Perturbative vs non-perturbative aspects of TMD phenomenology

Drell Yan processes
**Naive TMD approach**

Calculating a cross section which describes a hadronic process over the whole $q_T$ range is a highly non-trivial task.

**Let's consider Drell Yan processes** (for historical reasons)

Fixed order calculations cannot describe DY data at **small** $q_T$:
- At Born Level the cross section is vanishing
- At order $\alpha_s$ the cross section is divergent...

\[
q_T \to 0
\]

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2} \alpha_s \ln \left( \frac{M^2}{q_T^2} - \frac{3}{2} \right)
\]
Naive TMD approach

\[
\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \exp\left(-\frac{P_T^2}{\langle P_T^2 \rangle}\right) \frac{\langle P_T^2 \rangle}{\pi}
\]

- Considering the same DY process at different energies:

Each data set is Gaussian but with a different width
Drell-Yan phenomenology

Does the $q_T$ distribution behave like a Gaussian?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp\left(-\frac{P_T^2}{\langle P_T^2 \rangle}\right)}{\pi \langle P_T^2 \rangle}$$

Clearly this is not a Gaussian tail!
**Drell-Yan phenomenology**

- Fixed order calculations cannot describe correctly DY cross sections at small $q_T$

  ![Graph of Drell-Yan cross sections](image)

  - $\frac{d\sigma}{d^2P_T}$ (GeV$^2$)
  - $\sqrt{s} = 62$ GeV
  - $5 < m < 8$ GeV

- DY cross sections do not show a Gaussian behaviour at large $q_T$

  ![Graph showing CDF Run II data](image)
The cross section is written in $b_T$ space:

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2 b_T e^{i q_T \cdot b_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)
\]
The W term is designed to work well at low and moderate $q_T$, when $q_T << Q$. (Notice that W is devised to work down to $q_T \sim 0$, however collinear-factorization works up to $q_T > M$; therefore, TMD-factorization and collinear-factorization can be simultaneously applied only when $q_T >> M$).

The W term becomes unphysical at larger $q_T$, when $q_T \geq Q$, where it becomes negative (and large).

The Y term corrects for the misbehavior of W as $q_T$ gets larger, providing a consistent (and positive) $q_T$ differential cross section.

The Y term should provide an effective smooth transition to large $q_T$, where fixed order perturbative calculations are expected to work.
Example: the CSS resummation scheme:

\[
W_j(x_1, x_2, b_T, Q) = \exp \left[ S_j(b_T, Q) \right] \sum_{i,k} C_{ji} \otimes f_i(x_1, C_1^2/b_T^2) C_{jk} \otimes f_k(x_2, C_1^2/b_T^2)
\]

\[
S_j(b_T, Q) = - \int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[ A_j(\alpha_s(\kappa)) \ln \left( \frac{Q^2}{\kappa^2} \right) + B_j(\alpha_s(\kappa)) \right]
\]

At large \( b_T \) the scale \( \mu \) becomes too small!

Non-trivially connected to the physical region: \( Q^2 \gg q_T^2 \approx \Lambda_{QCD}^2 \)

All TMD evolution schemes require a model to deal with the non-perturbative region

Working in \( b_T \) space makes phenomenological analyses more difficult, as we lose intuition and direct connection with “real world experience”. (Experimental data are in \( q_T \) space).
This is a perturbative scheme. All the scales must be frozen when reaching the non-perturbative region:

\[ b_T \rightarrow b_* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\text{max}}^2}}} \]

\[ \mu = C_1 \frac{1}{b_T} \rightarrow \mu_b = C_1 / b_* \]

Then we define a non-perturbative function for large \( b_T \):

\[ \frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q) \]

\[ W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp[S_j(b_*, Q)] \left[ C_{ji} \otimes f_i(x_1, \mu_b) \right] \left[ C_{jk} \otimes f_k(x_2, \mu_b) \right] F_{NP}(x_1, x_2, b_T, Q) \]

\[ C_1 = 2 \exp(-\gamma_E) \]

For this scheme to work, 4 distinct kinematic regions have to be identified.

They should be large enough and well separated.

- $q_T \sim \lambda_{\text{QCD}}$
- $q_T \ll Q$
- $q_T \sim Q$
- $q_T \geq Q$

- **Intrinsic** $q_T$
- **Soft gluon radiation**
- **Hard gluon emission**

**TMD evolution**

**Matching region (Y factor)**

**Fixed Order collinear QCD**
CSS for DY processes

To perform phenomenological studies we need a non perturbative function.

\[ F_{NP}(x_1, x_2, b_T, Q) \]

Davies-Webber-Stirling (DWS)
\[ \exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) \right] b^2; \]

Ladinsky-Yuan (LY)
\[ \exp \left\{ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) b^2 - \left[ g_1 g_3 \ln(100x_1 x_2) \right] b \right\}; \]

Brock-Landry-Nadolsky-Yuan (BLNY)
\[ \exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 \ln(100x_1 x_2) \right] b^2 \]

