IMPACT OF NUCLEON STRUCTURE
ON THE
PROTON-NEUTRON MASS DIFFERENCE
AND THE
HYDROGEN SPECTRUM

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THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

@ 2nd Proton Mass Workshop
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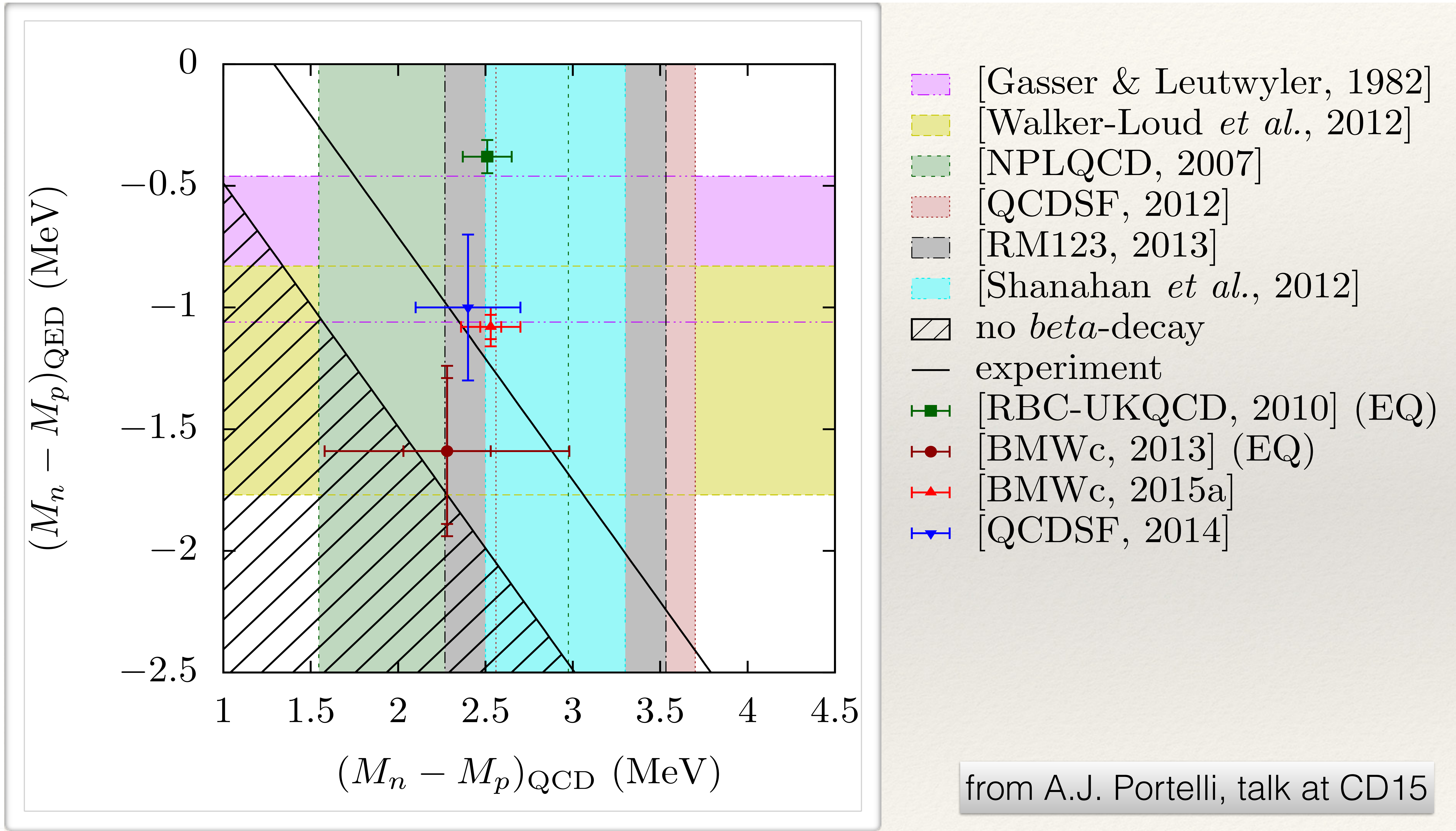
Isospin breaking of the nucleon mass

$$M_n - M_p = 1.2933322(4)\ \text{MeV}$$

	up	down	
Mass (MeV)	$2.3(^{+0.7}_{-0.5})$	$4.8(^{+0.5}_{-0.3})$	source: [PDG, 2013]
Charge (e)	2/3	-1/3	



from A.J. Portelli, talk at CD15

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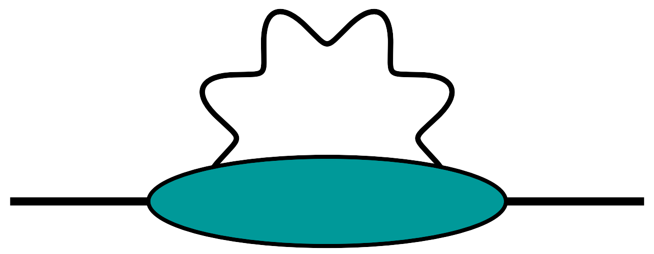

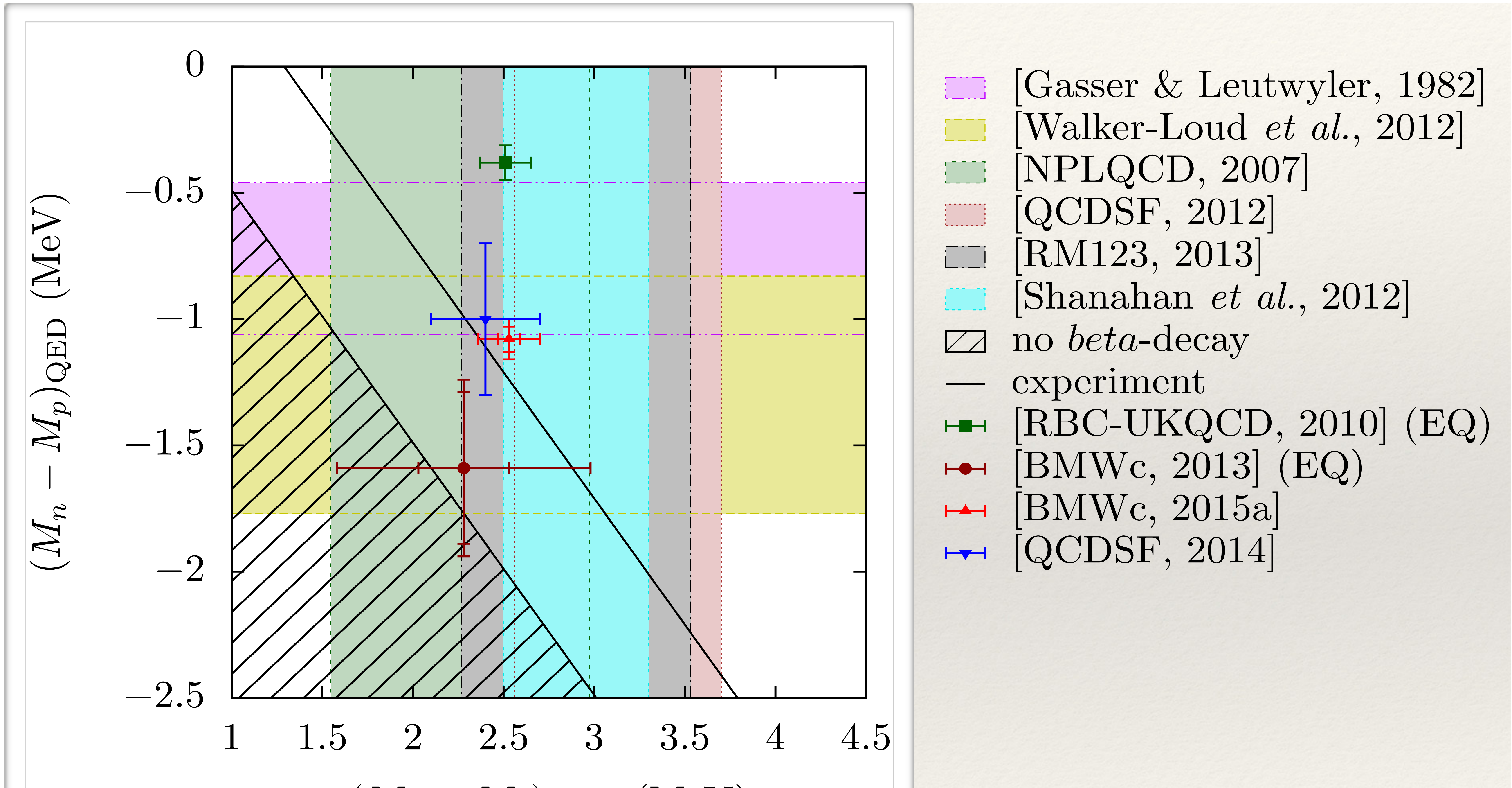

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Proton charge radius (historical perspective & the puzzle)

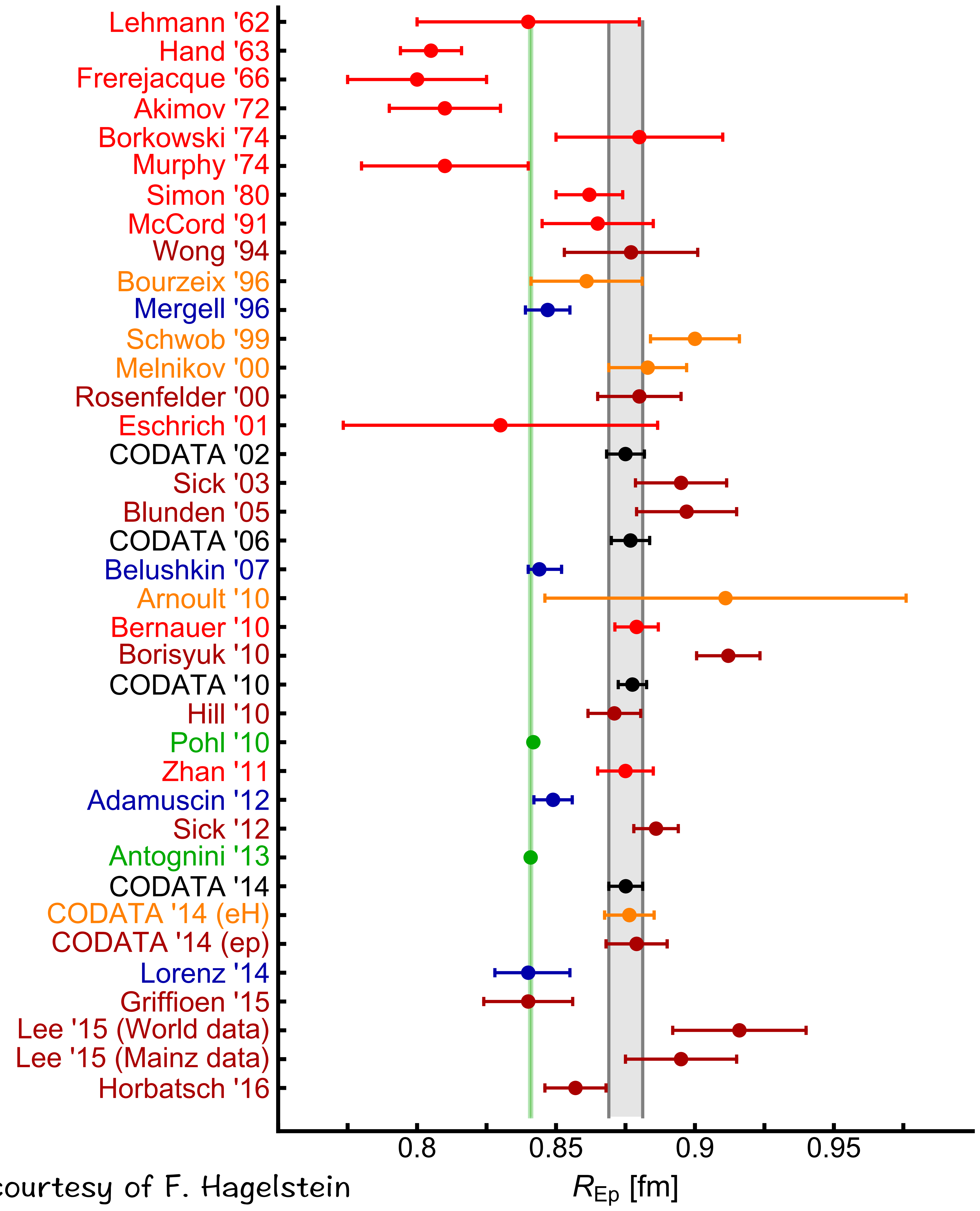
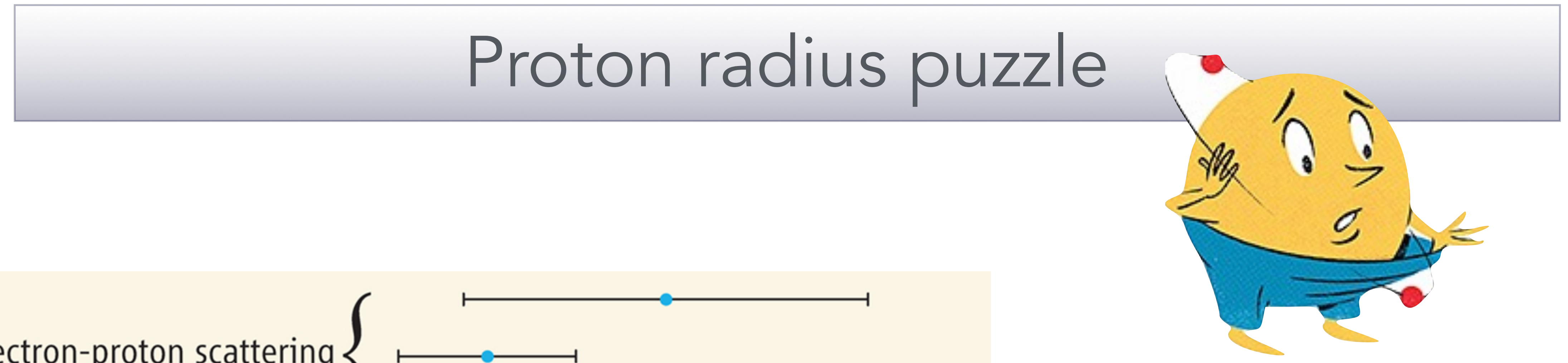
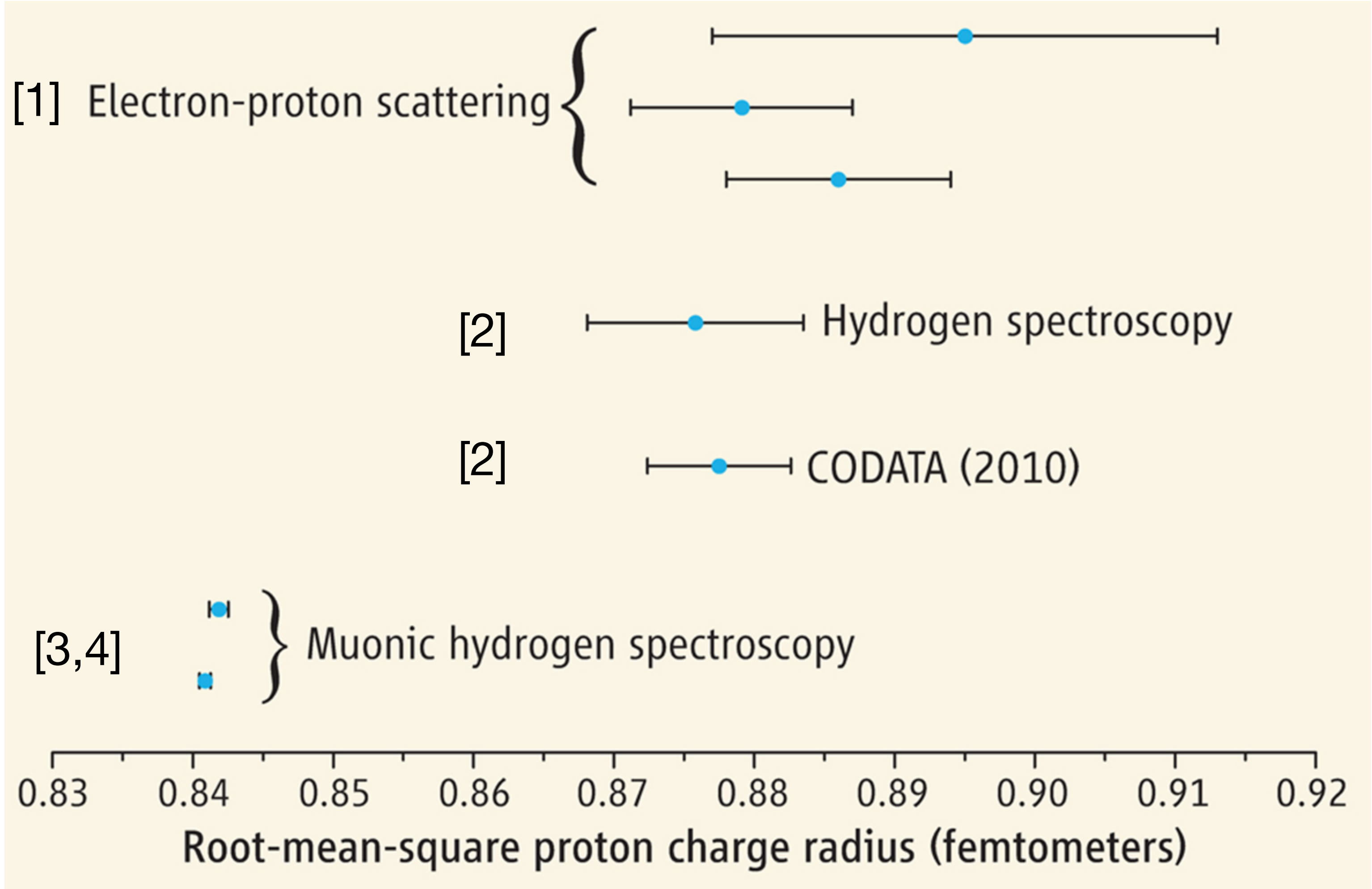


figure courtesy of F. Hagelstein





[1] J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).
[2] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).
[3] R. Pohl, A. Antognini *et al.*, Nature **466**, 213 (2010).
[4] A. Antognini *et al.*, Science **339**, 417 (2013).

7σ discrepancy

$$[R_E^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}] \longleftrightarrow [R_E^{\mathrm{CODATA\ 2010}} = 0.8775(51)\ \mathrm{fm}]$$

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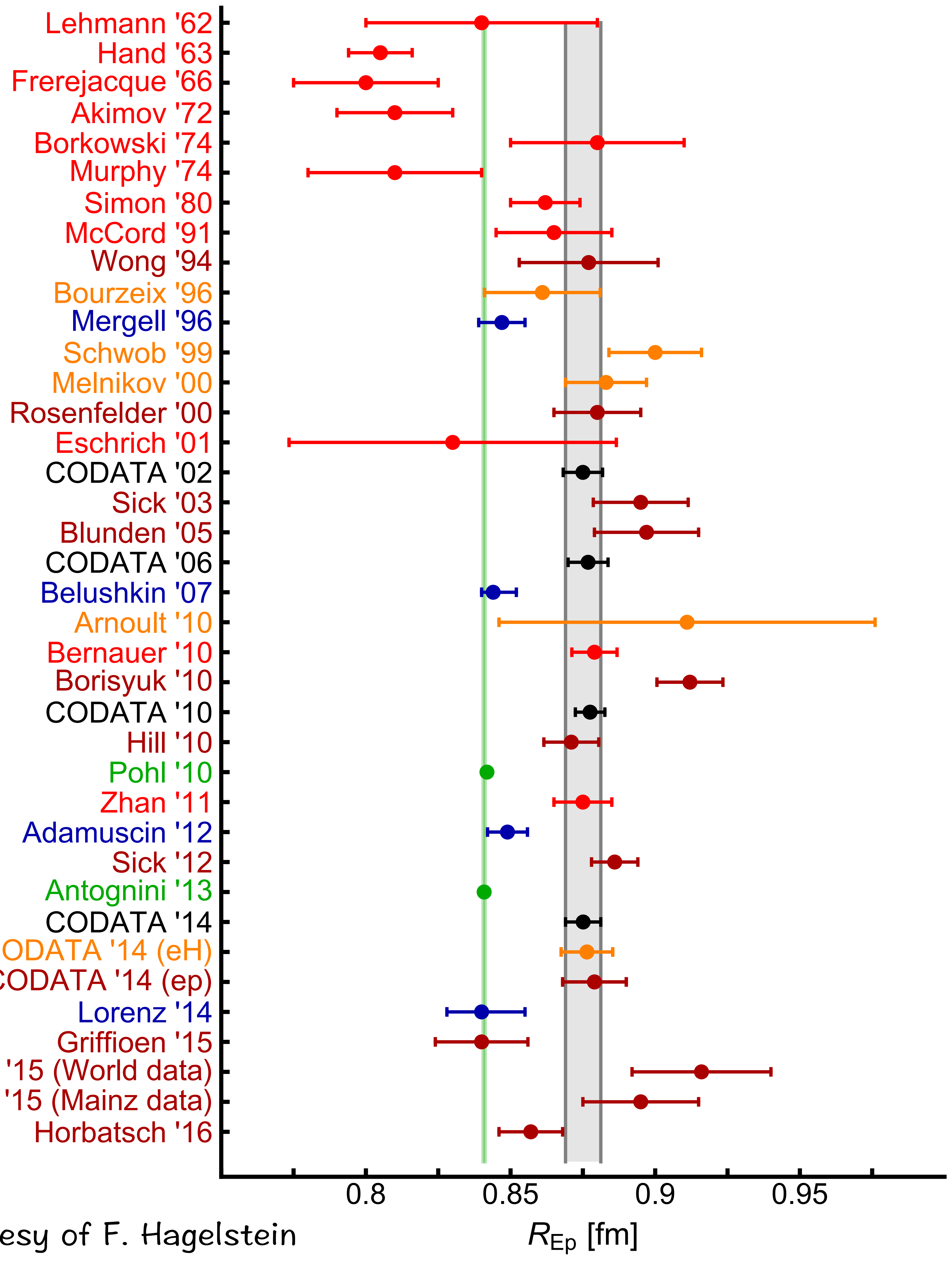
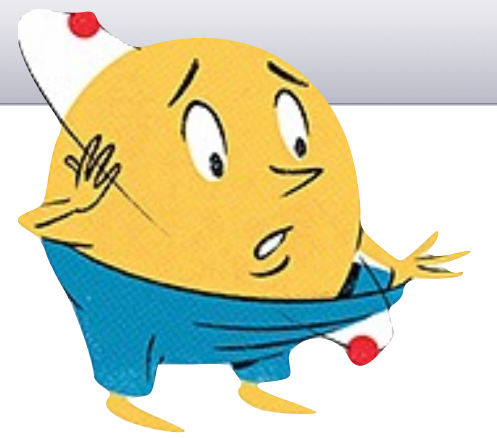
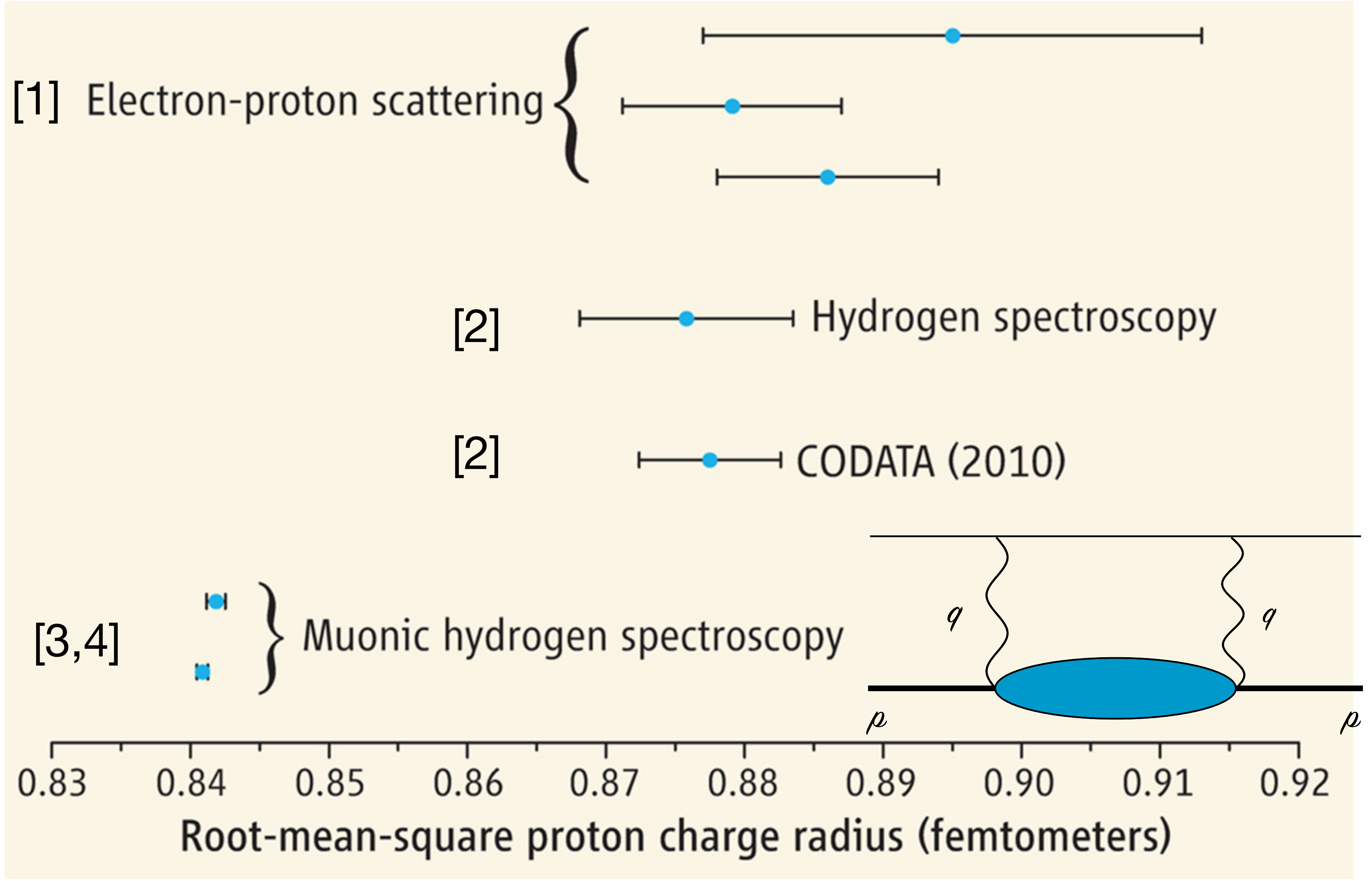


figure courtesy of F. Hagelstein

Proton radius puzzle





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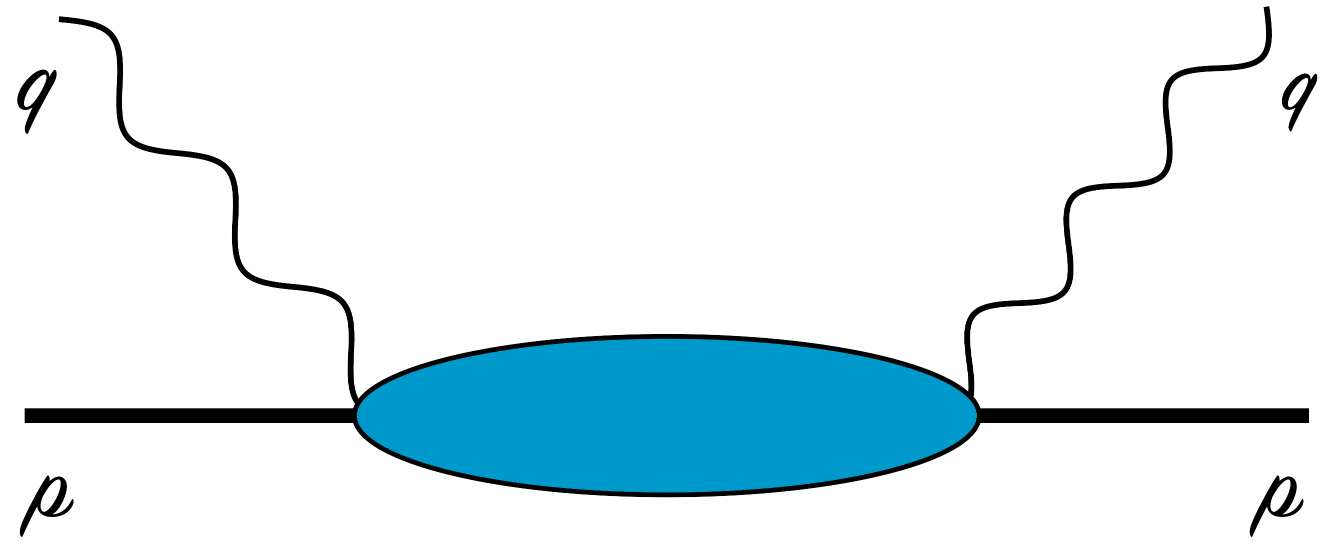
Forward doubly virtual Compton scattering



$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right) T_2(\nu,Q^2)$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\, S_1(\nu,Q^2) - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right) S_2(\nu,Q^2).$$

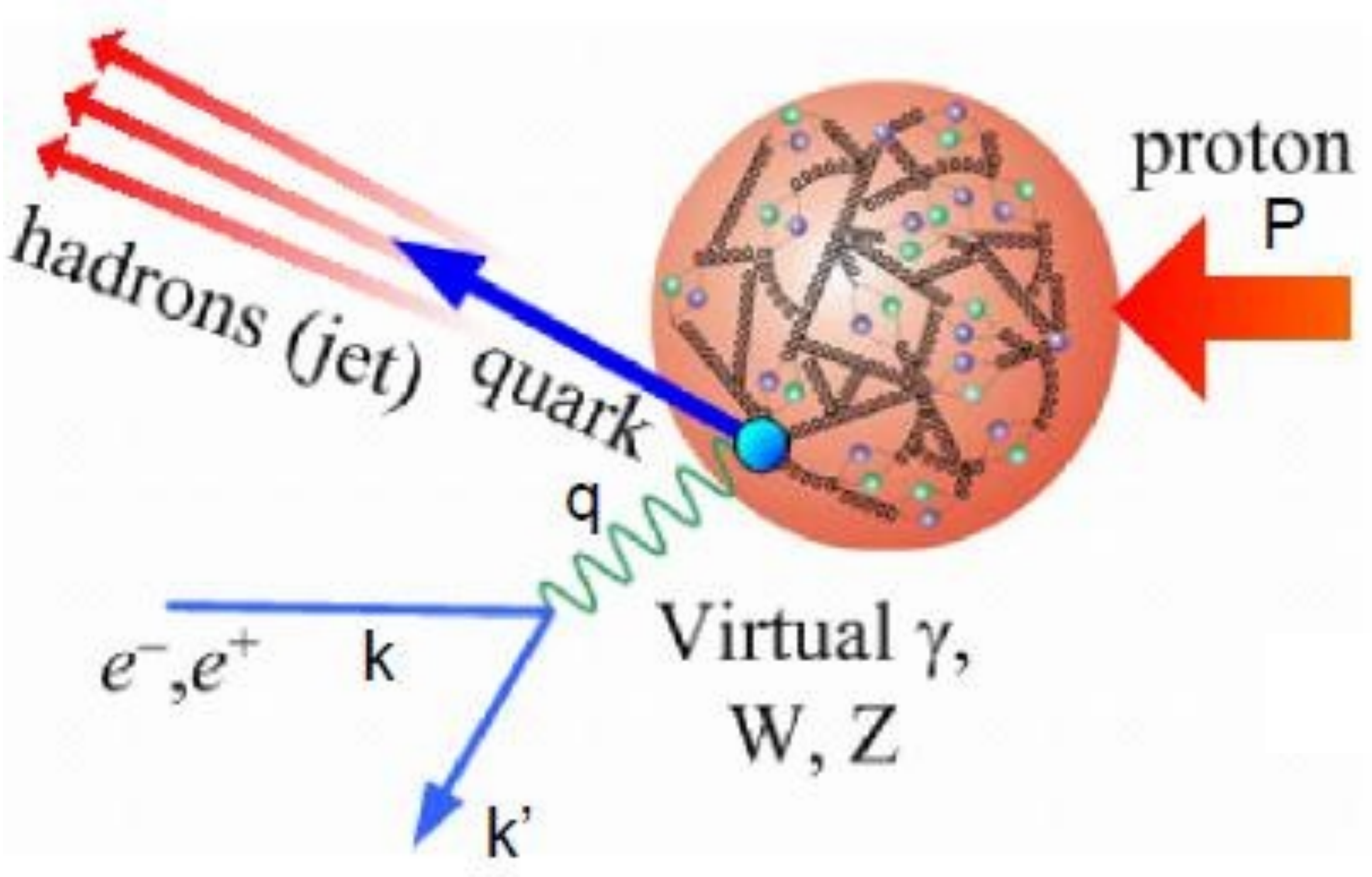
current conservation $\quad q_\mu T^{\mu\nu} = 0 = q_\nu T^{\mu\nu}$

optical theorem — unitarity:

$$\begin{aligned}
\mathrm{Im}\,T_1(\nu,Q^2) &= \frac{4\pi^2 Z^2\alpha}{M} f_1(x,Q^2) = \sqrt{\nu^2+Q^2}\,\sigma_T(\nu,Q^2),\\
\mathrm{Im}\,T_2(\nu,Q^2) &= \frac{4\pi^2 Z^2\alpha}{\nu} f_2(x,Q^2) = \frac{Q^2}{\sqrt{\nu^2+Q^2}}\left[\sigma_T+\sigma_L\right](\nu,Q^2),\\
\mathrm{Im}\,S_1(\nu,Q^2) &= \frac{4\pi^2 Z^2\alpha}{\nu} g_1(x,Q^2) = \frac{M\nu}{\sqrt{\nu^2+Q^2}}\left[\frac{Q}{\nu}\sigma_{LT}+\sigma_{TT}\right](\nu,Q^2),\\
\mathrm{Im}\,S_2(\nu,Q^2) &= \frac{4\pi^2 Z^2\alpha M}{\nu^2} g_2(x,Q^2) = \frac{M^2}{\sqrt{\nu^2+Q^2}}\left[\frac{\nu}{Q}\sigma_{LT}-\sigma_{TT}\right](\nu,Q^2)
\end{aligned}$$

