The dynamics of the gluon mass

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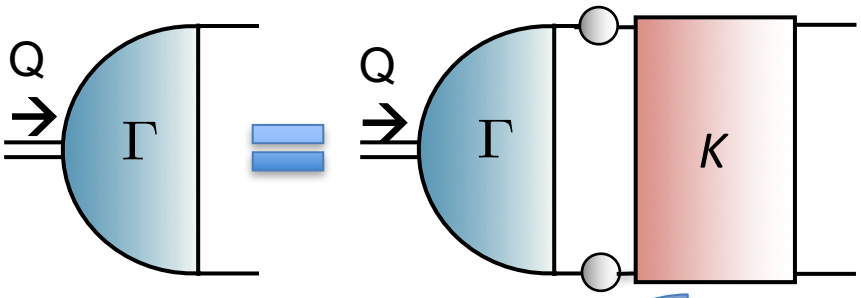
ECT*

Bound states in QCD

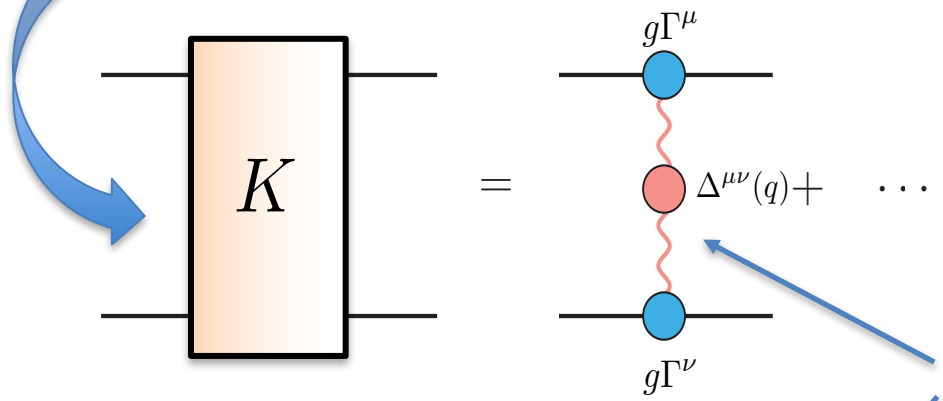


In the continuum :

Bethe-Salpeter eqs

Faddeev eqs

See talks of Ian Cloet and Craig Roberts



Gluon propagator

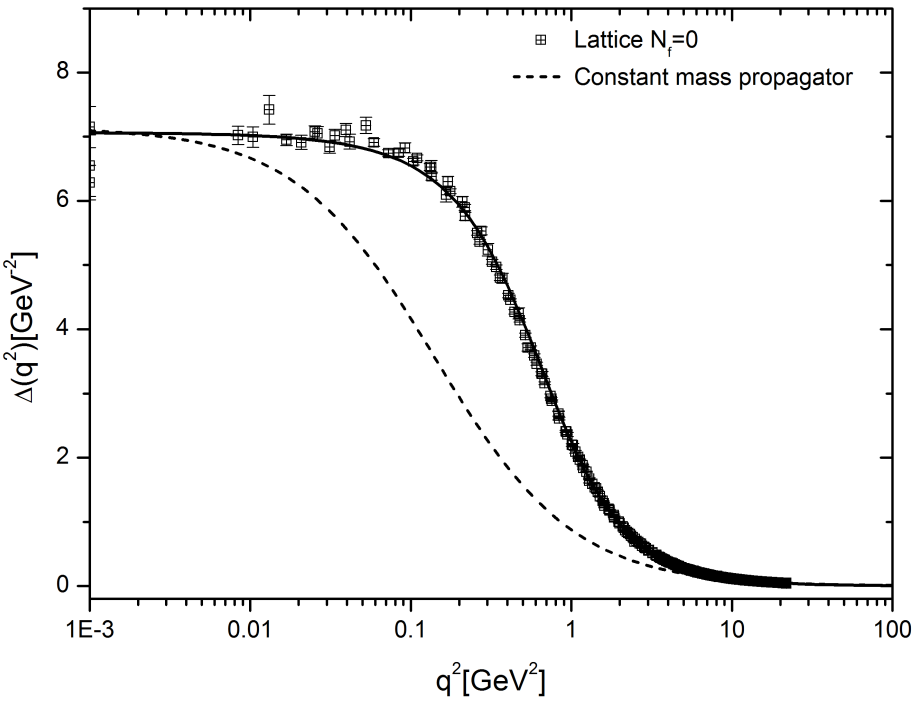
Dynamical generation of a constituent quark mass