Nadolsky et al.* analyzed successfully low energy DY data and $Z_0$ production data using different parametrizations.

\[ b_{\text{max}} = 0.5 \text{ GeV}^{-1} \]

\[ \chi^2 \]

\[ \chi^2/\text{DOF} \]

\[ 416, 407, 176 \]

\[ 3.47, 3.42, 1.48 \]

\[ g_1 \]

\[ 0.016, 0.02, 0.21 \]

\[ g_2 \]

\[ 0.54, 0.55, 0.68 \]

\[ g_3 \]

\[ 0.00, -1.50, -0.60 \]

\[ CDF \ Z \ Run-0 \]

\[ 1.00, 1.00, 1.00 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)}, \text{(fixed)}, \text{(fixed)} \]

\[ R209 \]

\[ 1.02, 1.01, 0.86 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)} \]

\[ E605 \]

\[ 1.15, 1.07, 1.00 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)} \]

\[ E288 \]

\[ 1.23, 1.28, 1.19 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)} \]

\[ D\O \ Z \ Run-1 \]

\[ 1.01, 1.01, 1.00 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)} \]

\[ CDF \ Z \ Run-1 \]

\[ 0.89, 0.90, 0.89 \]

\[ N_{\text{fit}} \]

\[ \text{(fixed)} \]

\[ *\text{Nadolsky et al., } \text{Phys.Rev. D67,073016 (2003)} \]
SIDIS processes
As mentioned above

- fixed order pQCD calculation fail to describe the SIDIS cross sections at small $q_T$,
- the cross section tail at large $q_T$ is clearly non-Gaussian.

**Need resummation of large logs and matching perturbative to non-perturbative contributions**


*ZEUS Collaboration (M. Derrick), Z. Phys. C 70, 1 (1996)*

*Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005*

As mentioned above:

- fixed order pQCD calculation fail to describe the SIDIS cross sections at small $q_T$,
- the cross section tail at large $q_T$ is clearly non-Gaussian.

The NLO collinear SIDIS cross section is not correctly normalized! (see talk of A. Bacchetta on Wednesday)
Naive TMD approach

Simple phenomenological ansatz can reproduce low $q_T$ data

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi^2_{dof} = 1.69$$


Anselmino et al., JHEP 1404 (2014) 005
Naive TMD approach

\[ F_{UU} = \sum_{q} e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \]

\[ \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle \]

\[ \langle k_{\perp}^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2 \]
\[ \langle p_{\perp}^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2 \]
\[ \chi^2_{\text{dof}} = 3.42 \]

Fit over 6000 data points with 2 free parameters!

\[ N_y = A + B y \]

“The point-to-point systematic uncertainty in the measured multiplicities as a function of \( p_{\perp}^2 \) is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the \( p_{\perp}^2 \)-integrated multiplicities depends on \( z \) and \( y \) and can be as large as 40%”.


Anselmino et al. JHEP 1404 (2014) 005
The dependence of HERMES data on $Q^2$ is... 

\[ F_{UU} = \sum_{q} e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \]

\[ \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h \langle k_{\perp}^2 \rangle \]

\[ \langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2 \]

\[ \langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2 \]

\[ \chi^2_{\text{dof}} = 1.69 \]

All four bins have been overlapped in the same panel.

Hard to decouple the $Q^2$ dependence from HERMES data alone.
Resummation of large logarithms

To ensure momentum conservation, write the cross section in the Fourier conjugate space

$$\delta^2(q_T - k_{1T} - k_{2T} - \ldots - k_{nT} + \ldots) = \int \frac{d^2b_T}{(2\pi)^2} e^{-ib_T \cdot (q_T - k_{1T} - k_{2T} - \ldots - k_{nT} + \ldots)}$$

$$\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \left[ \int \frac{d^2b_T e^{i q_T \cdot b_T}}{(2\pi)^2} X_{div}(b_T) \right] + Y_{reg}(q_T)$$

$$X_{div}(b_T) \rightarrow W(b_T) = \exp [S(b_T)] \times \text{(PDFs and Hard coefficients)}$$

$$\frac{d\sigma^{total}}{dx \ dy \ dz \ dq_T^2} = \pi \sigma_0^{DIS} \int \frac{d^2b_T e^{i q_T \cdot b_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q)$$

Resummed part

Regular part
Fit of HERMES and COMPASS data
Attempting “Resummation” in SIDIS ...

\[ \chi^2_{\text{tot}} = 1.17 \]

\[ \chi^2_{\text{HERMES}} = 1.32 \]

\[ \chi^2_{\text{COMPASS}} = 1.12 \]

\[ \frac{d\sigma}{dx dy dz dq_T^2} = \pi Q_0^{\text{DIS}} \left\{ \int d^2b_T e^{iQ_T b_T} \right\} \frac{W_{\text{SIDIS}}(x,z,b,T,Q)}{(2\pi)^2} F_{NP}^{\text{SIDIS}}(x,z,b_T,Q) \]

\[ F_{NP}^{\text{SIDIS}}(x,z,Q) = \exp \left\{ \left[ -\frac{g_1 + g_1 T^2}{2} - g_2 \ln(Q/(2Q_0)) - g_1 g_3 \ln(10x) \right] b_T^2 \right\} \]