Structure functions

Electron-proton scattering



$$Q^2 = -(k' - k)$$
$$x = Q^2/(2M_N\nu)$$

Yields 4 Structure functions:

$$f_1(\nu, Q^2),\ f_2(\nu, Q^2),\ g_1(\nu, Q^2),\ g_2(\nu, Q^2).$$

Lamb shift hyperfine splitting

(i) Elastic part given by form factors

$$\begin{aligned}
f_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} G_M^2(Q^2)\,\delta(1-x),\\
f_2^{\rm el}(\nu, Q^2) &= \frac{1}{1+\tau}\left[G_E^2(Q^2) + \tau G_M^2(Q^2)\right]\delta(1-x),\\
g_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} F_1(Q^2) G_M(Q^2)\,\delta(1-x),\\
g_2^{\rm el}(\nu, Q^2) &= -\tfrac{1}{2}\tau F_2(Q^2) G_M(Q^2)\,\delta(1-x),
\end{aligned}$$

where $\tau = Q^2/4M^2$ and $G_E(Q^2), G_M(Q^2)$ are the Sachs FFs

Proton Form Factors and RMS Radii

FF interpretation: Fourier transforms of charge and magnetization distributions

$$\rho(r) = \int \frac{d\boldsymbol{q}}{(2\pi)^3} G(\boldsymbol{q}^2) e^{-i\boldsymbol{q}\boldsymbol{r}}$$

$$G_E(Q^2) = 1 - 1/6\, R_E^2\, Q^2 + \cdots$$

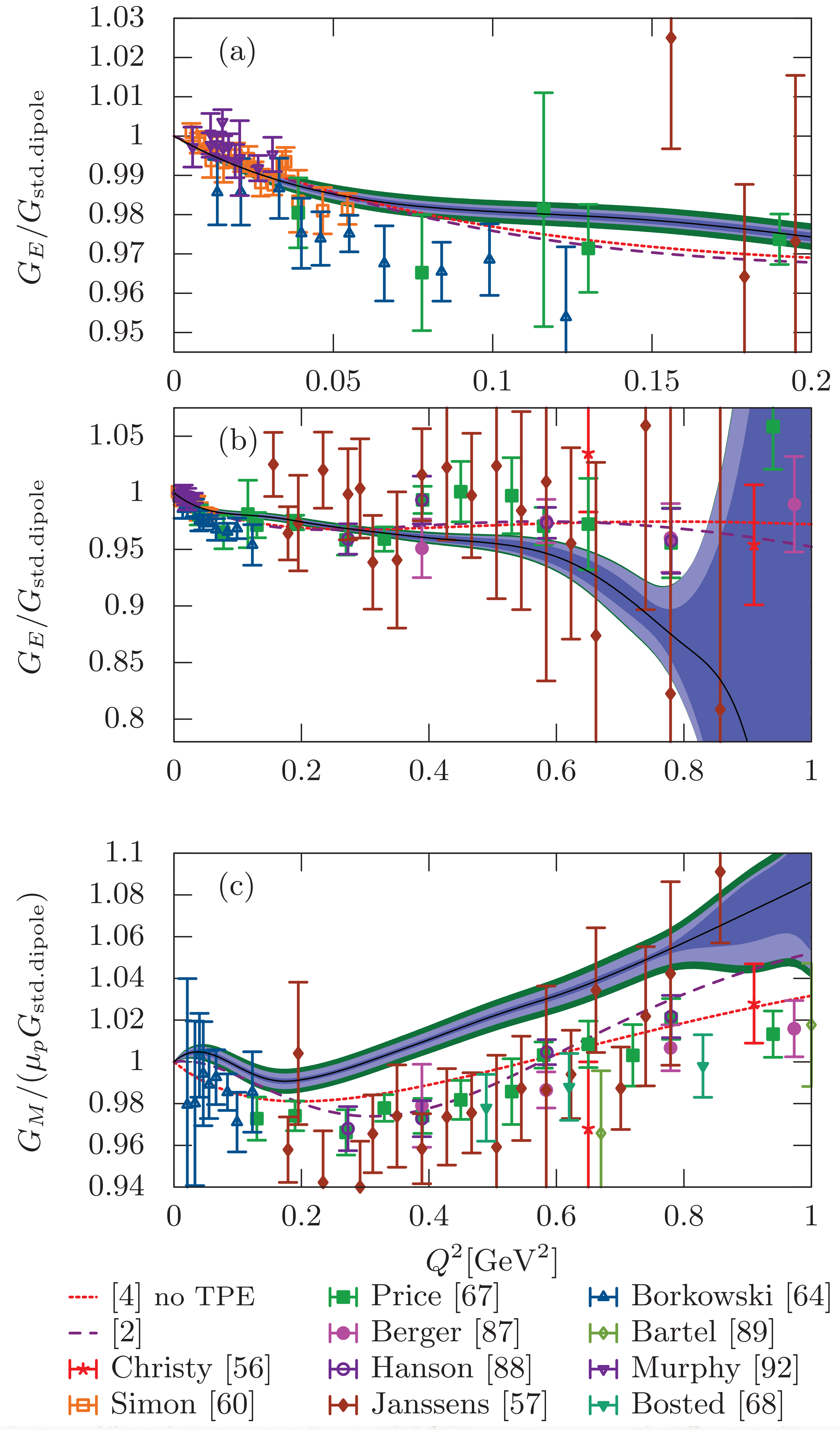
root-mean-square (rms) charge radius:

$$R_E = \sqrt{\langle r^2 \rangle_E}$$

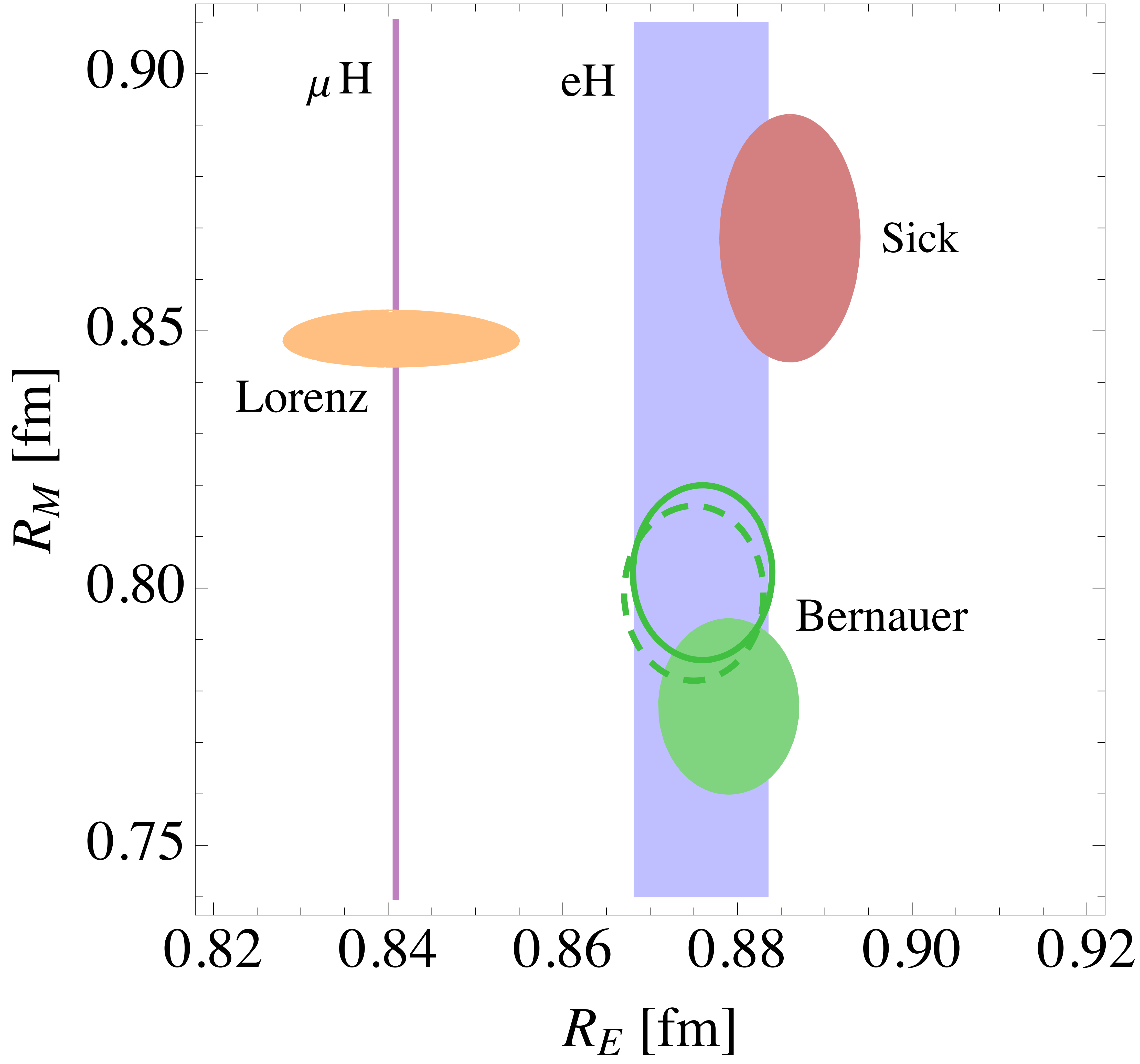
$$\langle r^2 \rangle_E \equiv \int d\boldsymbol{r}\, r^2\, \rho_E(\boldsymbol{r}) = -6 \frac{d}{dQ^2} G_E(Q^2)\Big|_{Q^2=0}$$

$$R_E = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}\ \text{fm},$$
$$R_M = 0.777(13)_{\text{stat}}(9)_{\text{syst}}(5)_{\text{model}}(2)_{\text{group}}\ \text{fm}.$$

J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).



Current status of the proton radii



compiled by Hagelstein, Miskimen & V.P., Prog Part Nucl Phys (2016)

H: $R_E = 0.8758(77)$ fm;

μH: $R_E = 0.84087(39)$ fm;

Sick: $R_E = 0.886(8)$ fm, $R_M = 0.868(24)$ fm;

Lorenz et al.: $R_E = 0.840\,[0.828\ldots0.855]$ fm, $R_M = 0.848\,[0.843\ldots0.854]$ fm;

Bernauer et al.:

$$\begin{aligned} R_E &= 0.879(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(4)_{\rm group}\ \text{fm},\\ R_E^{\text{TPE-a}} &= 0.876(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(5)_{\rm group}\ \text{fm},\\ R_E^{\text{TPE-b}} &= 0.875(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(5)_{\rm group}\ \text{fm},\\ R_M &= 0.777(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(2)_{\rm group}\ \text{fm},\\ R_M^{\text{TPE-a}} &= 0.803(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(3)_{\rm group}\ \text{fm},\\ R_M^{\text{TPE-b}} &= 0.799(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(3)_{\rm group}\ \text{fm}. \end{aligned}$$

[1] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).
[2] A. Antognini, et al., Science **339** (2013) 417– 420.
[3] I. Sick, Prog. Part. Nucl. Phys. **67** (2012) 473–478.
[4] I. Lorenz, et al., Phys. Rev. **D**91 (2015) 014023.
[5] J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).

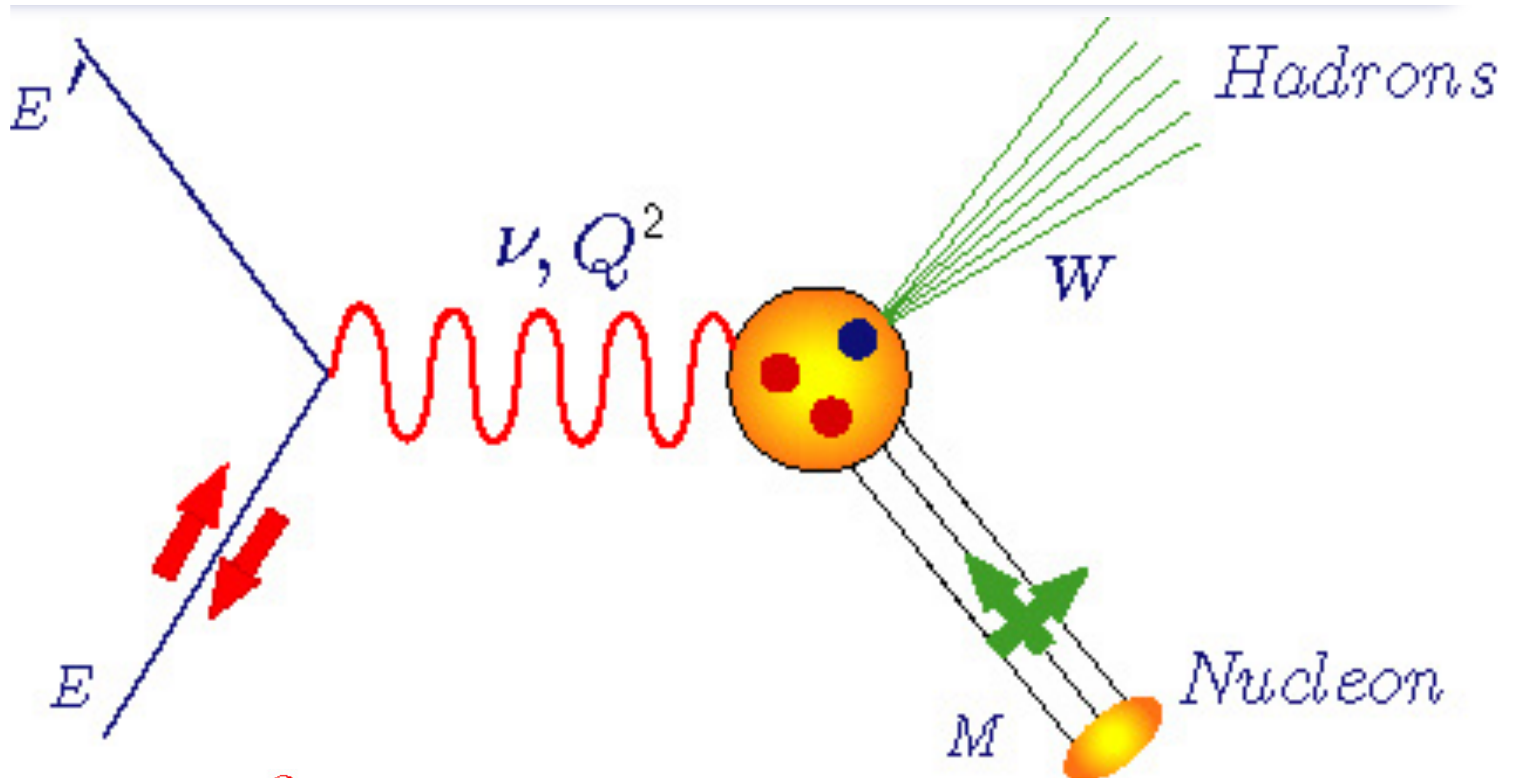
Inelastic structure functions

Polarized structure functions

$g_1(x,Q^2)$ *(parton model interpretation)*

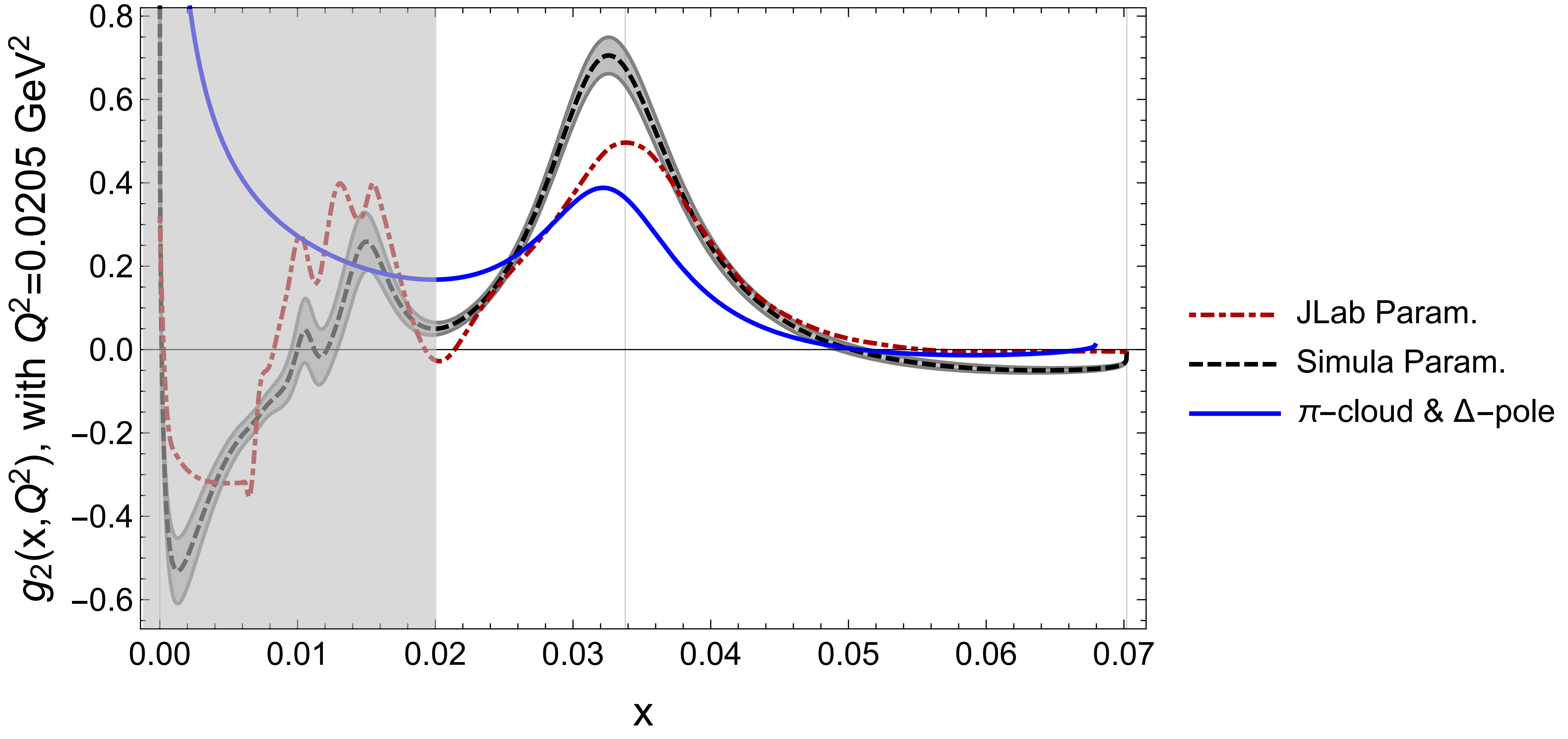
$g_2(x,Q^2)$ *(quark-gluon correlations)*

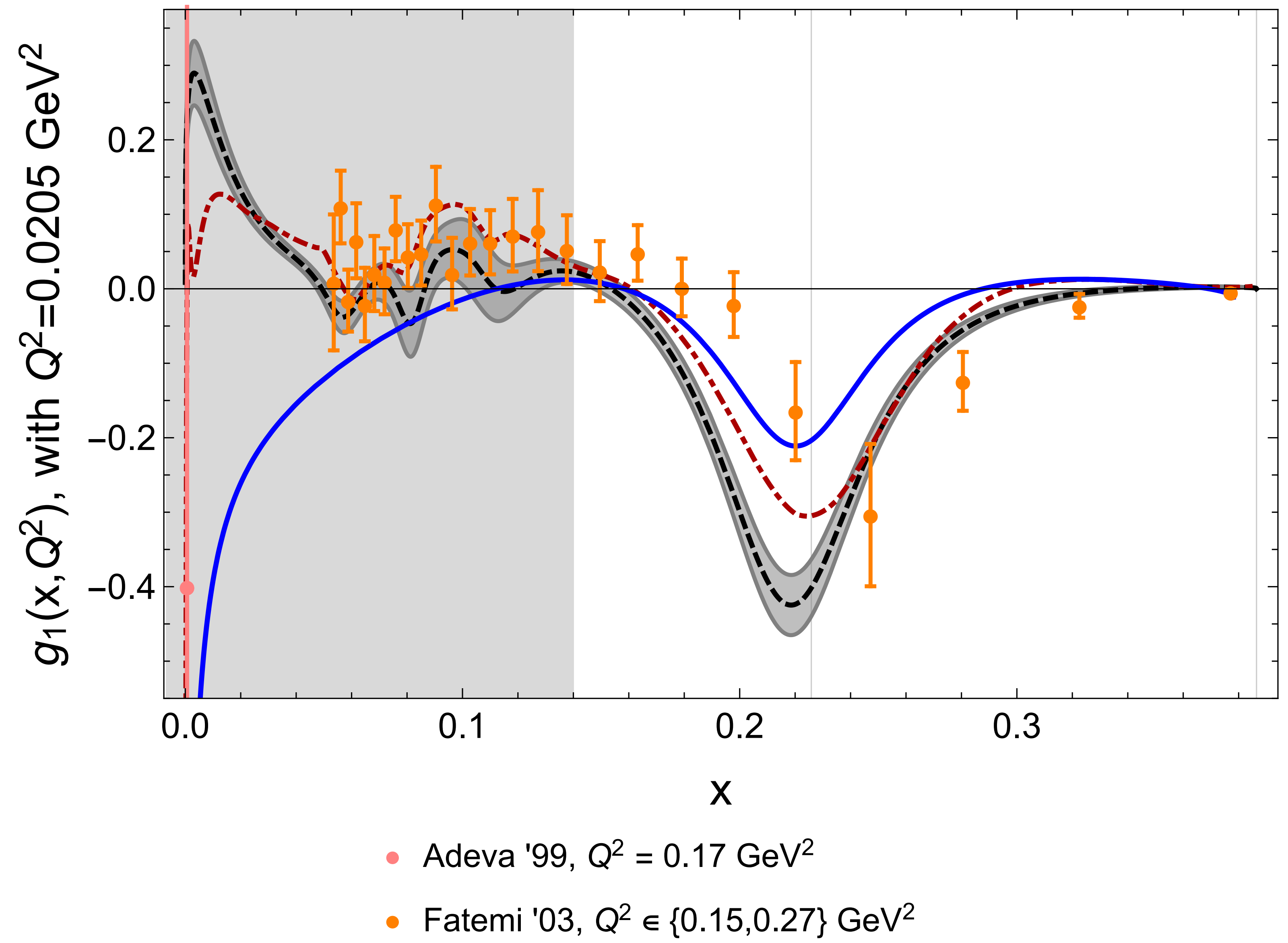
Q^2 : Four-momentum transfer
x : Bjorken variable
ν : Energy transfer
M : Nucleon mass
W : Final state hadrons mass



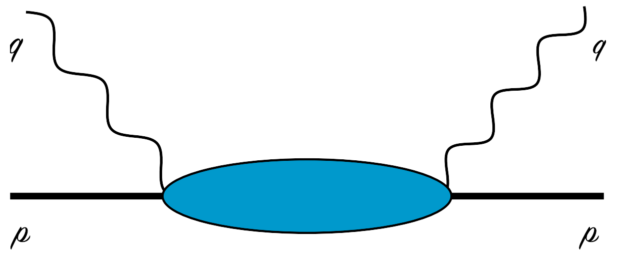
L $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E+E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

T $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2E g_2(x,Q^2)\right]$$





(unsubtracted) Dispersion relations for VVCS amplitudes — causality



Forward Compton scattering: $N(p) + \gamma(q) \to N(p) + \gamma(q)$, with either real or virtual photons.

$$T_1(\nu, Q^2) = \frac{8\pi\alpha}{M} \int_0^1 \frac{dx}{x} \frac{f_1(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

$$T_2(\nu, Q^2) = \frac{16\pi\alpha M}{Q^2} \int_0^1 dx \frac{f_2(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

$$S_1(\nu, Q^2) = \frac{16\pi\alpha M}{Q^2} \int_0^1 dx \frac{g_1(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

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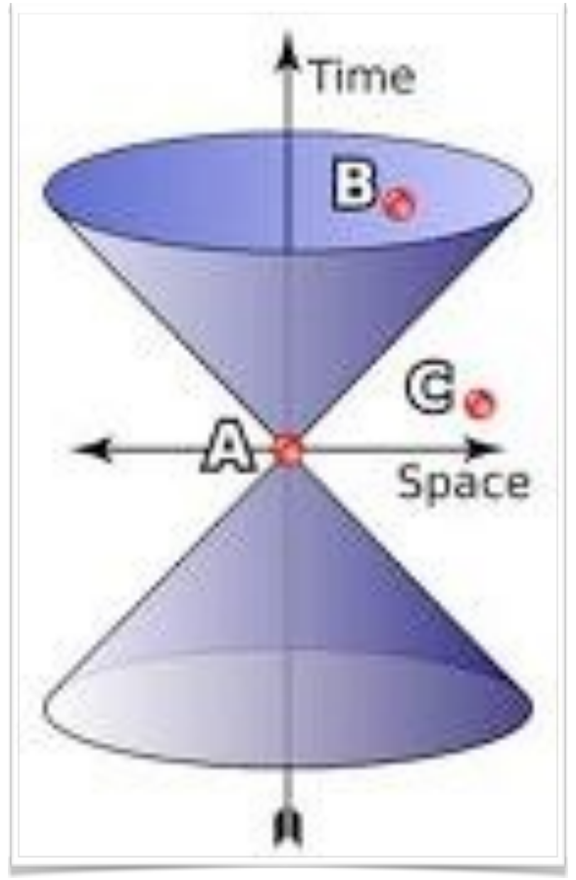

$$B(x) = \int dx' G(x - x') A(x')$$

$$G(x - x') = 0, \quad (x - x')^2 < 0$$

Split into Born (elastic form factors) and non-Born (polarizabilities)

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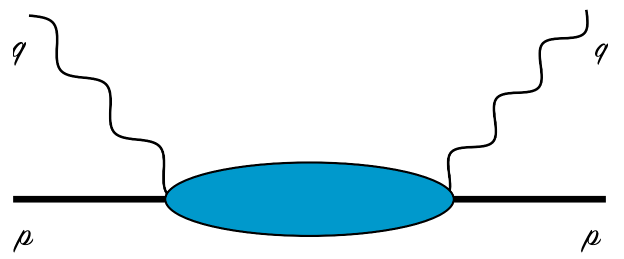

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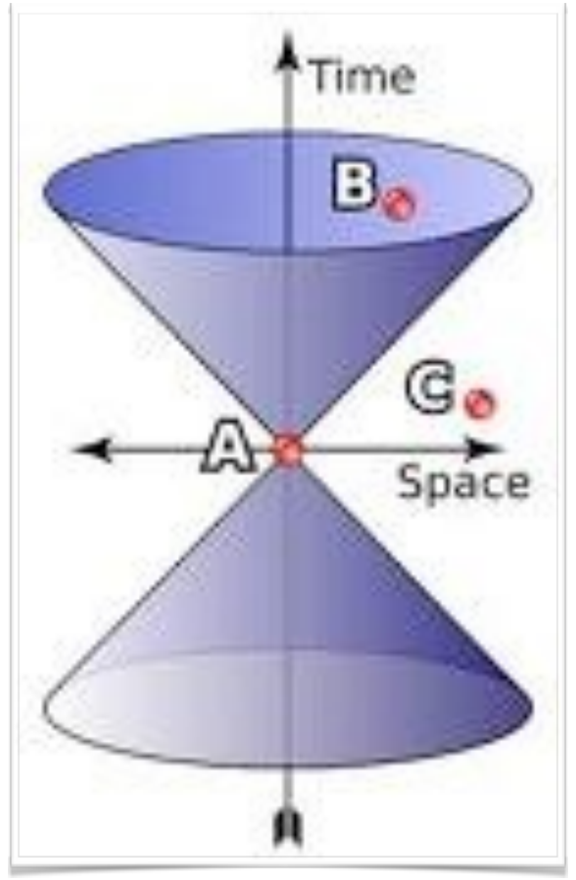

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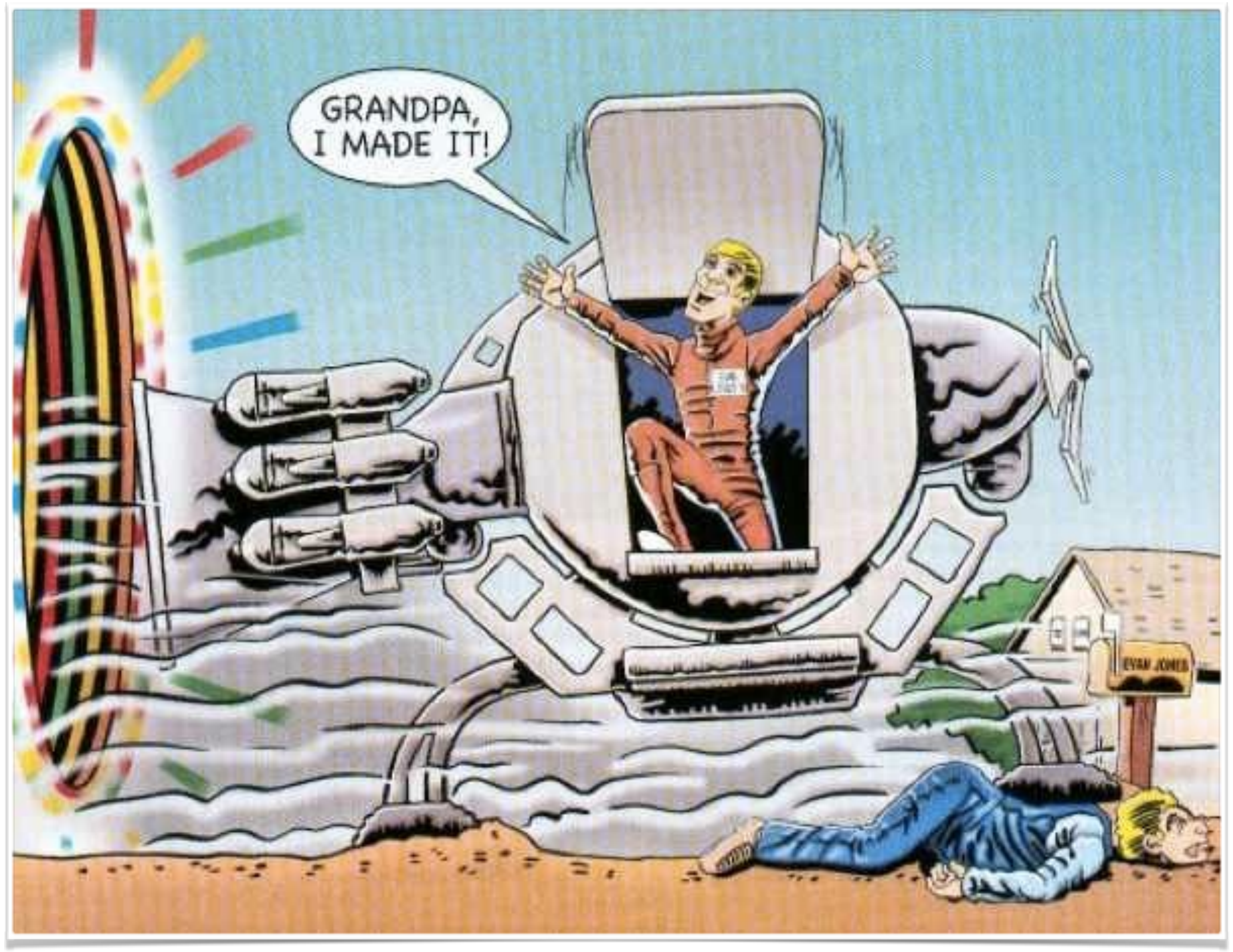

Split into Born (elastic form factors) and non-Born (polarizabilities)

Unfortunately T1 needs a subtraction!

