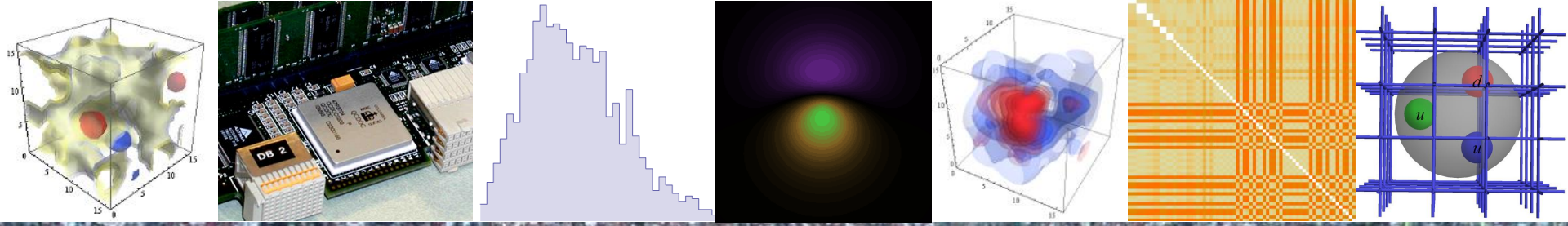


Scalar Charges from 2+1+1-Flavor Lattice QCD

Huey-Wen Lin

Lattice QCD 101

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

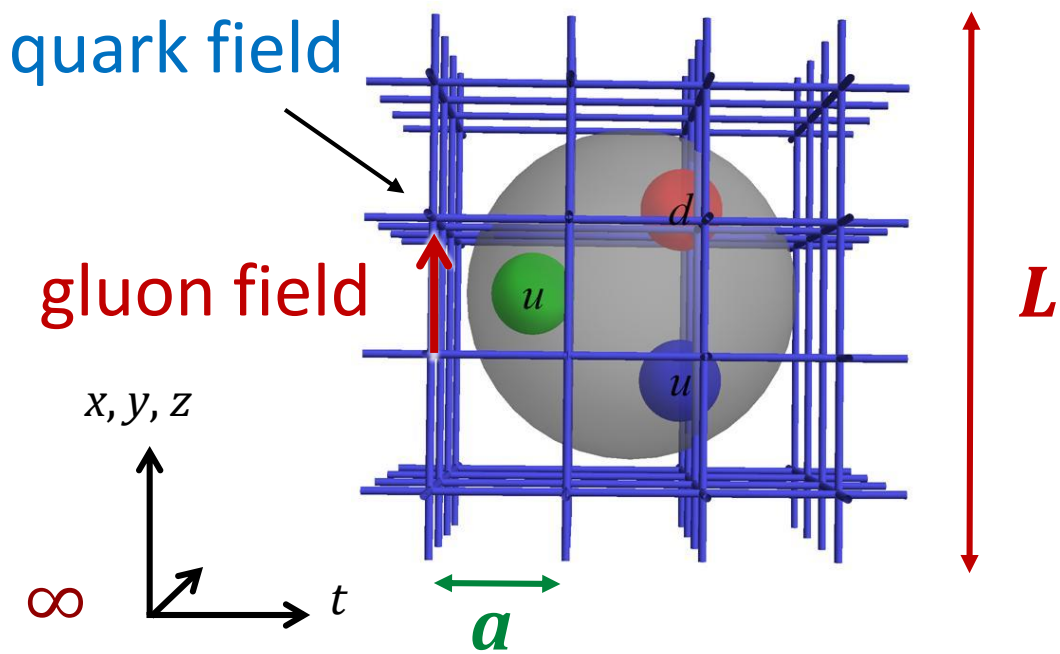
$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS(\bar{\psi},\psi,A)}\,O\,(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

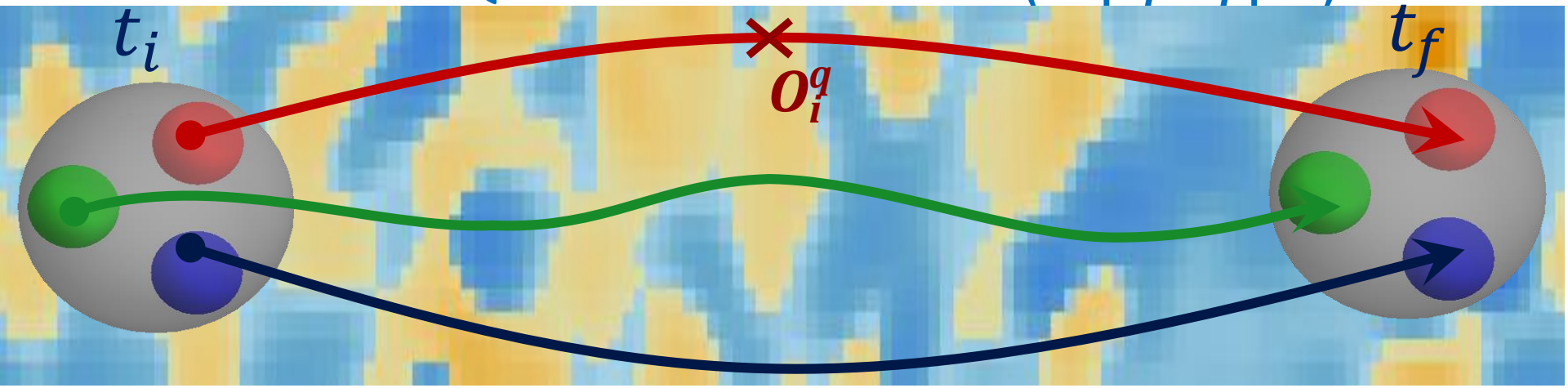
§ Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$



Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Control all systematic errors:

- Finite-volume effects $L \to \infty$
- Chiral extrapolations to physical u/d quark masses $m_\pi \to m_\pi^{\text{phys}}$
- Extrapolation to the continuum limit (lattice spacing $a \to 0$)
- Nonperturbative renormalization using the RI/SMOM scheme
- Contamination from excited states
- Statistical effects

Precision Nucleon Couplings

§ Much effort has been devoted to controlling systematics
§ A state-of-the art calculation (PNDME)

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	24k
0.12	$32^3 \times 64$	4.38	220	8,10,12	7.6k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14	7.0k
0.09	$48^3 \times 96$	4.79	220	10,12,14	7.1k
0.09	$64^3 \times 96$	3.90	130	10,12,14	84.7k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k
0.06	$96^3 \times 192$	3.80	135	16,18,20,22	52.5k

Systematic Control

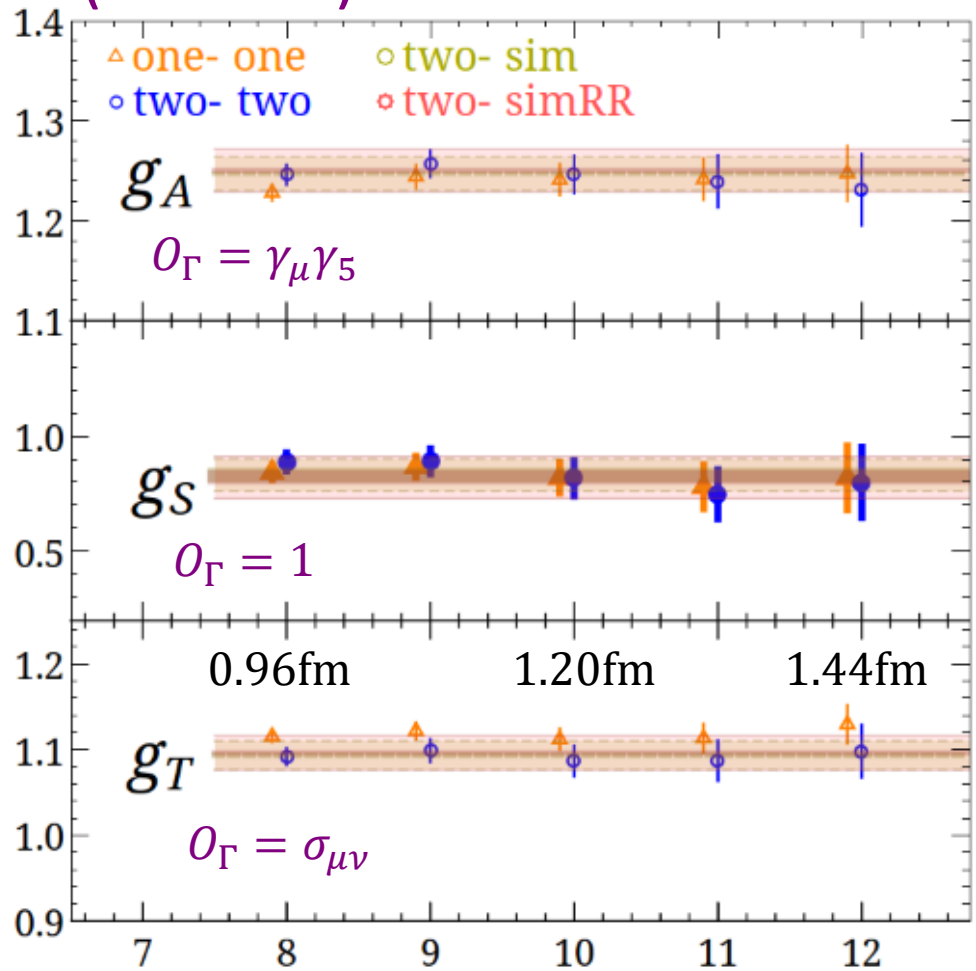
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME) $a = 0.12$ fm, 310-MeV pion

- Move the excited-state systematic into the statistical error

$$C^{3pt}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f - t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^* \langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1 \langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2 \langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

