Investigation of the Hadronic Decay
\[ \eta \rightarrow \pi^+ \pi^- \pi^0 \] with CLAS

01.02.2017 | 7th Workshop of the APS Topical Group Hadronic Physics 2017

Institute for Nuclear Physics - Jülich Research Center
One Meson, many Opportunities

Properties of the $\eta$-meson

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<th>Property</th>
<th>Value</th>
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<tr>
<td>$m_\eta$ [GeV/c^2]</td>
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<td>$\Gamma_\eta$ [keV]</td>
<td>$(1.31 \pm 0.05)$</td>
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<td>$\bar{\tau}$ [s]</td>
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- The $\eta$-meson is a $C$, $P$, $G$- and $CP$- eigenstate
- All strong and electromagnetic decays are forbidden to first order
- Access to rare decay processes
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$\eta$ meson decay modes:
- Radiative
- (Semi-)Leptonic
- Hadronic

Decay branches:
- $\eta \rightarrow \gamma \pi \pi \pi$
- $\eta \rightarrow \gamma (\pi^+ \pi^-)$
- $\eta \rightarrow (\pi^+ \pi^-)$
- $\eta \rightarrow \gamma l^+ l^-$
- $\eta \rightarrow QCD$ anomalies
- $\eta \rightarrow FSI$
- $\eta \rightarrow$ Transition Form Factor
- $\eta \rightarrow$ CP-violation

\[ L_{WZW} = \pi_0^0 \eta \eta' + \eta \eta' + \ldots \]

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- The $\eta$-meson is a $C$, $P$, $G$- and $CP$- eigenstate.
- All strong and electromagnetic decays are forbidden to first order.
- Access to rare decay processes.

- Quark mass ratio
- Isospin violation
- Transition Form Factor
- CP-violation
- QCD anomalies
- FSI

$\eta$, $\eta$, $0\pi$
$\rho$
$\pi^0\eta\eta'$
$L_{\pi\pi\pi\pi}$
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- Quark mass ratio
- Isospin violation
- Transition Form Factor
- CP-violation
- QCD anomalies
- FSI
- CP-allowed, and possible CP-forbidden amplitudes
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Focus of this Talk

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Decay Dynamics of $\eta \rightarrow \pi^+ \pi^- \pi^0$

| System                  | Isospin State $|I, I_z\rangle$ | C-Eigenvalue | G-Eigenvalue |
|-------------------------|-------------------------------|--------------|--------------|
| $\eta$                  | $|0, 0\rangle$                | +1           | +1           |
| $(\pi^+ \pi^- \pi^0)$  | $|0, 0\rangle$                | −1           | −1           |
| $(\pi^+ \pi^- \pi^0)$  | $|1, 0\rangle$                | +1           | −1           |

- Decay $\eta \rightarrow \pi^+ \pi^- \pi^0$ is G-violating $\Rightarrow$ Forbidden to first order
- Decay is driven by isospin breaking part of strong interaction $\Rightarrow$ C is conserved
- Decay width: $\Gamma \propto Q^{-4}$
  - with: $Q^2 = \left(\frac{m_s}{m_d}\right)^2 \times \left[1 - \left(\frac{m_u}{m_d}\right)^2\right]^{-1}$
- $\Rightarrow$ Determine decay width $\Gamma \Rightarrow$ Access to quark mass ratio
  - a) Measure $\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)$, e.g. via $\frac{\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)}{\Gamma(\eta \rightarrow \gamma \gamma)}$
  - b) Dalitz Plot Analysis
Dalitz Plot Analysis of $\eta \rightarrow \pi^+ \pi^- \pi^0$

- Describe three body decay by two variables (here: X and Y)
- Complete information about decay dynamics
- Parameterise decay width $\Gamma$:
  \[ \frac{d^2\Gamma}{dXdY} \propto (1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + \ldots) \]
  - $c \neq 0$ and $e \neq 0$:
    i) Imply C-violation
    ii) Cause asymmetries within the Dalitz Plot
- Compare Dalitz Plot parameters $a,b,d,f$ from experiment and theory
Recent Measurements I

Most recent result from the KLOE-Collaboration:\(^{(f)}\)

- \(\eta\)-Mesons produced via: \(e^+ e^- \rightarrow \Phi \rightarrow \eta \gamma\)
- \(\approx 4.7 \times 10^6 \ \eta \rightarrow \pi^+ \pi^- \pi^0\) events in the final data sample
- Fit function: \(\text{Norm} \times (1 + aY + bY + cX + dX^2 + eXY + fY^3)\)
- Determined asymmetries of the Dalitz Plot \(\Rightarrow\) Consistent with zero
  \(\Rightarrow\) No C-violation

\(^{(f)}\) KLOE coll., JHEP, 019, (2016)
Recent Measurements II

Result from the WASA-at-COSY Collaboration:\(^{(d)}\)