Real photons

$$T_1(\nu, 0) = \frac{2}{\pi} \int_0^\infty \mathrm{d}\nu' \, \frac{\nu'^{\,2} \sigma_T(\nu')}{\nu'^{\,2} - \nu^2 - i0^+},$$

$$S_1(\nu, 0) = \frac{2M}{\pi} \int_0^\infty \mathrm{d}\nu' \, \frac{\nu' \, \sigma_{TT}(\nu')}{\nu'^{\,2} - \nu^2 - i0^+}.$$

Real photons

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Polarizabilities:

$$H_{\rm eff}^{(2)} = -4\pi\left(\tfrac{1}{2}\alpha_{E1}\boldsymbol{E}^2+\tfrac{1}{2}\beta_{M1}\boldsymbol{H}^2\right),$$
$$H_{\rm eff}^{(3)} = -4\pi\left(\tfrac{1}{2}\gamma_{E1E1}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\dot{\boldsymbol{E}})+\tfrac{1}{2}\gamma_{M1M1}\boldsymbol{\sigma}\cdot(\boldsymbol{H}\times\dot{\boldsymbol{H}})-\gamma_{M1E2}E_{ij}\sigma_iH_j+\gamma_{E1M2}H_{ij}\sigma_iE_j\right)$$

Low-energy expansion

$$\frac{1}{4\pi}T_1(\nu,0) = -\frac{Z^2\alpha}{M}+(\alpha_{E1}+\beta_{M1})\nu^2+\left[\alpha_{E1\nu}+\beta_{M1\nu}+{}^1\!/_{12}\,(\alpha_{E2}+\beta_{M2})\right]\nu^4+O(\nu^6)$$
$$\frac{1}{4\pi}S_1(\nu,0) = -\frac{\alpha\varkappa^2}{2M}+M\gamma_0\nu^2+M\bar{\gamma}_0\nu^4+O(\nu^6),$$

Real photons

$$T_1(\nu,0) \;=\; \frac{2}{\pi}\int_0^\infty \mathrm{d}\nu'\, \frac{\nu'^{\,2}\sigma_T(\nu')}{\nu'^{\,2}-\nu^2-i0^+},$$
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Leading order:

$$-Z^2\alpha/M = (2/\pi)\int_0^\infty \mathrm{d}\nu\,\sigma_T(\nu)$$ — invalid

$$\frac{\alpha}{M^2}\varkappa^2 = -\frac{1}{\pi^2}\int_0^\infty \mathrm{d}\nu\,\frac{\sigma_{TT}(\nu)}{\nu}$$ GDH sum rule

Real photons

$$T_1(\nu,0) = \frac{2}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'^{\,2}\sigma_T(\nu')}{\nu'^{\,2}-\nu^2-i0^+},$$
$$S_1(\nu,0) = \frac{2M}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'\,\sigma_{TT}(\nu')}{\nu'^{\,2}-\nu^2-i0^+}.$$

Polarizabilities:

$$H^{(2)}_{\mathrm{eff}} = -4\pi\left(\tfrac{1}{2}\alpha_{E1}\boldsymbol{E}^2+\tfrac{1}{2}\beta_{M1}\boldsymbol{H}^2\right),$$
$$H^{(3)}_{\mathrm{eff}} = -4\pi\left(\tfrac{1}{2}\gamma_{E1E1}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\dot{\boldsymbol{E}})+\tfrac{1}{2}\gamma_{M1M1}\boldsymbol{\sigma}\cdot(\boldsymbol{H}\times\dot{\boldsymbol{H}})-\gamma_{M1E2}E_{ij}\sigma_iH_j+\gamma_{E1M2}H_{ij}\sigma_iE_j\right)$$

Low-energy expansion

$$\frac{1}{4\pi}T_1(\nu,0) = -\frac{Z^2\alpha}{M}+(\alpha_{E1}+\beta_{M1})\nu^2+\left[\alpha_{E1\nu}+\beta_{M1\nu}+{}^1\!/_{12}\,(\alpha_{E2}+\beta_{M2})\right]\nu^4+O(\nu^6)$$
$$\frac{1}{4\pi}S_1(\nu,0) = -\frac{\alpha\varkappa^2}{2M}+M\gamma_0\nu^2+M\bar{\gamma}_0\nu^4+O(\nu^6),$$

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Next-to-leading order:

Baldin sum rule $$\alpha_{E1}+\beta_{M1} = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}\nu\,\frac{\sigma_T(\nu)}{\nu^2}.$$

Forward spin polarizability: $$\gamma_0 = -(\gamma_{E1E1}+\gamma_{M1M1}+\gamma_{E1M2}+\gamma_{M1E2})$$
$$= \frac{1}{4\pi^2}\int_{\nu_0}^\infty d\nu\,\frac{\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)}{\nu^3}$$

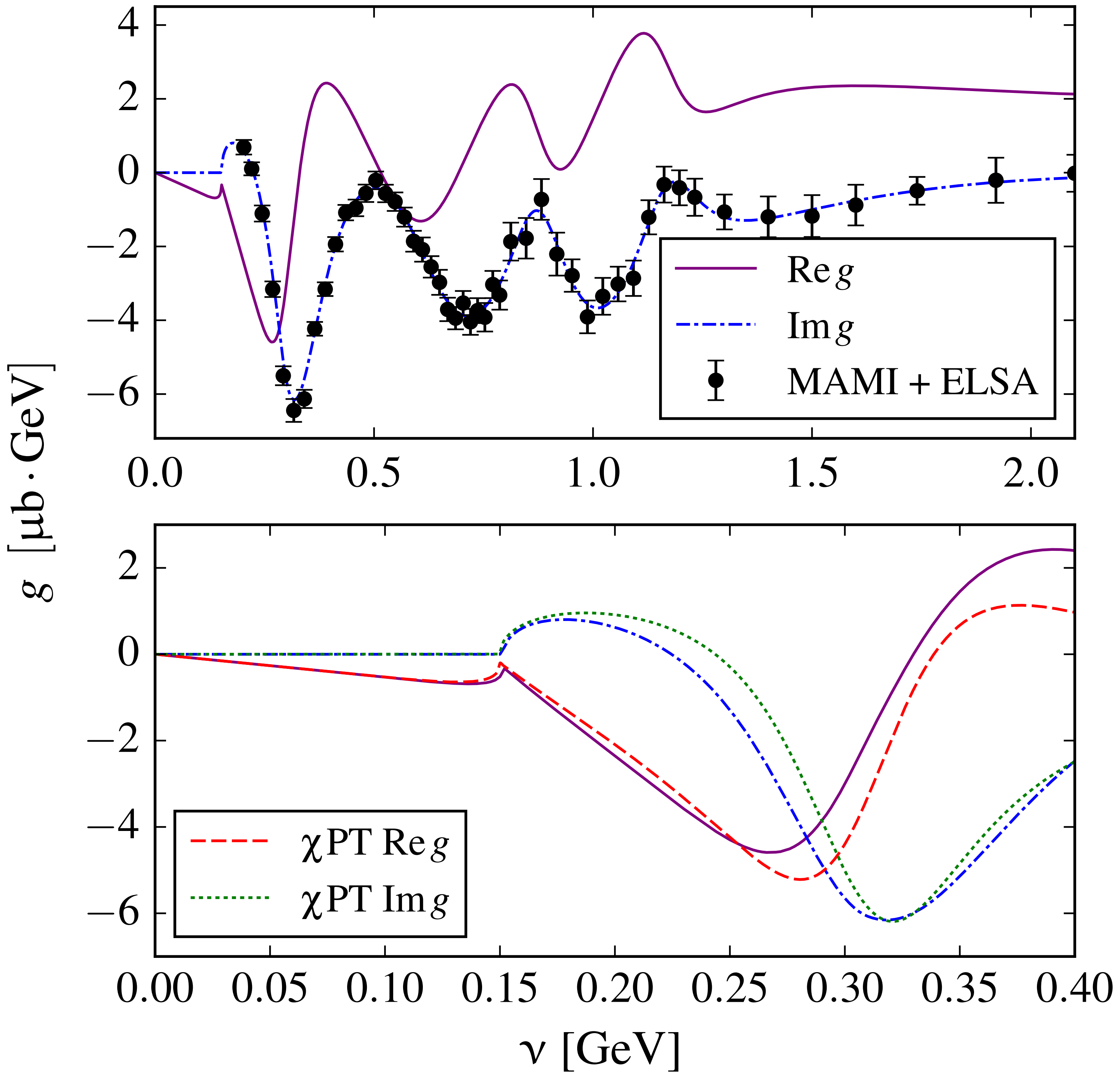
Empirical evaluation of forward Compton scattering

$$f(\nu) = -\frac{Z^2\alpha}{M} + \frac{\nu^2}{2\pi^2}\int_0^\infty d\nu'\,\frac{\sigma_T(\nu')}{\nu'^2-\nu^2-i0^+}$$
$$g(\nu) = \frac{\nu}{2\pi^2}\int_0^\infty d\nu'\,\frac{\nu'\sigma_{TT}(\nu')}{\nu'^2-\nu^2-i0^+}.$$

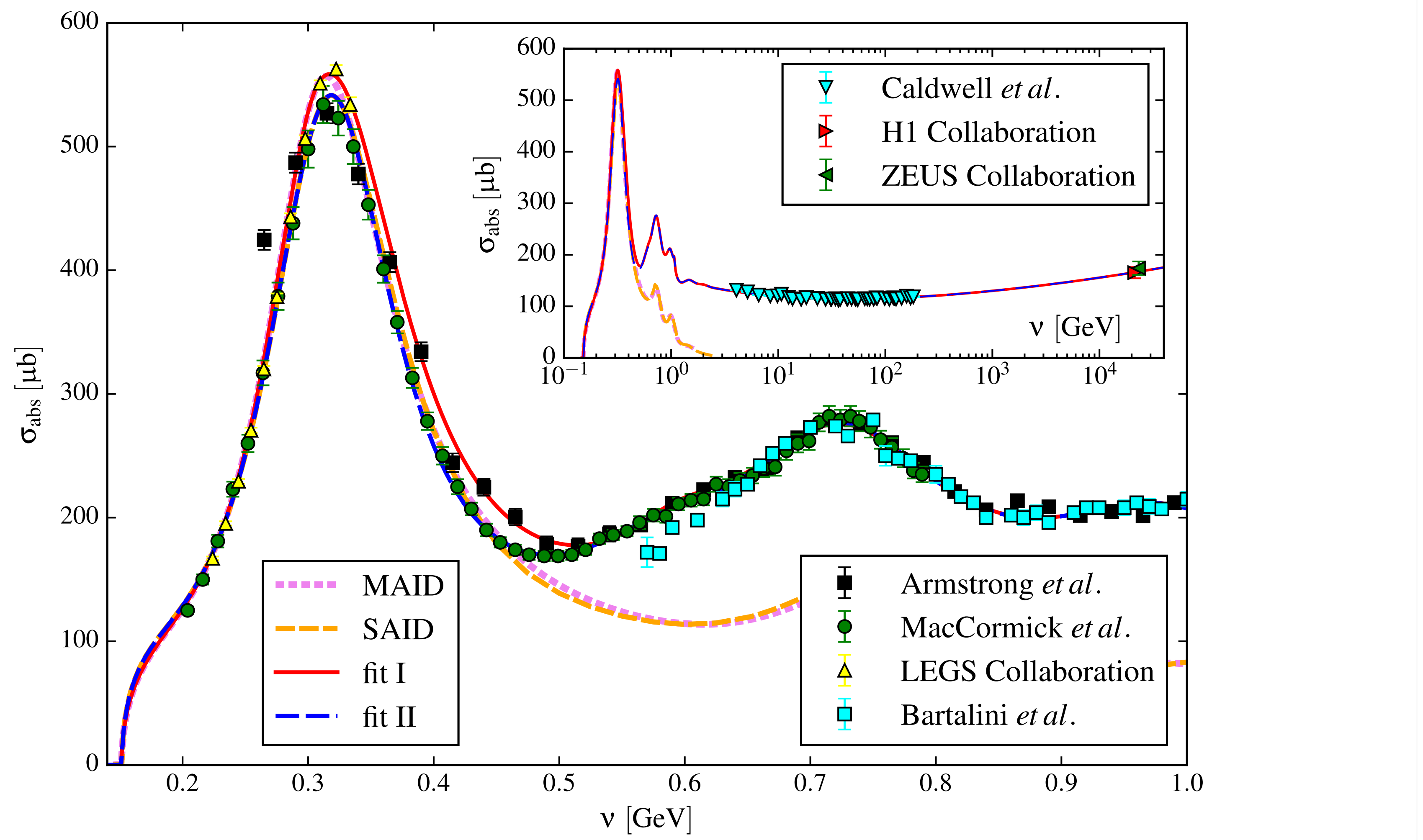
$O(\nu)$ $O(\nu^3)$ $O(\nu^5)$

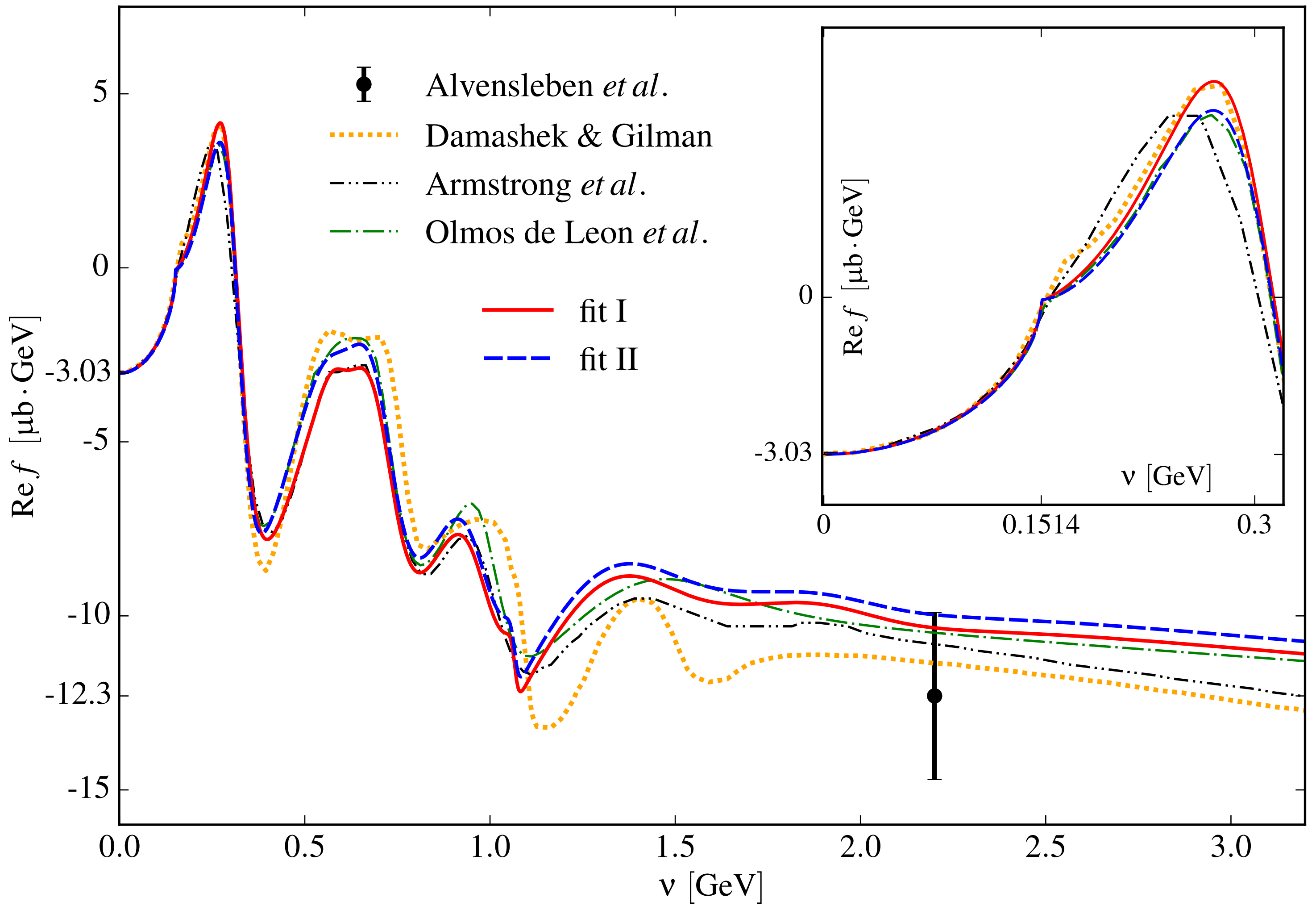
	I_{GDH} (μb)	γ_0 (10^{-6} fm^4)	$\bar{\gamma}_0$ (10^{-6} fm^6)
GDH & A2 [9, 11]	≈ 212	≈ -86	
Helbing [21]	$212 \pm 6 \pm 12$		
Bianchi-Thomas [24]	207 ± 23		
Pasquini *et al.* [12]	$210 \pm 6 \pm 14$	$-90 \pm 8 \pm 11$	$60 \pm 7 \pm 7$
This work	204.5 ± 21.4	-92.9 ± 10.5	48.4 ± 8.2
GDH sum rule	204.784481(4)[a]		
BχPT [15]		-90 ± 140	110 ± 50
HBχPT [17]		-260 ± 190	

Gryniuk, Hagelstein & V.P., PRD (2015), (2016)



EVALUATION OF THE FORWARD COMPTON SCATTERING ... PHYSICAL REVIEW D **92,** 074031 (2015)





Cottingham formula and TPE contribution in hydrogen

$$\delta M = -\int \frac{\mathrm{d}^4 q}{(2\pi)^4\, i}\, \frac{g_{\mu\nu} T^{\mu\nu}(q,p)}{q^2 + i0^+}$$

$$\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, \frac{\left(Q^2 - 2\nu^2\right) T_1(\nu, Q^2) - \left(Q^2 + \nu^2\right) T_2(\nu, Q^2)}{Q^4 (Q^4 - 4m^2\nu^2)}$$

$$\frac{E_{\mathrm{HFS}}(nS)}{E_F(nS)} = \frac{4m}{\mu}\, \frac{1}{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, \frac{1}{Q^4 - 4m^2\nu^2} \left\{ \frac{(2Q^2 - \nu^2)}{Q^2} S_1(\nu, Q^2) + 3\frac{\nu}{M} S_2(\nu, Q^2) \right\}$$

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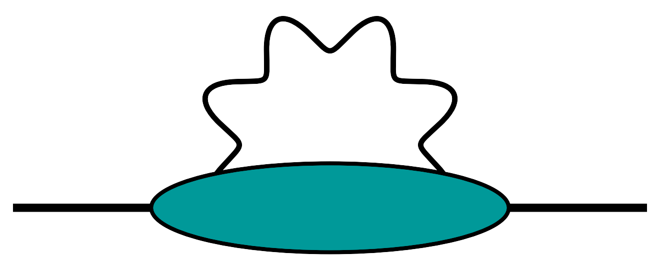

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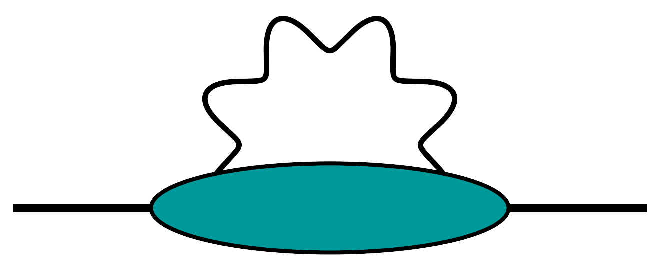
Cottingham formula and TPE contribution in hydrogen



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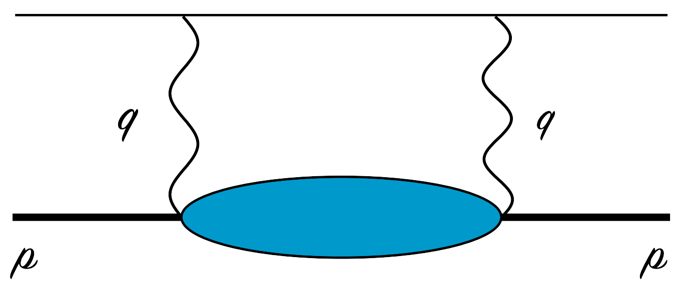

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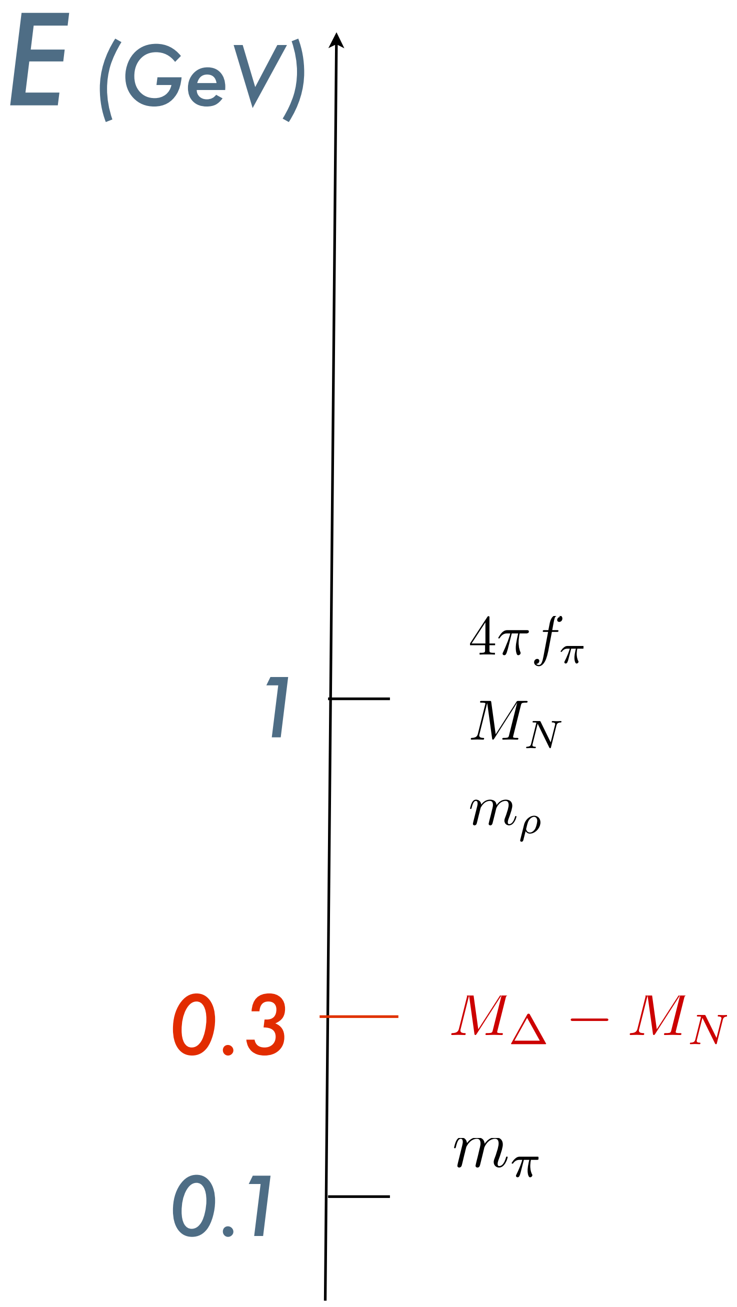
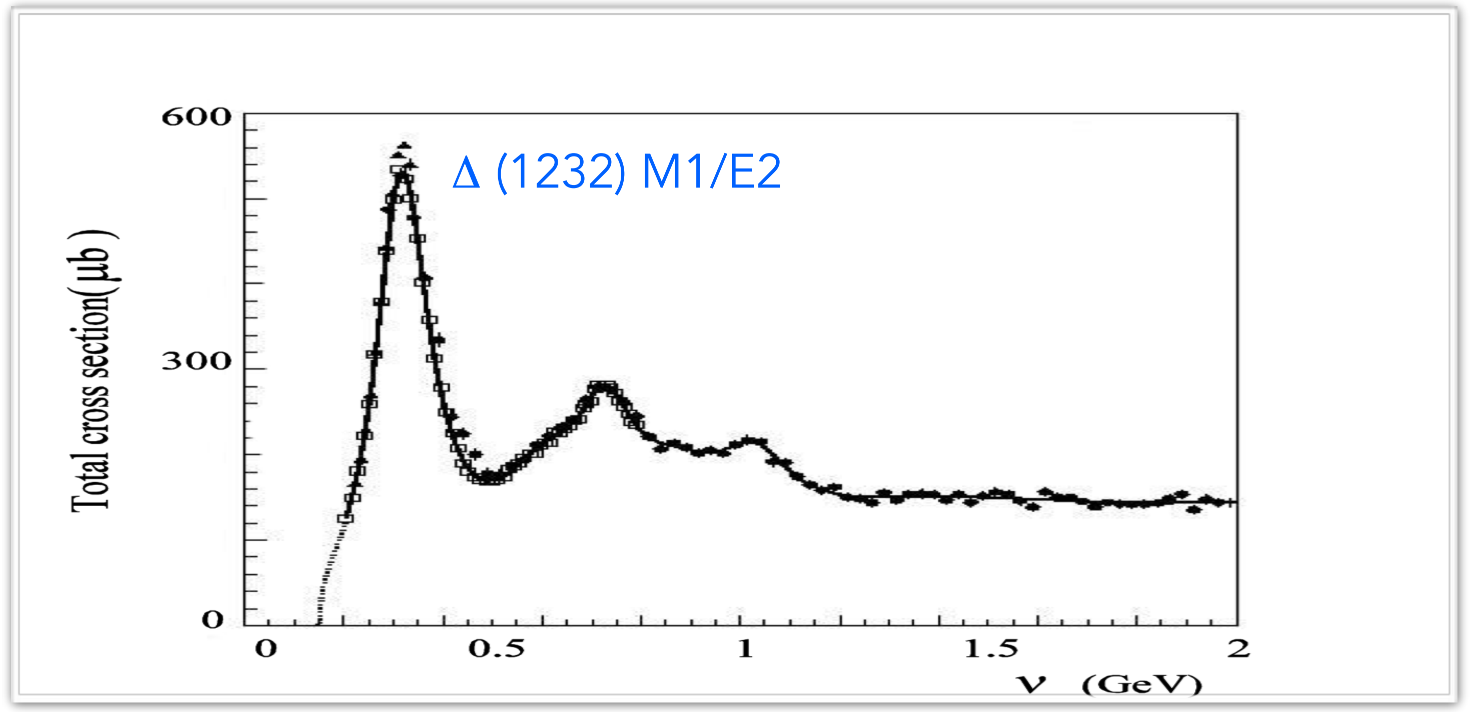
Baryon ChPT

pion cloud + Delta(1232) excitation

Jenkins & Manohar, PLB (1991)
Hemmert, Holstein, Kambor, JPhysG (1998)
V.P. & Phillips, PRC (2003)





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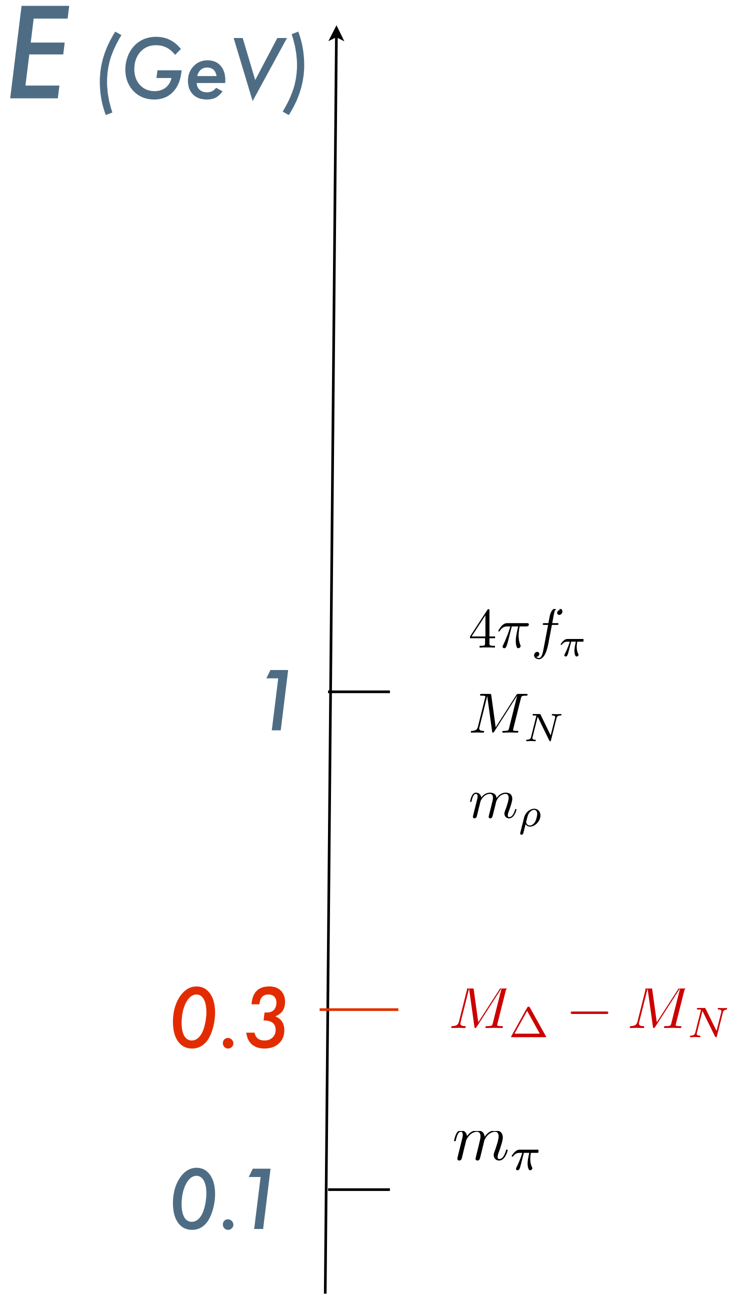
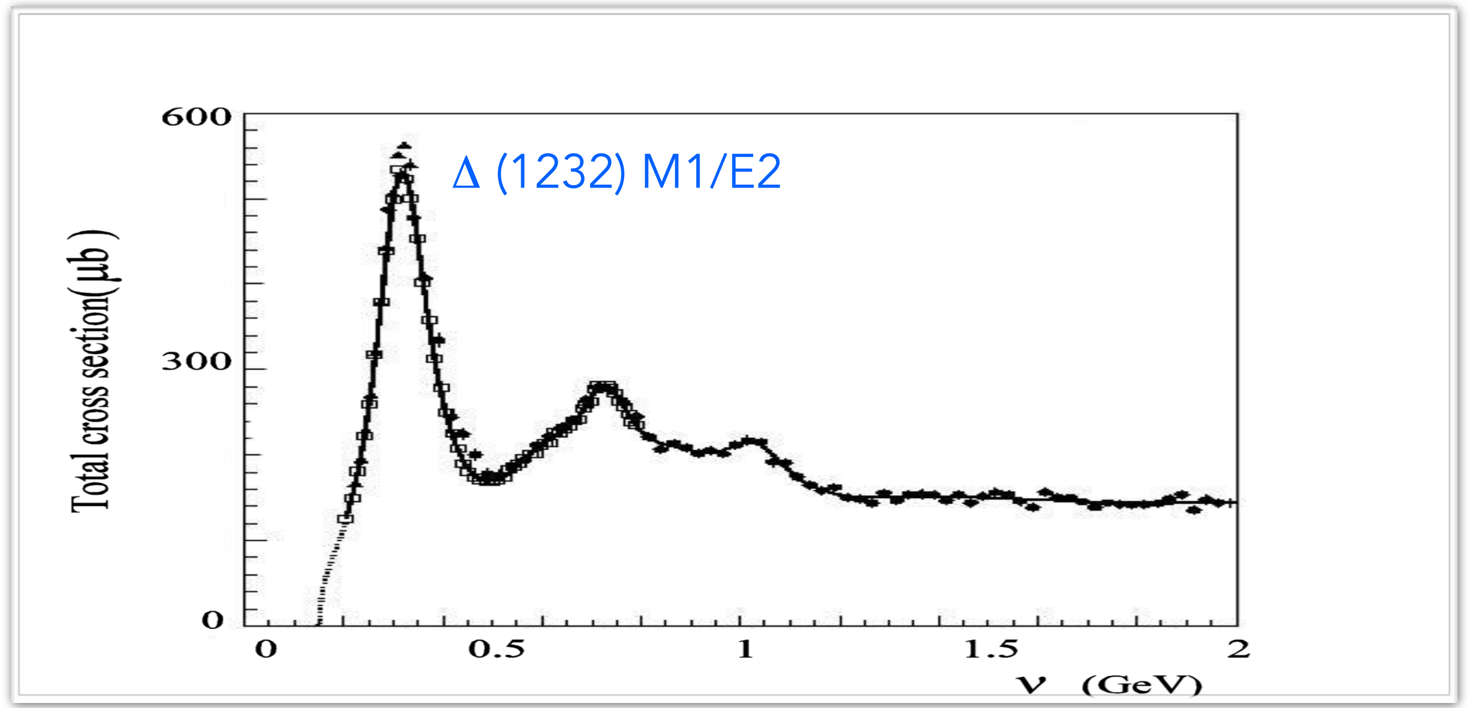
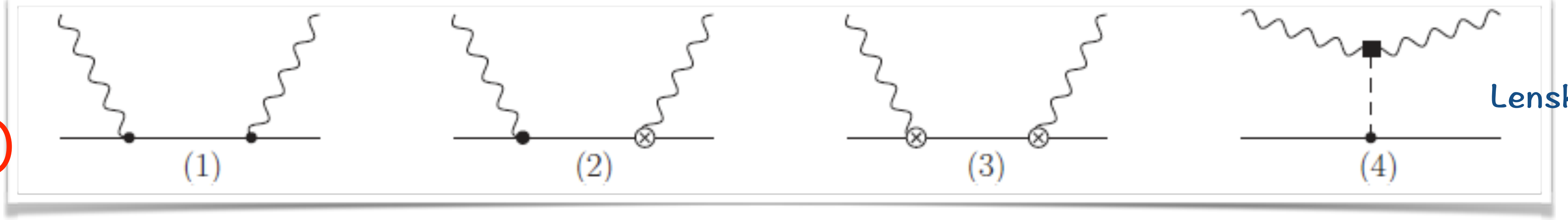




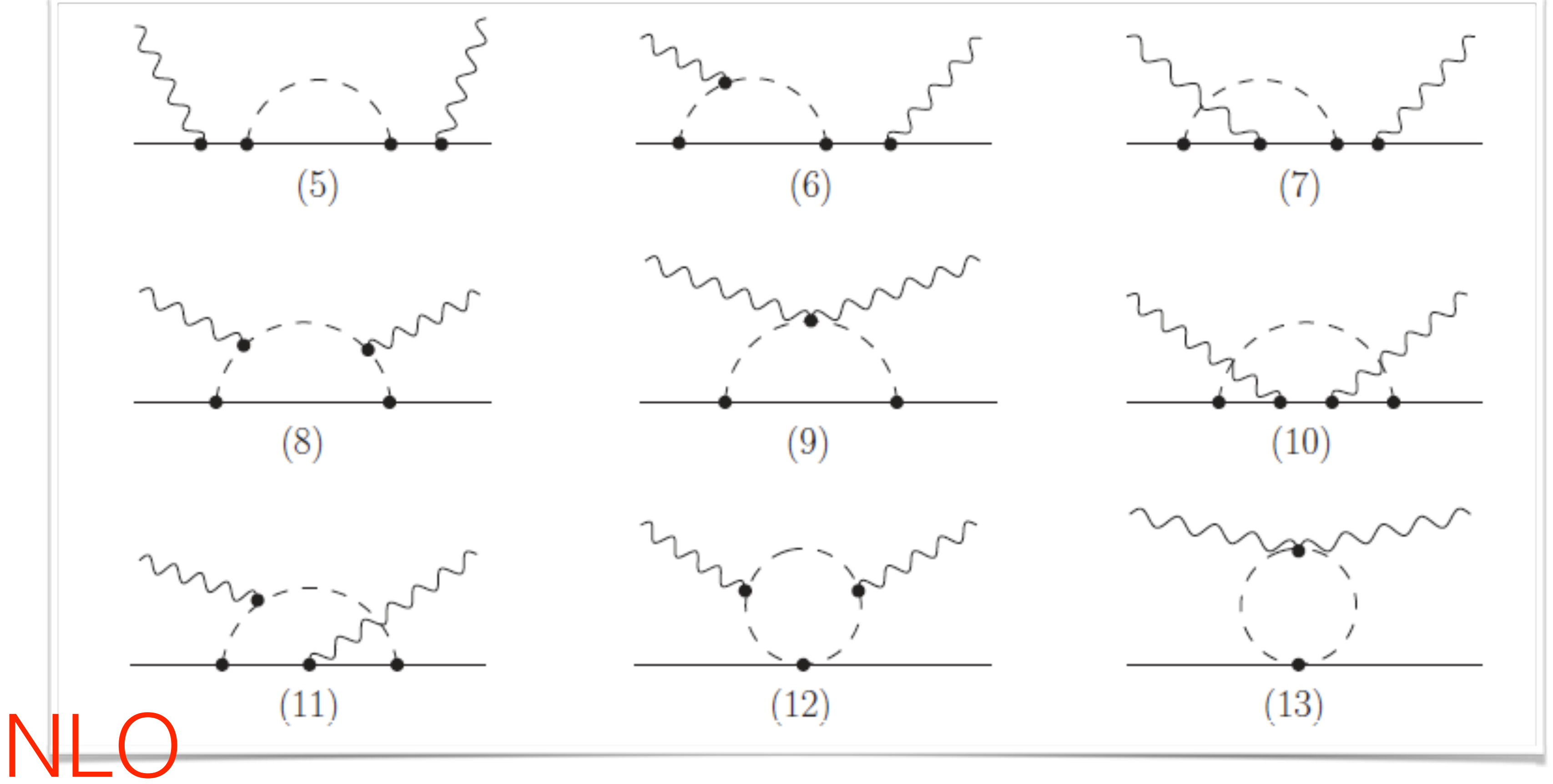
- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE, ``delta-counting”) depends on what chiral order is assigned to the excitation scale.

