmenting jet function (FJF):**

\[
\mathcal{G}_g^h(E, R, \mu, z)
\]
inclusive hadron production: \textbf{fragmentation functions}

\[
\frac{1}{\sigma_0} \frac{d\sigma^h}{dz} (e^+ e^- \to h X) = \sum_i \int_z^1 \frac{dx}{x} C_i(E_{cm}, x, \mu) D_i^h(z/x, \mu)
\]

jet cross sections: \textbf{jet functions}

\[
\frac{d\sigma^h}{dz}(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell}
\]

\[
G^h_g (E, R, \mu, z) \rightarrow D^h_i(z/x, \mu), J_{\ell}
\]
relationship to jet function:

\[ \sum_h \int_0^1 dz D_j^h(z, \mu) = 1 \]

\[ J_i(E, R, z, \mu) = \frac{1}{2} \sum_h \int \frac{dz}{(2\pi)^3} z \mathcal{G}_i^h(E, R, z, \mu) \]

cross section for jet w/ identified hadron from jet cross section

\[ \frac{d\sigma}{dE} = \int d\Phi_N \text{tr} [H_N S_N] \prod_\ell J_\ell J_i(E, R, \mu) \]

\[ \frac{d\sigma}{dE dz} = \int d\Phi_N \text{tr} [H_N S_N] \prod_\ell J_\ell \mathcal{G}_i^h(E, R, z, \mu) \]
relationship to fragmentation functions

\[ g_i^h(E, R, z, \mu) = \sum_i \int_z^1 \frac{dz'}{z'} J_{ij}(E, R, z', \mu) D_j^h\left(\frac{z}{z'}, \mu\right) \left[ 1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{4E^2 \tan^2(R/2)}\right)\right] \]

matching coefficients calculable in perturbation theory

\[ \frac{J_{gg}(E, R, z, \mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi} \left[ \left( L^2 - \frac{\pi^2}{24} \right) \delta(1-z) + \hat{P}_{gg}(z) L + \hat{J}_{gg}(z) \right] \]

\[ \hat{J}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z) \ln z & z \leq 1/2 \\ \frac{2(1-z+z^2)^2}{z} \ln(1-z) & z \geq 1/2. \end{cases} \]

scale for \( J_{ij}(E, R, z, \mu) \)

sum rule for matching coefficients

\[ \sum_j \int_0^1 dz z J_{ij}(R, z, \mu) = 2(2\pi)^3 J_i(R, \mu) \]
\[
\sigma(gg \rightarrow J/\psi + X) = \sum_n \sigma(gg \rightarrow c\bar{c}(n) + X) \langle O^{J/\psi}(n) \rangle
\]

\[n = 2S+1 L_j^{(1,8)}\]

do double expansion in \(\alpha_s, \nu\)

NRQCD long-distance matrix element (LDME)

\[
\langle O^{J/\psi}(3 S_1^{[1]}) \rangle \sim \nu^3
\]

\[
\langle O^{J/\psi}(3 S_1^{[8]}) \rangle, \langle O^{J/\psi}(1 S_0^{[8]}) \rangle, \langle O^{J/\psi}(3 P_j^{[8]}) \rangle \sim \nu^7
\]

CSM - lowest order in \(\nu\)

color-octet mechanisms
Global Fits with NLO CSM + COM

Butenschoen and Kniehl, PRD 84 (2011) 051501

\[ e^+ e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X \]

fit to 194 data points, 26 data sets
NLO: CSM + COM Required to Fit Data

\[ \text{ep} \rightarrow J/\psi + X \]

\[ \gamma^* \gamma^* \rightarrow J/\psi + X \]
Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

extracted LDME satisfy NRQCD v-scaling

\[ \langle O^{J/\psi}(3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3 \]

\[
\begin{array}{|c|c|}
\hline
\langle O^{J/\psi}(1S_0^{[8]}) \rangle & (4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3 \\
\langle O^{J/\psi}(3S_1^{[8]}) \rangle & (2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3 \\
\langle O^{J/\psi}(3P_0^{[8]}) \rangle & (-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5 \\
\hline
\end{array}
\]

\[ \chi^2_{d.o.f.} = 857/194 = 4.42 \]
Polarization of $J/\psi$ at LHCb

LDME from Global fits

CSM

$\lambda_\theta$

LHCb $\sqrt{s} = 7$ TeV

2.5 < $y$ < 4.0

$p_T(J/\psi)$ [GeV/c]
Polarization of $\Upsilon(nS)$ at CMS
Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_t production, polarization