- \(\eta\)-Mesons produced via: \(pd \rightarrow ^3\text{He}\eta\)
- \(\approx 120 \text{ k}\ \eta \rightarrow \pi^+\pi^-\pi^0\) events in the final data sample
- Translate each pair \((X,Y)\) into a global bin \(i(X, Y)\)
  \(\rightarrow\) Obtain one dimensional Dalitz Plot
- Fit function: \(\text{Norm} \times (1 + aY + bY + cX + dX^2 + eXY + fY^3)\)

\(^{(d)}\) WASA-at-COSY coll., Phys. Rev., C90(045207), 2014
Recent Measurements and Theoretical Predictions

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<tr>
<th>Parameter:</th>
<th>– a</th>
<th>b</th>
<th>d</th>
<th>f</th>
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<tr>
<td>Exp. KLOE (08)</td>
<td>1.090(5) (+8_19)</td>
<td>0.124(6)(10)</td>
<td>0.057(6)(+7_16)</td>
<td>0.14(1)(2)</td>
</tr>
<tr>
<td>WASA (d)</td>
<td>1.144(18)</td>
<td>0.219(19)(47)</td>
<td>0.086(18)(15)</td>
<td>0.115(37)</td>
</tr>
<tr>
<td>KLOE (16)</td>
<td>1.104(3)(2)</td>
<td>0.142(3)(5_4)</td>
<td>0.073(3)(+4_3)</td>
<td>0.154(6)(+4_5)</td>
</tr>
<tr>
<td>Theor. ChPT (NNLO)</td>
<td>1.271(75)</td>
<td>0.394(102)</td>
<td>0.055(57)</td>
<td>0.025(160)</td>
</tr>
<tr>
<td>NREFT (c)</td>
<td>1.213(14)</td>
<td>0.308(23)</td>
<td>0.050(3)</td>
<td>0.083(19)</td>
</tr>
<tr>
<td>PWA (e)</td>
<td>1.116(32)</td>
<td>0.188(12)</td>
<td>0.063(4)</td>
<td>0.091(3)</td>
</tr>
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</table>

(a) KLOE coll., JHEP, 05, (2008)  
(b) J. Bijnens and K. Ghorbani., JHEP, 11, (2007)  
(c) S- P. Schneider et al., JHEP, 028, (2011)  
(d) WASA-at-COSY coll., Phys. Rev, C90(045207), 2014  
(f) KLOE coll., JHEP, 019, (2016)

- WASA-at-COSY results used for Partial Wave Analysis (PWA) from JPAC* group
  ⇒ Direct calculation of: \( Q = 21.4 \pm 0.4 \text{ stat} \)  
- Dalitz Plot Analysis and determination of \( Q \) for \( \gamma p \rightarrow p\eta[\eta \rightarrow \pi^+\pi^-\pi^0] \) with CLAS

* See plenary talk "Hadron Spectroscopy at JPAC“ by Alessandro Pilloni at 4.40 p.m.
Small Outlook: Dalitz Plot Analysis for $\omega \rightarrow \pi^+\pi^-\pi^0$ with WASA-at-COSY

- Theoretical description of this decay: VMD Model, Lagrangian Approach\textsuperscript{(a)}, Dispersive Analysis\textsuperscript{(b),(c)} → Input from experiment needed
- Look at: $\frac{d^2\Gamma}{dZd\Phi} \propto (1 + 2\alpha Z + 2\beta Z^{3/2} \sin^3 \Phi + \mathcal{O}(Z^2) + \ldots)$
- Analysis ongoing\textsuperscript{(d),(e)} for the reaction: $pd \rightarrow ^3He\omega[\omega \rightarrow \pi^+\pi^-\pi^0]$ and $pp \rightarrow pp\omega[\omega \rightarrow \pi^+\pi^-\pi^0]$

\textsuperscript{(c)} I.V. Danilkin et al., Phys. Rev. D91, 094029, (2015) \hspace{1cm} \textsuperscript{(d)} PhD.-Project of Lena Heijkenskjöld \hspace{1cm} \textsuperscript{(e)} PhD.-Project of Siddhesh Sawant
Small Outlook: Dalitz Plot Analysis for \( \eta' \rightarrow \pi^+\pi^-\eta \) with CLAS

- The decay \( \eta' \rightarrow \pi^+\pi^-\eta \) allows to probe the low energy regime of QCD
  \( \Rightarrow \) Test ChPT
- Compare theory and experiment by checking decay kinematics
  \( \Rightarrow \) Dalitz Plot
- Analysis* performed using the CLAS g12 data set
- \( \approx 87 \, k \) events reconstructed

* PhD-Project of S. Ghosh
The CLAS g12 $\gamma p \rightarrow pX$ Data Set

- Photon beam: $E_{\gamma,\text{beam}} \in [1.1\, \text{GeV}, 5.45\, \text{GeV}]$
- (Main) Contributions from:
  - Direct pion production (e.g. $\gamma p \rightarrow p\pi^+\pi^-\pi^0$)
  - $\pi^0$, $\eta$, $\omega$ and $\rho$ decays
Reconstruction of $\eta \rightarrow \pi^+\pi^-\pi^0$ Events