\( N \sim 2 \) (One overall normalization parameter is required)
\( g_1 \sim 0.5 \) (too large compared to the value extracted from DY data)
\( g_2 \sim 0.5 \)
\( g_3 \sim -0.03 \)
For this scheme to work, 4 distinct kinematic regions have to be identified

They should be large enough and well separated

<table>
<thead>
<tr>
<th>TMD evolution</th>
<th>Matching region (Y factor)</th>
<th>Fixed Order collinear QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_T \sim \lambda_{QCD}$</td>
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<td>$q_T \sim Q$</td>
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For this scheme to work, 4 distinct kinematic regions have to be identified.

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**Does not work in SIDIS!**

### TMD evolution

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<tr>
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<td>$q_T \approx Q$</td>
<td>$q_T$</td>
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</table>

$\lambda_{QCD}$
What's wrong ???
The Y factor is very large (even at low $q_T$)

However, it could be affected by large theoretical uncertainties

Boglione, Gonzalez, Melis, Prokudin, JHEP 02 (2015) 095

The Y factor cannot be neglected !!!

See talk by A. Bacchetta on Wednesday

New prescription for Y factor, $b^*$ and W


\[
\sigma^{ASY} = \frac{Q^2}{q_T^2} [A \ln(Q^2/q_T^2) + B + C]
\]
Other issues related to TMD regions ...

TMD regions are defined in terms of $q_T$ and not in terms of $P_T$.
This fit gives a very high quality description of a wide amount of data points.

However, there are a few issues that are worth mentioning:

★ The NLL SIDIS cross section is not correctly normalized → $N \sim 2$

★ The Y factor has been neglected

★ More work required to include Drell-Yan data into the fit
"The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant (‘leading-order’) channel, and not to the ‘genuine’ increase in the partonic cross section [...]. The dominance of the new channel is due to the size of the gluon distribution at small $x_B$ and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \rightarrow g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the gg component represents less than 25% of the total NLO contribution at small $x_B$."

Kinematics of current region

Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato

Need a quantitative way to identify the region of validity of TMD factorization (current region)

Fracture Functions

TMDs

(Breit frame)
Kinematics of current region

$y_h \equiv \frac{1}{2} \log \frac{P^+_h}{P^-_h}$

Current and fragmentation regions should be well separated in the observed hadron rapidity.

These beautiful drawings are courtesy of Osvaldo Gonzalez.
Kinematics of current region

These beautiful drawings are courtesy of Osvaldo Gonzalez
Kinematics of current region

Factorization implies power counting for the momenta

\[ P_h \cdot k_f = O(m^2) \]
\[ P_h \cdot k_i = O(Q^2) \]

Small mass

Current region

Collinearity must be small in the current region

\[ R(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

These beautiful drawings are courtesy of Osvaldo Gonzalez
Kinematics of current region

\[ R(y_h, z_h, x_{bj}, Q) \ll 1 : \text{collinear to outgoing quark}, \]

\[ R(y_h, z_h, x_{bj}, Q)^{-1} \ll 1 : \text{collinear to incoming quark}. \]

\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

\[ y_h \equiv \frac{1}{2} \log \frac{P^+_h}{P^-_h} \]

\[ y_T = -\ln \frac{Q}{M_{TT}} \]

\[ y_i = \ln \frac{Q}{M_{iT}} \]
Kinematics of current region

- Colored points belong to the current fragmentation region.
- Gray points are likely to be outside of current region.

Beautiful work on TMD phenomenology with R cuts will be presented by M. Albright, at 3:15 pm. STAY TUNED!
Kinematics of soft region

\[ y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-} \]

However, this neglects the soft fragmentation region

(No factorization theorem for this region)
Conclusions

Phenomenological studies of TMD factorization and evolution have come a long way. Many aspects of the interplay between perturbative and non-perturbative contributions are now better understood.

Some issues remain open and need further investigation, especially as far as phenomenology is concerned:

- Difficult to work in $b_T$ space where we lose phenomenological intuition
- F.T. involves integration of an oscillating function over $b_T$ up to infinity:
  upon integration one loses track of what was small $b_T$ and what was large $b_T$.
- ...

$P_T$ distributions of SIDIS cross sections over the full $P_T$ range will have to be further investigated.

Simultaneous fits of SIDIS, Drell-Yan and $e^+e^-$ annihilation data are highly recommended, but they should be performed within a consistent and solid framework where they can be implemented.

Data selection is crucial in global fitting:
- not too many
  (only data within the ranges where the TMD evolution schemes work should be considered)
- not too few
  (too strict a selection can bias the fit results and neglect important information from experimental data) → see our new criteria to select current fragmentation region events!