Recent Attempts to Resolve $J/\psi$ Polarization Puzzle

i) large $p_t$ production at CDF

ii) resum logs of $p_t/m_c$ using AP evolution

iii) fit COME to $p_t$ spectrum, predict basically no polarization

Extracted COME inconsistent with global fits

$$\langle O^{J/\psi}(1S^0_0) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$$

$$\langle O^{J/\psi}(3S^1_1) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$$

$$\langle O^{J/\psi}(3P^0_0) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$$
Recent Attempts to Resolve $J/\psi$ Polarization Puzzle

argue for $^{1}S_{0}^{(8)}$ dominance in both $\psi(2S)$ & $\Upsilon(3S)$ production
NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230
Braaten, Chen, PRD 54 (1996) 3216
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable at the scale $2m_c$

\[
D^{(8)}_g(z, 2m_c) = \frac{\pi \alpha_s(2m_c)}{3M^3_{\psi}} \langle O^\psi(3S_1^{(8)}) \rangle \delta(1 - z).
\]

\[
D^{(1)}_g(z, 2m_c) = \frac{5 \alpha_s^3(2m_c)}{648\pi^2} \langle O^\psi(3S_1^{(1)}) \rangle \int_0^z \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2} dy (1-y)^2(y-r)^2(y^2-r)^2
\]

\[
\sum_{i=0}^2 z^i \left( f_i(r, y) + g_i(r, y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r + \sqrt{y^2-r}}{y-r - \sqrt{y^2-r}} \right),
\]

Altarelli-Parisi evolution: $2m_c$ to $2E \tan(R/2)$
FJJF in terms of fragmentation function

\[
G_{g}^{\psi}(E, R, z, \mu) = D_{g \to \psi}(z, \mu) \left( 1 + \frac{C_{A}\alpha_{s}}{\pi} \left( L_{1-z}^{2} - \frac{\pi^{2}}{24} \right) \right) + \frac{C_{A}\alpha_{s}}{\pi} \left[ \int_{z}^{1} \frac{dy}{y} \hat{P}_{gg}(y)L_{1-y}D_{g \to \psi} \left( \frac{z}{y}, \mu \right) 
+ 2 \int_{z}^{1} dy \frac{D_{g \to \psi}(z/y, \mu) - D_{g \to \psi}(z, \mu)}{1-y} L_{1-y} 
+ \theta \left( \frac{1}{2} - z \right) \int_{z}^{1/2} \frac{dy}{y} \hat{P}_{gg}(y) \ln \left( \frac{y}{1-y} \right) D_{g \to \psi} \left( \frac{z}{y}, \mu \right) \right]
\]

\[
L_{1-z} = \ln \left( \frac{2E \tan(R/2)(1-z)}{\mu} \right)
\]

For large E, FJJF \sim NRQCD frag. function (at scale 2E \tan(R/2))

\[
G_{g}^{h}(E, R, \mu = 2E \tan(R/2), z) \to D_{g}^{\psi}(z, 2E \tan(R/2)) + O(\alpha_{s})
\]
NRQCD FF’s (at scale $2m_c$)

Evolution to $2E \tan(R/2)$ will soften discrepancies
FJF’s at Fixed Energy vs. $z$

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

**$E = 50$ GeV**

**$E = 200$ GeV**
FJF’s at Fixed $z$ vs. Energy

1S$_0^{(8)}$ dominance predicts negative slope for $z$ vs. $E$ if $z > 0.5$

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003
Transverse Momentum Dependent FJFs

jets with identified hadron: hadron $z$, $p_T$ are both measured

Jet Energy: $E$

$\rho_H = z\rho_{\text{jet}}$

transverse momentum measured w/ rspt. to jet axis

jet axis ~ parton initiating jet if out of jet radiation is ultrasoft

$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\text{QCD}}$
Scales in TMDFJF

\[ p_c \sim \omega(\lambda^2, 1, \lambda) \quad p_{cs} \sim p_h^\perp(r, 1/r, 1) \quad p_{us} \sim \Lambda(1, 1, 1) \]

\[ \lambda = p_h^\perp / \omega \]
Factorization Theorem

\[ D_{q/h}(p_\perp, z, \mu) = H_+(\mu) \times \left[ D_{q/h} \otimes_{\perp} S_C \right](p_\perp, z, \mu) \]