The mass of the visible matter depends on the non-perturbative behavior of the gluon propagator

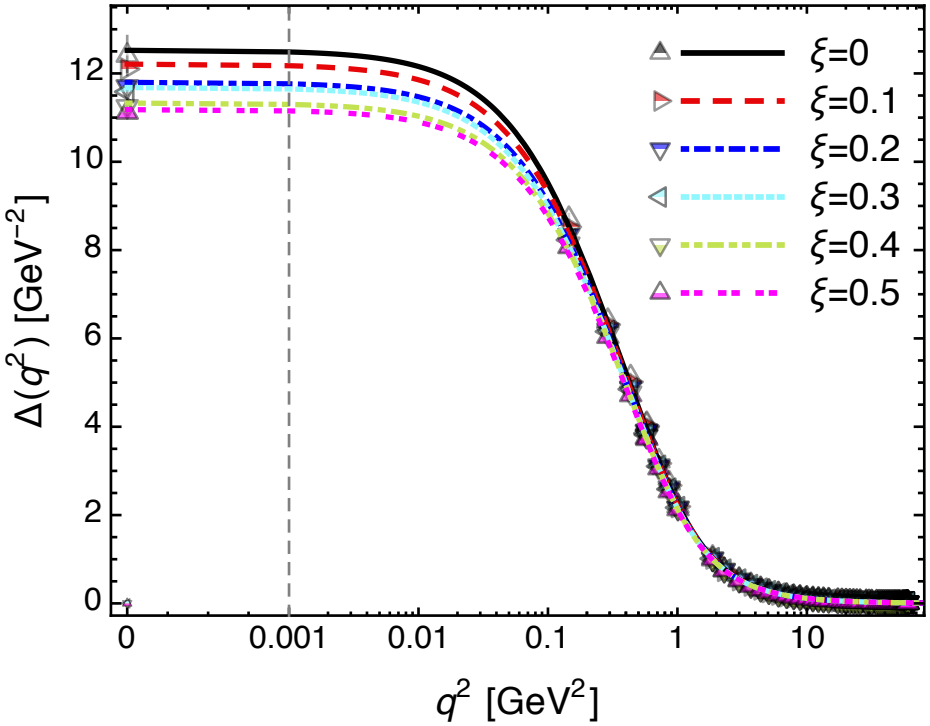
Large volume lattice simulations

The gluon propagator saturates in the deep infrared,
both in the Landau gauge and away from it !

I.L.Bogolubsky, et al , PoS LAT2007, 290 (2007)



P. Bicudo et al, Phys. Rev.D 92, 114514 (2015)



Saturation suggests some sort of
"dynamical gluon mass generation"

$$\Delta^{-1}(0) = m^2(0)$$

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

SDEs in the PT-BFM framework

The Background Field Method (BFM) is a powerful quantization scheme

B. S. DeWitt, Phys. Rev. 162 (1967) 195—1239
G. 't Hooft, In *Karpacz 1975, Proceedings, Acta Universitatis Wratislaviensis No.368, 1976, 345-369
L. F. Abbott, Nucl. Phys. B185 (1981) 189

Split the gauge field: $A^a_\mu \to \tilde{A}^a_\mu + A^a_\mu$

background quantum

Residual invariance after special gauge-fixing leads to ghost-free Slavnov-Taylor identities

The Pinch Technique (PT) is a diagrammatic method: systematic rearrangement of off-shell Green's functions, starting from any gauge-fixing scheme

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)
J. M. Cornwall and J.P., Phys. Rev. D40 (1989)
D. Binosi and J.P. , Phys.Rept. 479 (2009)