Chiral EFT of Compton scattering off protons

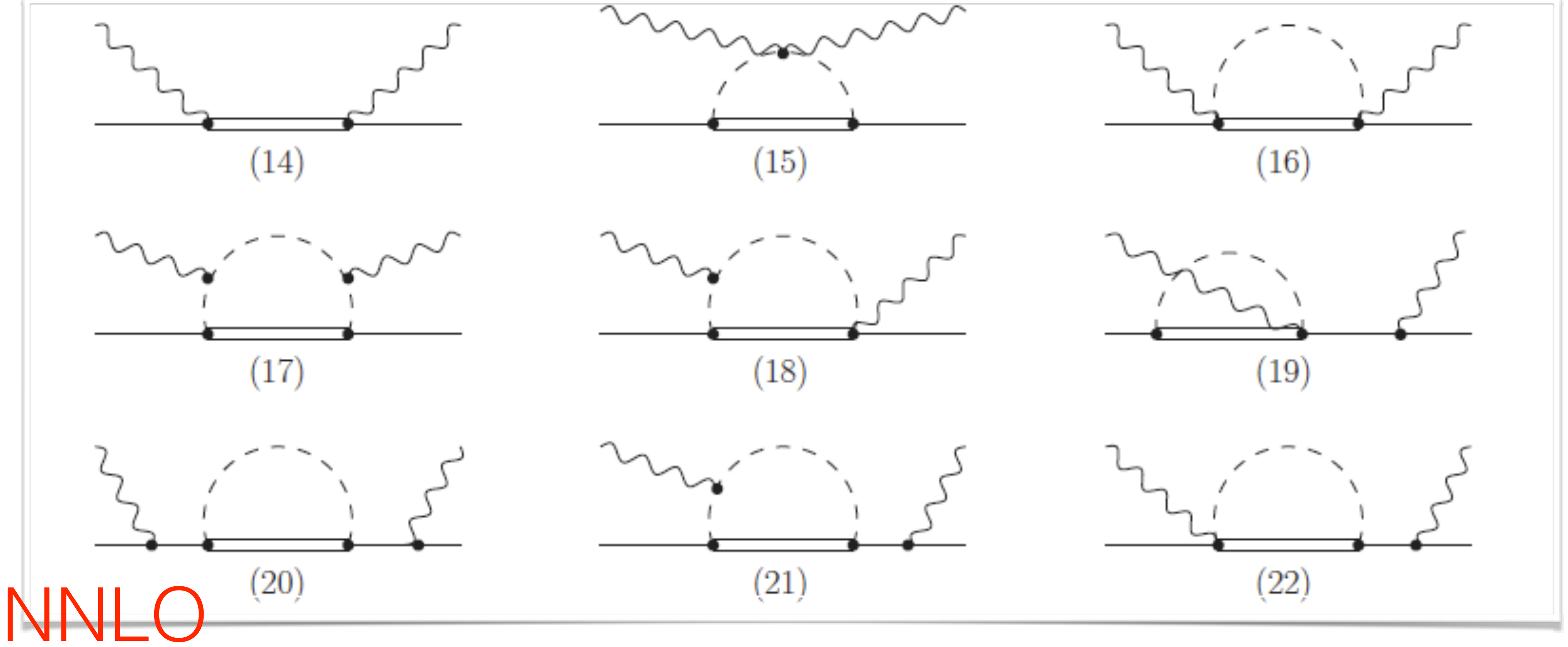
LO



Lensky & V.P., EPJC (2010);
Lensky, McGovern & V.P., EPJC (2015)

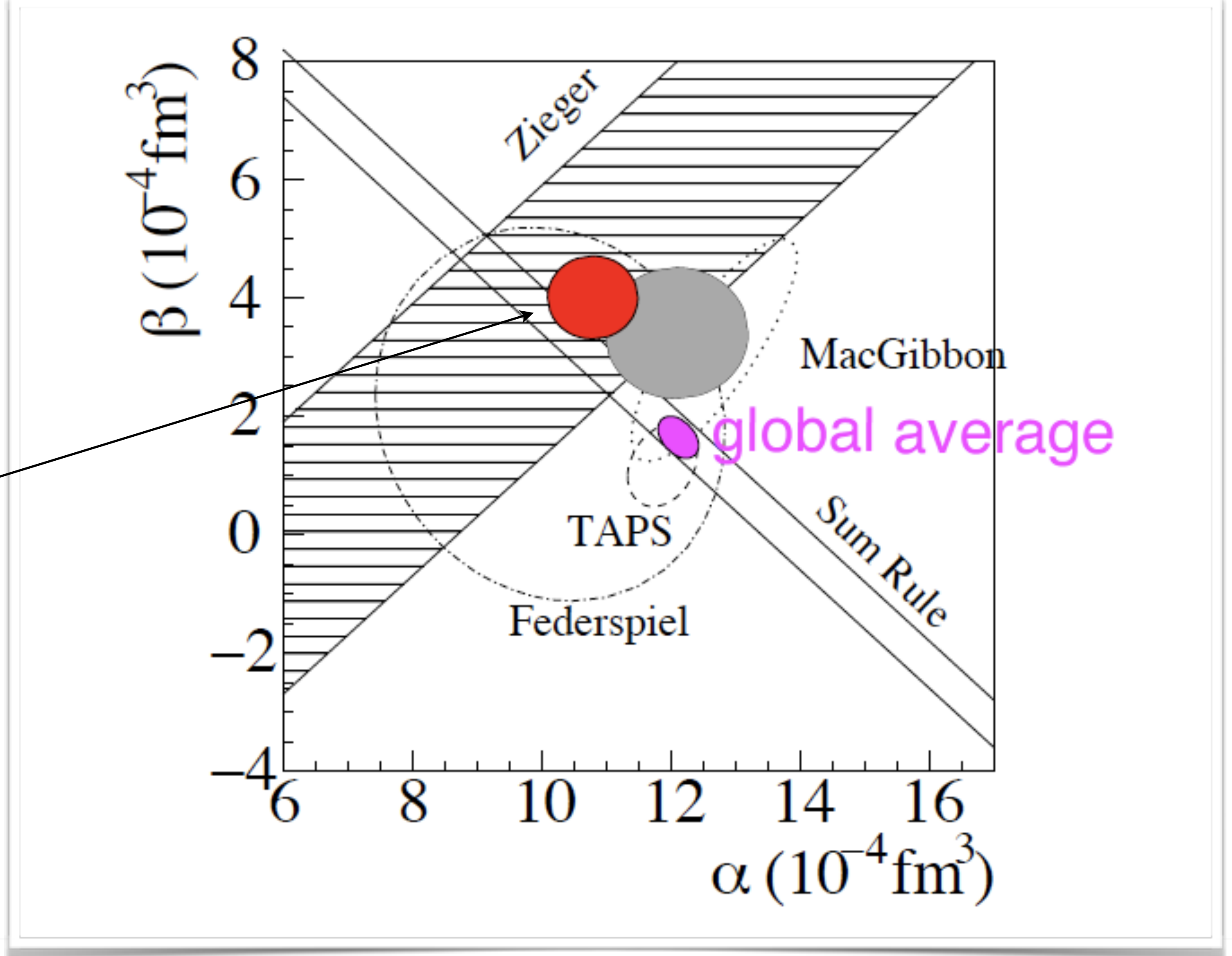


NLO

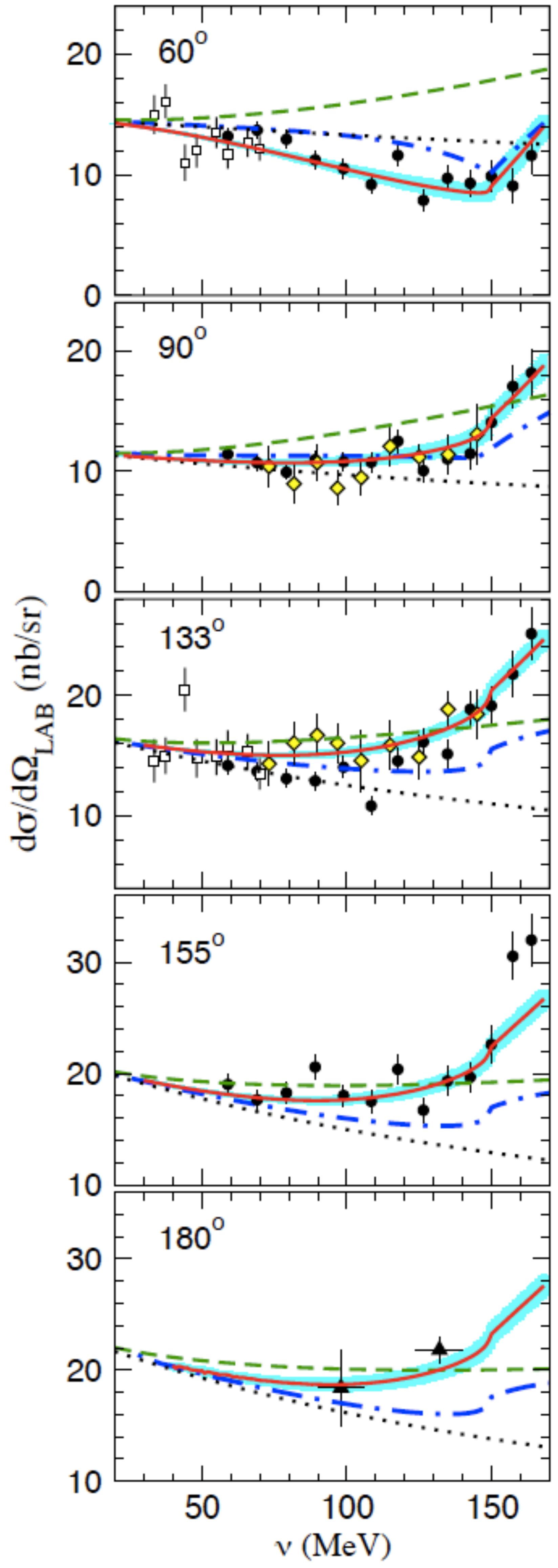


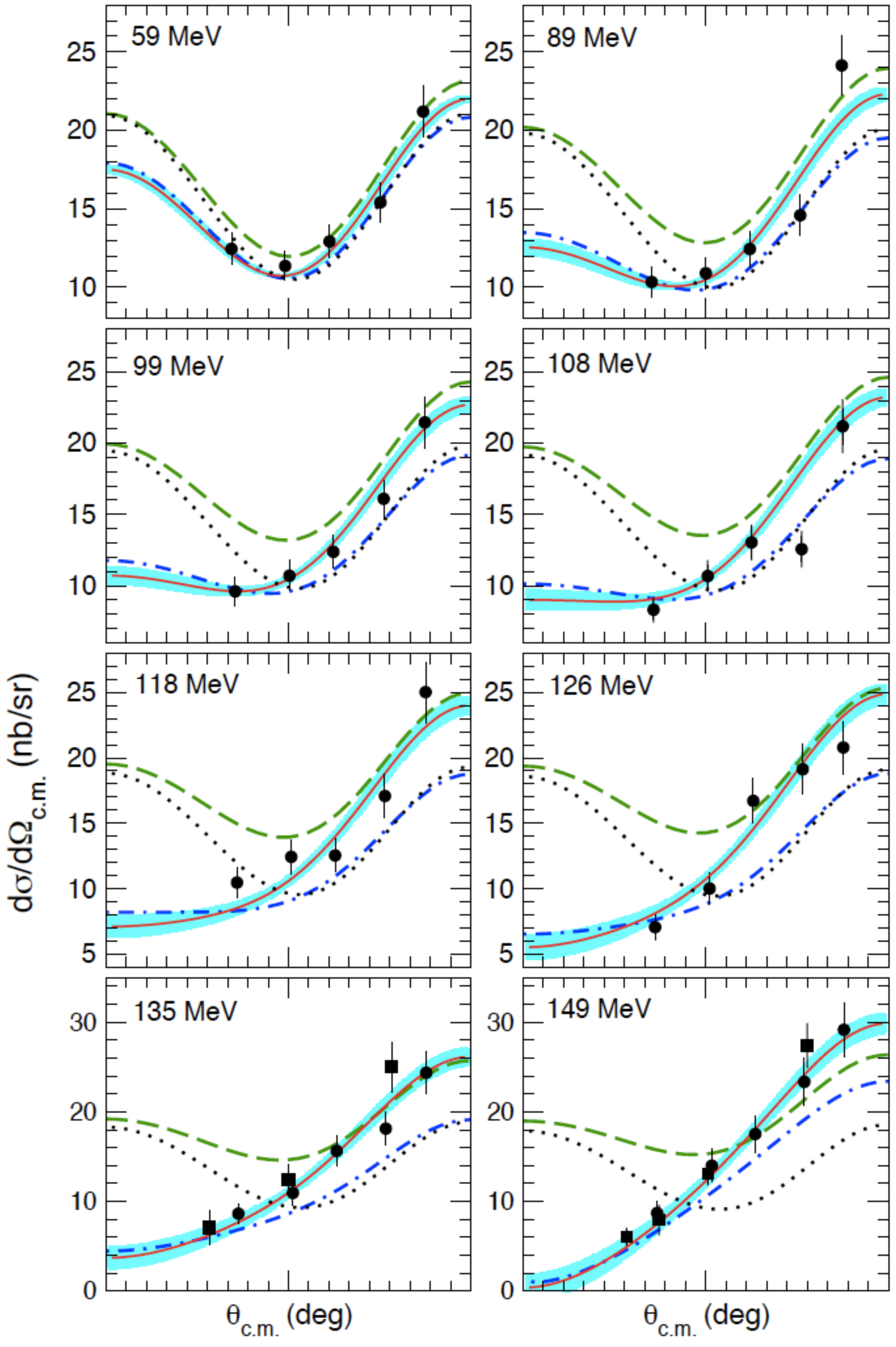
NNLO

$\mathscr{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$fm
$\mathscr{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathscr{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathscr{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$ size of the red blob



Unpolarized cross sections for RCS





Data points:
MAMI/TAPS (2001)
SAL (1993)

Klein-Nishina

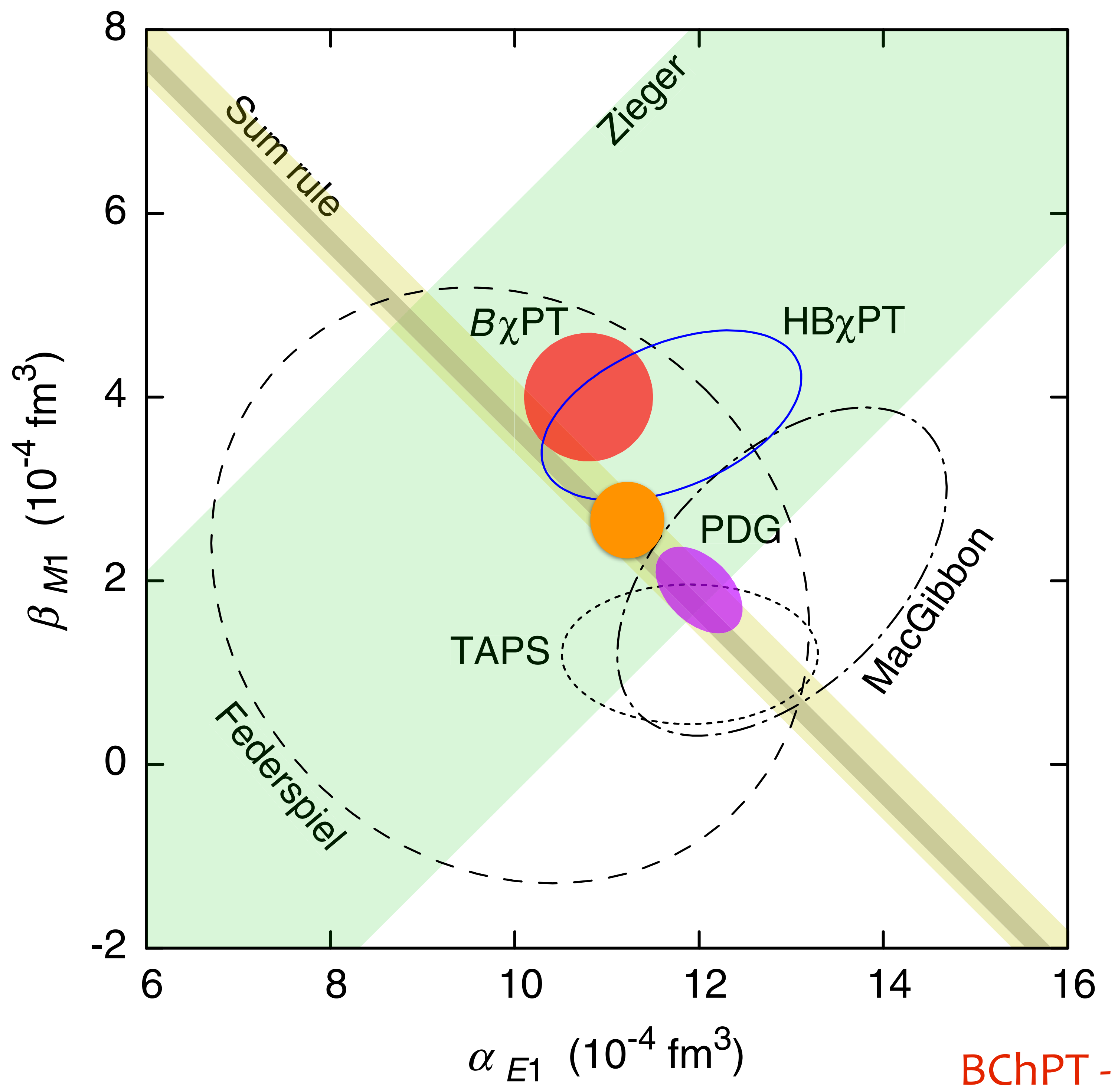
Born + WZW

+ p-qube

Total NNLO

Lensky & V.P.,EPJC (2010)

Proton polarizabilities from Compton scattering

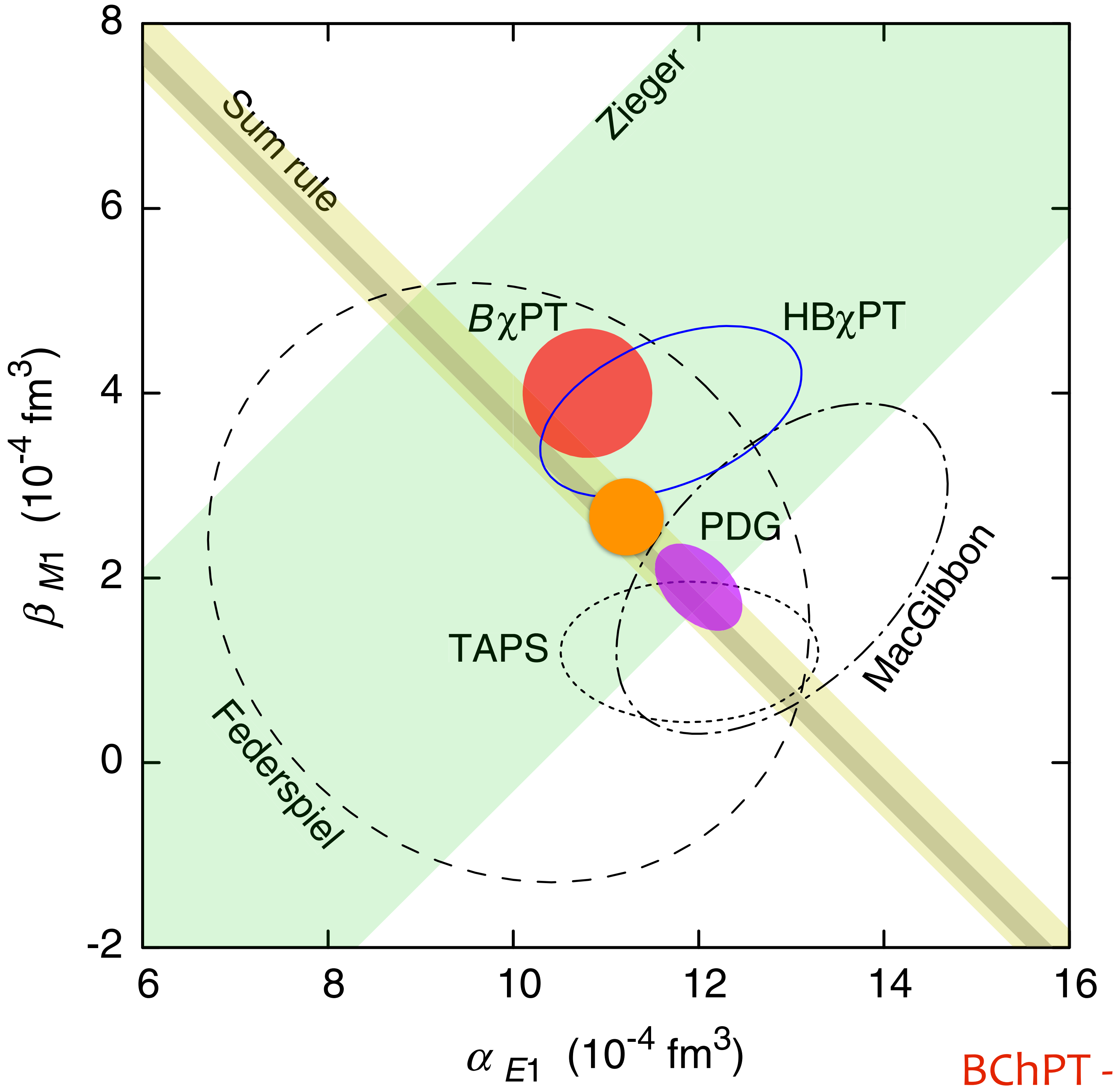


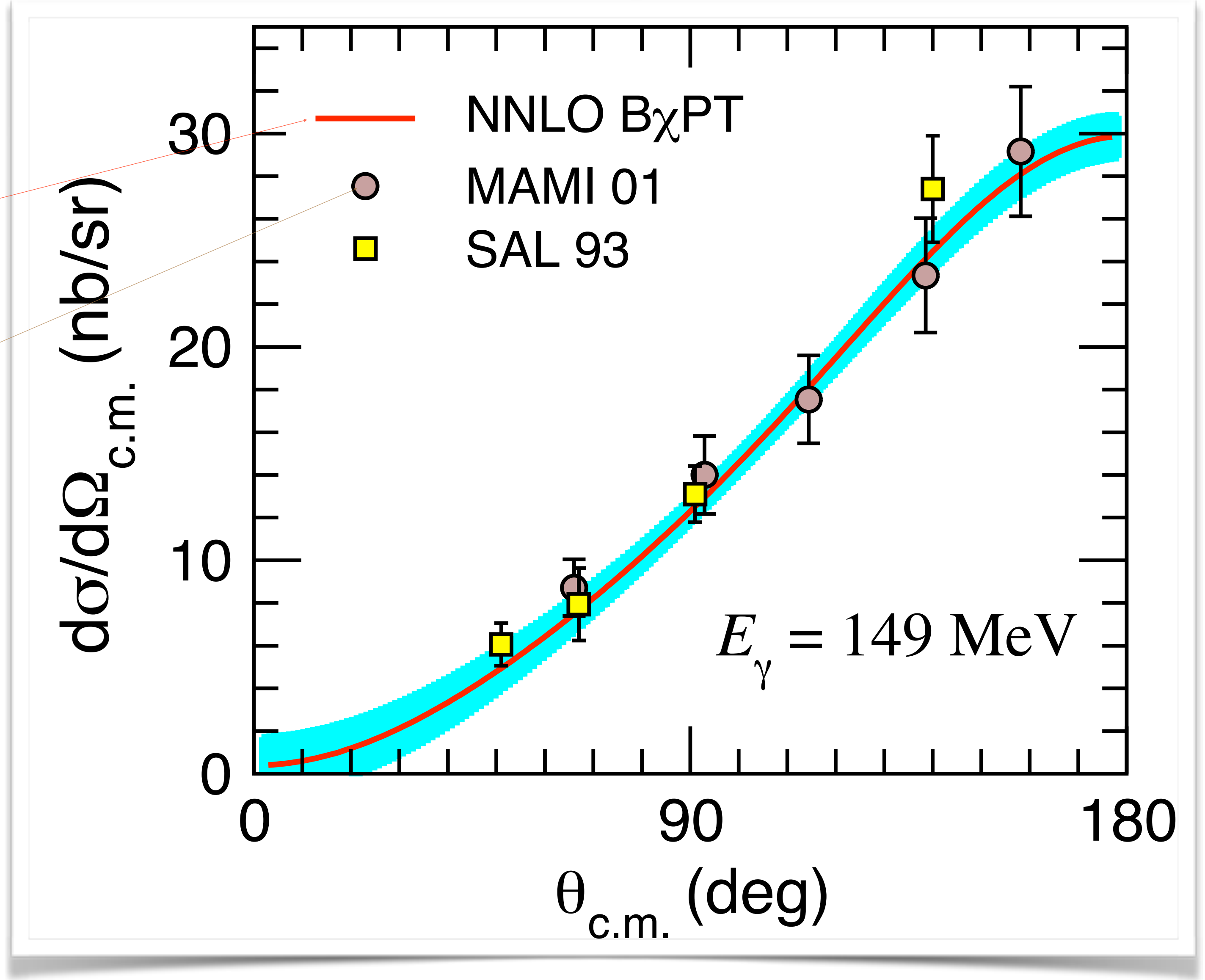
BChPT - Lensky & V.P., EPJC (2010)
HBChPT - Griesshammer, McGovern, Phillips, EPJA (2013)

Mainz program to determine scalar polarizabilities:

Krupina & V.P., PRL (2013)
Sokhoyan et al [A2 Collaboration], EPJA 53 (2017)
Sokhoyan et al (2016), Accepted Proposal
Krupina, Lensky & V.P, PWA of RCS, in progress.

Proton polarizabilities from Compton scattering





BChPT - Lensky & V.P., EPJC (2010)
HBChPT - Griesshammer, McGovern, Phillips, EPJA (2013)

$$\beta_{M1} = (1.9 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{PDG}]$$

$$\beta_{M1} = (4.0 \pm 0.7) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{BChPT@NNLO}]$$

PDG adjusted values
from 2012 edition (purple) to
2013 on-line edition (orange)

Mainz program to determine scalar polarizabilities:

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Moments of structure functions

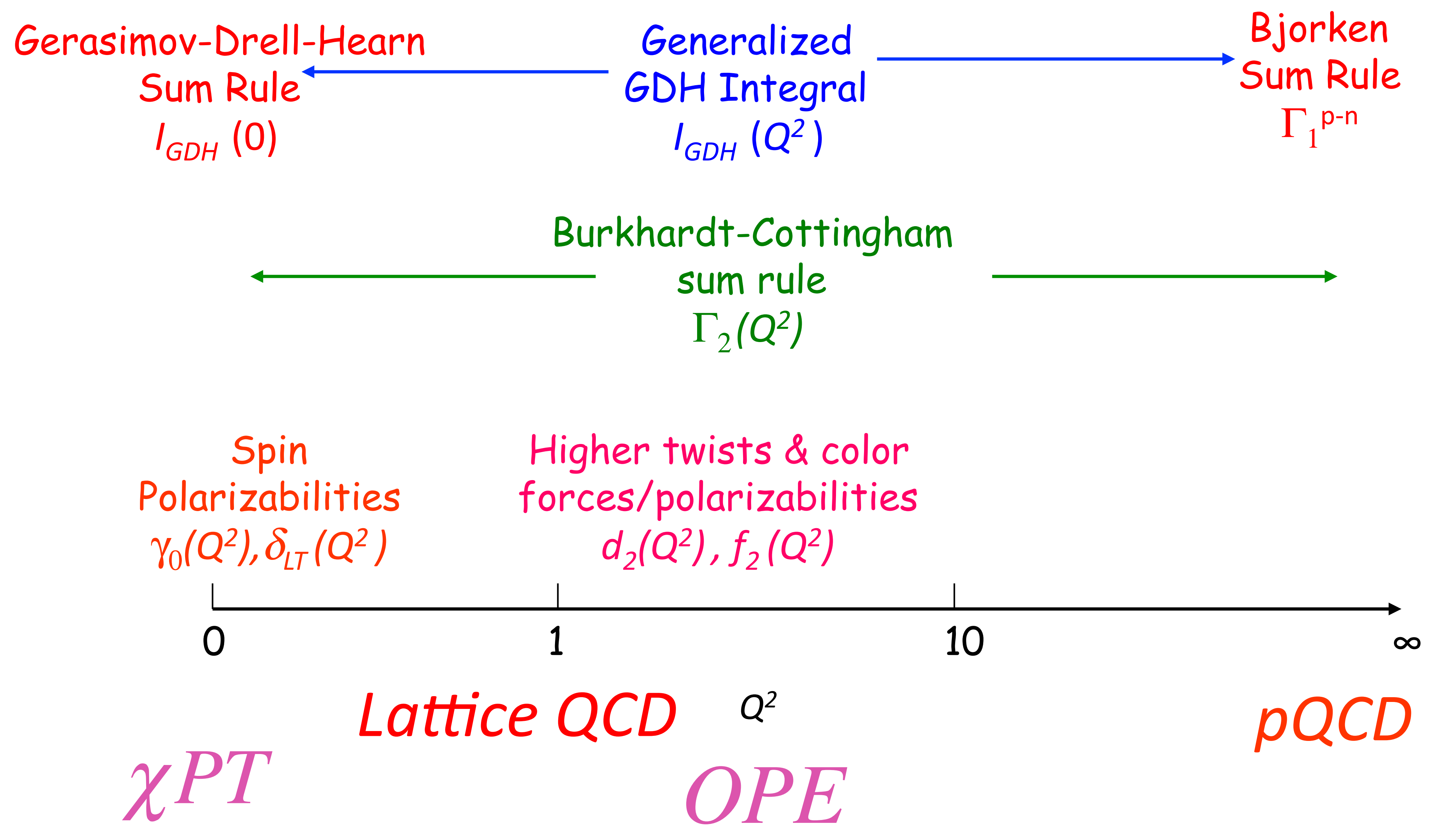


figure from Z.E. Meziani

low Q:
how nucleon
spin affects
atomic systems

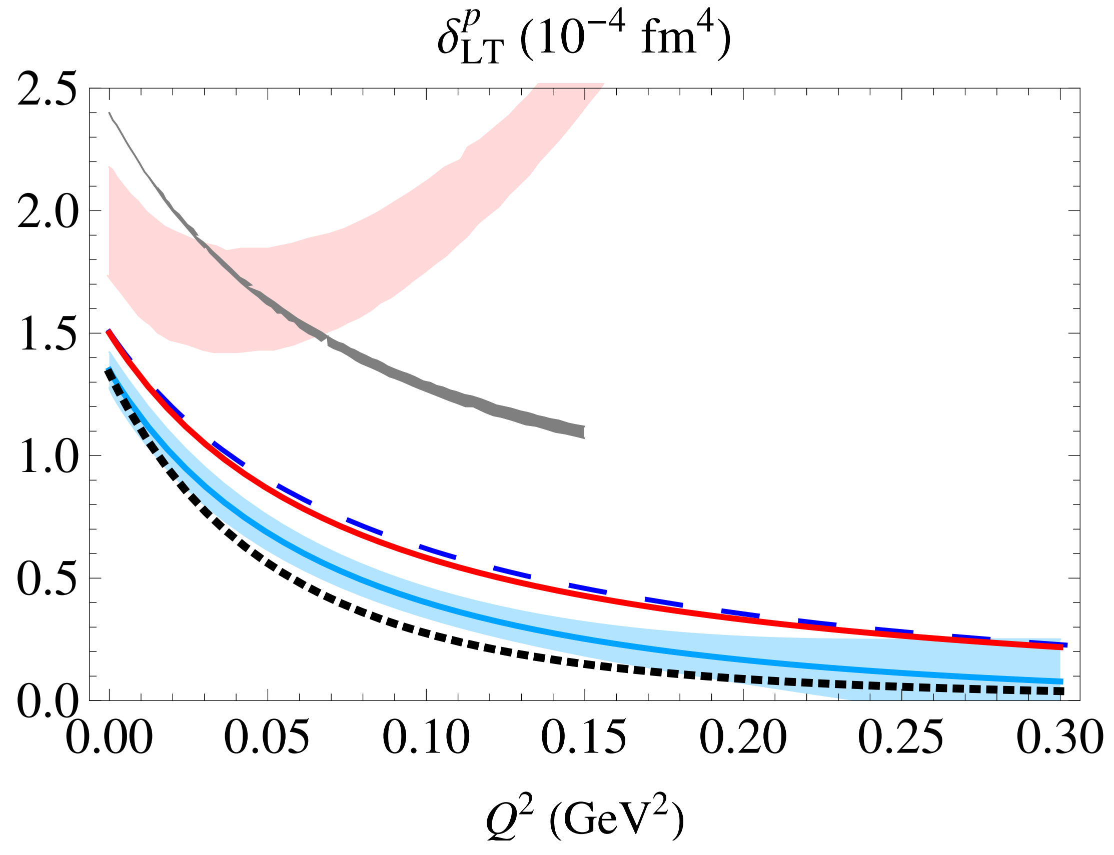
high Q:
how it is distributed
among constituents,
quarks and gluons