\[ H_+(\mu) = (2\pi)^2 N_c \ C_+^\dagger(\mu) C_+(\mu) \]

\[ D_{q/h}(p_{\perp}^D, z) \equiv \frac{1}{z} \sum_{X_n} \frac{1}{2N_c} \delta(p_{X_{h;r}}^-) \delta^{(2)}(p_{X_{h;r}}^\perp) \text{Tr} \left[ \frac{i\hbar}{2} \langle 0 | \delta_{\omega, \bar{\rho}} \chi_n(0) \delta^{(2)}(\mathcal{P}_{\perp}^X + p_{\perp}^D) | X_n h \rangle \right. \\
\left. \times \langle X_n h | \bar{\chi}_n(0) | 0 \rangle \right] \]

\[ D_{i/h}(p_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x} \ J_{i/j}(\frac{z}{x}, \mu, \nu) D_{j/h}(\frac{z}{x}, \mu) + O \left( \frac{\Lambda^2_{QCD}}{|p_\perp|^2} \right) \]

\[ S_C(p_{\perp}^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \text{Tr} \left[ \langle 0 | V_{n}^\dagger(0) U_{n}(0) \delta^{(2)}(\mathcal{P}_{\perp} + p_{\perp}^S) | X_{cs} \rangle \langle X_{cs} | U_{n}^\dagger(0) V_{n}(0) | 0 \rangle \right] \]
Anomalous Dimensions for RGE, RRGE

**RGE**

\[
\gamma^S_C (\nu) = \frac{\alpha_s C_i}{\pi} \ln \left( \frac{\mu^2}{r^2 \nu^2} \right)
\]

\[
\gamma^D (\nu) = \frac{\alpha_s C_i}{\pi} \left( \ln \left( \frac{\nu^2}{\omega^2} \right) + \bar{\gamma}_i \right)
\]

\[
\gamma^D (\nu) + \gamma^S_C (\nu) = \gamma^J = \frac{\alpha_s C_i}{\pi} \left( \ln \left( \frac{\mu^2}{r^2 \omega^2} \right) + \bar{\gamma}_i \right)
\]

**Rapidity Renormalization Group**

\[
\gamma^S_C (p_\perp, \mu) = +(8\pi) \alpha_s C_i \mathcal{L}_0 (p_\perp, \mu^2)
\]

\[
\gamma^D (p_\perp, \mu) - (8\pi) \alpha_s C_i \mathcal{L}_0 (p_\perp, \mu^2)
\]

\[
\gamma^D (p_\perp, \mu) + \gamma^S (p_\perp, \mu) = 0
\]


Solution to Evolution Eqs. in Fourier Space

\( D_{i/h}(p_\perp, z, \mu) = (2\pi)^2 p_\perp \int_0^\infty db \, b J_0(b p_\perp) U_{SC}(\mu, \mu_{SC}, m_{SC}) U_D(\mu, \mu_D, 1) \)

\[ \times V_{SC}(b, \mu_{SC}, \nu_D, \nu_{SC}) \mathcal{F} \mathcal{T} \left[ D_{i/h}(p_\perp, z, \mu_D, \nu_D) \otimes_\perp S^i_C(p_\perp, \mu_{SC}, \nu_{SC}) \right] \]

\( \mu = \omega r \)

\( \mu_D = \mu_{SC} = \mu_c(b) = 2e^{-\gamma E / b} \)

\( \nu_{SC} = \mu_c(b) / r \)

\( \nu_D = \omega \)
Application to Quarkonium Production

$E_J = 100 \text{ [GeV]}, \ z = 0.3$

$E_J = 500 \text{ GeV}, \ z = 0.3$

$E_J = 100 \text{ [GeV]}, \ z = 0.9$

$E_J = 500 \text{ GeV}, \ z = 0.9$
Application to Quarkonium Production

$E_J = 100 \text{ GeV}, \ p_\perp = 10 \text{ GeV}$

$E_J = 500 \text{ GeV}, \ p_\perp = 10 \text{ GeV}$

$D_{g/J/\psi}(p_\perp, z)$

- $^3S_1^{[8]} \times 10^6$
- $^1S_0^{[8]} \times 10^6$
- $^3P_J^{[8]} \times 3 \times 10^5$
- $^3S_1^{[1]} \times 4 \times 10^5$
Application to Quarkonium Production

\[ \langle \theta \rangle(z) \sim \frac{2 \int dp_\perp p_\perp D_{g/h}(p_\perp, z, \mu)}{z \omega \int dp_\perp D_{g/h}(p_\perp, z, \mu)} \equiv f_\omega^h(z) \]

\begin{align*}
E_J &= 100 \text{ GeV} \\
5 \text{ GeV} < p_\perp < 20 \text{ GeV}
\end{align*}

\begin{align*}
\ln(f(x)) &= g(x; C_0, C_1) \text{ s.t. } g(x = 0) = C_0 \\
g_2(x) &= C_0 \exp(-C_1 x)
\end{align*}
Recent Observations of Quarkonia within Jets