Decay Specific Analysis Steps

i) Kinematic fit with reaction hypothesis: $\gamma p \rightarrow p \pi^+\pi^- (\pi^0)$

ii) Kinematic Limit: $M(\pi^+,\pi^-) \leq m_\eta + 3\sigma_{\eta, \text{res}} - m_{\pi^0}$

$\approx 700 \text{ k}\eta \rightarrow \pi^+\pi^-\pi^0$ events reconstructed so far
Towards the Dalitz Plot

Preliminary

i) Look at $M_x(p)$-spectrum as a function of the global bin $i(X, Y)$

ii) Subtract non-resonant background

iii) Correct for contributions from $\eta \rightarrow \pi^+ \pi^- \gamma$
Current Status

- **Left:** Dalitz Plot after background subtraction and correction for $\eta \rightarrow \pi^+ \pi^- \gamma$ events (see previous slide)
- **Right:** One Dimensional Dalitz Plot after applying efficiency correction
- Fit function: $\text{Norm} \times (1 + aY + bY + dX^2 + fY^3)$
- Systematic checks and validation of current results ongoing
Summary and Outlook

1. Features of the hadronic decay: $\eta \rightarrow \pi^+ \pi^- \pi^0$
   - C-conserving
   - Isospin-violating
   - Decay amplitude driven by quark mass ratio

2. Determine decay amplitude via Dalitz Plot Analysis
   - Kinematics of a three body decay described by two variables
   - Dalitz Plot Parameters: a,b,c,d,e and f
   - Asymmetries in Dalitz Plot $\leftrightarrow c$ and $e$ are non-zero $\leftrightarrow$ C-Violation

3. Several experimental and theoretical efforts to determine Dalitz Plot
   - $c = e = 0$ confirmed by experiments
   - Latest measurements done by WASA-at-COSY and KLOE:
     i) KLOE confirmed Dalitz Plot asymmetries to be consistent with zero
     ii) WASA-at-COSY results used in PWA (JPAC-group) to determine the quark mass ratio
   - JPAC: Partial Wave Analysis as a tool to fit experimental Dalitz Plot distributions

4. Dalitz Plot Analysis of $\eta \rightarrow \pi^+ \pi^- \pi^0$ with CLAS
   - $\approx 700 k$ events reconstructed so far
   - Final (one dimensional) Dalitz Plot distribution
   - To do:
     i) Systematics checks (ongoing)
     ii) Determination of quark mass ratio
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Three Body Decays and the Dalitz Plot

Reconstruction of $\eta \rightarrow \pi^+ \pi^- \pi^0$

The Dalitz Plot in one Dimension

Features of the 1D Dalitz Plot
Backup: Three Body Decays and the Dalitz Plot

Kinematic Constraints | NDF
---|---
3 Lorentz-Vectors | 12
Momentum Conservation | −3
Energy Conservation | −1
3 Masses | −3
3 Euler Angles | −3
**Total*** | 2

* Valid for all particles being scalars

**Dalitz Plot**

- Two* variables sufficient to describe three body decay
- Display decay kinematics
- Show possible resonances
Backup: Reconstruction of $\eta \rightarrow \pi^+ \pi^- \pi^0$

I) The Kinematic Fit

- $\sim 2/3$ of the available CLAS g12 data set analysed so far
- Use least squares kinematic fit
  - Reaction hypothesis: $\gamma p \rightarrow p \pi^+ \pi^- (\pi^0)$
  - Reject events with a probability < 10%
Backup: Reconstruction of $\eta \rightarrow \pi^+\pi^-\pi^0$

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II) Reject Contributions from $\omega \rightarrow \pi^+ \pi^- \pi^0$

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Backup: Reconstruction of $\eta \rightarrow \pi^+ \pi^- \pi^0$

II) Reject Contributions from $\omega \rightarrow \pi^+ \pi^- \pi^0$

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- Use least squares kinematic fit
- Use kinematic limit: $M(\pi^+, \pi^-) \leq m_\eta + 3\sigma_{\eta, \text{rec}} - m_{\pi^0}$
Follow WASA-at-COSY analysis\(^{(a)}\):

- Divide Dalitz Plot in \(N \times N\) bins
- Translate each pair \((X,Y)\) into a global bin number \(i \in [0, N^2 - 1]\) (e.g. \(X = Y = 0 \equiv i = 60\))
- Take kinematic boundaries (see black line in left plot) into account

\(^{(a)}\) WASA-at-COSY coll., Phys. Rev., C90(045207), 2014
Backup: Features of the 1D Dalitz Plot

- Look at 1D Dalitz Plot with WASA-at-COSY values for a, b, d and f
- Center of Dalitz Plot is at $i(0, 0) = 60$
- $\frac{d^2\Gamma}{dxdy} \propto (1 + aY + bY^2 + cX + dX^2 + eXY + fY^3)$
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