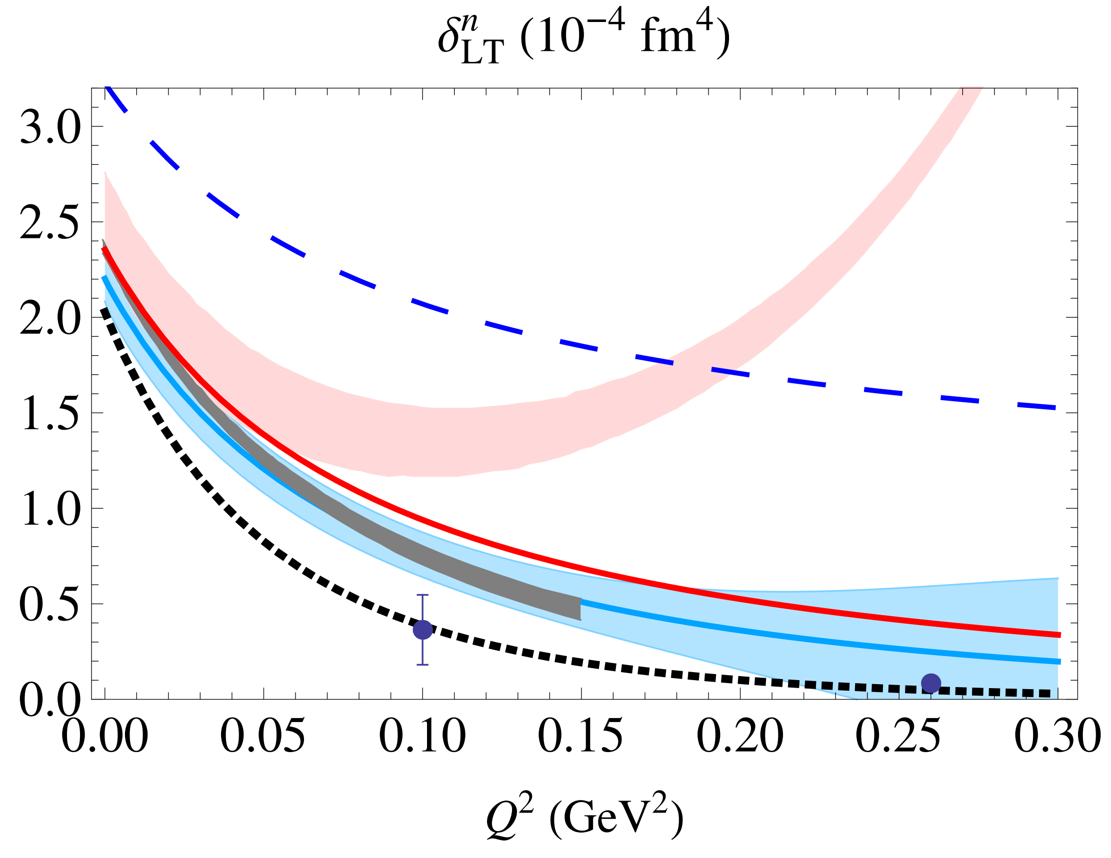
Relation to polarizabilities:

$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4}\int_0^{x_0} \mathrm{d}x\, x F_1(x,Q^2),$$

$$\gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 g_{TT}(x,Q^2), \qquad g_{TT} = g_1 - (4M_N^2 x^2/Q^2) g_2$$

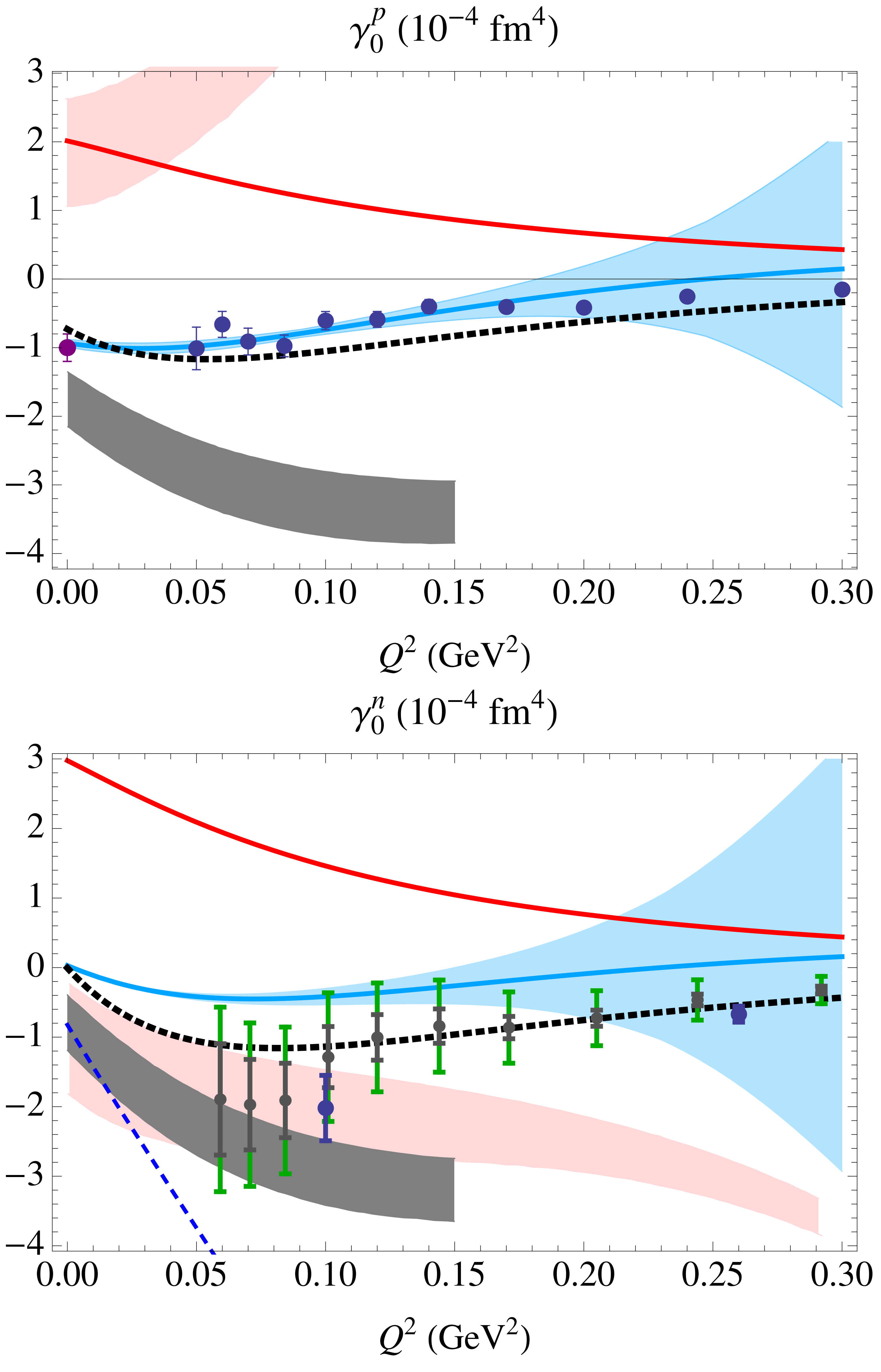
$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 \Big[g_1(x,Q^2) + g_2(x,Q^2)\Big]$$





DeltaLT puzzle — none of chiral PT calculation describe neutron deltaLT.

Forward spin polarizability at low Q



figures from Lensky, Alarcon & V.P., PRC (2014)
and Alarcon, Hagelstein, Lensky & V.P., in progress

Curves:

MAID (empir.)

LO-HBChPT

NLO-HBChPT

NLO- IRBChPT
[Bernard et al (2006)]

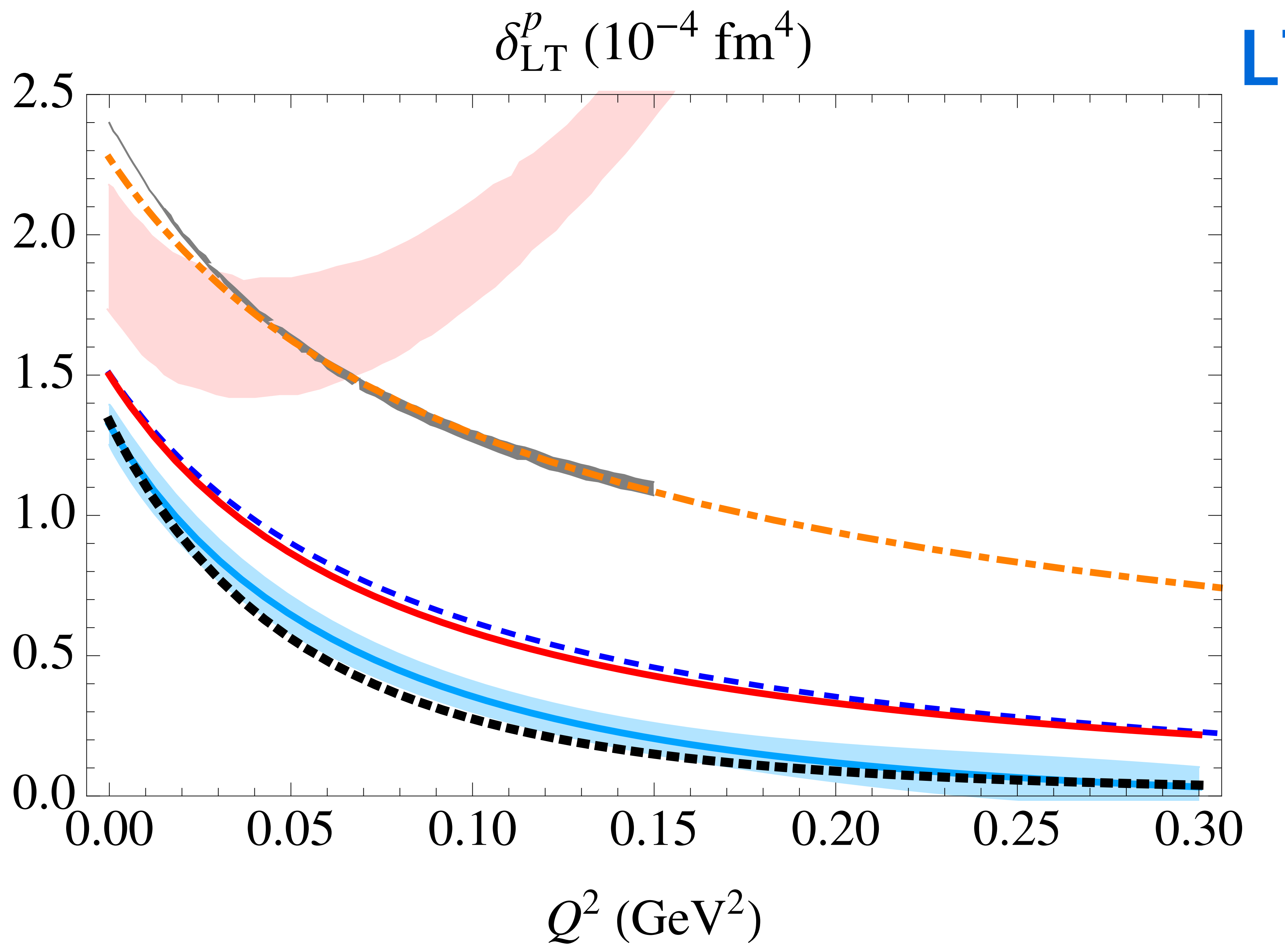
LO-BChPT

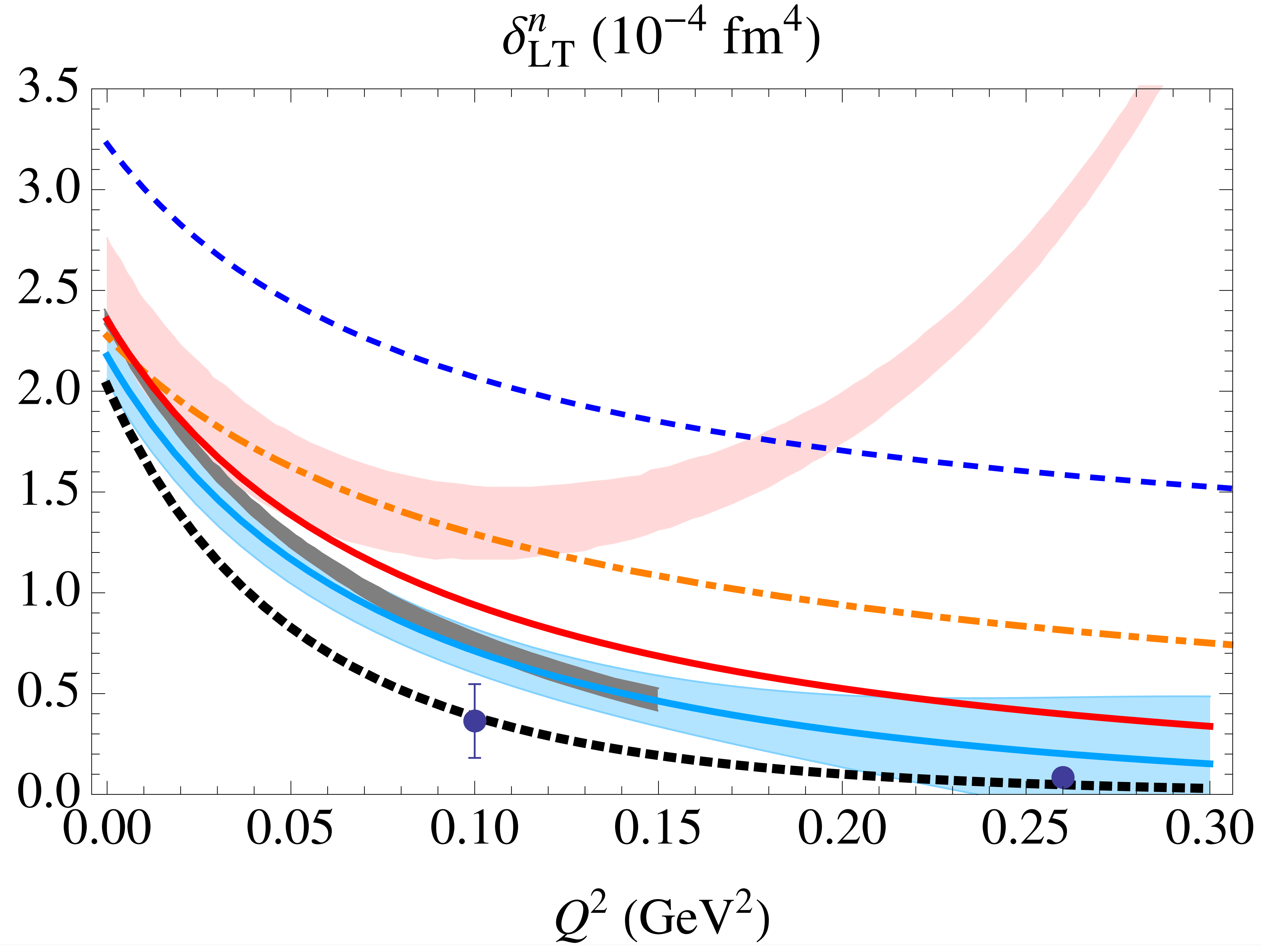
NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

NLO-BChPT
[Bernard et al (2013)]

Data points:
K. Slifer, J.-P. Chen, S. Kuhn, A. Deur et al
[Jefferson Lab Spin Program]

LT spin polarizability





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K. Slifer, J.-P. Chen, S. Kuhn, A. Deur et al
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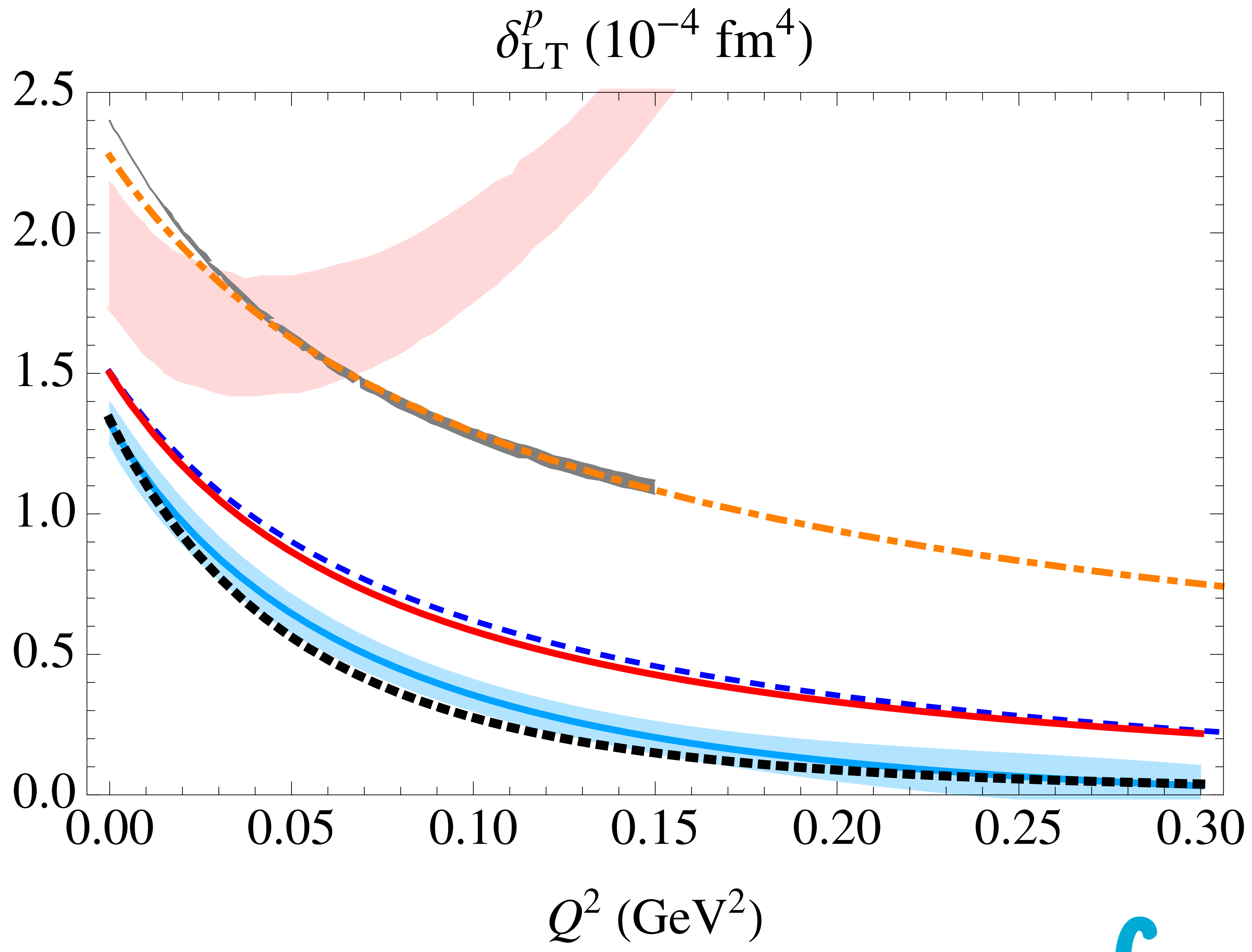
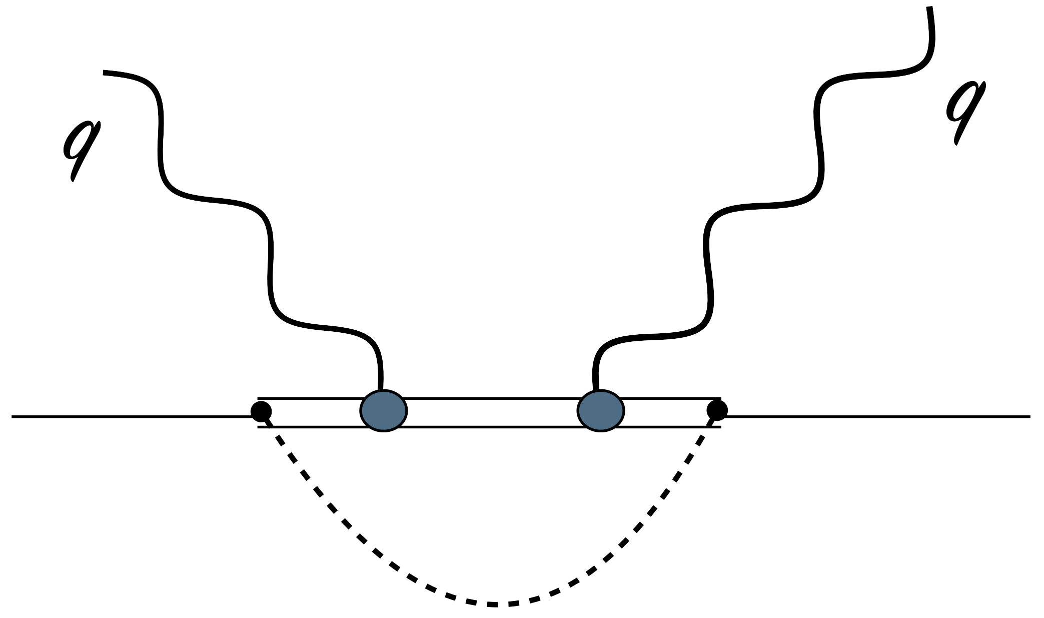
The difference



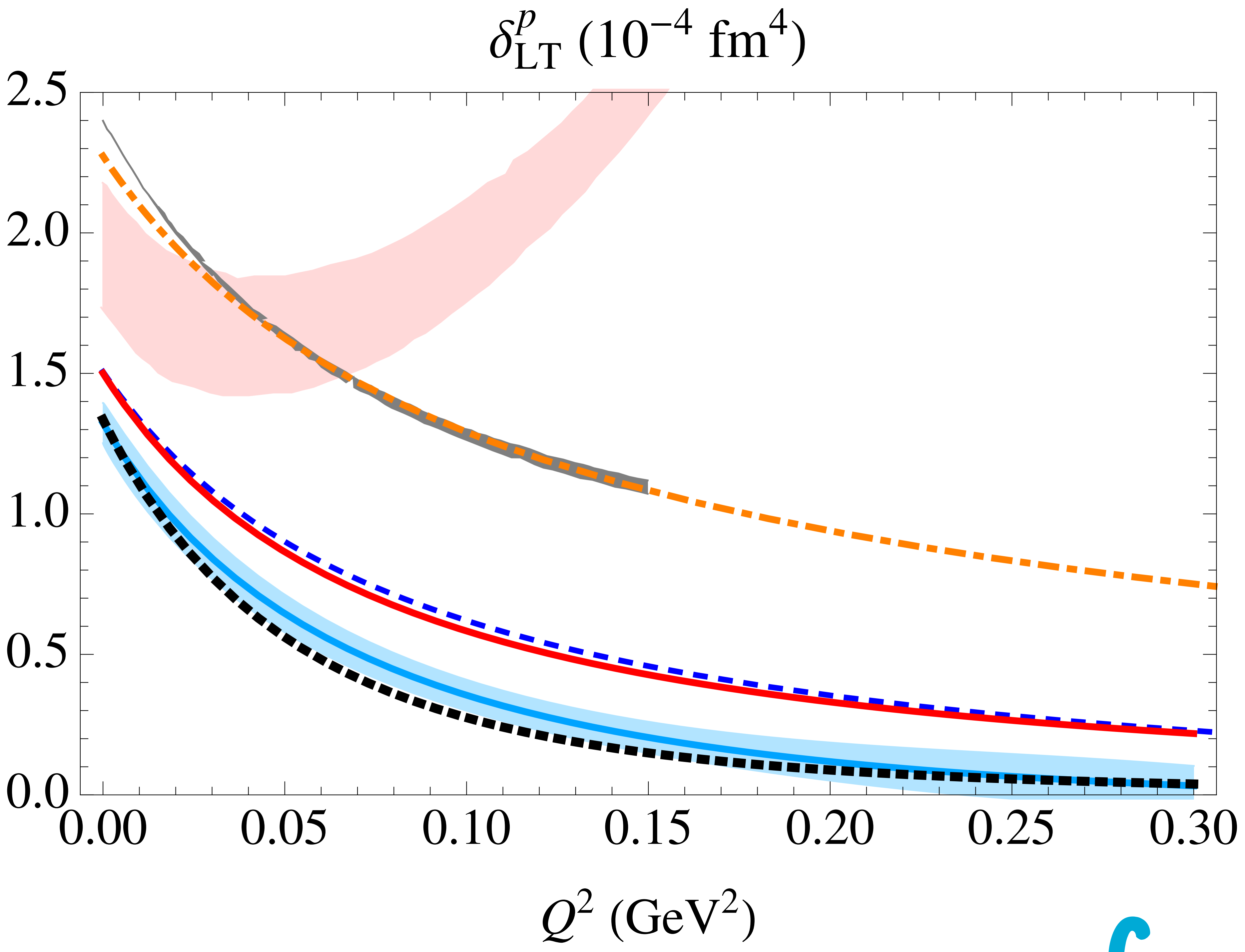
NLO-BChPT
[Bernard, Epelbaum, Krebs, Meissner, EPJA (2013)]

NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

comes from



The difference



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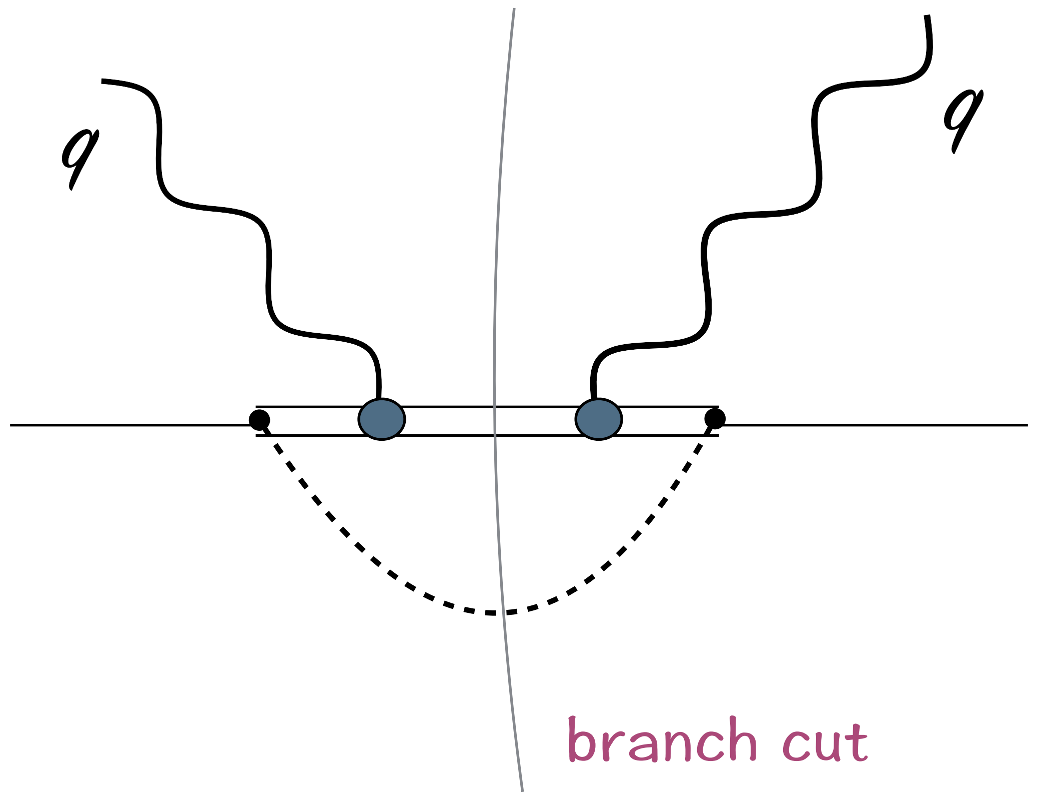

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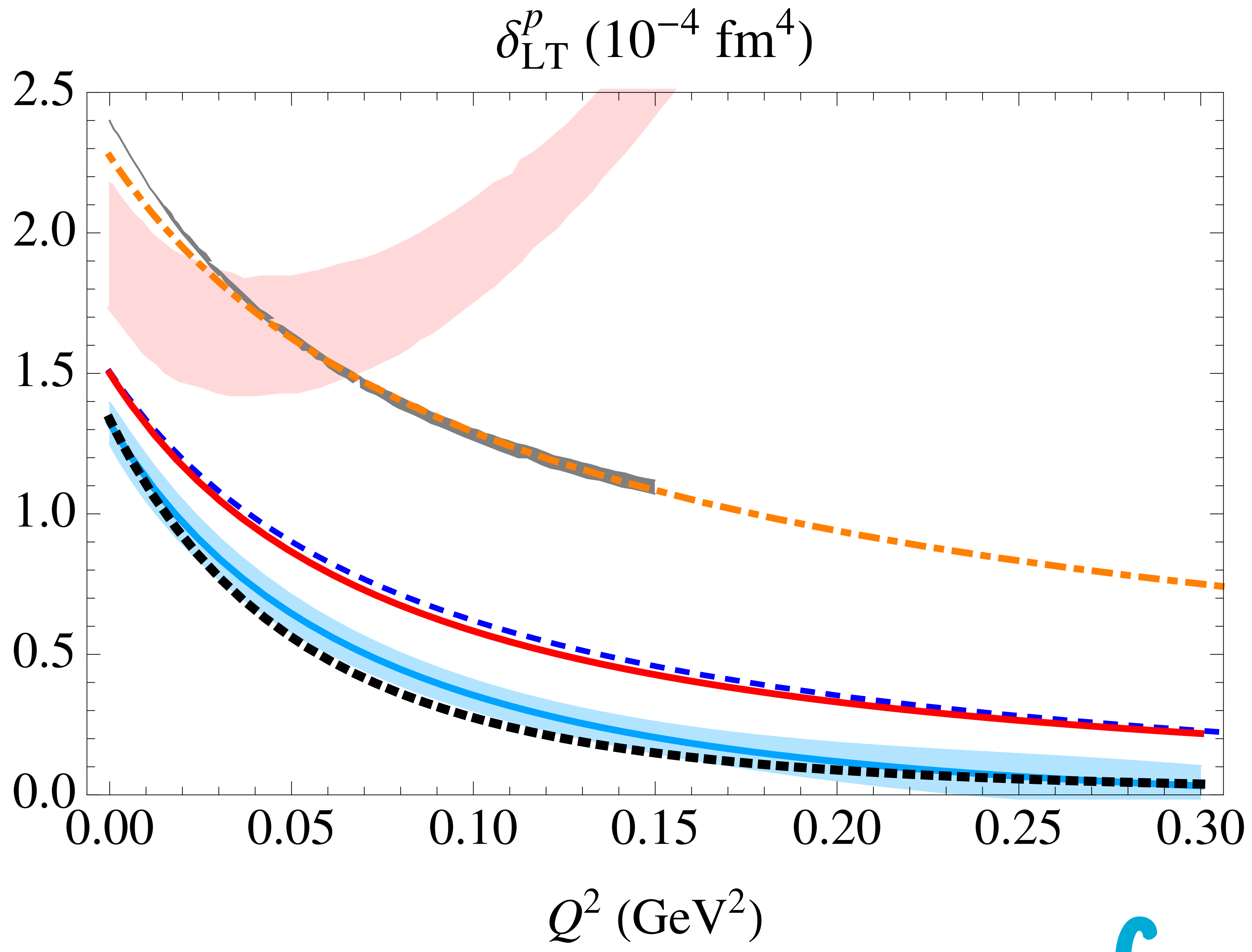