Formal equivalence between PT and BFM → *PT-BFM*

D. Binosi and J.P., Phys. Rev. D77, 061702 (2008) ; JHEP 0811, 063 (2008)

Improved truncation scheme for SDEs

Background and quantum propagators are related :

$$\widetilde{\Delta}(q) = [\mathbf{1} + \mathbf{G}(\mathbf{q})]\Delta(q)$$

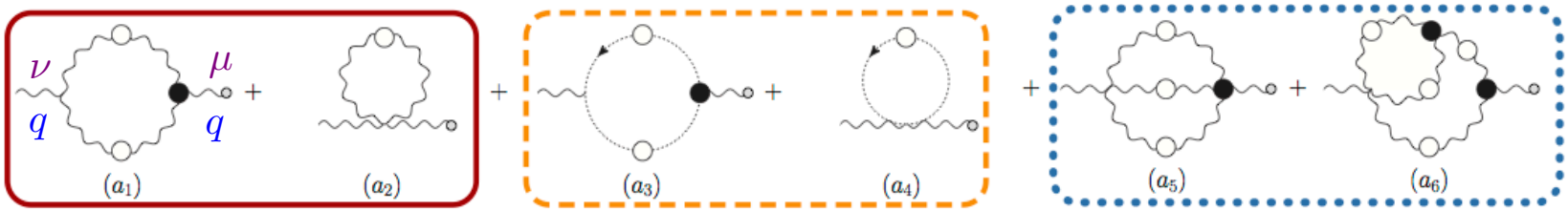
$$\widehat{\Delta}(q) = [\mathbf{1} + \mathbf{G}(\mathbf{q})]^{\mathbf{2}}\Delta(q)$$

Known function in the Landau gauge

P. A. Grassi, T. Hurth and M. Steinhauser, Annals Phys. 288 , 197 (2001)
D. Binosi and J. P., Phys. Rev. D66, 025024 (2002)

The "quantum-background" gluon self-energy

$\widetilde{\Pi}_{\mu\nu}(q) =$



$$q^{\mu}\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p) \quad \Rightarrow \quad q^{\mu}[(a_1)+(a_2)]_{\mu\nu} = 0$$

$$q^{\mu}\widetilde{\Gamma}_{\mu}(q,r,-p) = D^{-1}(p) - D^{-1}(r) \quad \Rightarrow \quad q^{\mu}[(a_3)+(a_4)]_{\mu\nu} = 0$$

$$q^{\mu}\widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma} = f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma} + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha} + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta} \quad \Rightarrow \quad q^{\mu}[(a_5)+(a_6)]_{\mu\nu} = 0$$

$\Downarrow$

Transversality is enforced separately for gluon and ghost loops, and order-by-order in the "dressed-loop" expansion!

A.C. Aguilar and J.P. , JHEP 0612, 012 (2006)
D. Binosi and J. P. , Phys.Rev. D 77, 061702 (2008); JHEP 0811:063,2008.

First ingredient : From Takahashi to Ward identities

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Taylor expansion of both sides around $q = 0$

assuming no poles $1/q^2$ in the form factors of $\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p)$

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

Exact analogy with the Ward identity of spinor QED !

$$\Gamma_\mu(0,r,-r) = \frac{\partial S^{-1}(r)}{\partial r^\mu}$$

Second ingredient: Seagull identity

In dimensional regularization, any function that satisfies the criterion of Wilson

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \int_0^\infty dy y^{\frac{d}{2}-1} f(y) = \text{finite}, \qquad \text{for } 0 < d < d^*$$

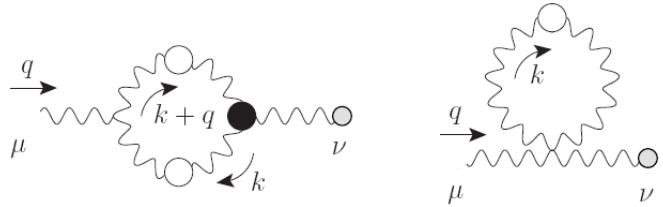
$$\longrightarrow \quad \int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2} \int_k f(k^2) = 0$$