- No obvious contamination between 0.96 and 1.44 fm separation



Systematic Control

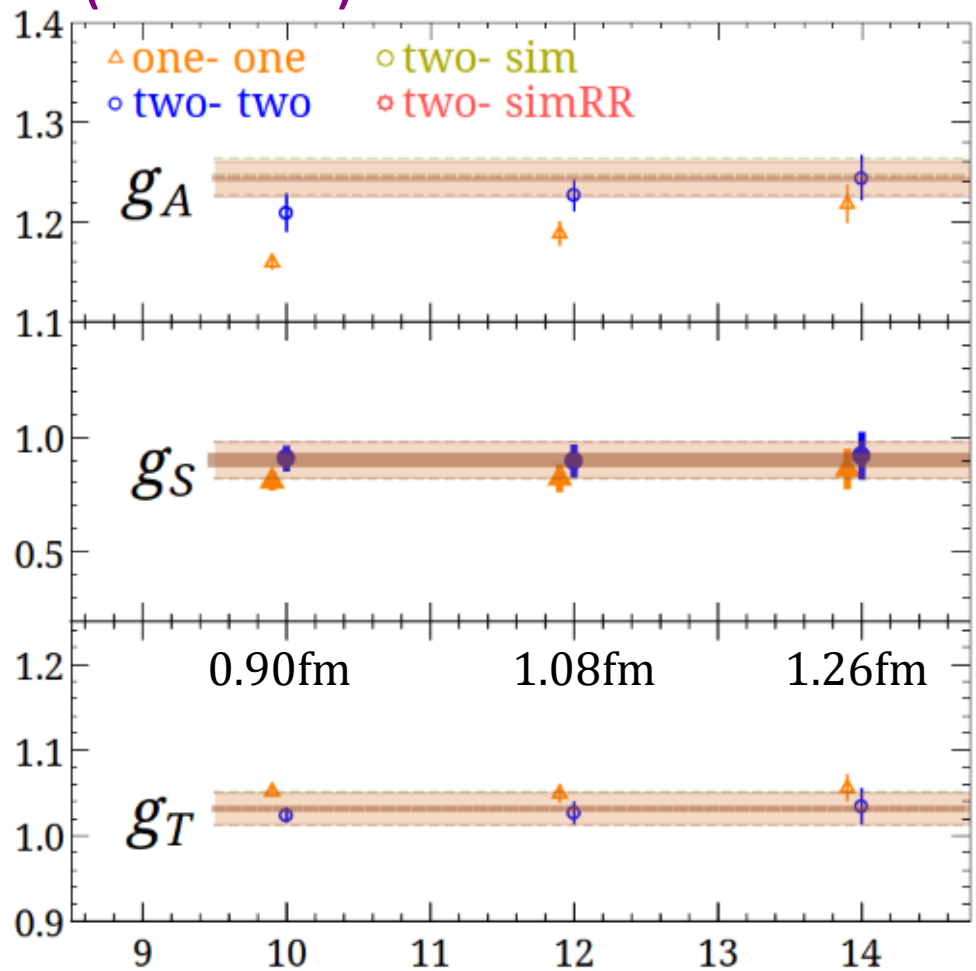
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME) $a = 0.09$ fm, 310-MeV pion

- Move the excited-state systematic into the statistical error

$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f - t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^* \langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1 \langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2 \langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case



Systematic Control

§ Much effort has been devoted to controlling systematics

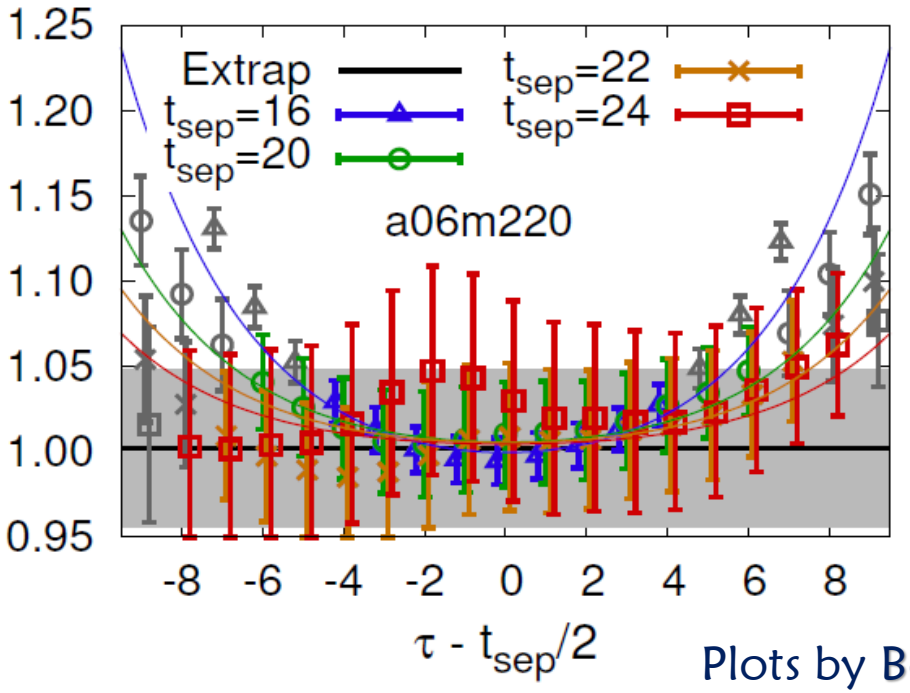
§ A state-of-the art calculation (PNDME)

$a = 0.06$ fm, 220-MeV pion

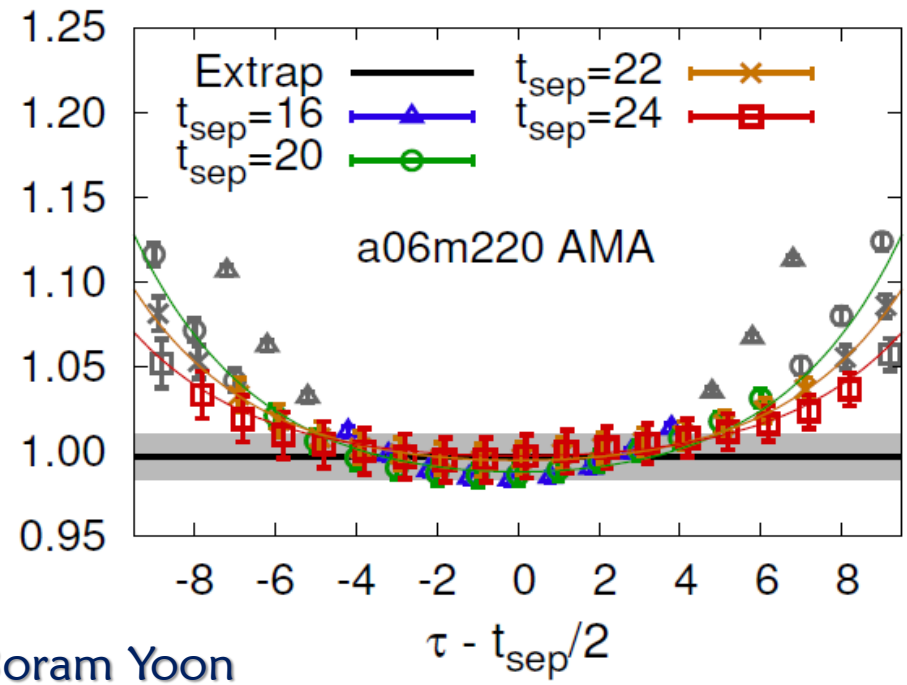
Statistical effect

g_T^{bare}

2.6k



41.6k



Plots by Boram Yoon

Systematic Control

§ Much effort has been devoted to controlling systematics

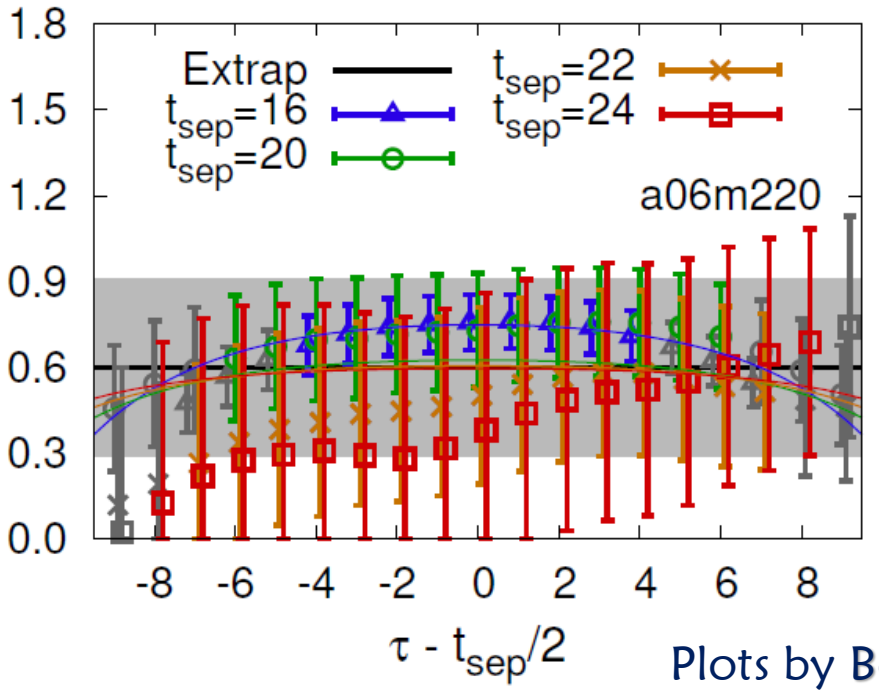
§ A state-of-the art calculation (PNDME)