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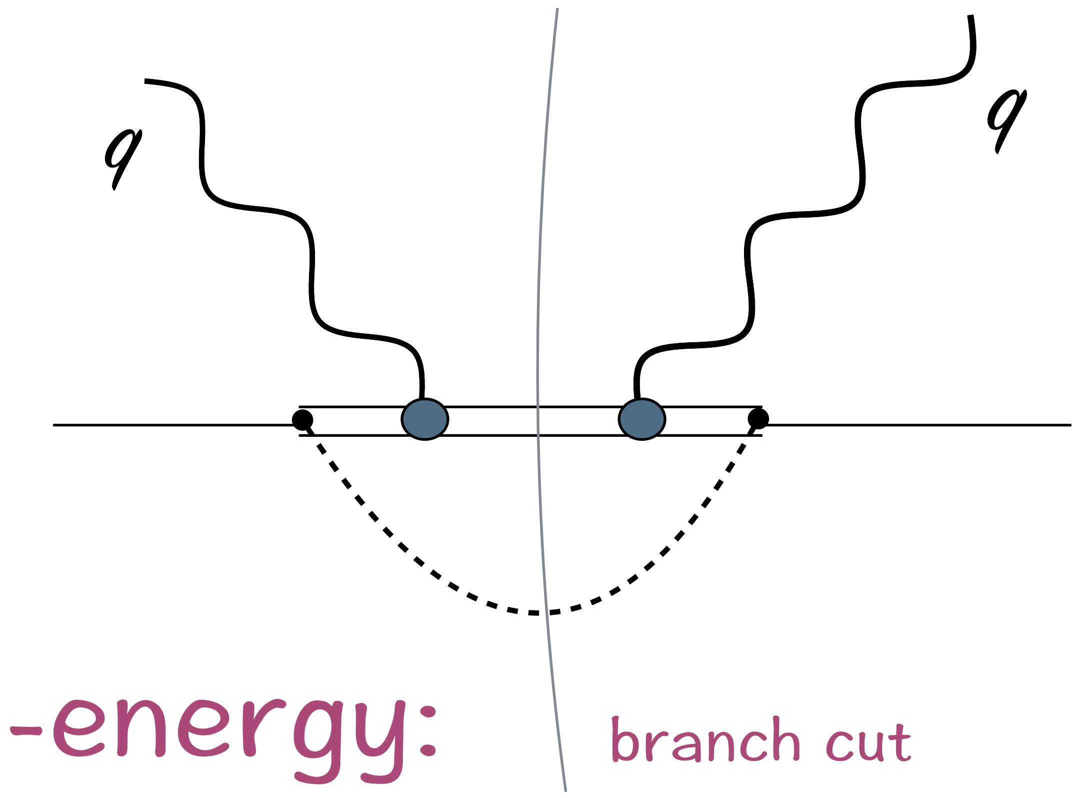
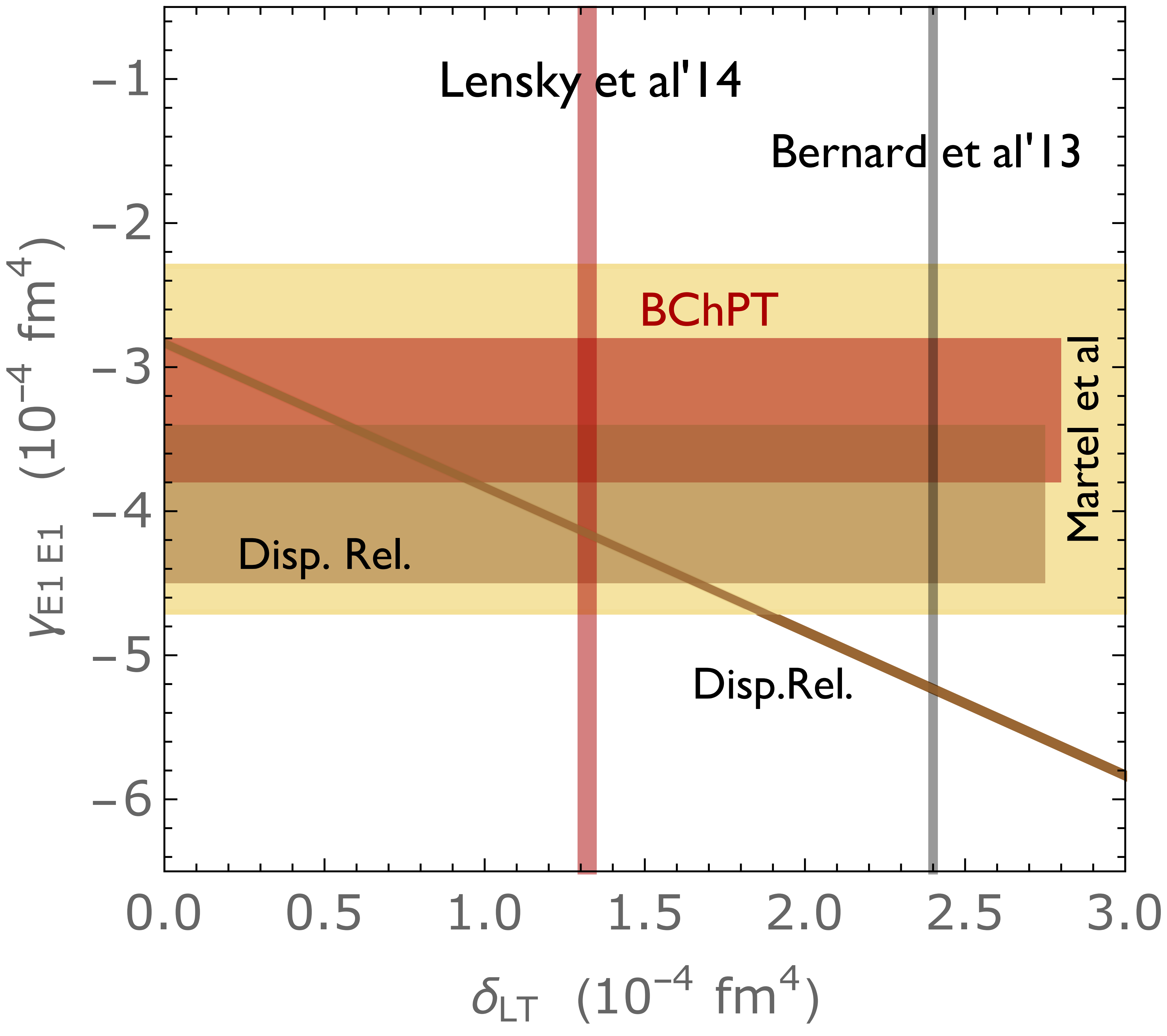


normally suppressed at low-energy:

pi-Delta production << pi-nucleon production

Relations among spin polarizabilities

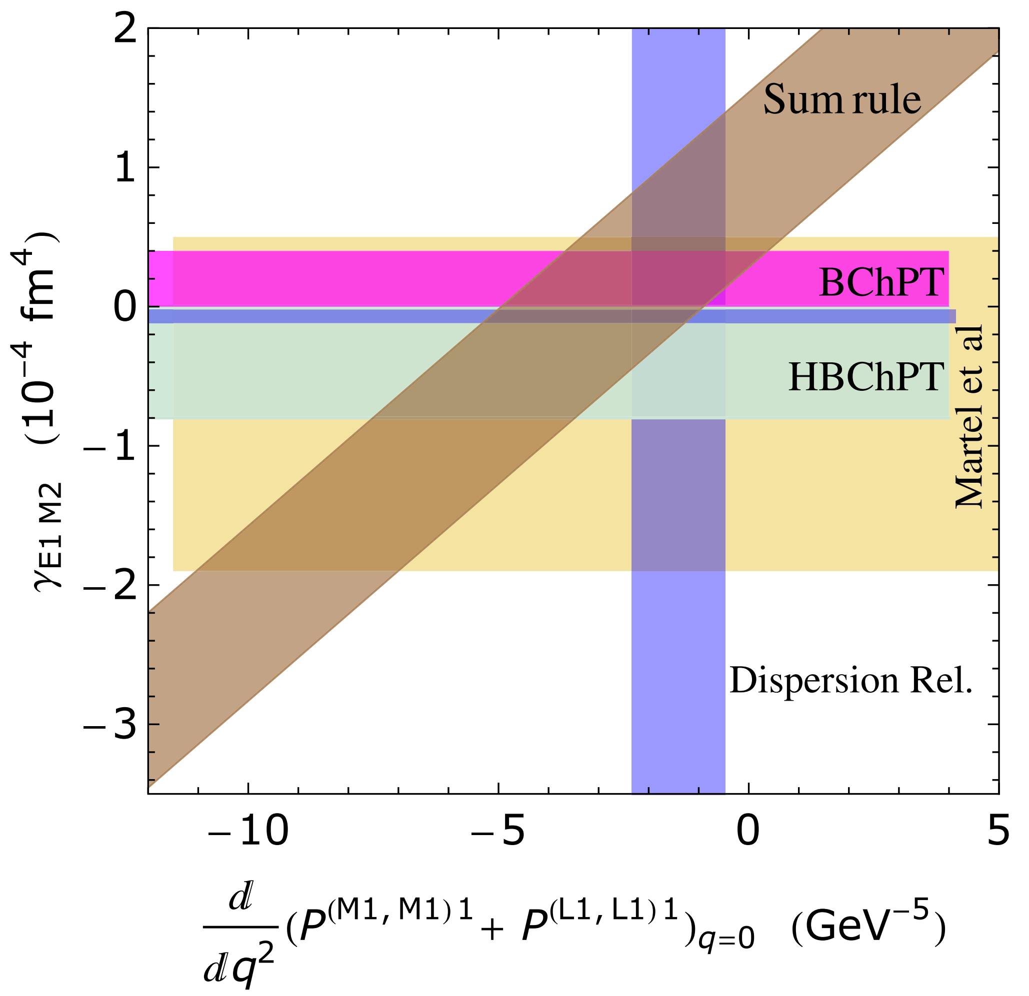
$$\delta_{LT} = -\gamma_{E1E1} + \text{VCS spin GPs}$$

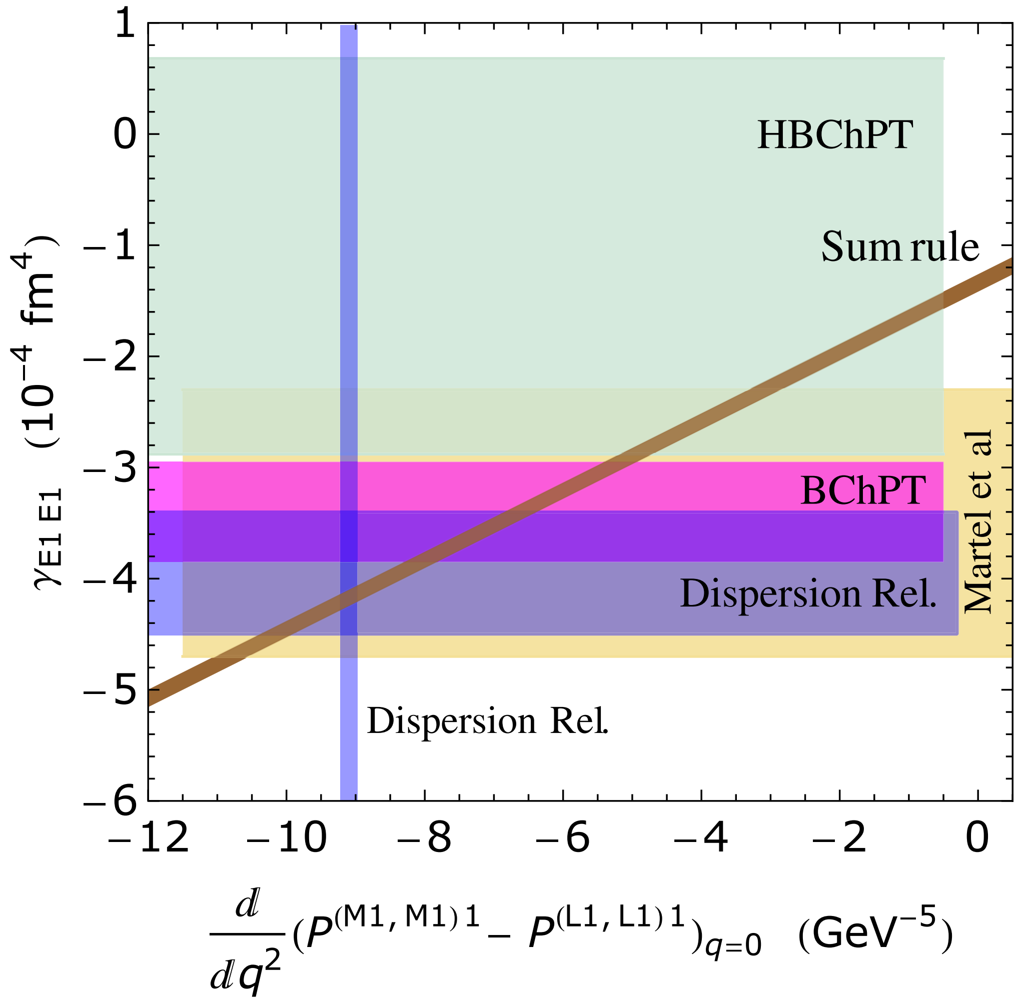


RAPID COMMUNICATIONS

VLADIMIR PASCALUTSA AND MARC VANDERHAEGHEN

PHYSICAL REVIEW D **91,** 051503(R) (2015)





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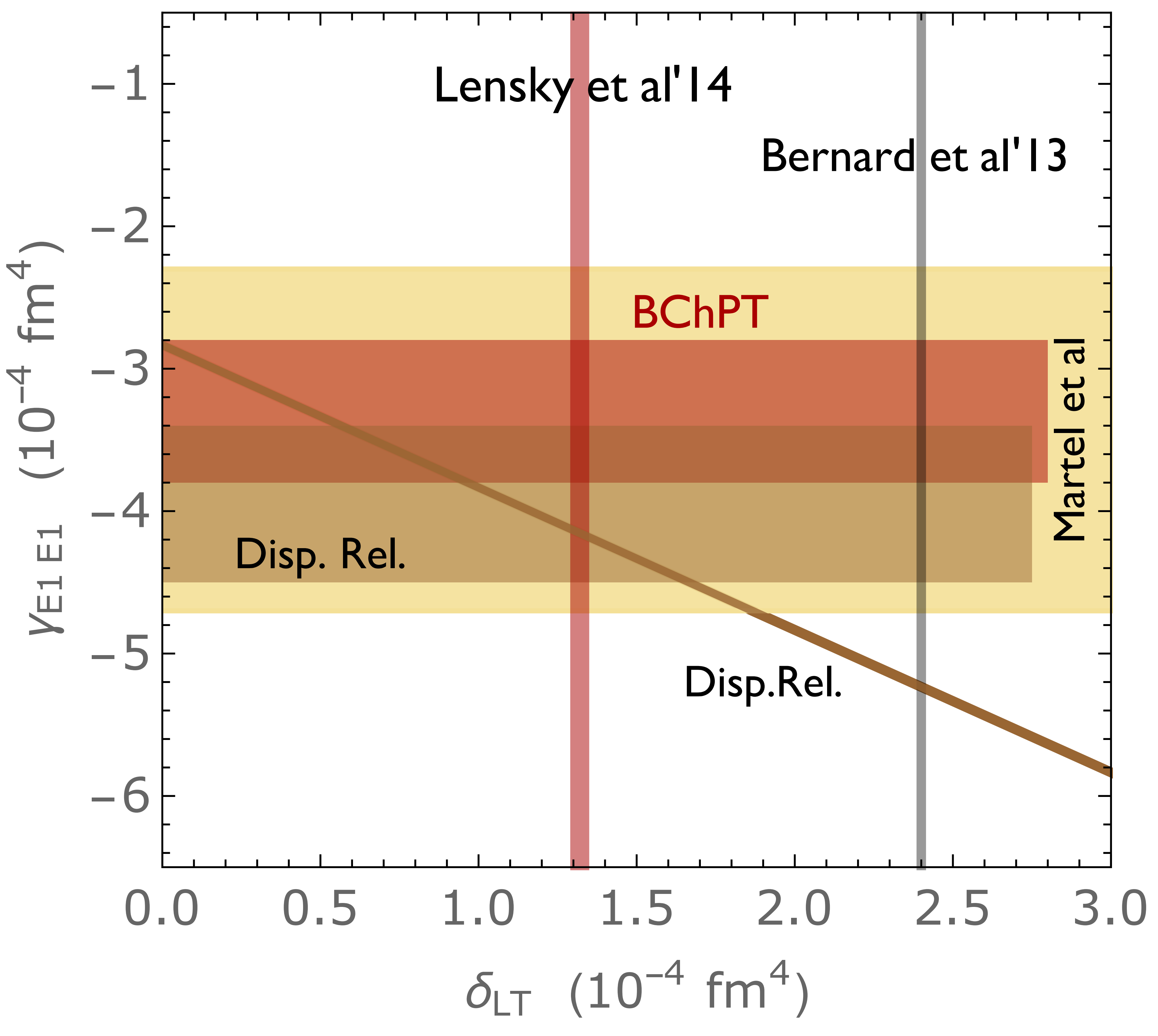
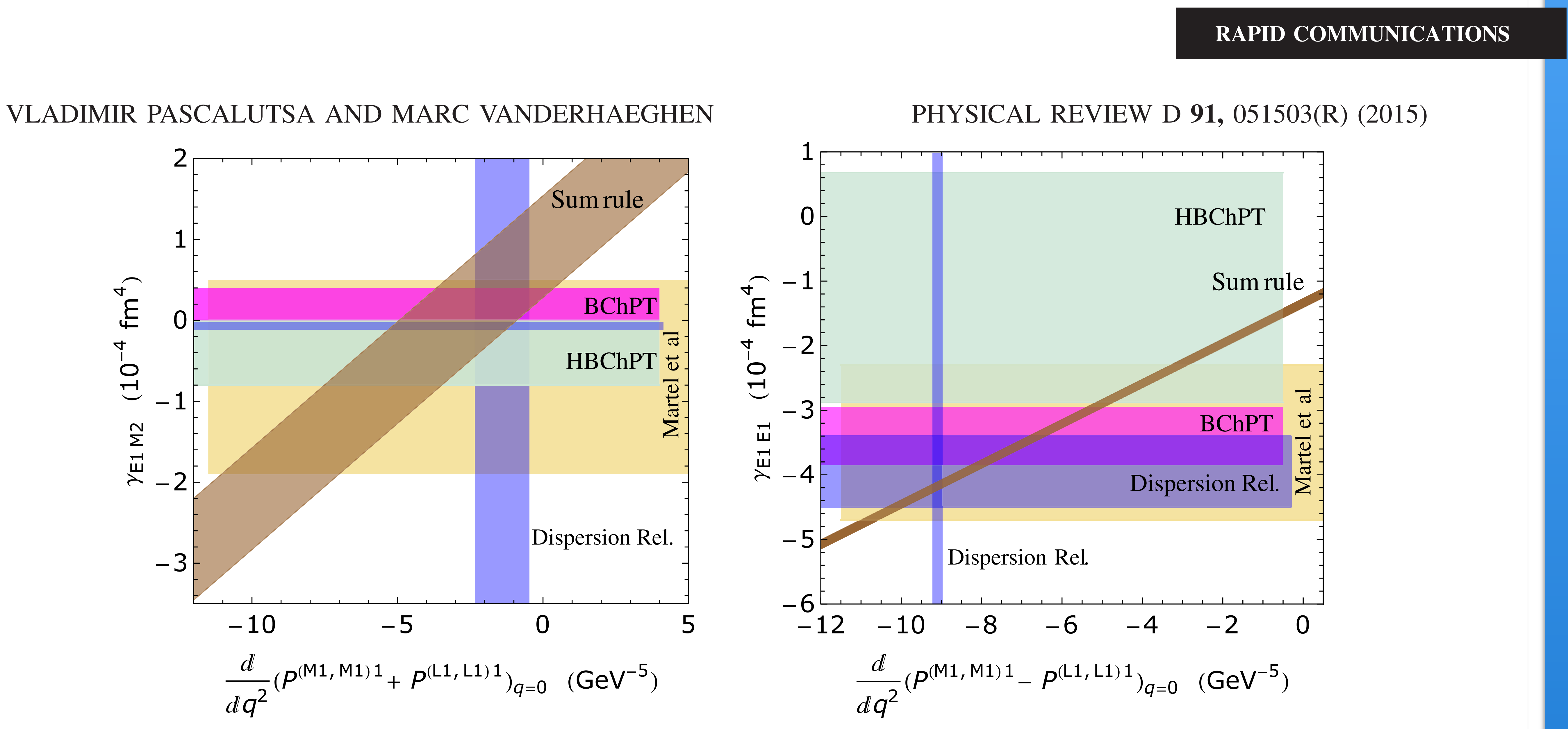




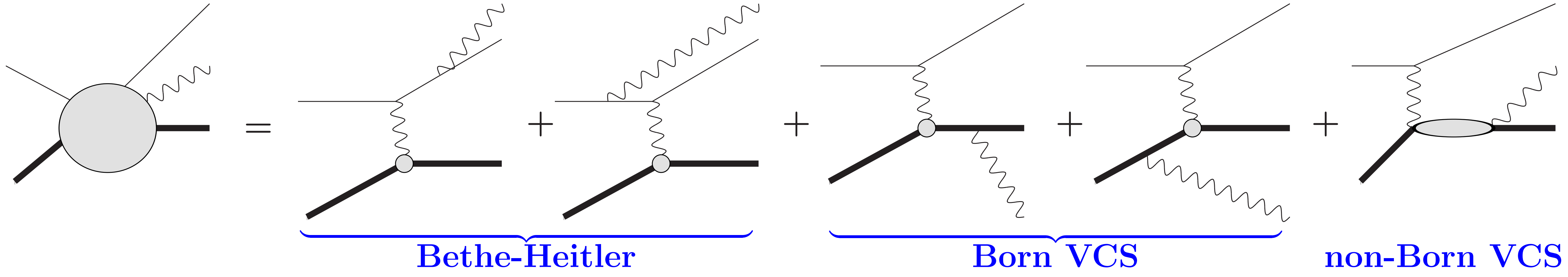
1) Disp. Rel. (Pasquini et al) uses MAID as input for RCS and VCS and is consistent with MAID value of δ_{LT}

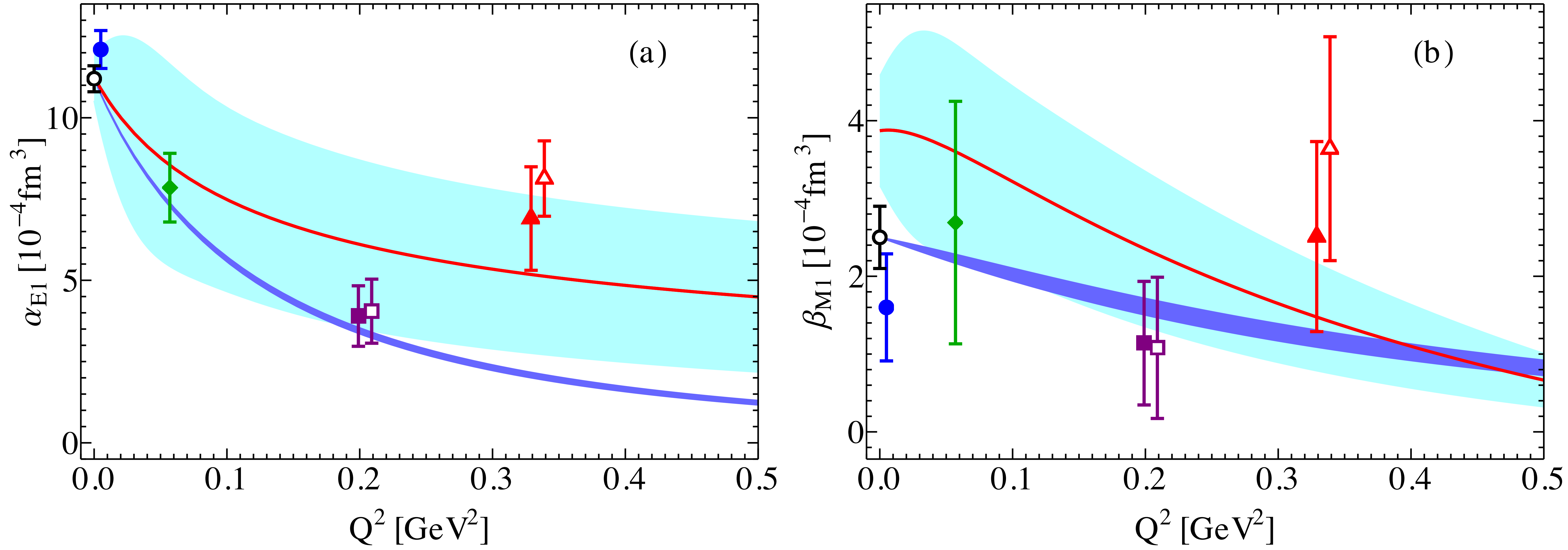
2) Lensky, Kao, Vanderhaeghen & V.P, arXiv:1701.01947 : verify the relation $\delta_{LT} = -\gamma_{E1E1} + \text{VCS spin GPs}$

in baryon and heavy-baryon ChPT.

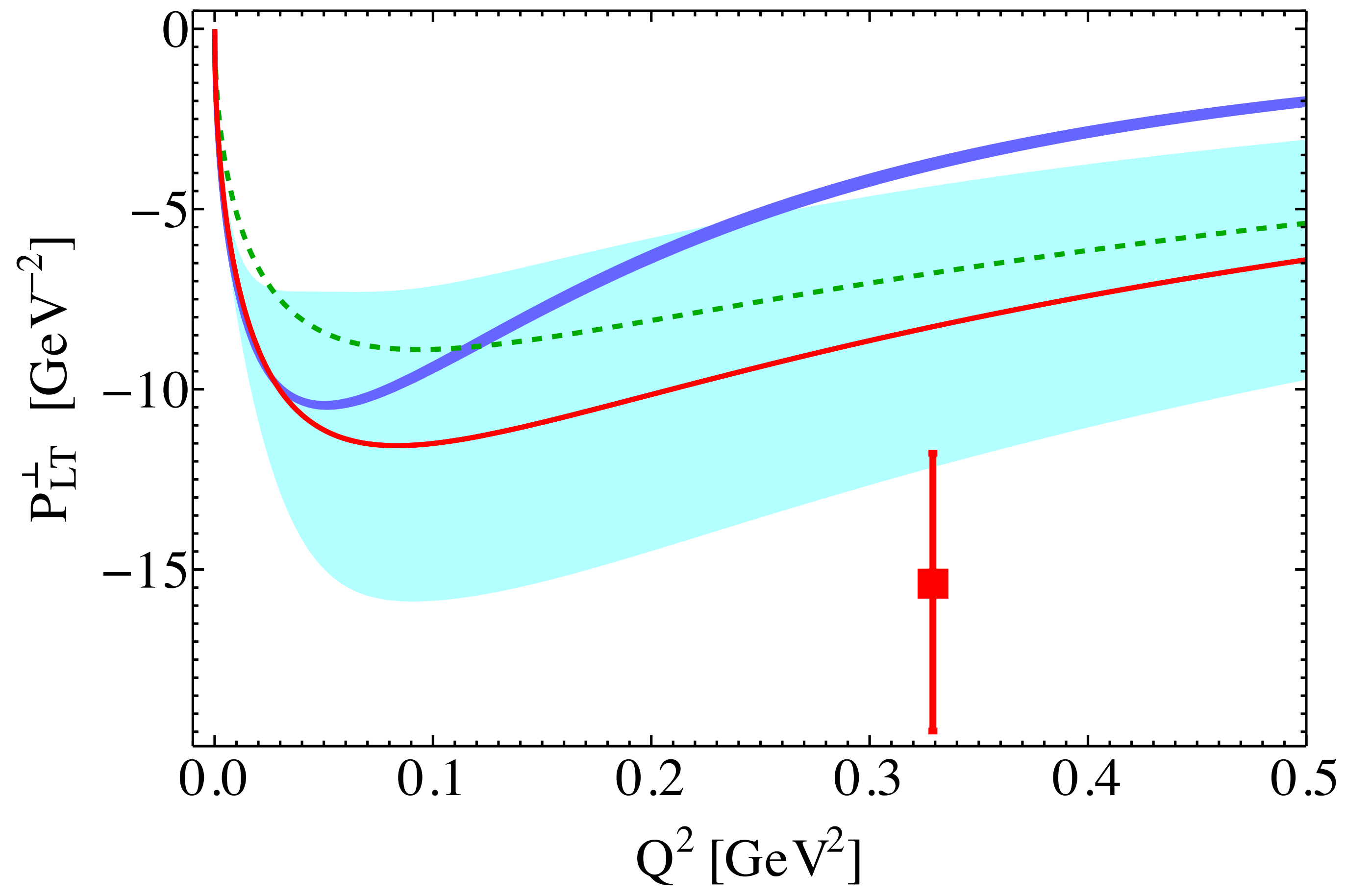
Virtual Compton scattering (VCS) and generalized polarizabilities (GPs)

Lensky, Vanderhaeghen & V.P, EPJC (2017)[arXiv:1612.08626]



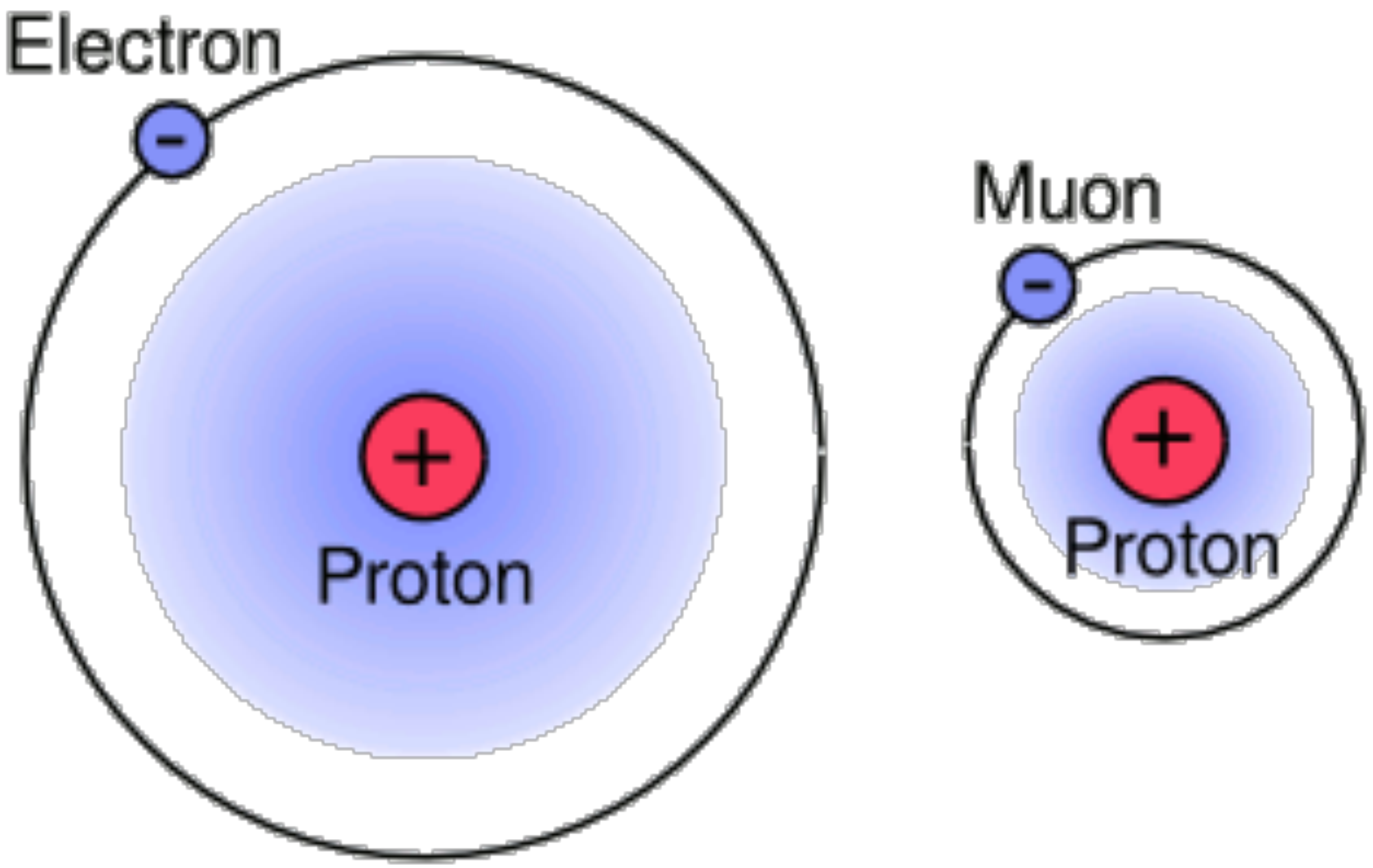


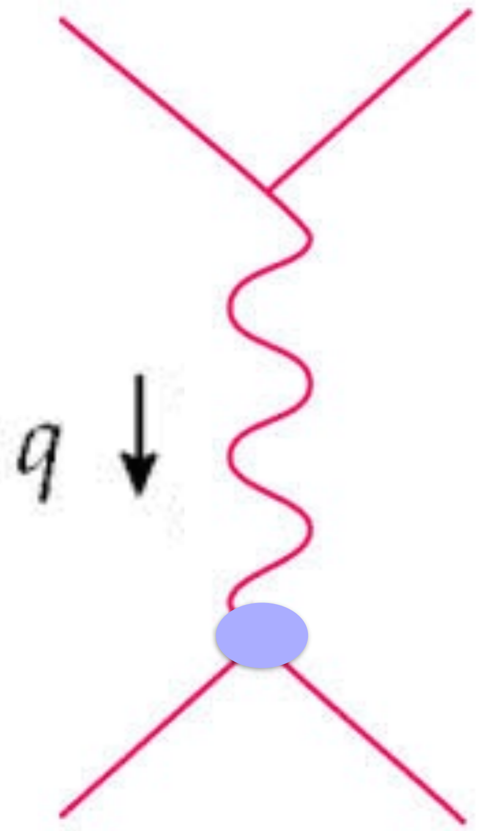
open circle, PDG 2014 [61]; blue circle, Olmos de León et al [62]; green diamond, MIT-Bates (DR) [7, 8]; green open diamond, MIT-Bates (LEX) [7, 8]; purple solid square, MAMI (DR) [13]; purple open square, MAMI (LEX) [13]; red solid triangle, MAMI1 (LEX) [9]; red solid inverted triangle, MAMI1 (DR) [11]; red open triangle, MAMI2 (LEX) [10]. Some of the data points are shifted to the right in order to enhance their visibility; namely, Olmos de León, MIT-Bates (LEX), MAMI LEX, MAMI1 DR and MAMI2 LEX sets have the same values of Q^2 as PDG, MIT-Bates (DR), MAMI DR, and MAMI1 LEX, respectively.



preliminary MAMI data:
L. Corea, H. Fonvieille,
H. Merkel et al. [A1 Coll.]