Integrate by parts, drop the surface term, do analytic continuation

Gluon propagator at the origin

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^{\mu}} \quad \Rightarrow \quad \Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\widetilde{\Gamma}^{\mu}_{\sigma\rho}(0,k,-k) = \frac{\partial\Delta^{\alpha\beta}(k)}{\partial k^{\mu}}$$

$$\Delta^{-1}(0) = \lim_{q\to 0}\mathrm{Tr}\Bigg\{$$



$$\Bigg\} \sim \int_k k^2\frac{\partial\Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$

Ward Identities (No poles) **Seagull identity**

BFM Ward identites + Seagull identity $\Rightarrow$ $\Delta^{-1}(0) = 0$

No "gluon mass"

Exact result from BFM Schwinger-Dyson equation !

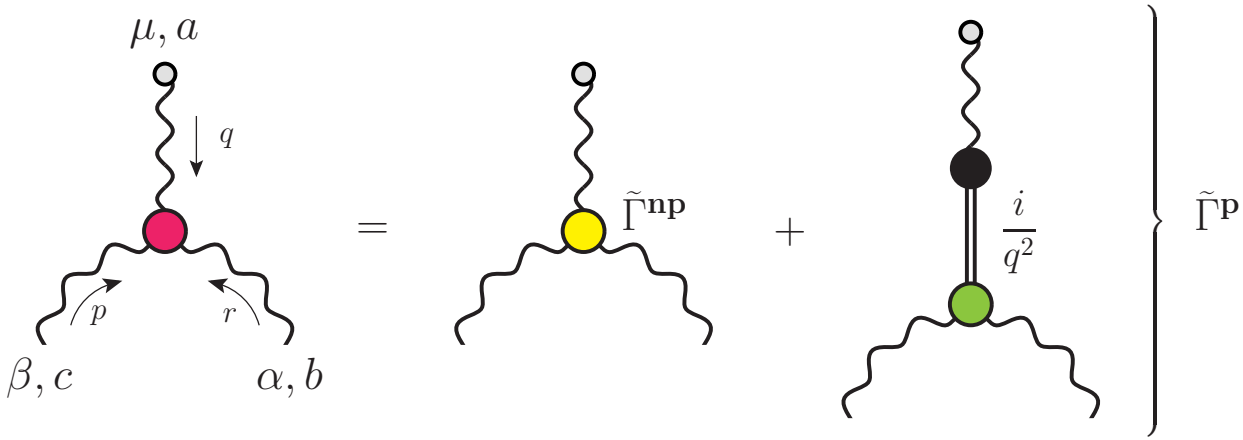
Vertices with massless poles

To evade the previous result, one must relax one of the underlying assumptions

In particular, the derivation of the WI hinges on the absence of poles $1/q^2$

Therefore, let us introduce poles in $\widetilde{\Gamma}_{\mu\alpha\beta}$

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



Explicit implementation of the Schwinger mechanism in Yang-Mills theories