$a = 0.06$ fm, 220-MeV pion

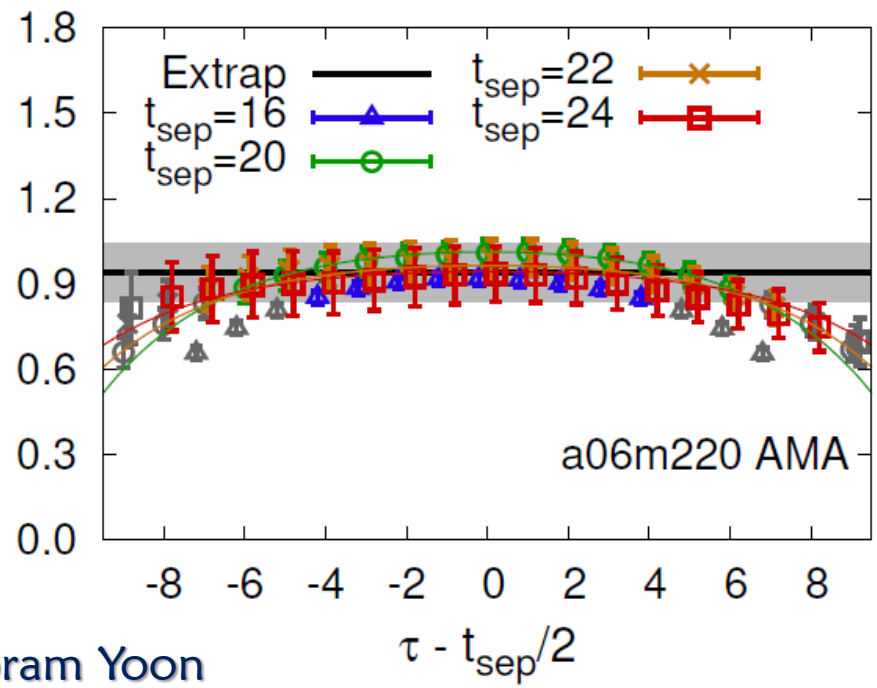
Statistical effect

g_S^{bare}

2.6k



41.6k



Plots by Boram Yoon

Systematic Control

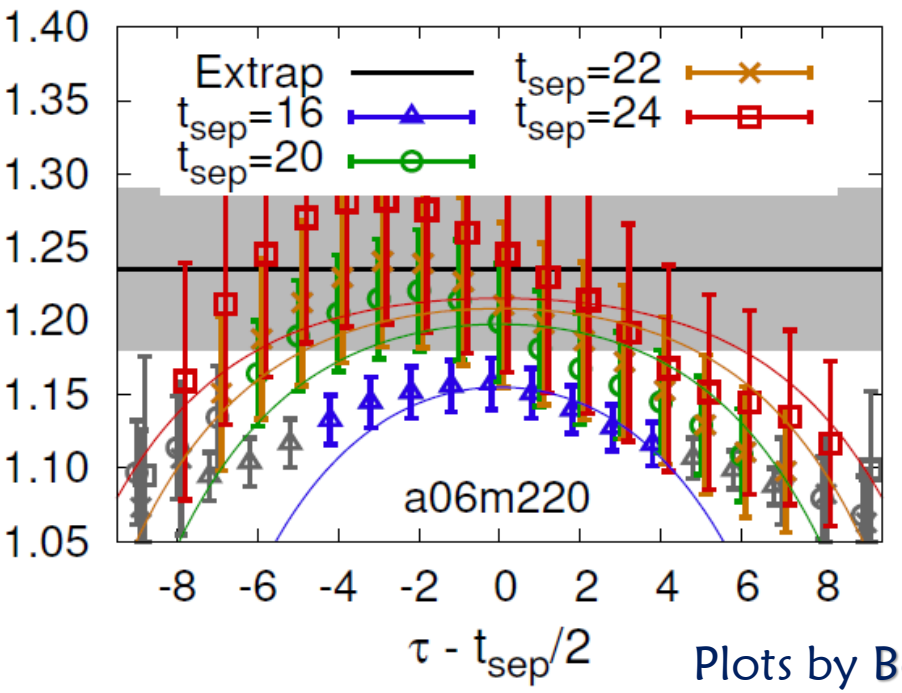
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME)

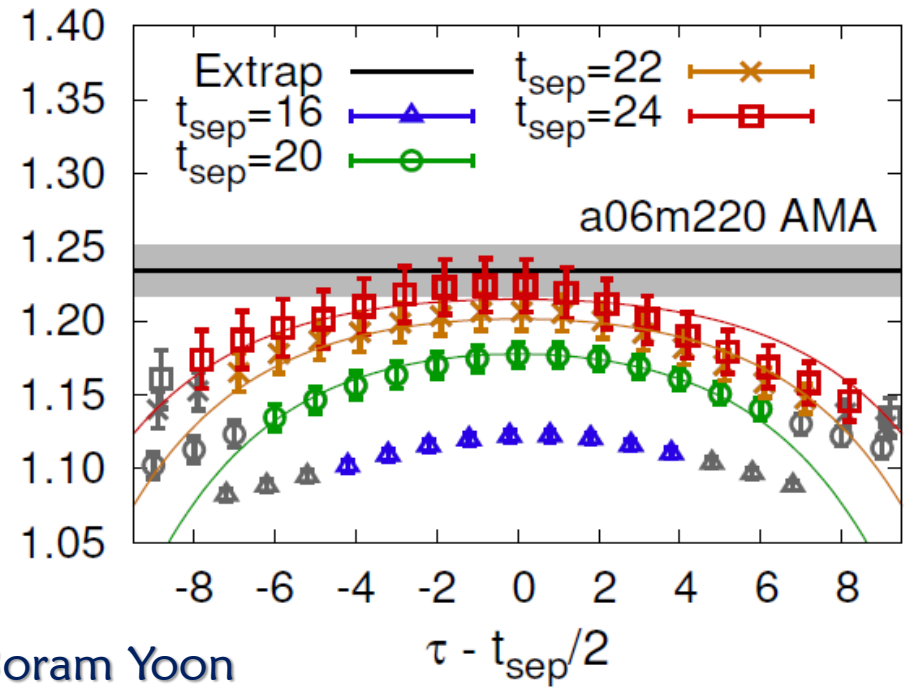
- Statistical effect (worst case) $a = 0.06$ fm, 220-MeV pion

g_A^{bare}

2.6k



41.6k



Plots by Boram Yoon

Systematic Control

§ Much effort has been devoted to controlling systematics

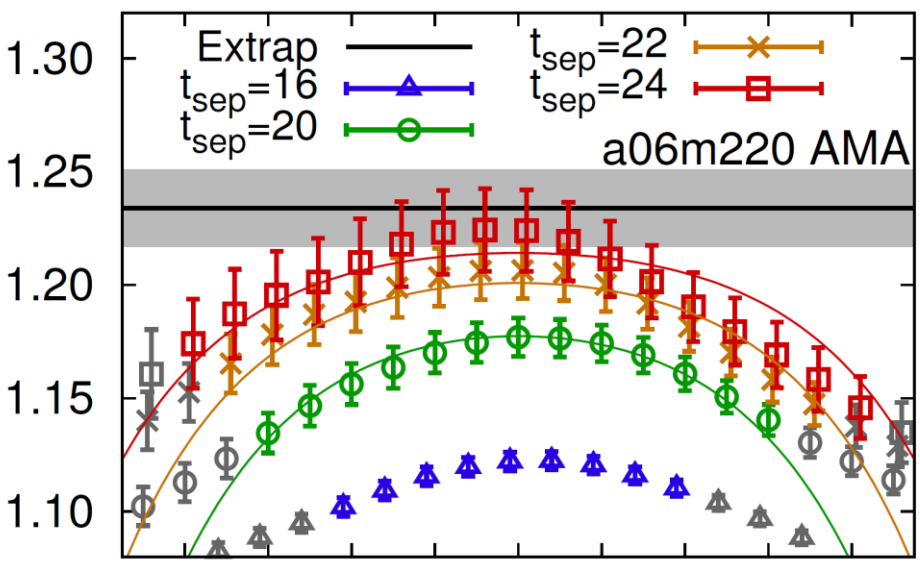
§ A state-of-the art calculation (PNDME)