LHCb collaboration, arXiv:1701.05116

cuts: \(2.5 < \eta_{\text{jet}} < 4.0\), \(p_{T,jet} > 20\text{ GeV}\), \(p(\mu) > 5\text{ GeV}\)
NLL' FJF vs. Pythia


\[ \sigma(\tau_0=0.004, z) \quad \sigma(\tau_0=0.005, z) \quad \sigma(\tau_0=0.006, z) \]

\[ ^3S_1^{(8)} \quad ^1S_0^{(8)} \quad S_1^{(1)} \]

\[ e^+ e^- \rightarrow \overline{q} q g \quad E_{CM} = 250 \text{ GeV} \quad \tau_0 = s / \omega^2 \]

\[ \downarrow \text{jet w/ } J/\psi \]
Explaining difference between NLL’ vs Pythia

PYTHIA’s model for showering color-octet cc pairs:

Physical picture of analytical calculation

Pythia z distributions much harder than NLL’ calculations
Gluon Fragmentation Improved PYTHIA (GFIP)

**Madgraph 5**

\[ e^+e^- \rightarrow b\bar{b}g \]

**PYTHIA + Convolution**

Arbitrary gluon fragments

shower gluon with PYTHIA down to scale \( \sim 2m_c \), no hadronization

convolve final state gluon distribution w/ NRQCD FFs
NLL', PYTHIA, and GFIP

\[
\sigma(\tau_0=0.004, \mathbf{z}) \quad \sigma(\tau_0=0.005, \mathbf{z}) \quad \sigma(\tau_0=0.006, \mathbf{z})
\]

\[\begin{align*}
\text{Pythia} & \quad \text{GFIP} & \quad \text{NLL'} \\
\text{\(3S_1^{(8)}\)} & \quad \text{\(1S_0^{(8)}\)} & \quad \text{\(3S_1^{(1)}\)}
\end{align*}\]
LDME from Global fit (no P-wave)

color singlet $g, c$ fragmentation dominate

weak sensitivity to color-octet

NRQCD: good agreement with data
Conclusions

measuring $Q\bar{Q}$ within jets, and using jet observables should provide insights into $Q\bar{Q}$ production

If $^1S_0^{(8)}$ mechanism dominates high $p_T$ production FJF should have negative slope for $z(E)$, for $z>0.5$

$p_T$-dependent quarkonium fragmenting jet functions (TMDFJFs)

$p_T$, theta of quarkonium in jet sensitive to NRQCD production mechanism

Preliminary analysis of recent LHCb data
Backup
fragmentation function (QCD)

\[ D^h_q(z) = z \int \frac{dx^+}{4\pi} e^{ik^-x^+/2} \frac{1}{4N_c} \text{Tr} \sum_X \langle 0 | \bar{\Psi}(x^+, 0, 0_\perp) | X h \rangle \langle X h | \bar{\Psi}(0) | 0 \rangle \rvert_{p^+_h = 0} \]

fragmentation function (SCET)

\[ D^h_q \left( \frac{p^-_h}{\omega}, \mu \right) = \pi \omega \int dp^+_h \frac{1}{4N_c} \text{Tr} \sum_X \bar{\Psi} \langle 0 | [\delta_{\omega, \bar{\rho}} \delta_{0, \rho_\perp} \chi_n(0)] | X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle \]

Jet function (SCET)

\[ J_u(k^+\omega) = -\frac{1}{\pi \omega} \text{Im} \int d^4x \ e^{ik \cdot x} \ i \langle 0 | T \bar{\chi}_{n,\omega,0_\perp}(0) \frac{\bar{\Psi}}{4N_c} \chi_n(x) | 0 \rangle \]

fragmentation jet function (SCET)

\[ g^h_{q,\text{bare}}(s, z) = \int d^4y e^{ik^+y^-/2} \int dp^+_h \sum_X \frac{1}{4N_c} \text{tr} \left[ \frac{\bar{\Psi}}{2} \langle 0 | [\delta_{\omega, \bar{\rho}} \delta_{0, \rho_\perp} \chi_n(y)] | X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle \right] \]

\[ \delta(p^+/z - P^+_H) \rightarrow \delta(p^+/z - P^+_H) \delta(p^- - s/p^+) \]