Hydrogens sensitive to proton e.m. structure





$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}\left[G_E(-\vec{q}^{\,2}) - 1\right] = \frac{2}{3}\pi\alpha\, r_E^2 + O(\vec{q}^{\,2})$$

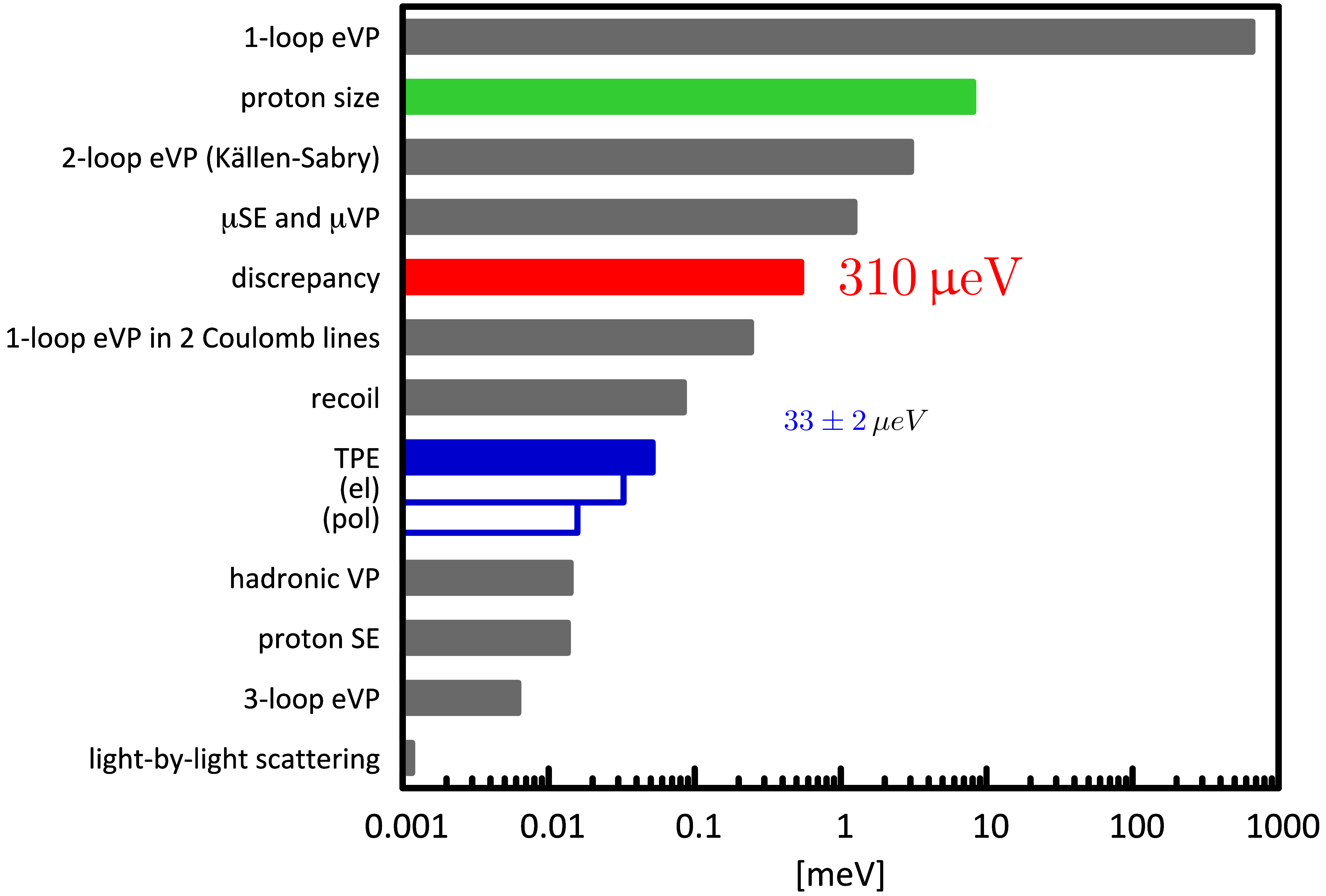
$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm \right| \delta V^{(1\gamma)} \left| nlm \right\rangle = \delta_{l0}\,\frac{2}{3}\pi\alpha\, r_E^2\; \underbrace{\frac{\alpha^3 m_r^3}{\pi n^3}}_{\text{wave function at origin}} + O(\alpha^5)$$

Muonic Hydrogen Lamb shift

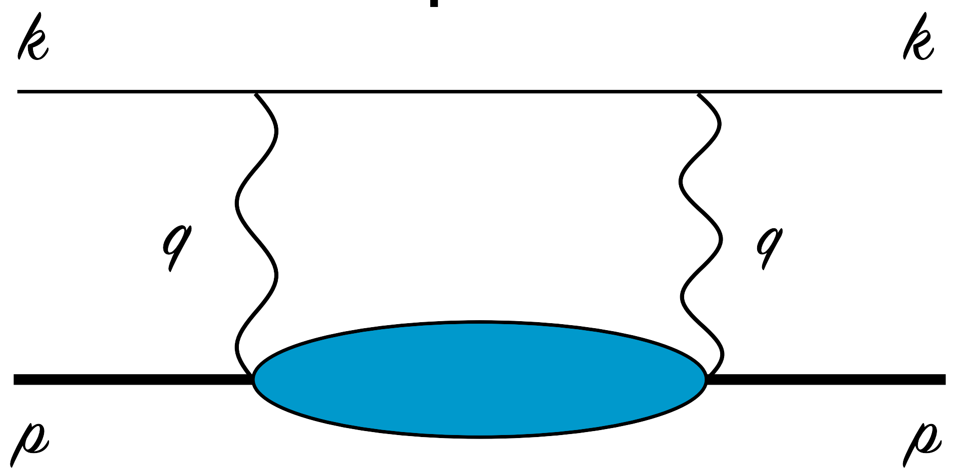
$$\Delta E_{\rm LS}^{\rm th} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

theory uncertainty: $2.5\ \mu eV$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).



subleading effects of proton structure proposed to resolve the puzzle



$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

A. De Rujula, Phys. Lett. B693 (2010)

G. A. Miller, Phys. Lett. B718 (2013)

Polarizability contribution in ChPT

Eur. Phys. J. C (2014) 74:2852
DOI 10.1140/epjc/s10052-014-2852-0

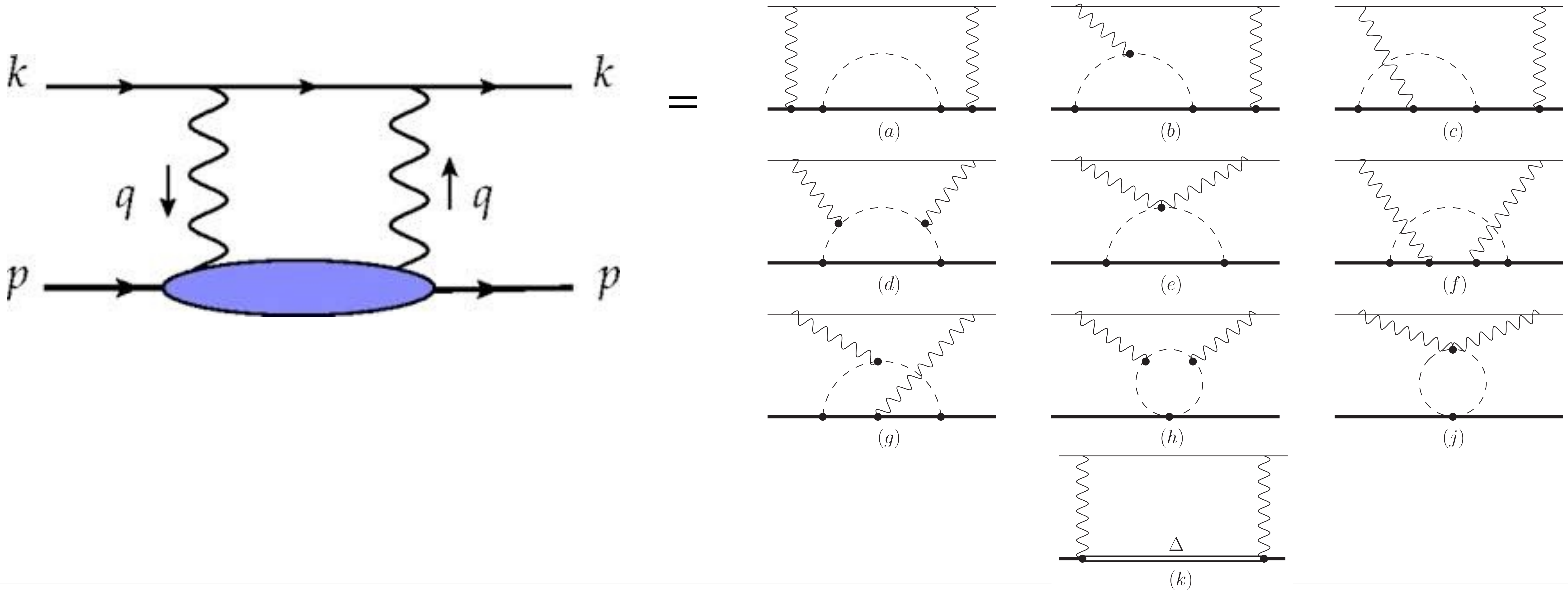
THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

Jose Manuel Alarcón[1,a], **Vadim Lensky**[2,3], **Vladimir Pascalutsa**[1]

[1] Cluster of Excellence PRISMA Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz 55099, Germany
[2] Theoretical Physics Group, School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, UK
[3] Institute for Theoretical and Experimental Physics, Bol'shaya Cheremushkinskaya 25, 117218 Moscow, Russia



with corrections to elastic proton FFs subtracted, i.e. "polarizability" alone

Proton polarizability effect in mu-H

			Heavy-Baryon (HB)ChPT				[Alarcon, Lensky & VP, EPJC (2014)]
(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\text{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\text{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\text{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
[b] Taken from Ref. [12]

[9] K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
[10] A. P. Martynenko, Phys. Atom. Nucl. **69**, 1309 (2006).
[11] D. Nevado and A. Pineda, Phys. Rev. C **77**, 035202 (2008).
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[13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A **48**, 120 (2012).
[14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, **052501** (2013).

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$$\Delta E_{2S}^{(\mathrm{pol})}(\text{LO-HB}\chi\text{PT}) \approx \frac{\alpha_{\mathrm{em}}^5 m_r^3 g_A^2}{4(4\pi f_\pi)^2}\frac{m_\mu}{m_\pi}(1-10G+6\log 2) = -16.1\ \mu\mathrm{eV},$$

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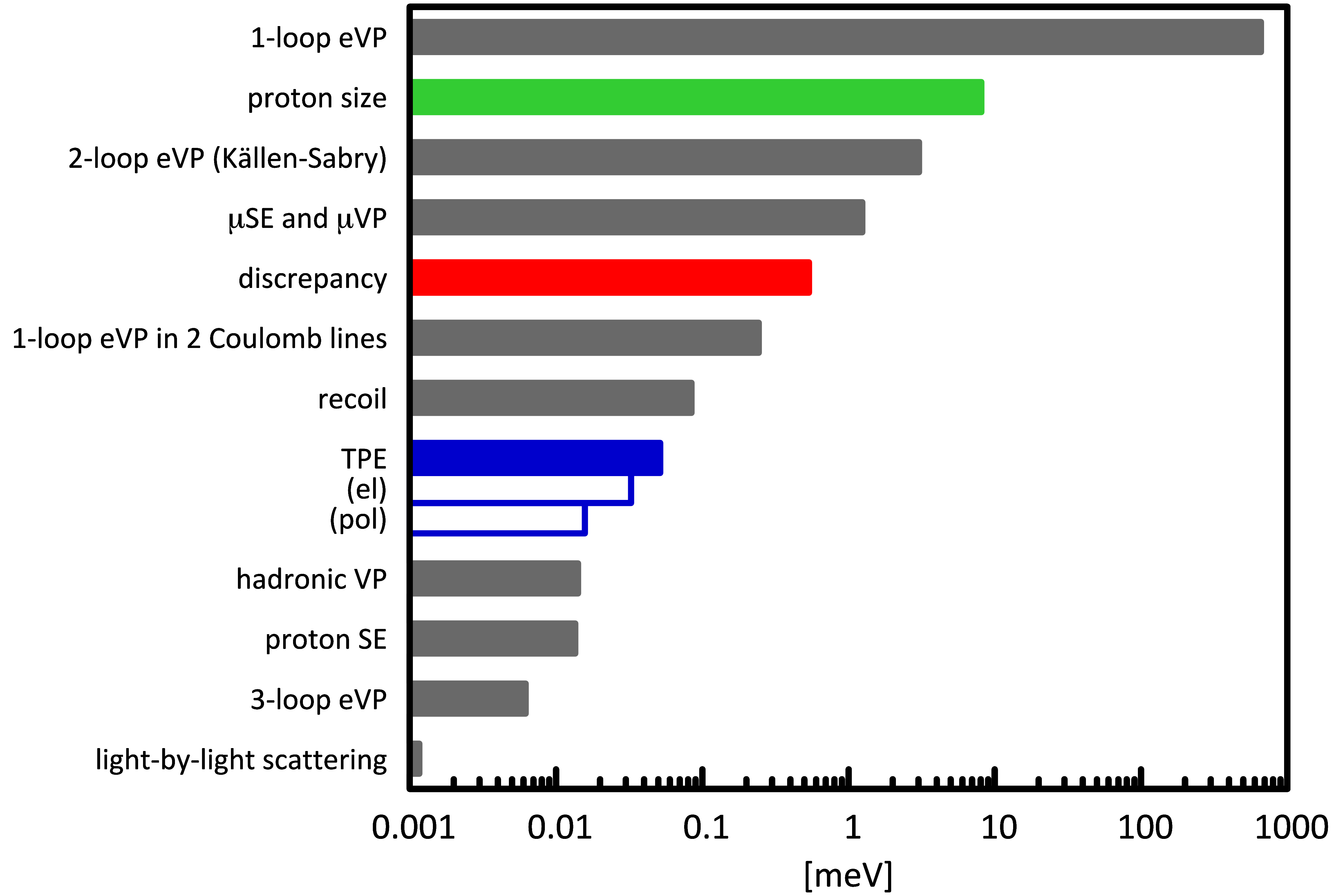
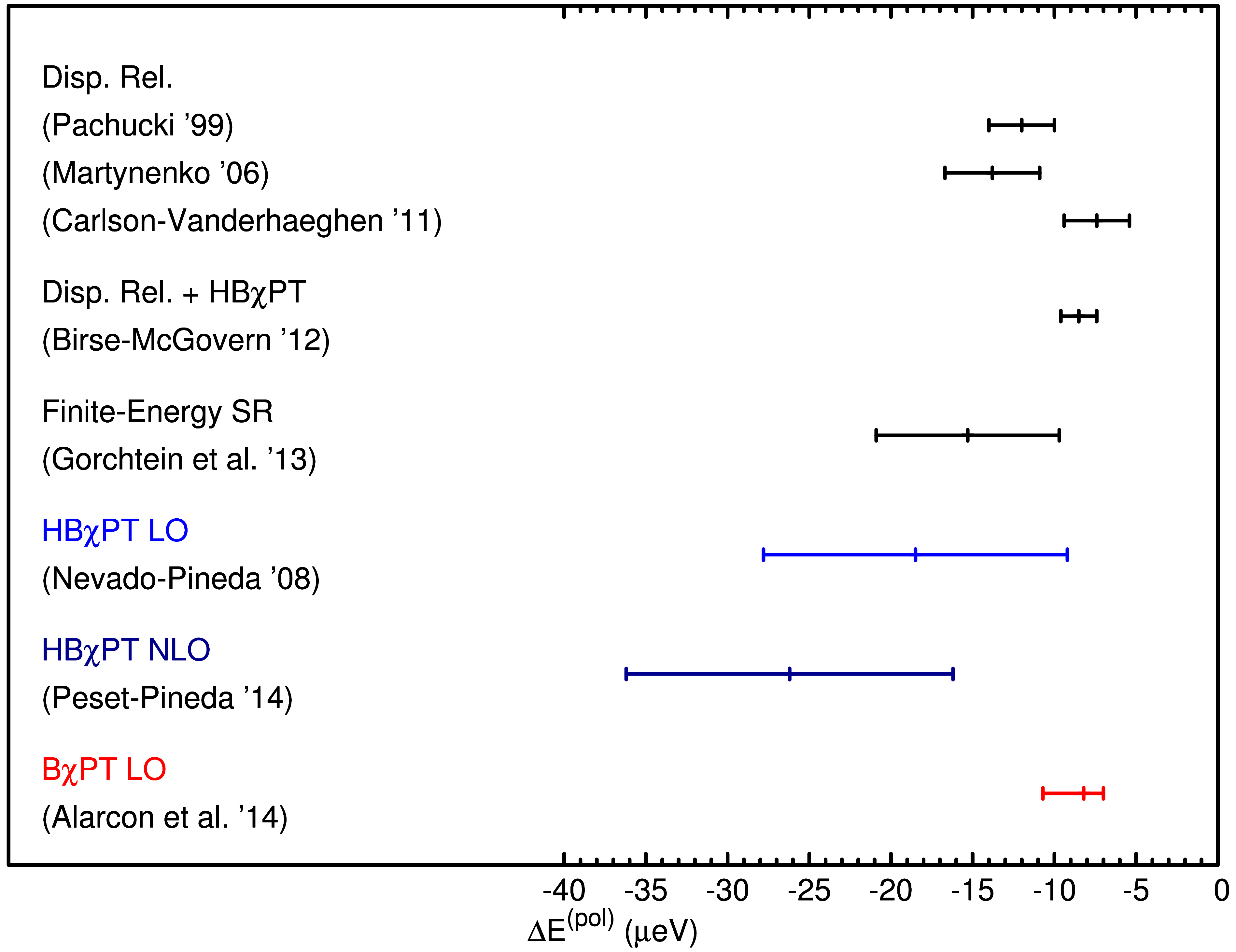
$G \simeq 0.9160$ is the Catalan constant.

Summary of polarizability contribution to mu-H Lamb shift





more in

Progress in Particle and Nuclear Physics 88 (2016) 29–97

Contents lists available at ScienceDirect

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journal homepage: www.elsevier.com/locate/ppnp

Review

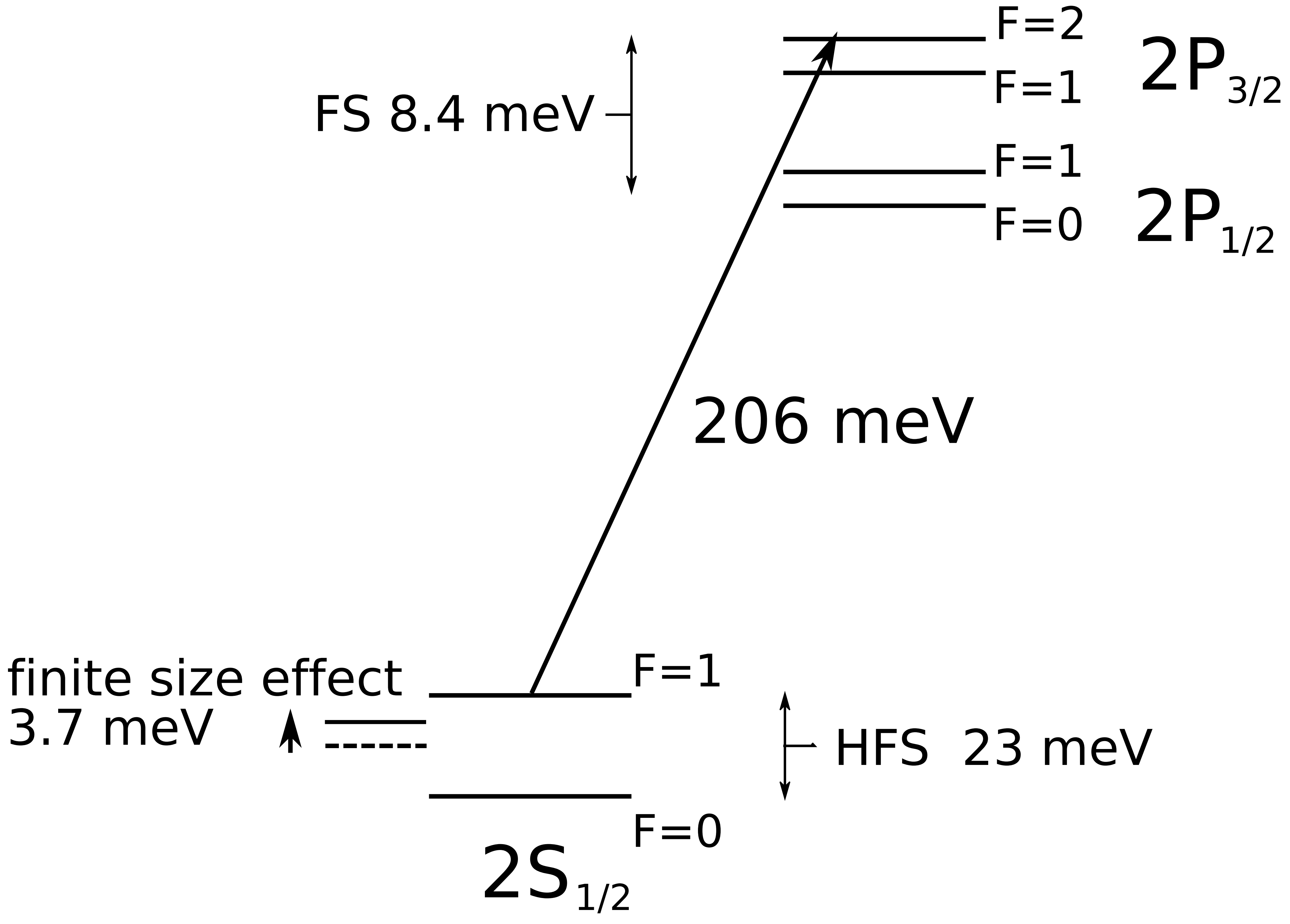
Nucleon polarizabilities: From Compton scattering to hydrogen atom

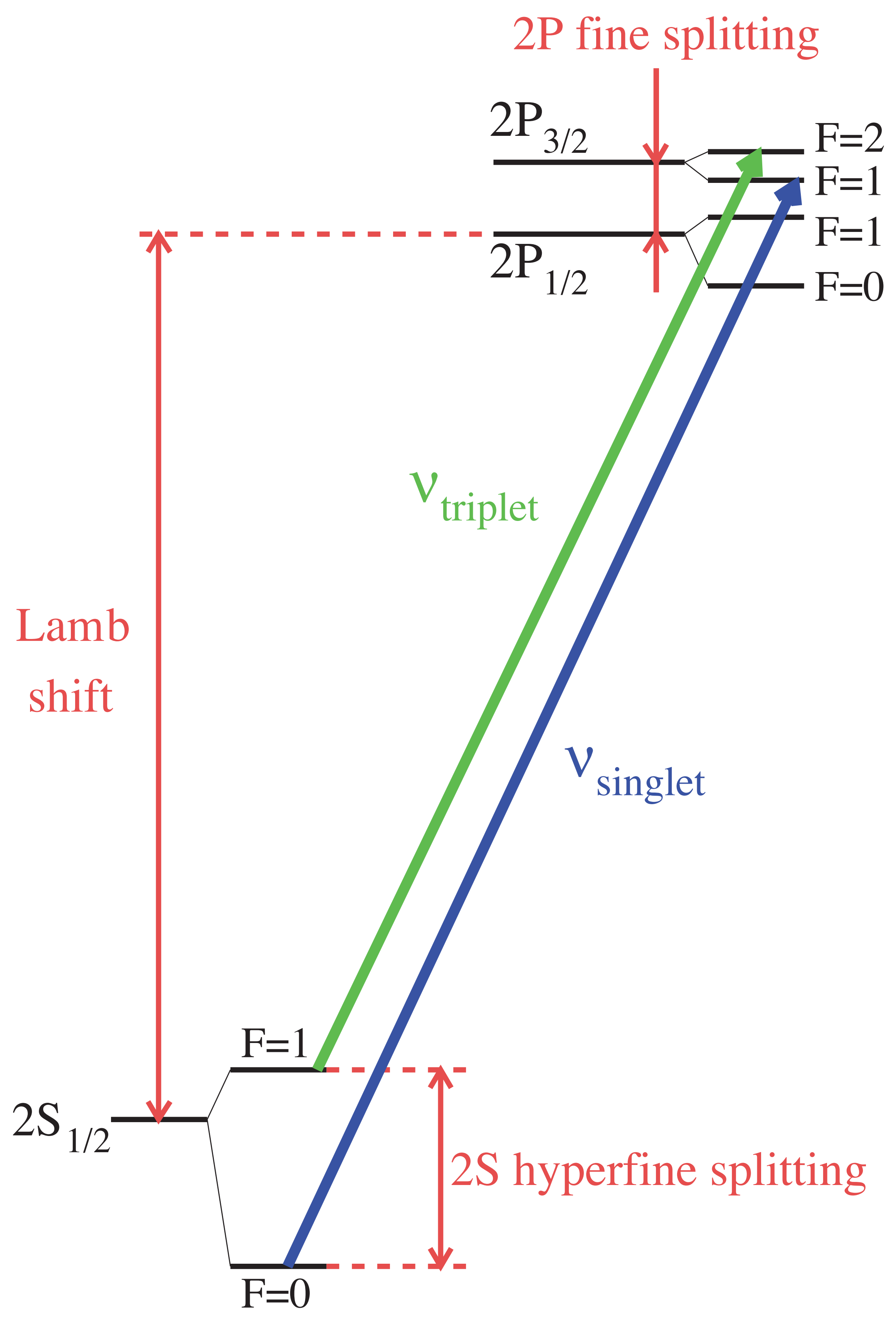
Franziska Hagelstein [a], Rory Miskimen [b], Vladimir Pascalutsa [a,*]

[a] Institut für Kernphysik and PRISMA Excellence Cluster, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany
[b] Department of Physics, University of Massachusetts, Amherst, 01003 MA, USA

Muonic hydrogen theory and experiment

CREMA Collaboration measured 2 transitions in muonic H:
Pohl et al., Nature (2010)
Antognini et al., Science (2013)





2S hyperfine splitting and Zemach radius

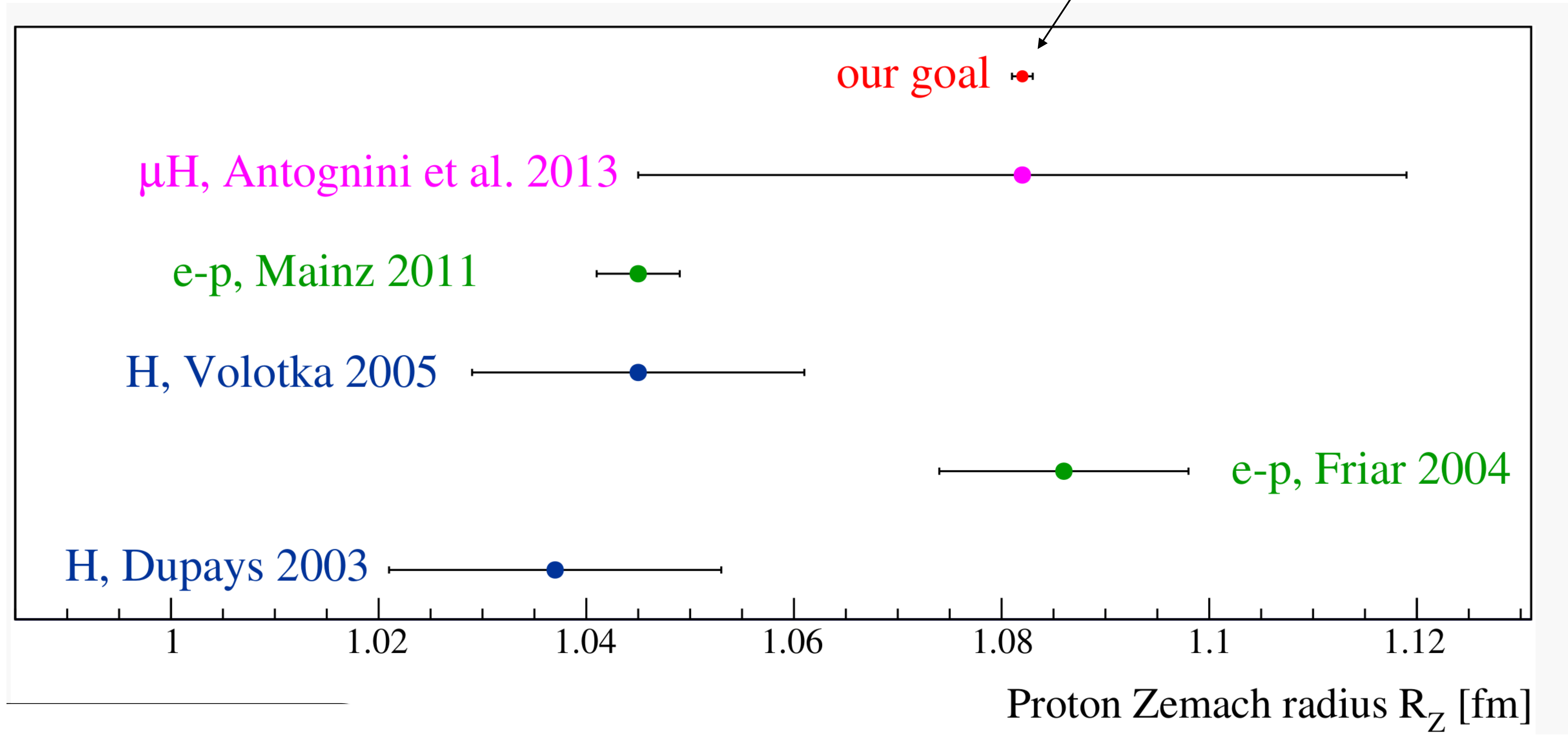
$$\Delta E^{\rm exp}_{\rm HFS} = 22.8089(51)\,{\rm meV}$$

$$\Delta E^{\rm th}_{\rm HFS} = 22.9763(15) - 0.1621(10)\,(R_Z/{\rm fm}) + \Delta E^{(\rm pol)}_{\rm HFS}$$

$$R_Z = -\frac{4}{\pi}\int_0^\infty \frac{{\rm d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\varkappa} - 1\right]$$

from 2S HFS:
$1.082(37)$ [fm]

Approved PSI experiment to measure 1S HFS



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Zemach radius: $R_Z = -\frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\varkappa} - 1\right]$

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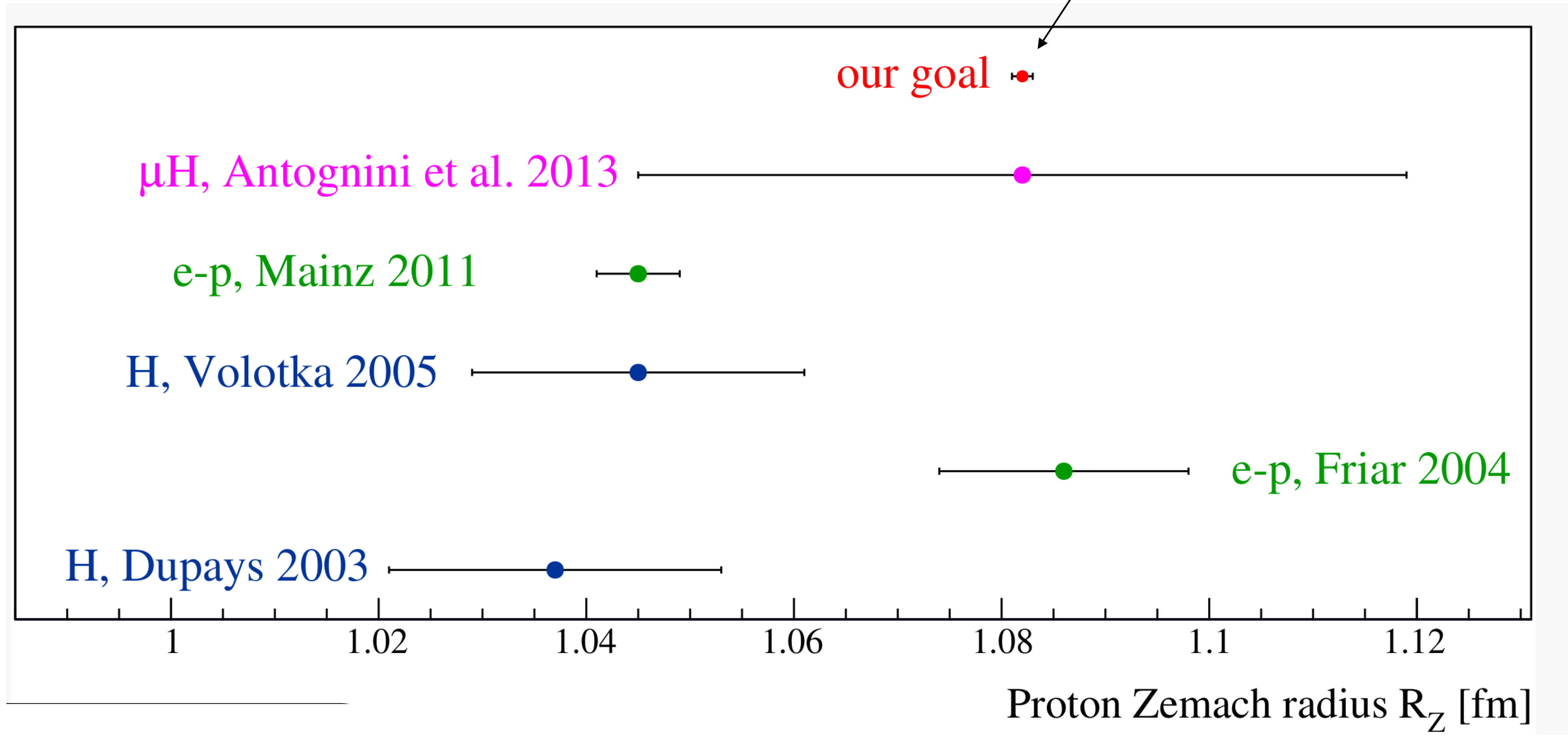


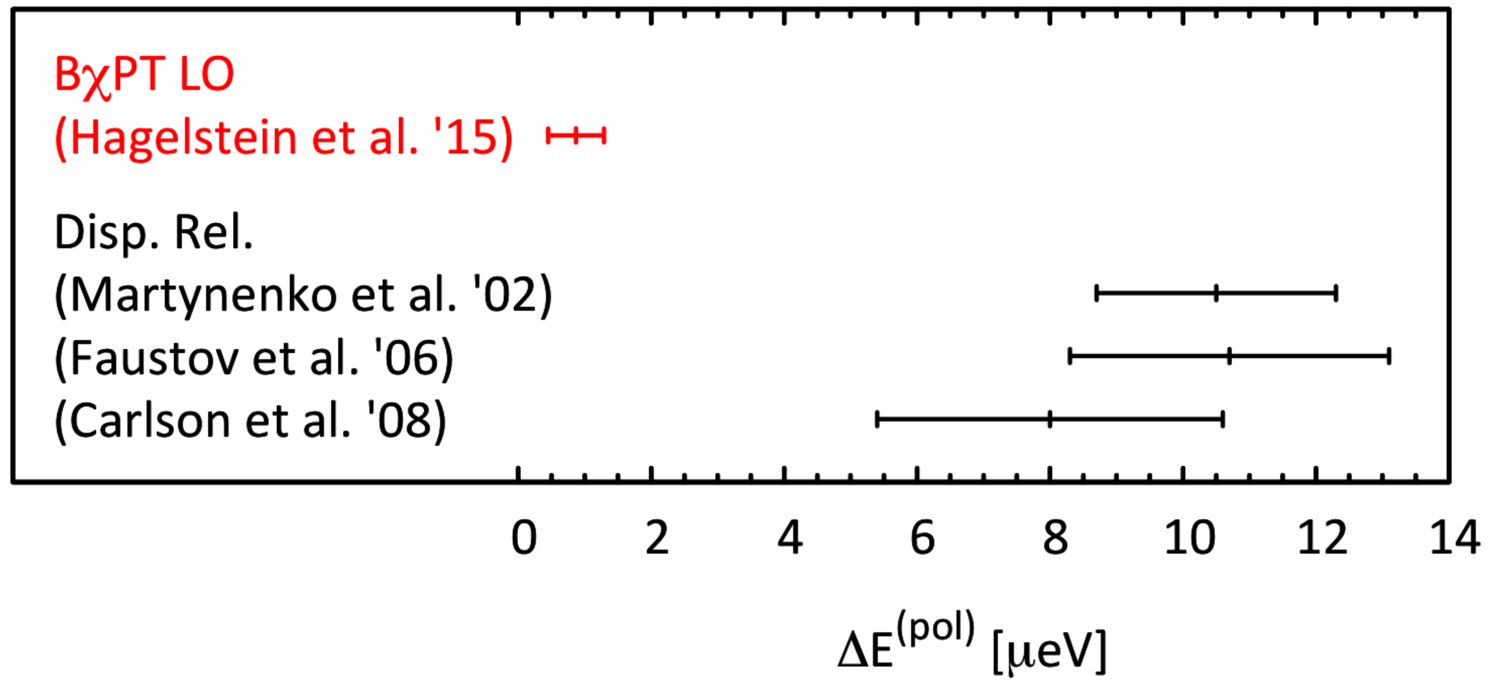
1S HFS: New experiment (approved)