- Robustness of the 2-state fit

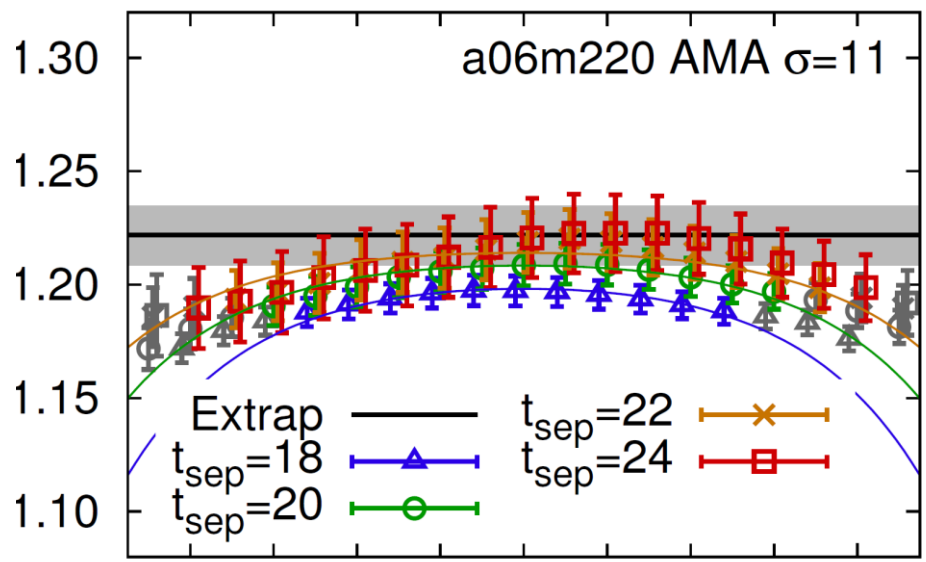
$a = 0.06$ fm, 220-MeV pion

g_A^{bare}

2.6k



41.6k



Plots by Boram Yoon

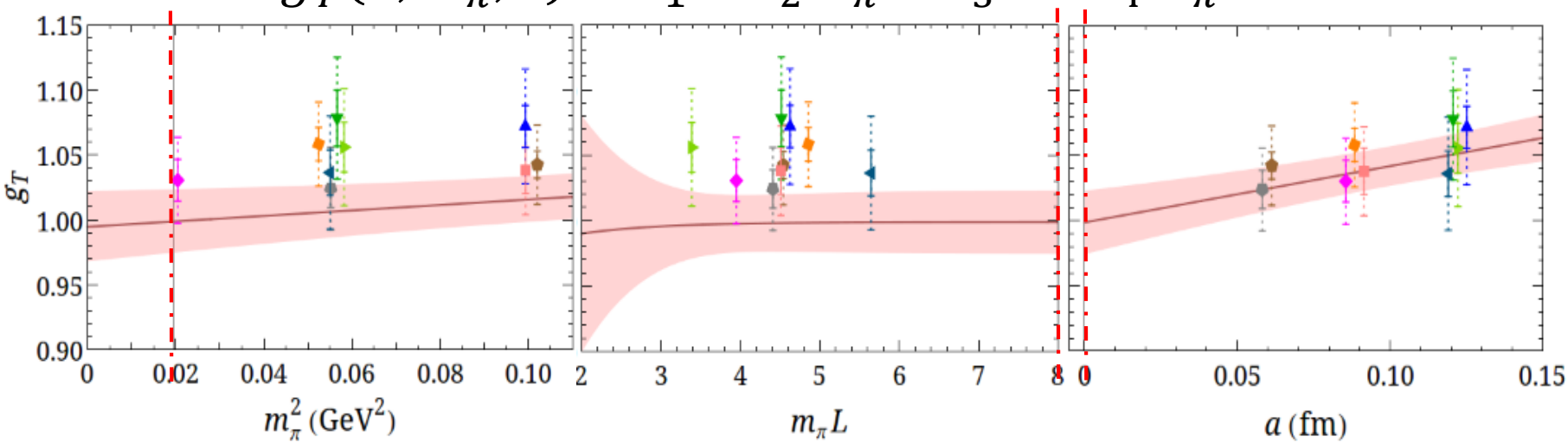
Precision Nucleon Couplings

§ Much effort has been devoted to controlling **systematics**

§ A state-of-the art calculation (PNDME)

- Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$



First extrapolation to the physical limit
of a nucleon matrix element!

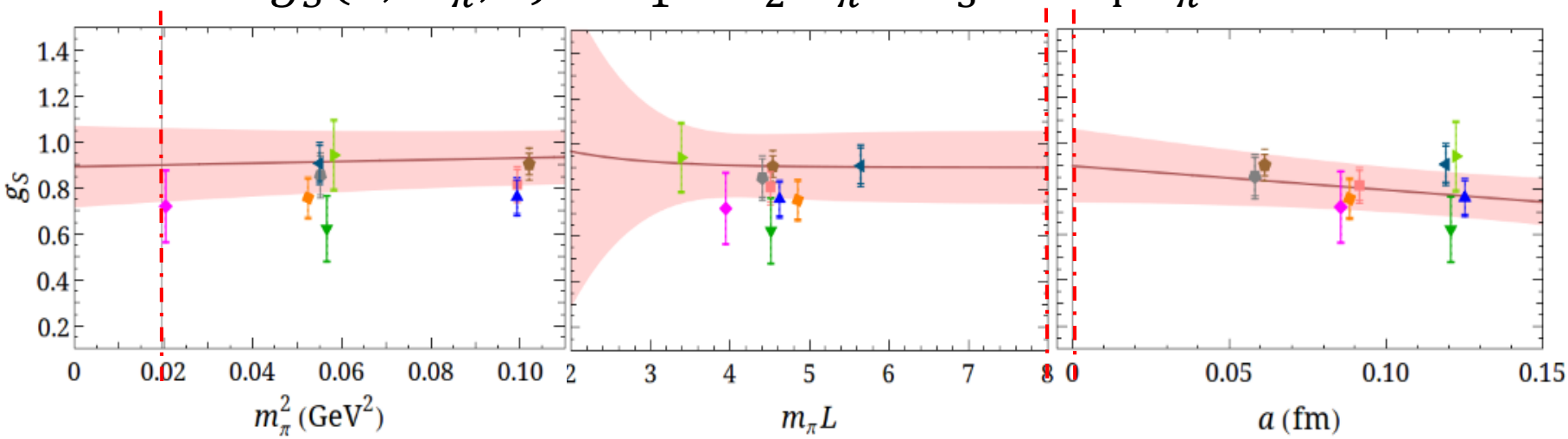
Precision Nucleon Couplings

§ Much effort has been devoted to controlling **systematics**

§ A state-of-the art calculation (PNDME)

- Extrapolate to the physical limit

$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$



First extrapolation to the physical limit
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Precision Nucleon Couplings

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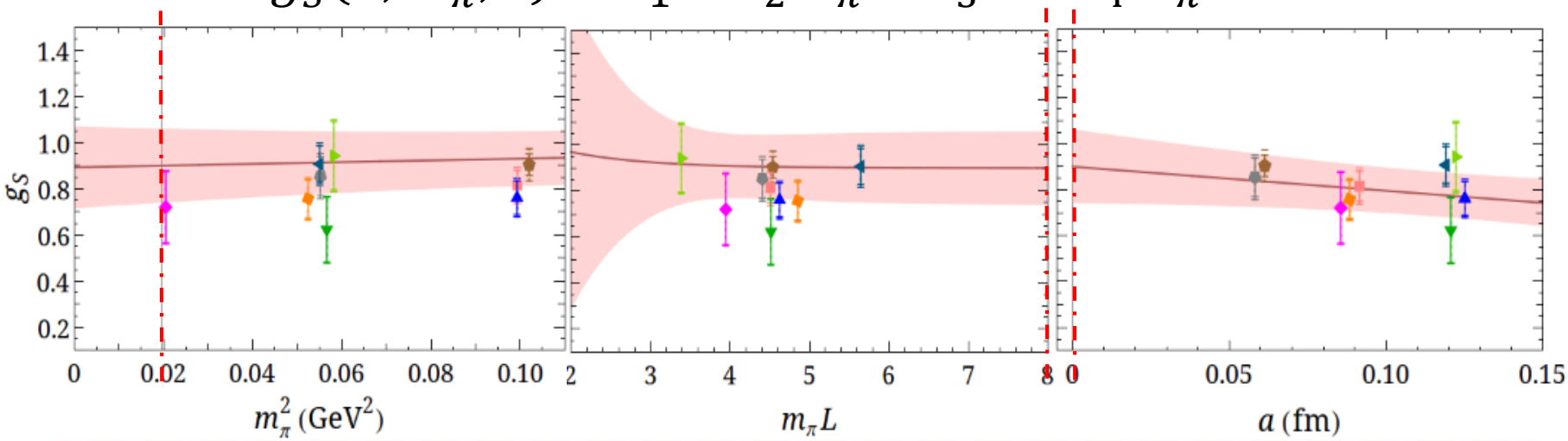
$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$



Using conserved vector current relation:

$$\left(M_n - M_p\right)^{\mathrm{QCD}} = (g_S/g_V)(m_u - m_d)^{\mathrm{QCD}}_{\mathrm{FLAG}} = 2.59(49)\ \mathrm{MeV}$$

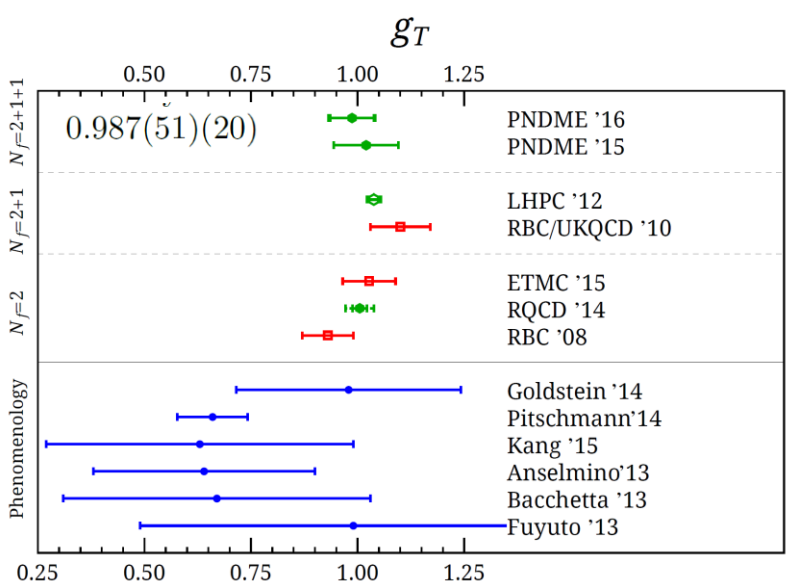
Precision Nucleon Couplings

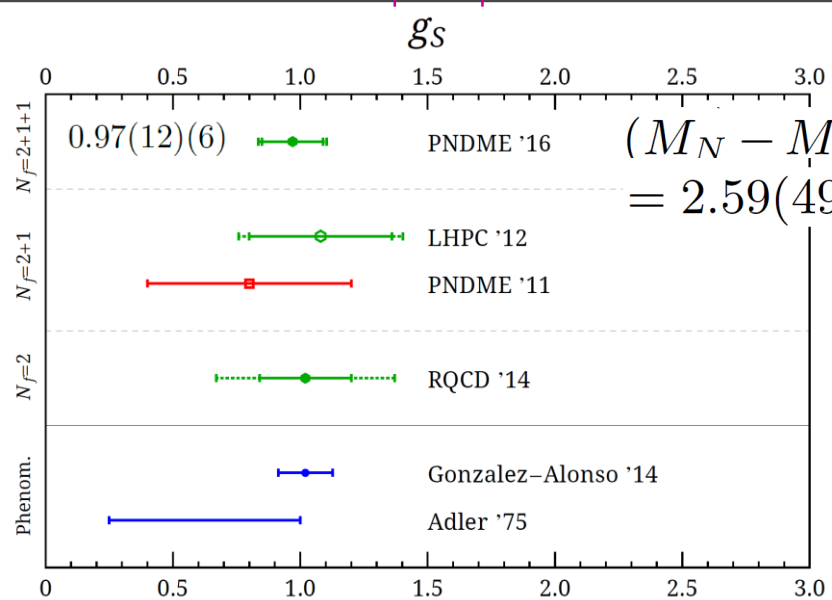
FLAG rating system

PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]