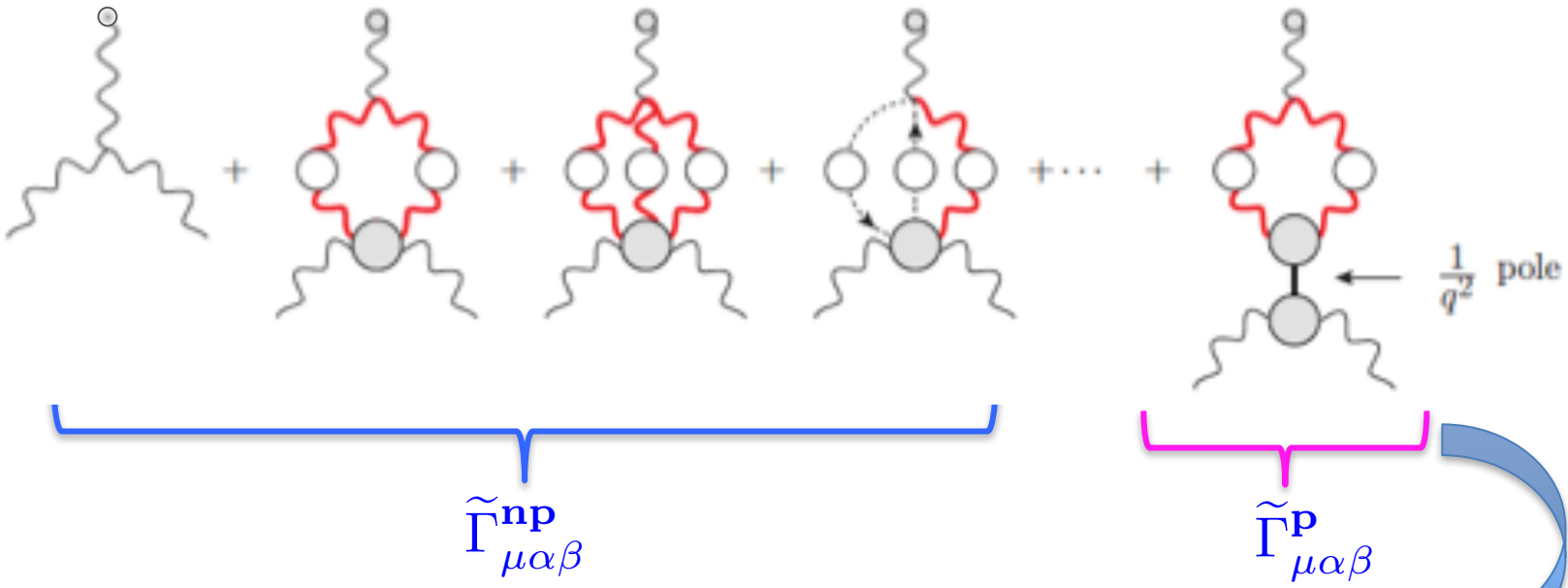
J.S. Schwinger
Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)

E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

The poles correspond to massless bound-state (colored) excitations

$\widetilde{\Gamma}_{\mu\alpha\beta} =$



The poles are longitudinally coupled

$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\underbrace{\widetilde{C}_{\alpha\beta}(q,r,p)}_{\text{contains 5 form factors}}$$

Bose symmetry $\Rightarrow$ $\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$

Landau gauge: $\widetilde{C}_{\alpha\beta}(q,r,p) = \widetilde{C}_1(q,r,p)g_{\alpha\beta} + \ldots$ $\Rightarrow$ $\widetilde{C}_1(0,r,-r) = 0$

Ward identities in the presence of poles

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Same ST identity !

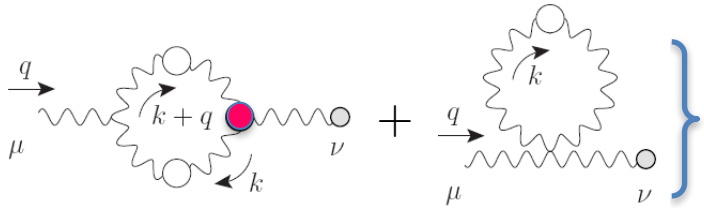
$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),$$

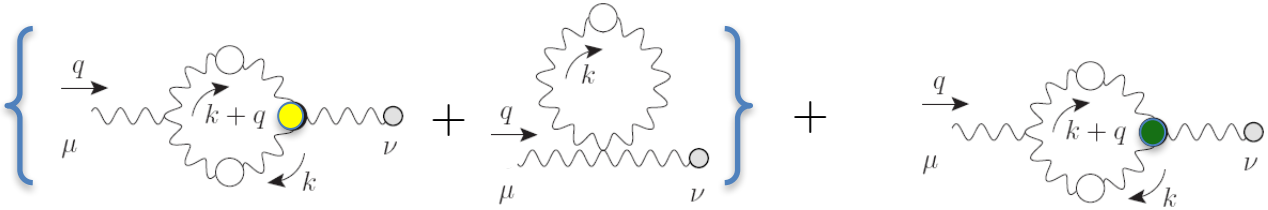
Expand around $q = 0$

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} - \left\{ \frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p) \right\}_{q=0}$$