HFS theory status

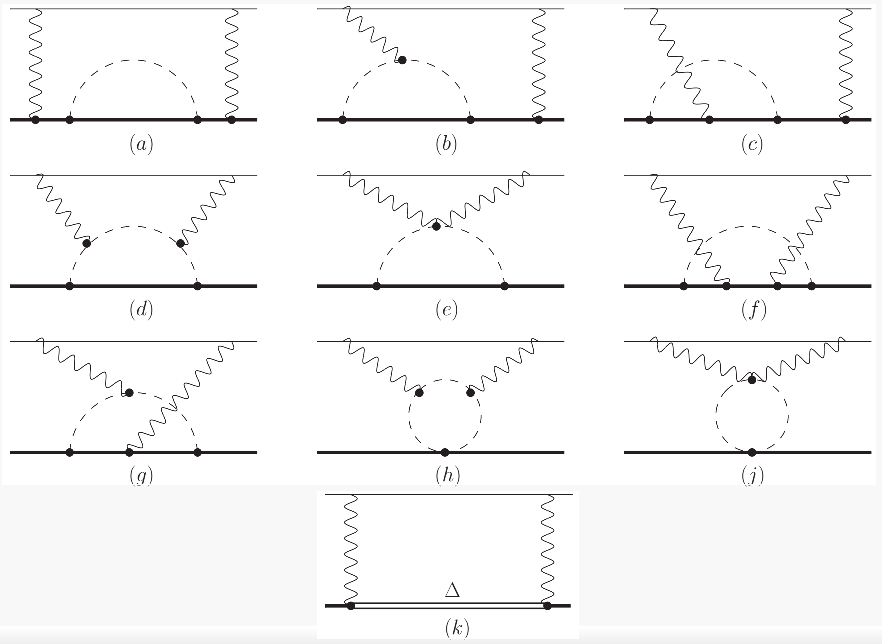
$$\Delta E_{\mathrm{HFS}}(1S) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak+hVP}} + \underbrace{\Delta_{\mathrm{Zemach}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}}_{\Delta_{\mathrm{TPE}}}]\Delta E_0^{\mathrm{HFS}}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

	μp		$\mu\,^3$He$^+$		
	Magnitude	Uncertainty	Magnitude	Uncertainty	
$\Delta E_0^{\mathrm{HFS}}$	182.443 meV	0.1×10^{-6}	1370.725 meV	0.1×10^{-6}	
Δ_{QED}	1.1×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}	
$\Delta_{\mathrm{weak+hVP}}$	2×10^{-5}	2×10^{-6}			
Δ_{Zemach}	7.5×10^{-3}	7.5×10^{-5}	3.5×10^{-2}	2.2×10^{-4}	$\leftarrow G_E(Q^2), G_M(Q^2)$
Δ_{recoil}	1.7×10^{-3}	10^{-6}	2×10^{-4}		$\leftarrow G_E, G_M, F_1, F_2$
Δ_{pol}	4.6×10^{-4}	8×10^{-5}	$(3.5 \times 10^{-3})^*$	$(2.5 \times 10^{-4})^*$	$\leftarrow g_1(x, Q^2), g_2(x, Q^2)$



HFS calculation in ChPT versus dipersive evaluations



Summary and Conclusion

QED contribution to proton-neutron mass difference and two-photon exchange contributions to electron scattering and muonic hydrogen Lamb shift share the VVCS amplitude which (up to a subtraction function) is determined by the nucleon structure functions

In view of muonic hydrogen Lamb shift and (upcoming HFS) experiments, **low-Q data on (spin) structure functions are in high demand!**

Collaborators

Oleksii Gryniuk (Mainz)

Franziska Hagelstein (Mainz —> Bern)

Jose Alarcon (JLab)

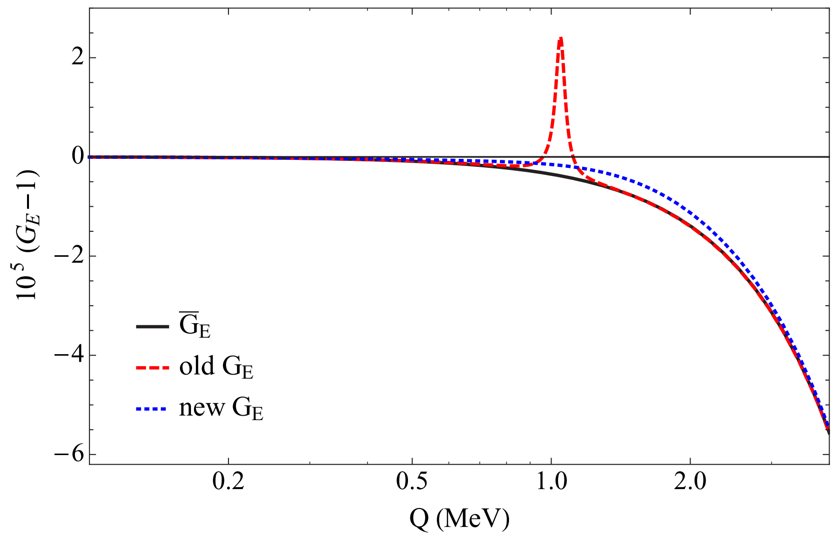
Vadim Lensky (Mainz)

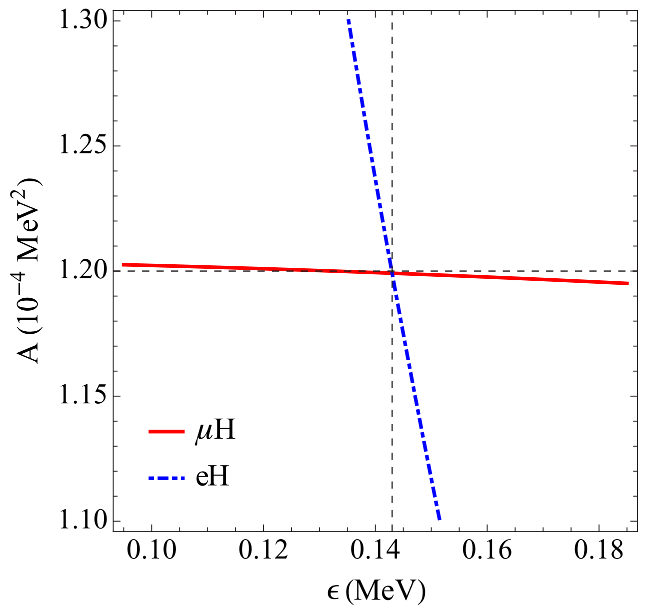
Marc Vanderhaeghen (Mainz)

Backup slides

Soft effects in FFs

$$\widetilde{G}_E(Q^2) = \frac{A\, Q_0^2 Q^2 [Q^2 + \epsilon^2]}{[Q_0^2 + Q^2]^4}$$





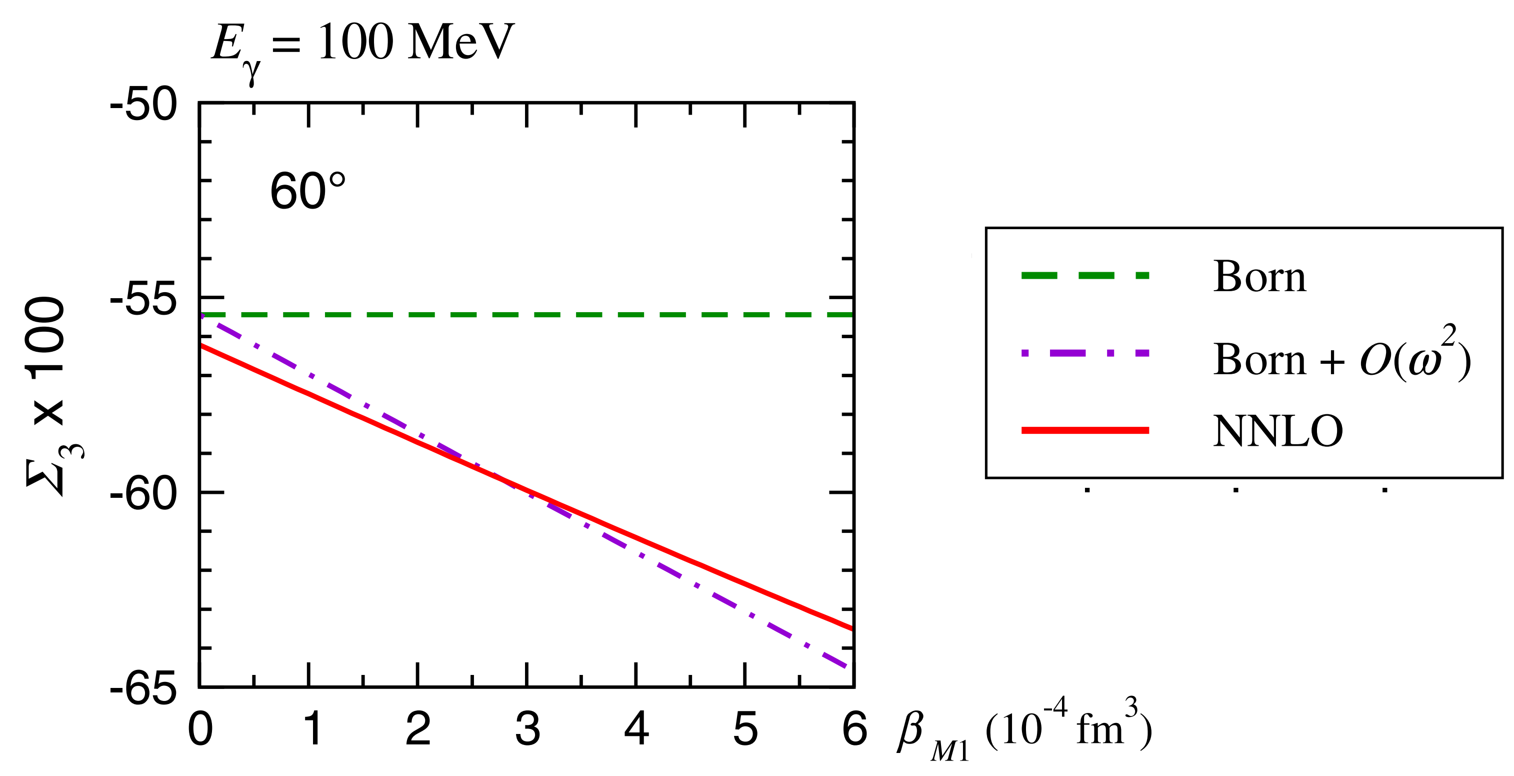
From beam asymmetry

PRL 110, 262001 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Separation of Proton Polarizabilities with the Beam Asymmetry of Compton Scattering

Nadiia Krupina and Vladimir Pascalutsa
PRISMA Cluster of Excellence Institut für Kernphysik, Johannes Gutenberg–Universität Mainz, 55128 Mainz, Germany
(Received 3 April 2013; published 25 June 2013)

$$\Sigma_3 \equiv \frac{d\sigma_{||} - d\sigma_\perp}{d\sigma_{||} + d\sigma_\perp} \overset{\text{LEX}}{=} \Sigma_3^{(\text{Born})} - \frac{4\beta_{M1}}{Z^2\alpha_{em}} \frac{\cos\theta \sin^2\theta}{(1+\cos^2\theta)^2}\,\omega^2 + O(\omega^4)$$



Vladimir Pascalutsa — A few moments in ChPT — Workshop on Tagged Structure Functions — JLab, Jan 16-18,

Saving De Rujula's scenario

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **91**, 040502(R) (2015)

Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution

Franziska Hagelstein and Vladimir Pascalutsa
Institut für Kernphysik, Cluster of Excellence PRISMA, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany

(Received 13 February 2015; published 20 April 2015)

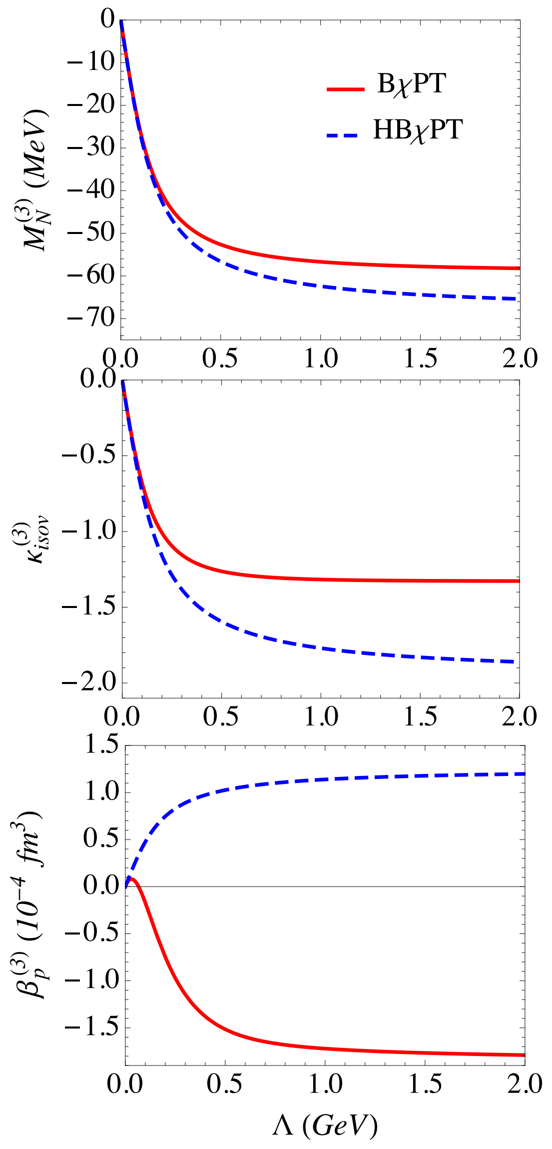
$$E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{1}{3}\pi(Z\alpha)^4 m_r^3 \int_0^\infty \mathrm{d}r\, r^4 e^{-r/a} \rho_E(r)$$

with

$$\rho_E(r) = \frac{1}{(2\pi)^2\, r} \int_{t_0}^\infty \mathrm{d}t\, \mathrm{Im}\, G_E(t)\, e^{-r\sqrt{t}}$$

$a = 1/(Z\alpha m_r)$ Bohr radius

UV dependence in HB- vs B-ChPT



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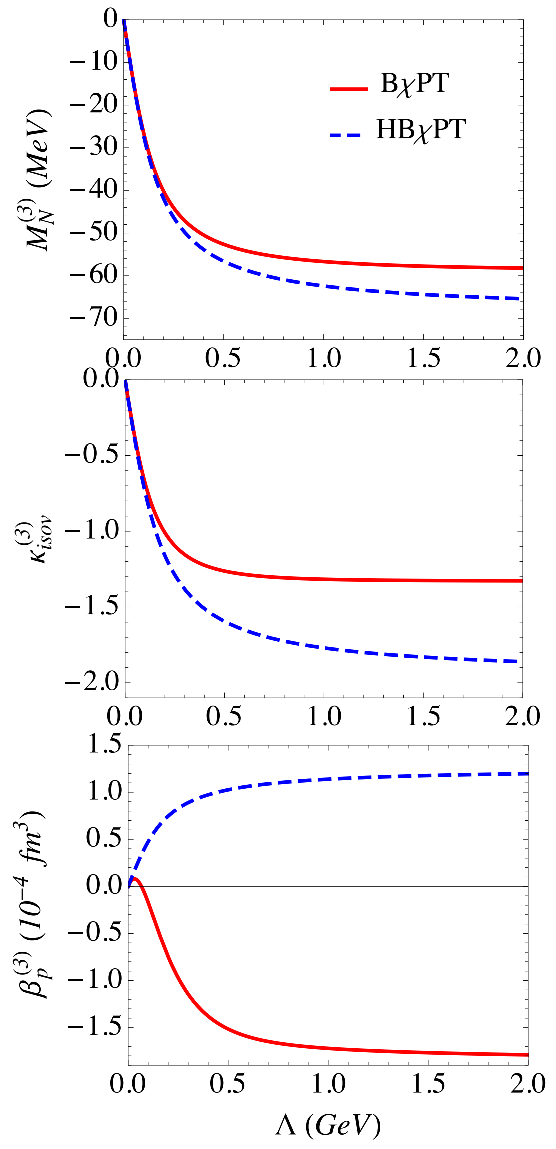

$$M_N \sim m_\pi^3$$
$$\kappa \sim m_\pi$$
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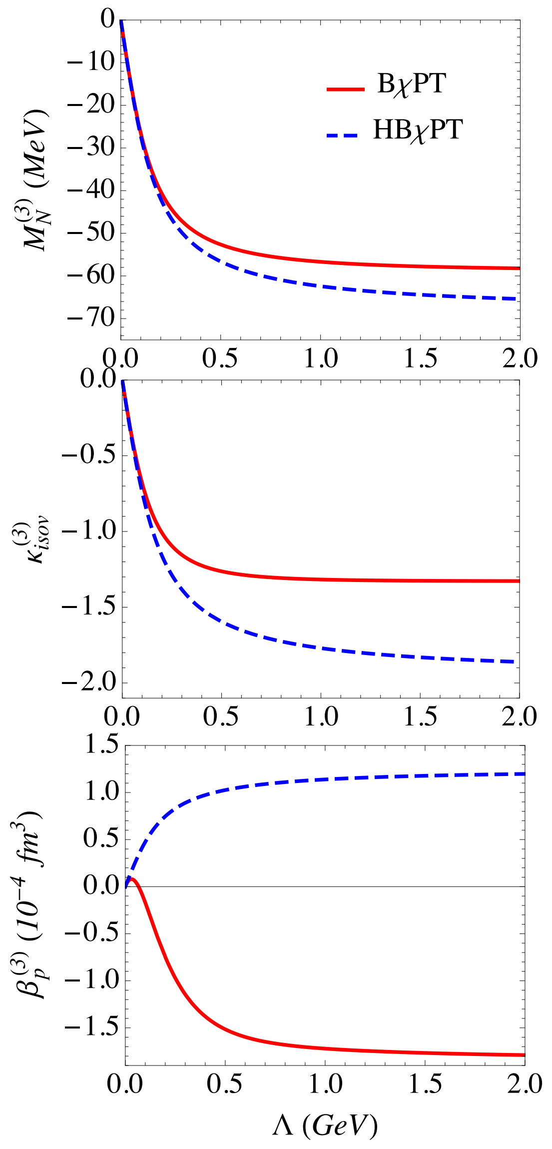

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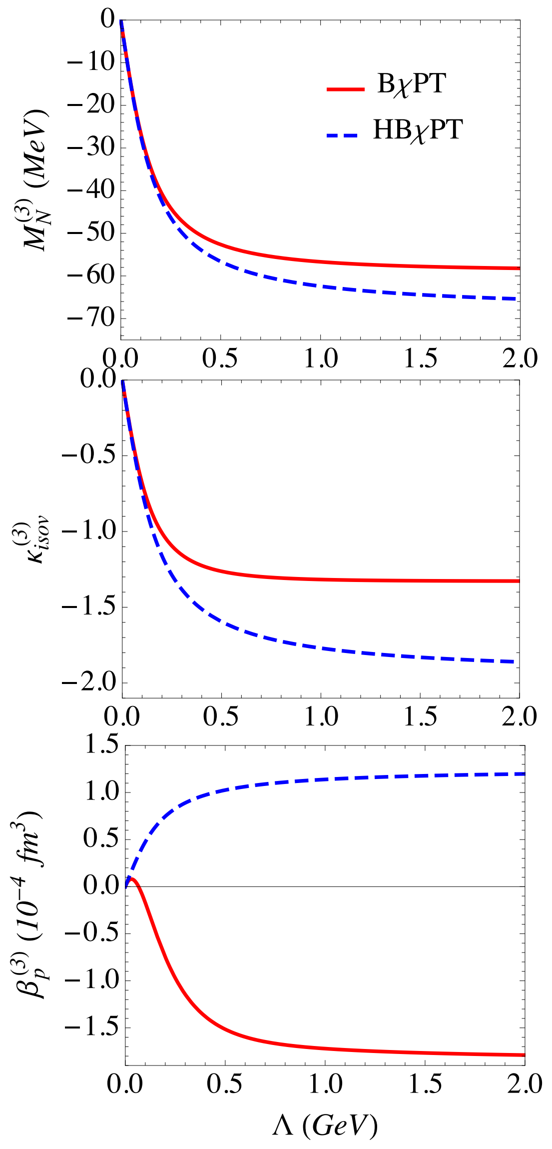

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Heavy-Baryon expansion fails for quantities where the leading chiral-loop effects scales with a negative

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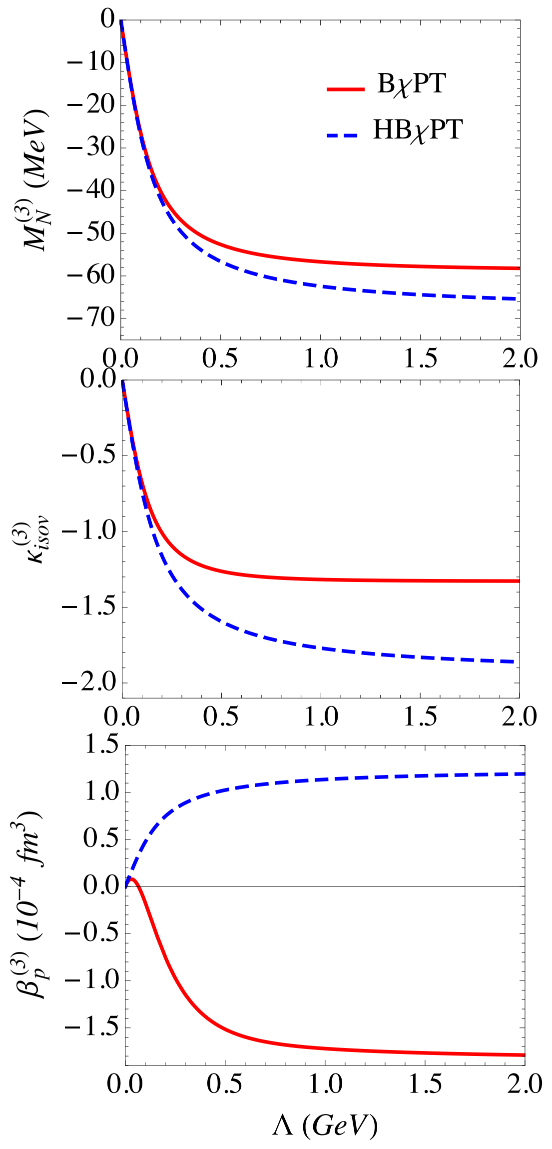

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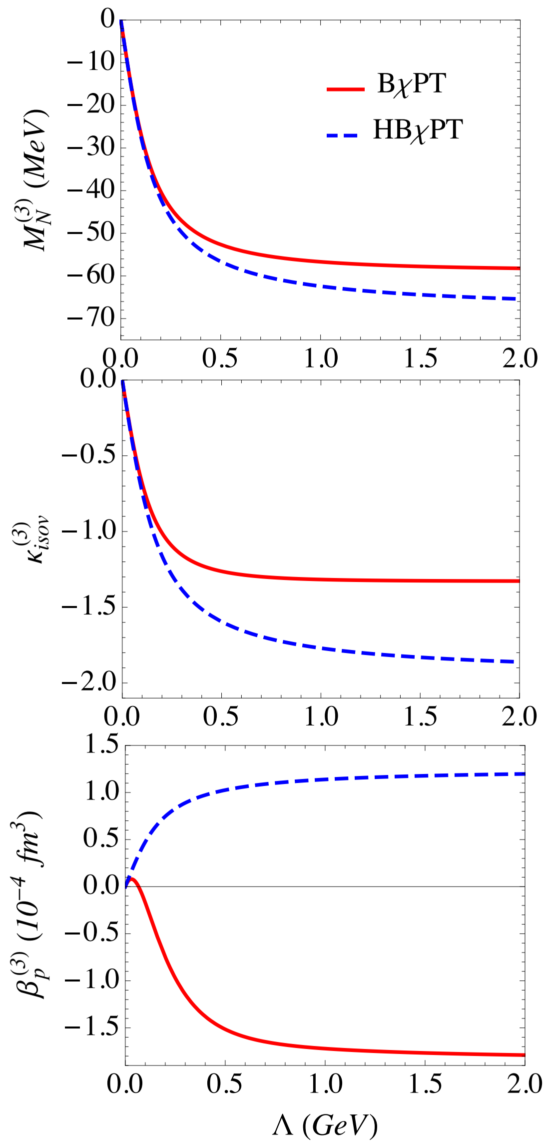

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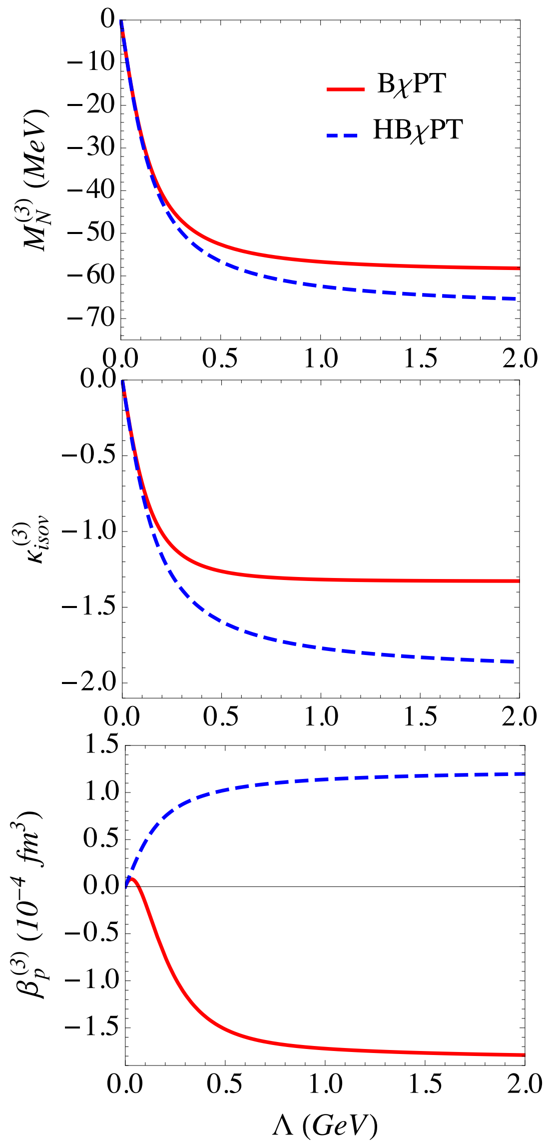

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E.g.: the effective range parameters of the NN force

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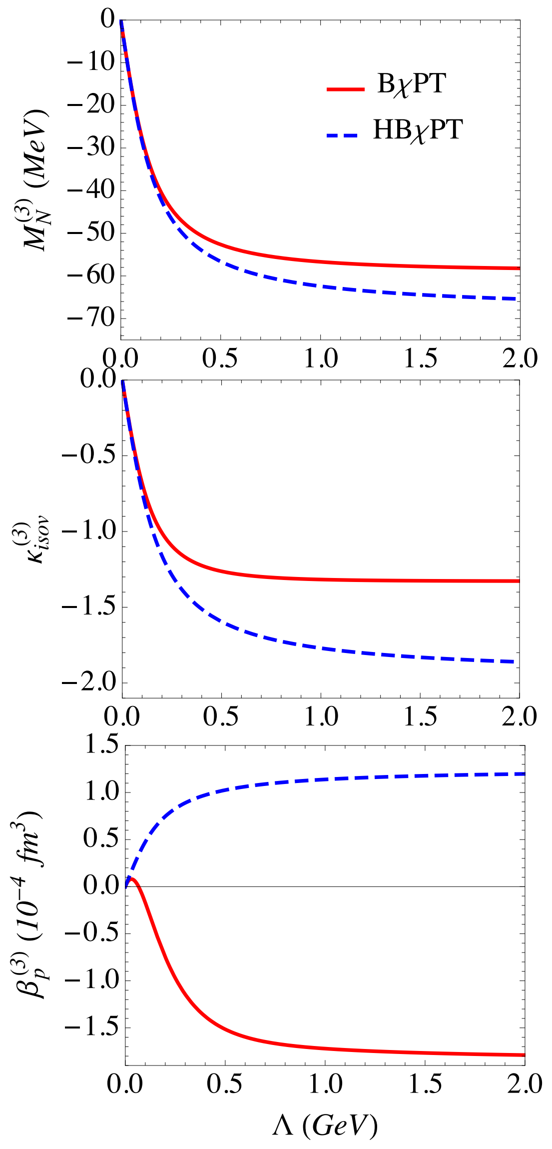
UV dependence in HB- vs B-ChPT



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E.g.: the effective range parameters of the NN force
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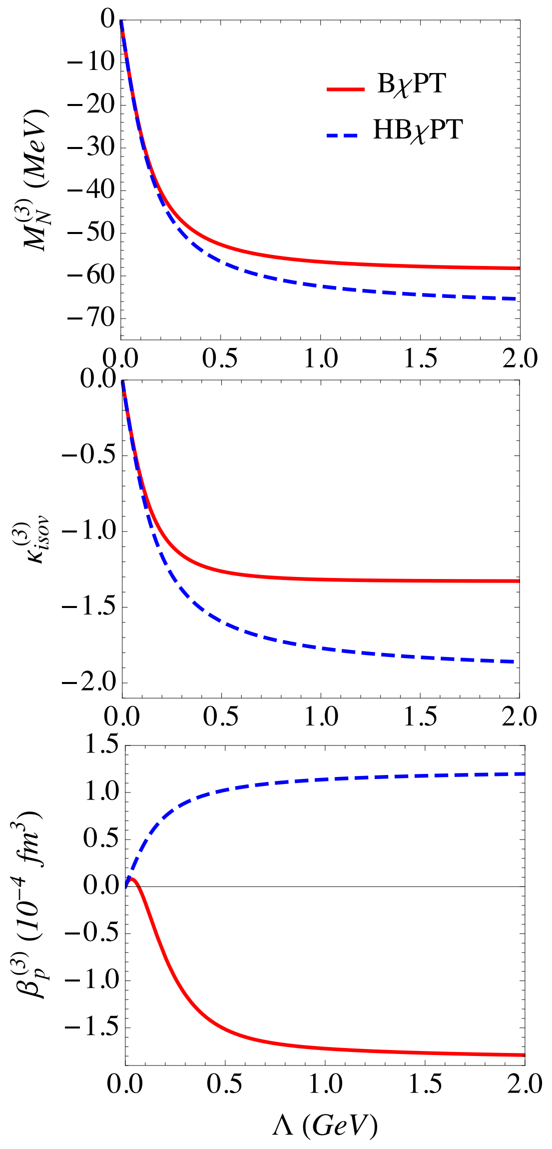
UV dependence in HB- vs B-ChPT



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Predictions of HBChPT vs BChPT

HBChPT@LO

Bernard, Keiser, Meissner Int J Mod Phys (1995)

$$\alpha_p = \alpha_n = \frac{5\pi\alpha}{6m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 12.2\times 10^{-4}\ \text{fm}^3,$$

$$\beta_p = \beta_n = \frac{\pi\alpha}{12m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 1.2\times 10^{-4}\ \text{fm}^3,$$

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BChPT@NLO

Lensky & V.P., EPJC (2010)

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$$\beta = \underbrace{-1.8}_{\mathscr{O}(p^3)} + \underbrace{7.1 - 1.3}_{\mathscr{O}(p^4/\Delta)} = 4.0\,.$$

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$$\beta = \underbrace{-1.8}_{\mathcal{O}(p^3)} + \underbrace{7.1 - 1.3}_{\mathcal{O}(p^4/\Delta)} = 4.0\,.$$

$\mu = m_\pi/M_N$

$$\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu + \frac{121}{6})\mu^2 + \mathcal{O}(\mu^3)\right]$$

Predictions of HBChPT vs BChPT

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BChPT@NLO

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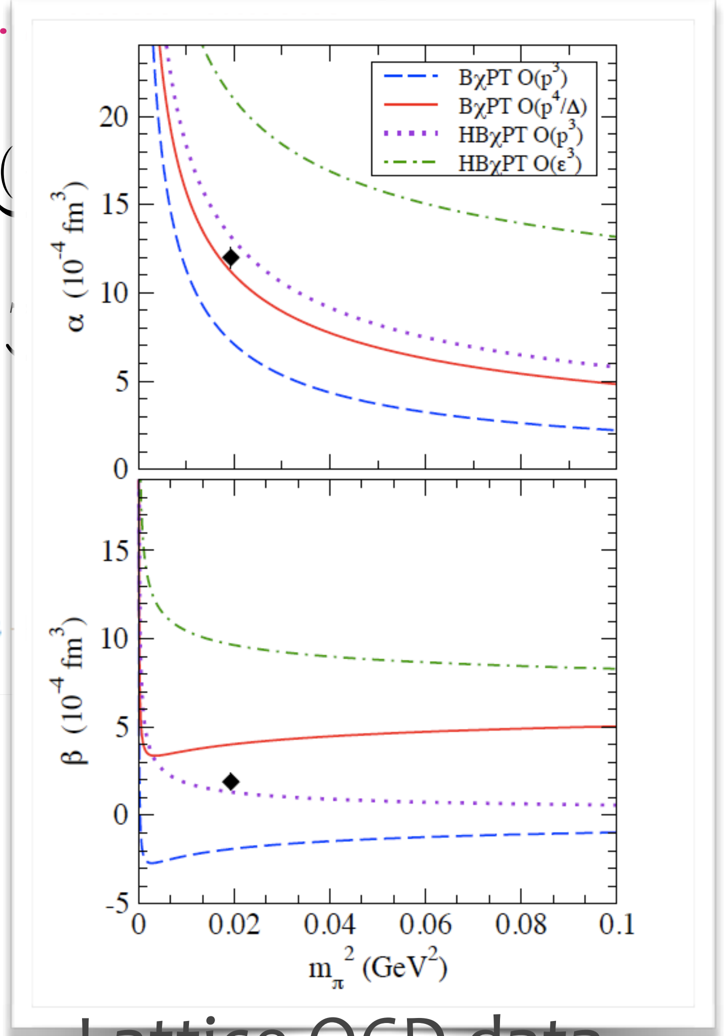

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Lattice QCD data expected soon