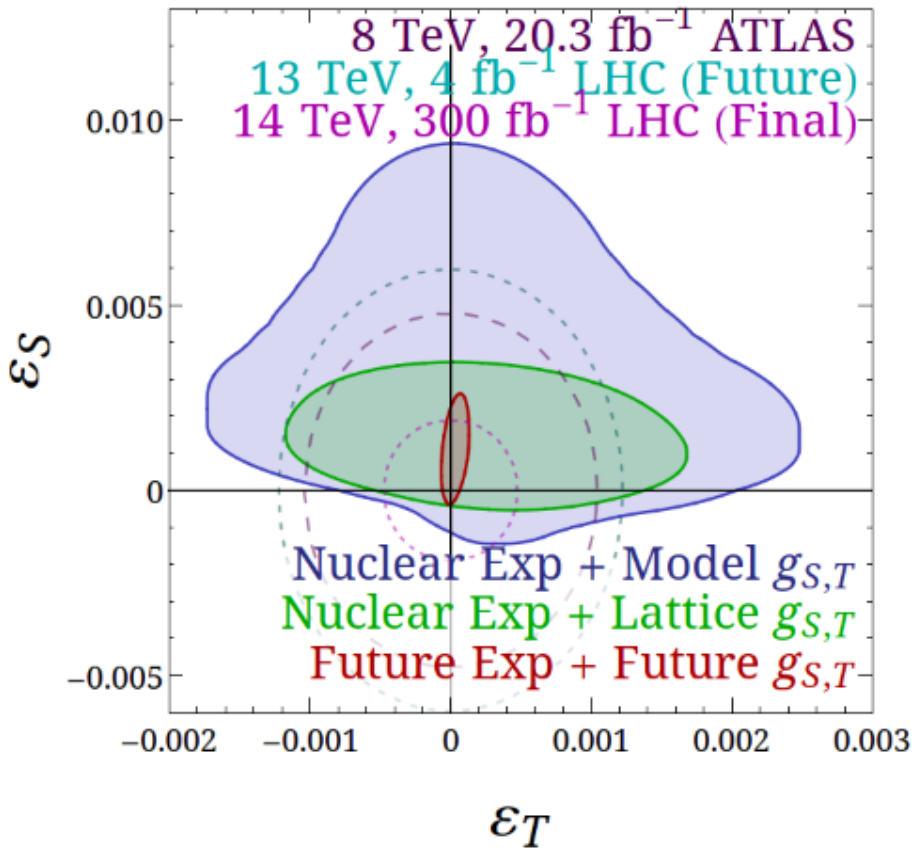


$$(M_N - M_P)^{\rm QCD} = 2.59(49)\ \text{MeV}$$

Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Low-Energy Expt → $O_{BSM} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)



$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$$|B_1 - b|_{BSM} < 10^{-3}$$
$$|b|_{BSM} < 10^{-3}$$

CENPA: ^{6}He(b_{GT}) at 10^{-3}

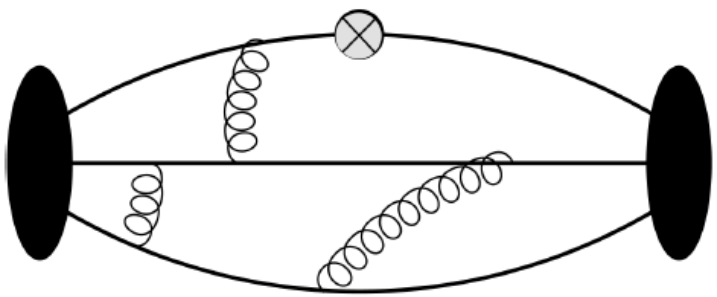
PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049

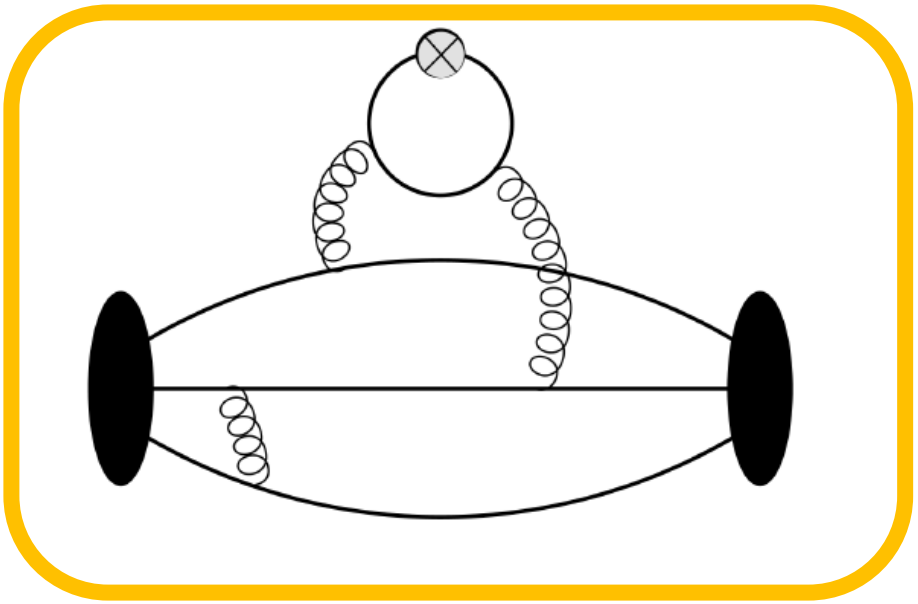
$$\Lambda_S > 7 \text{ TeV}$$
$$\Lambda_T > 13 \text{ TeV}$$

Flavor Decomposition

§ Disconnected diagram

- Multiple ways to calculate this notorious contribution…
- Truncated solver, hopping-parameter expansion, hierarchical probing, …