FF  FJJF
Ratios of Moments

$E \tan(R/2) < \mu < 4E \tan(R/2)$
Ratios of Moments

\[
\frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \big|_{3P_j^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \big|_{3S_1^{(8)}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \big|_{1S_0^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \big|_{c-\text{quark}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \big|_{3S_1^{(1)}}
\]
Gluon FJF for different extractions of LDME

fix z, vary energy

$z = 0.3$

$z = 0.5$

$z = 0.8$


Bodwin, et. al. arXiv:1403.3612

Chao, et. al. PRL 108, 242004 (2012)
Gluon FJF for different extractions of LDME

fix energy, vary $z$

$E = 50$ GeV

$E = 200$ GeV
Scales in Jet Cross section

```
\begin{align*}
\mu_H &= \omega \\
\mu_J^{\text{unmeas}} &= \omega \tan \frac{R}{2} \\
\mu_J^{\text{meas}} &= \omega \tau_a^{\frac{2-a}{1-a}} \\
\gamma_J^{\text{unmeas}} &= \gamma_J^{\text{meas}} \\
\gamma_J &= \gamma_S \\
\end{align*}
```

<table>
<thead>
<tr>
<th>EFT counting</th>
<th>matching/matrix element</th>
<th>( \Gamma_{\text{cusp}} )</th>
<th>( \gamma_{H,J,S} )</th>
<th>( \beta[\alpha_s] )</th>
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<td>1-loop</td>
<td>3-loop</td>
<td>2-loop</td>
<td>3-loop</td>
</tr>
</tbody>
</table>

\begin{align*}
\mu_\Lambda &= 2\Lambda \\
\mu_{\Lambda R} &= 2\Lambda \tan(R/2) \\
\mu_S \approx \omega \tau_a / \tan^{1-a}(R/2) \\
\mu_S^{\text{meas}} &= \omega \tau_a / \tan^{1-a}(R/2) \\
\end{align*}
Color-Octet $^3S_1$ fragmentation function, $F_{JF}$

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

1. For $E = 50$ GeV
2. For $E = 100$ GeV
3. For $E = 200$ GeV

$10^4 \times F_8^\psi$ vs. $z$

- **Blue line**: Fragmentation function
- **Red line**: Fragmenting jet function
$3S_1^{[8]}$ fragmentation at large $p_T$ predicts transversely polarized $J/\psi, \psi'$

Braaten, Kniehl, Lee, 1999
\[ D_{q/h}(p_{\perp}, z, \mu) = \frac{1}{z} \sum_{X} \frac{1}{2N_c} \delta(p_{Xh,r}) \delta^{(2)}(p_{\perp} + p_{\perp}^X) \text{Tr} \left[ \frac{\hbar}{2} \langle 0 | \delta_{\omega, P} \chi_n^{(0)}(0) | Xh \rangle \langle Xh | \chi_n^{(0)}(0) | 0 \rangle \right] \]

\[ \int d^2p_{\perp}^{h} D_{q/h}(p_{\perp}^{h}, z, \mu) = D_{q/h}(z, \mu) \]
Transverse Momentum Dependent FJFs

\[ \omega \gg p^\perp_h \gg \Lambda \]

Jet 1

Jet 2

ultra-soft radiation

ultra-soft radiation

\[
\begin{align*}
D_{i/h} (z, p^\perp_h, \mu) \\
p_c &\sim \omega(\lambda^2, 1, \lambda) \\
p_{cs} &\sim p^\perp_h (r, 1/r, 1) \\
p_{us} &\sim \Lambda (1, 1, 1) \\
\lambda &\equiv p^\perp_h / \omega
\end{align*}
\]
Profile Functions

\[
\mu_S(\text{GeV})
\]

\[
\mu_J(\text{GeV})
\]

<table>
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<tr>
<th>Canonical</th>
<th>Traditional</th>
<th>Profile</th>
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<tr>
<td>(\epsilon_{S/J}=+1/2\ (+50%))</td>
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