Evading the seagull identity

$$\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}\Big\{$$





Triggers seagull identity exactly as before

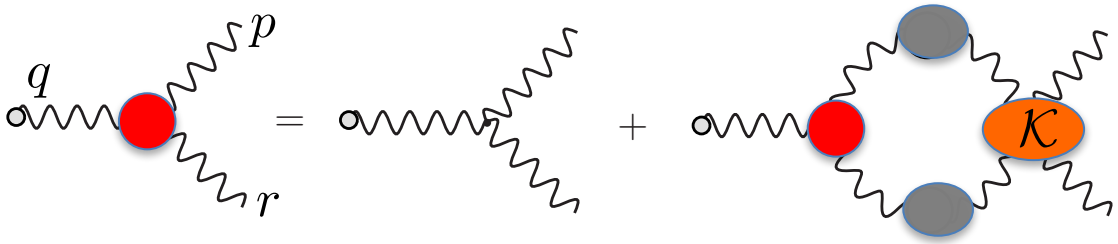
Vanishes identically

$$\Delta^{-1}(0) \sim \int_k k^2 \Delta^2(k^2) \widetilde{C}_1'(k^2)$$

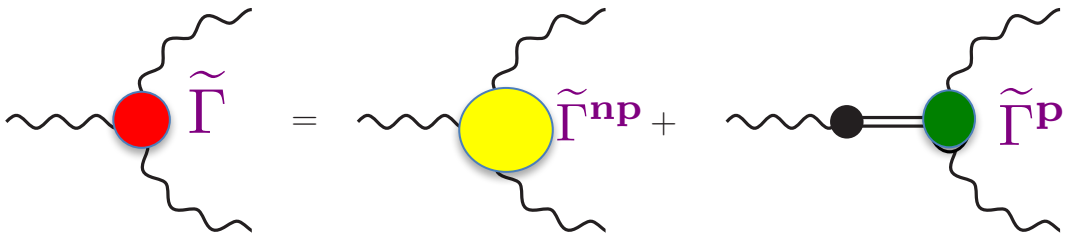
A.C.Aguilar, D.Binosi, C.T.Figueiredo and J.P., Phys. Rev. D 94, no. 4, 045002 (2016)

Dynamical formation of massless poles

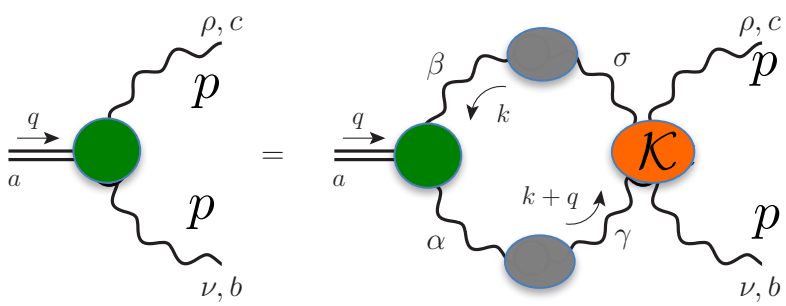
Schwinger-Dyson equation for the full vertex



Substitute:



$q \to 0$



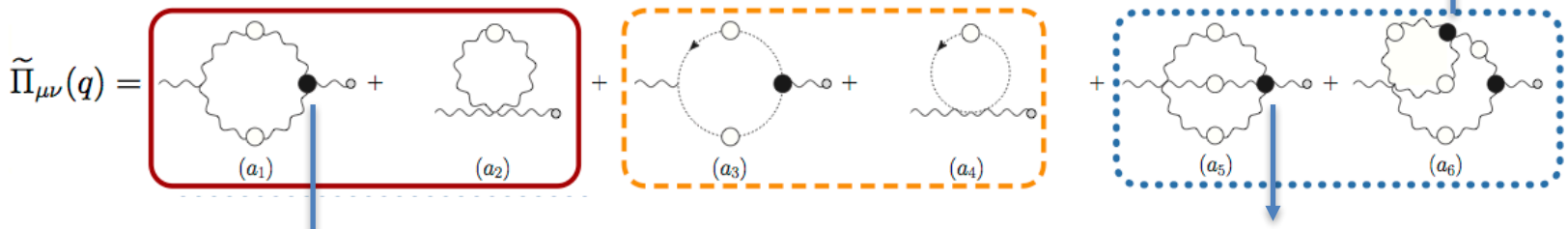
Equate terms linear in q

Bethe-Salpeter equation for the bound state wave function

$$\widetilde{C}_1'(p^2) = \lambda \int_k \widetilde{C}_1'(k^2)\Delta^2(k)\Delta(k+p)\,\mathcal{K}(k,p)$$

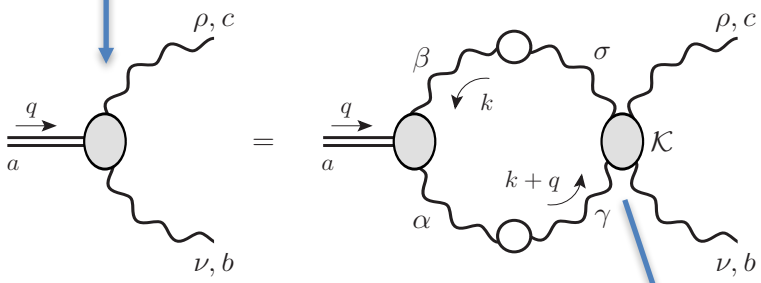
The SDE-BSE system (with. D.Binosi)