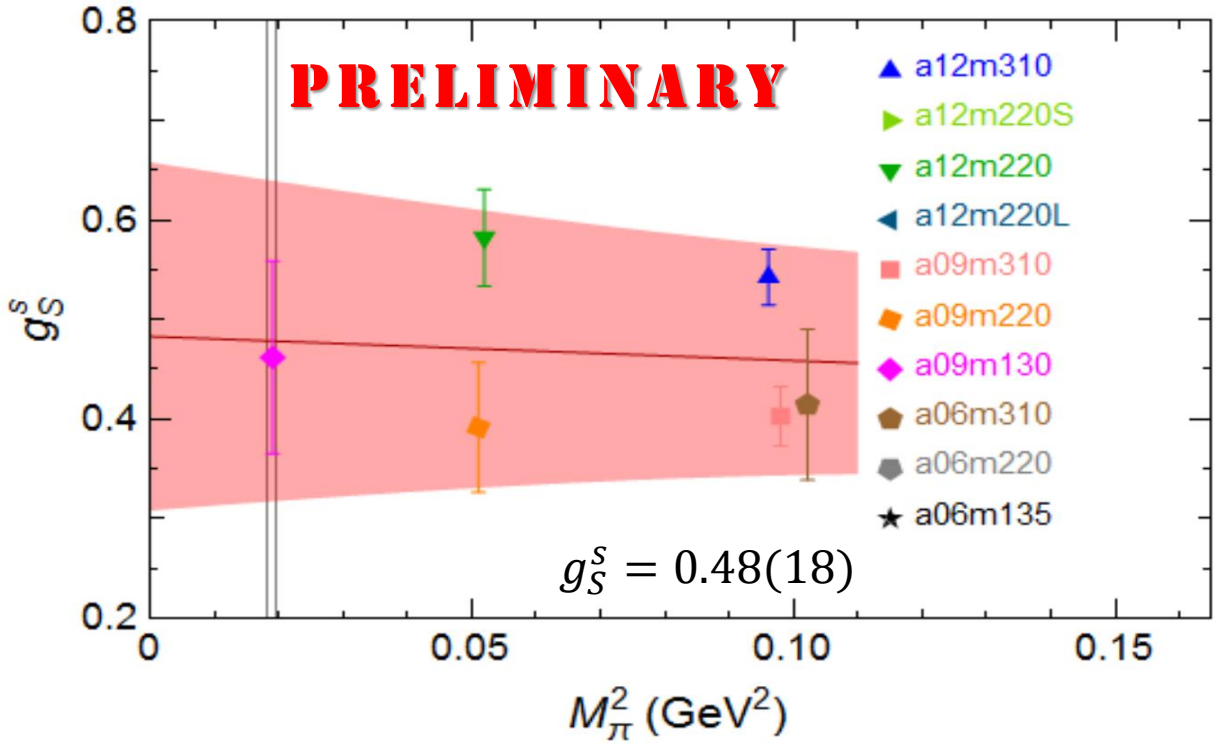


Strange Contribution

§ $\langle N|s\bar{s}|N\rangle$

- Purely disconnected contribution

$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



0.42(13) χQCD
0.35(15) RQCD
0.33(9) ETMC

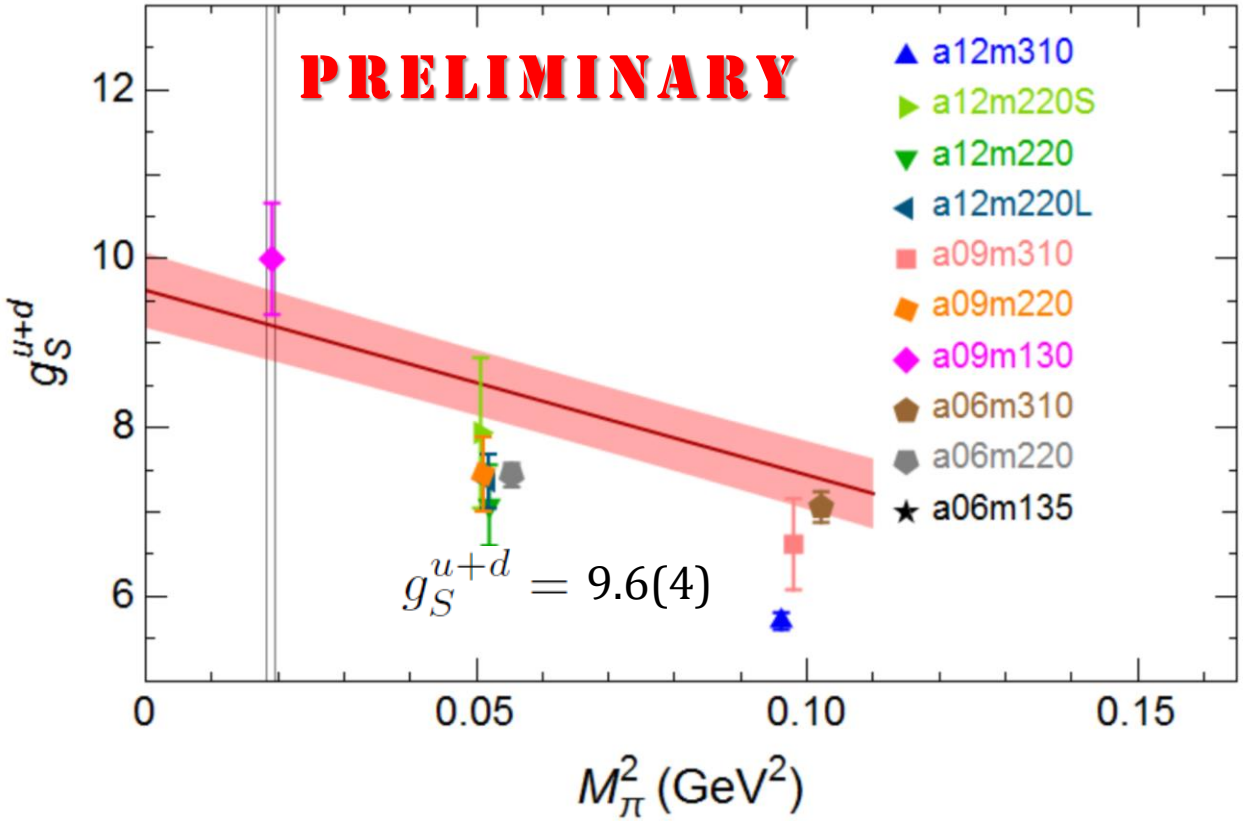
Yang this Monday,
1603.00827,1703.08788

Up/Down Contribution

§ $\langle N|u\bar{u} + d\bar{d}|N\rangle$

- Including the disconnected contribution

$$g_S(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



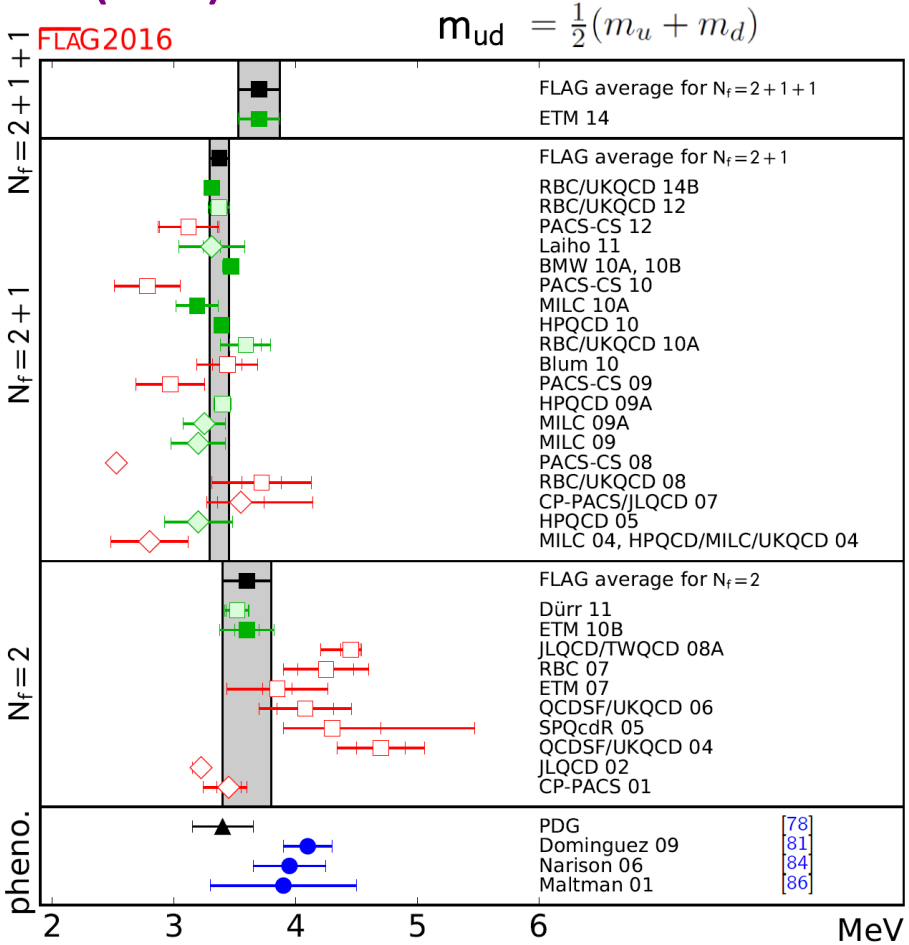
9.3(1.8) RQCD
~9.5(6) ETMC

1603.00827,1703.08788

Sigma Term

§ Convert to sigma term using FLAG 2+1+1f m_{ud}

35.5(2.2) MeV



$$m_{ud} = 3.70(17)\ \text{MeV}$$

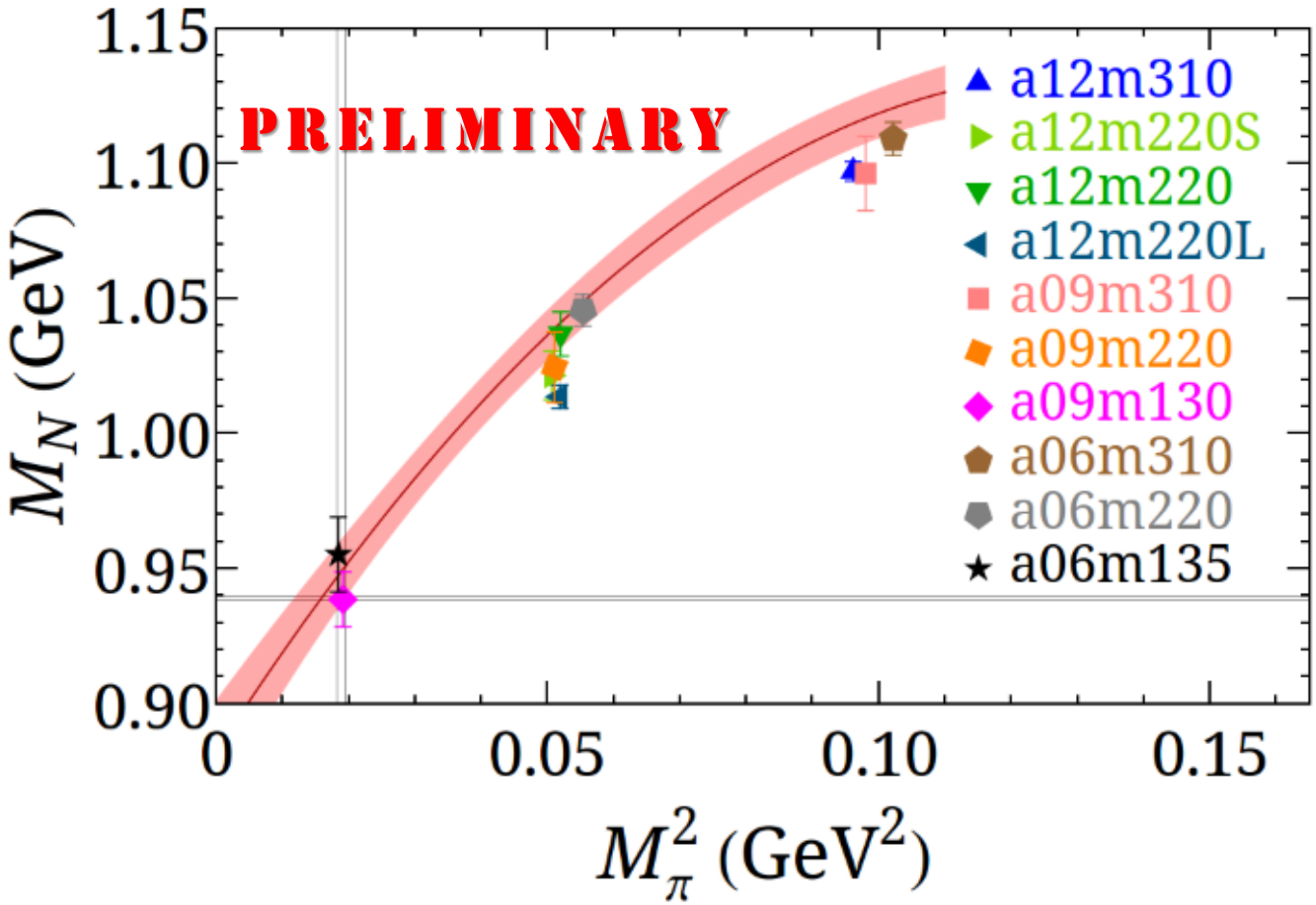
$$m_{ud} = 3.373(80)\ \text{MeV}$$

$$m_{ud} = 3.6(2)\ \text{MeV}$$

Nucleon Mass

§ Using Feynman-Hellman Theorem to get sigma term

$$M_N(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_2' m_\pi^4 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$

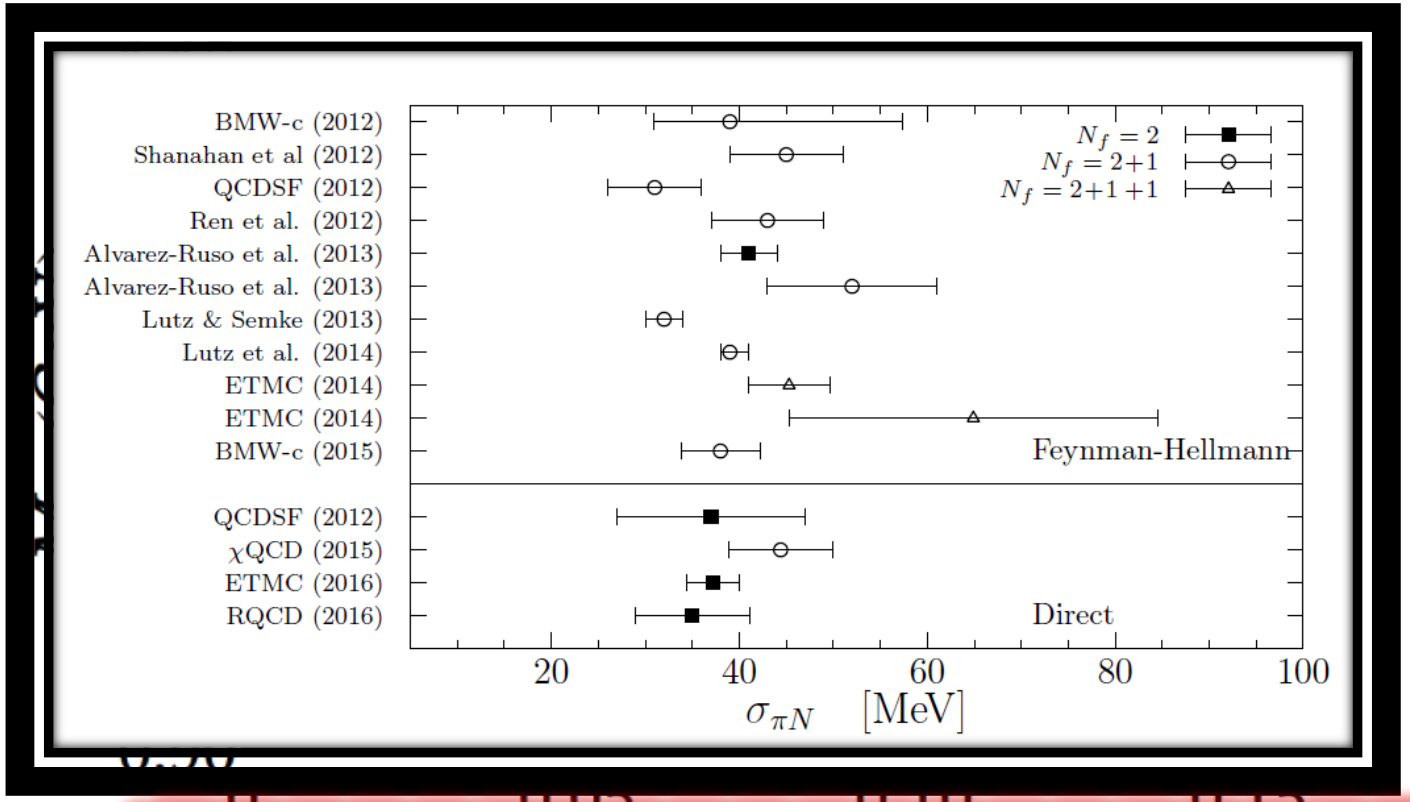


$\sigma_{\pi N} =$ 63(8) MeV

Nucleon Mass

§ Using Feynman-Hellman Theorem to get sigma term

$$M_N(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_2' m_\pi^4 + c_3 a + c_4 m_\pi^2 e^{-m_\pi L}$$