tree-level

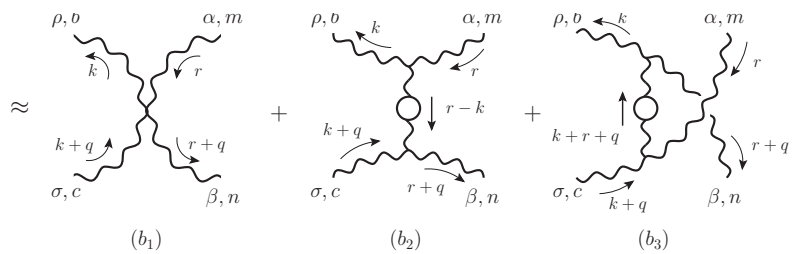


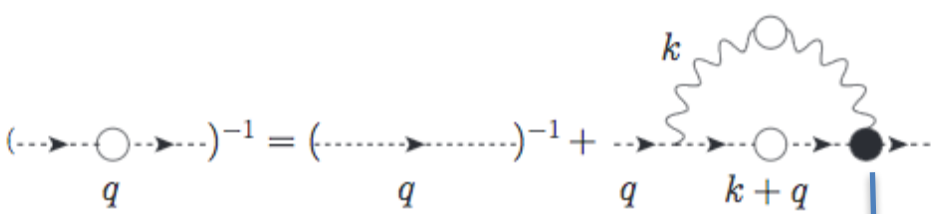
GLUON SDE

tree-level



MASSLESS POLE BSE





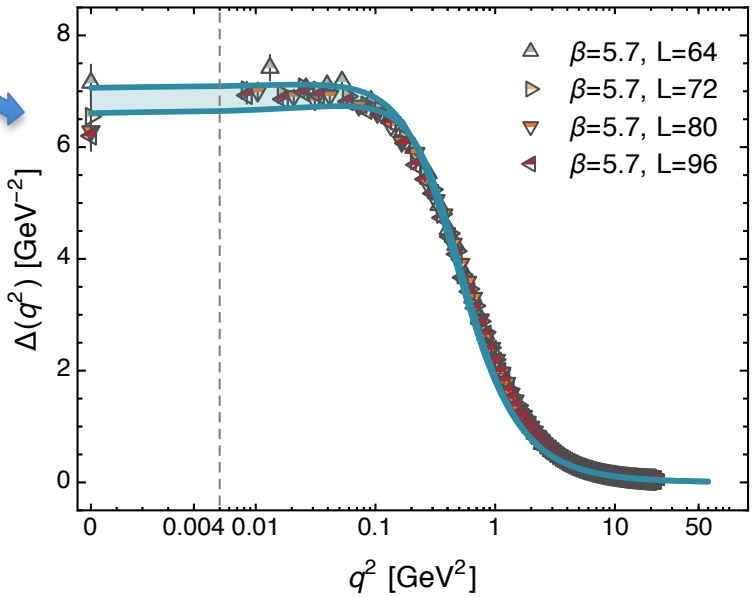
GHOST SDE

Determined from the ghost-gluon vertex SDE

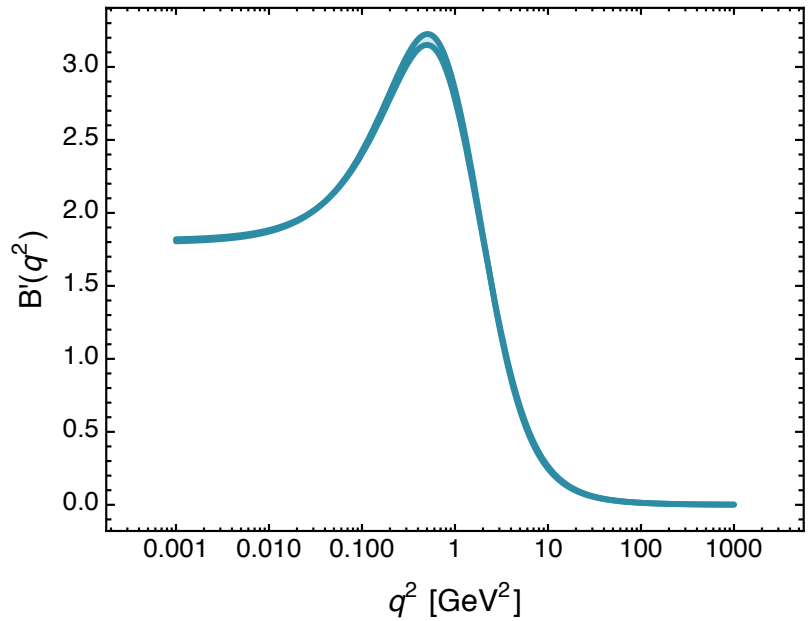
Preliminary Results

$\Delta(0)$ "sets the scale"

Gluon propagator



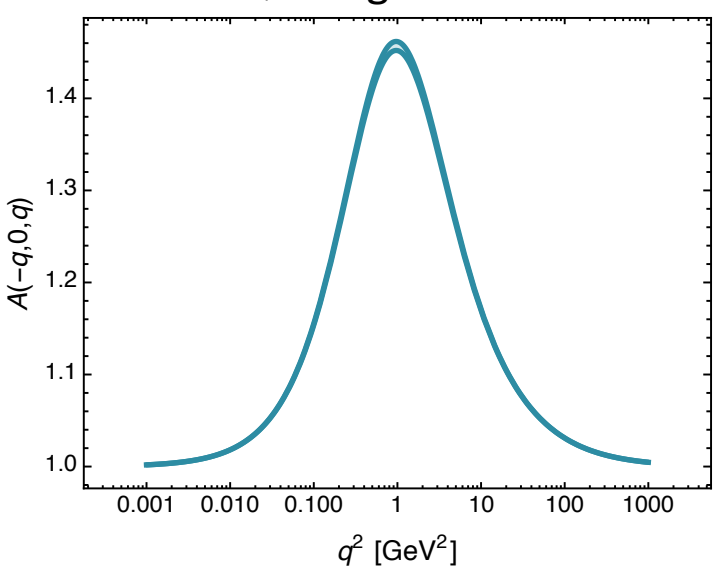
$B_1'(q^2) = \widetilde{C}_1'(q^2)$ normalized from $\Delta(0)$



$\alpha_s \approx 0.24$

$\mu = 4.3$ GeV

Quark-gluon vertex



Ghost dressing function $F(q^2) = q^2 D(q^2)$