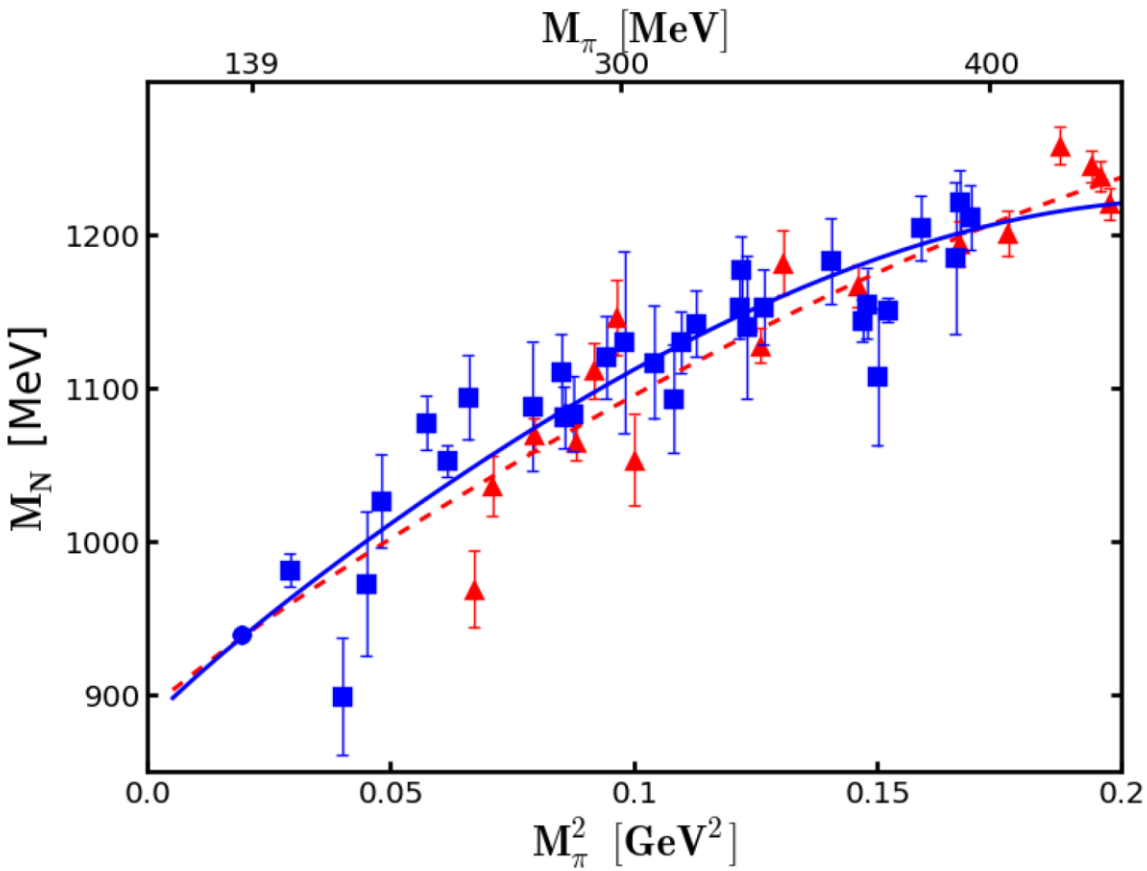


$\sigma_{\pi N} =$ 63(8) MeV

Overall trend: Direct calculation has smaller $\sigma_{\pi N}$

Nucleon Mass

§ Sea-flavor dependence: 2f vs 2+1f



$$\sigma_{\pi N}^{N_f=2+1} = 52\,(3)\,(8) \quad \text{MeV}$$

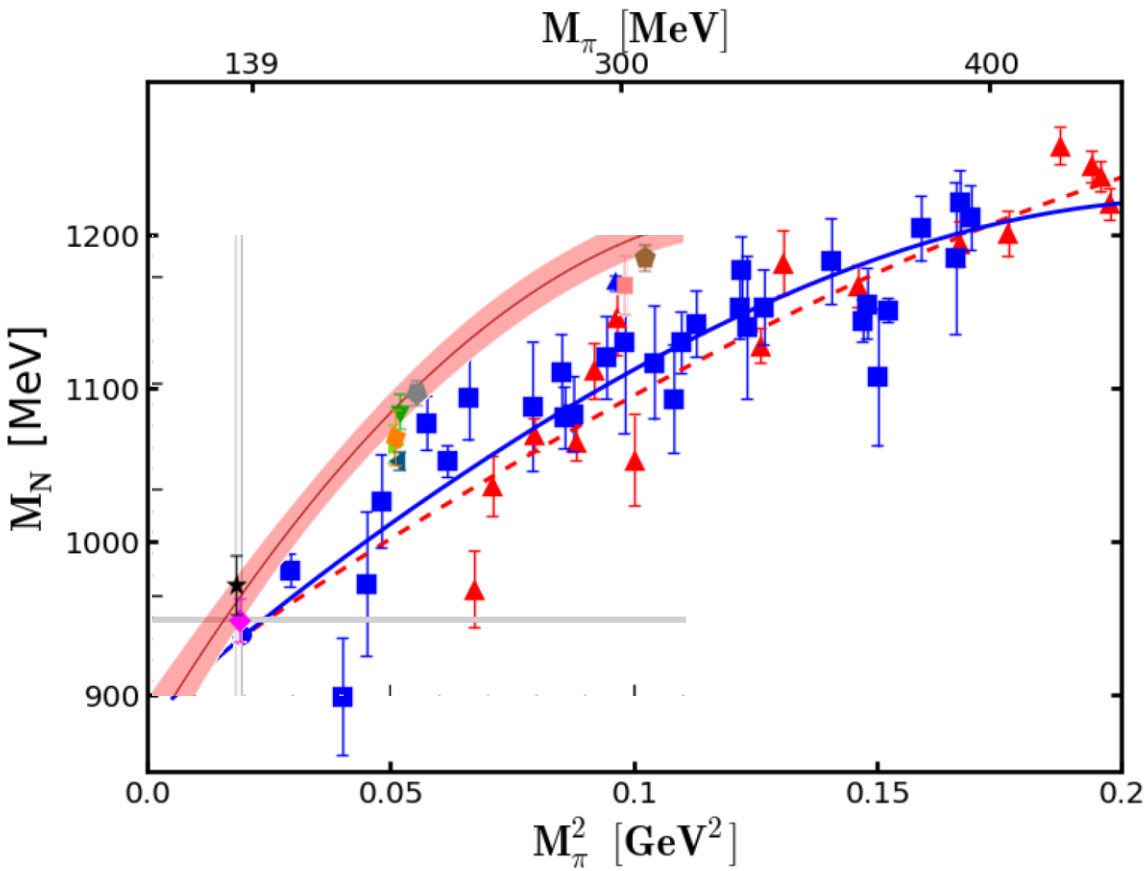
$$\sigma_{\pi N}^{N_f=2} = 41\,(5)\,(4) \quad \text{MeV}$$

L. Alvarezz-Ruso et al, 1402.3991

$$M_N^{(4)}(M_\pi^2) = M_0 - c_1 4M_\pi^2 + \frac{\bar{\alpha} M_\pi^4}{2} + \frac{c_1 M_\pi^4}{8\pi^2 f_\pi^2} \ln \frac{M_\pi^2}{M_0^2} + \Sigma_{\text{loops}}^{(3+4)}(M_\pi^2) + O(p^5)$$

Nucleon Mass

§ Sea-flavor dependence: 2f vs 2+1f vs 2+1+1f



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"b" Term

§ On Monday, Jianwei mentioned

✧ **Quark mass contribution:**

$$H_m = \int d^3\vec{x}\ \bar{\psi} m \psi \qquad M_m = \left.\frac{\langle P|H_m|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = b\, M_p$$

✧ **Trace anomaly contribution:**

$$H_a = \int d^3\vec{x}\, \frac{9\alpha_s}{16\pi}\left(\mathbf{E^2} - \mathbf{B^2}\right) \qquad M_a = \left.\frac{\langle P|H_a|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-b)\,\frac{1}{4}\, M_p$$

§ $bM_p = 2m_{ud}\langle N|u\bar{u} + d\bar{d}|N\rangle + m_s\langle N|s\bar{s}|N\rangle = 80(16)\ \text{MeV}$

$$\Rightarrow b = 0.086(17)$$

➰ Quark-mass contribution to proton mass: 8.6%

§ Trace anomaly contribution: 22.8%

How about Nuclei?

§ Only $A \leq 4$ light nuclei are calculated on the lattice

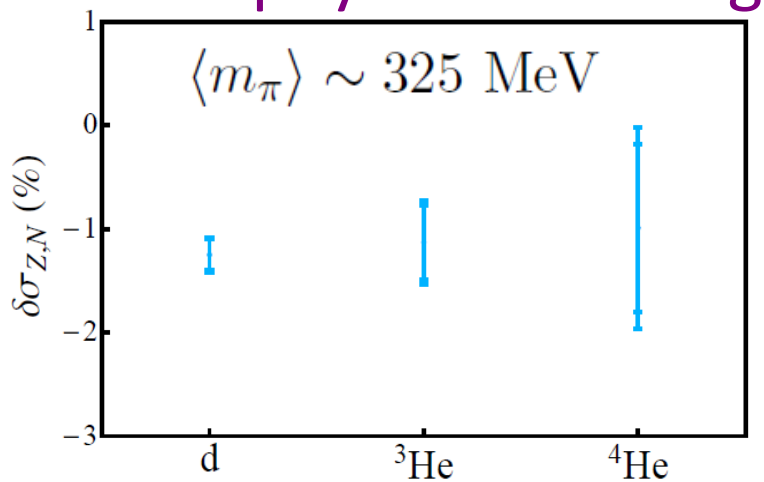
NPLQCD, 1306.6939

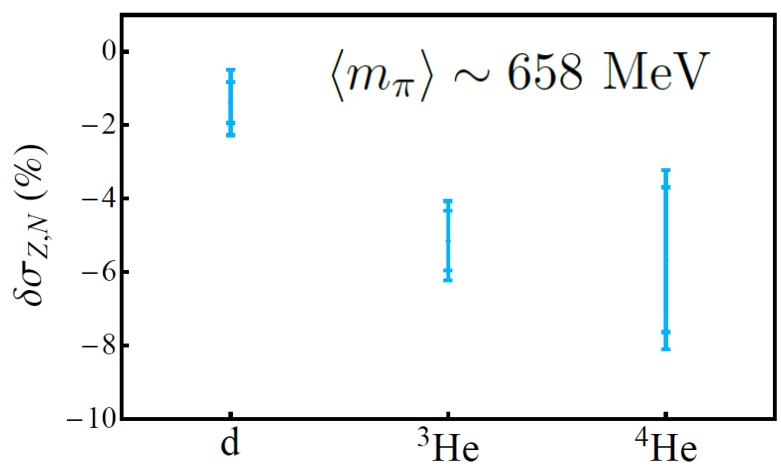
- Assume isospin symmetry

$$\sigma_{Z,N} = \bar{m}\langle Z,N|\bar{u}u + \bar{d}d|Z,N\rangle = \bar{m}\frac{d}{d\bar{m}}E_{Z,N} \approx \frac{M_\pi}{2}\frac{d}{dM_\pi}E_{Z,N}$$

- Express in terms of the nuclear binding energies $E = AM_N - B$
- Impulse correction is $\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\bar{u}u+\bar{d}d|Z,N(\mathrm{gs})\rangle}{A\langle N|\bar{u}u+\bar{d}d|N\rangle} - 1$

§ Lattice + physical data suggest effects are O(10%) or less





Backup Slides

Strangeness

§ Importance of g_S^s

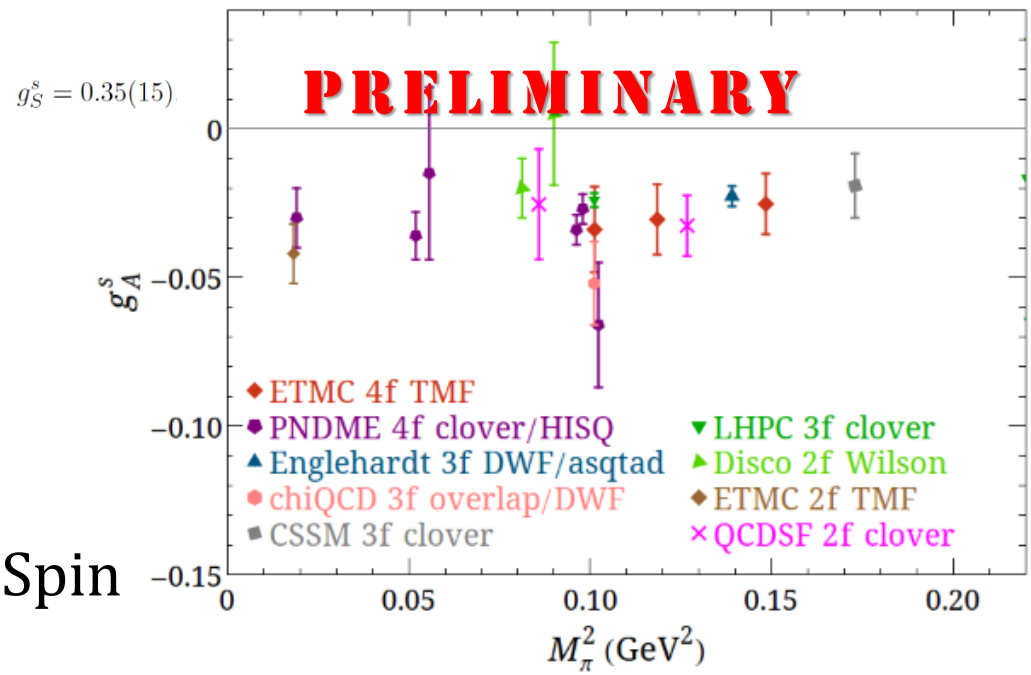
- Strange-quark intrinsic-spin contribution to proton
- Astrophysics application: the CCSN "problem"

  3D explosions require $g_A^s \approx -0.2$ Janka, Melson, & Summa (2016)

§ Global fit: $g_A^s \approx -0.1$

assumptions often used:

$$\Delta\bar{s}(x,Q^2) = \Delta\bar{u}(x,Q^2) = \Delta\bar{d}(x,Q^2) = \frac{1}{2}\Delta s^+(x,Q^2)$$



§ Lattice status

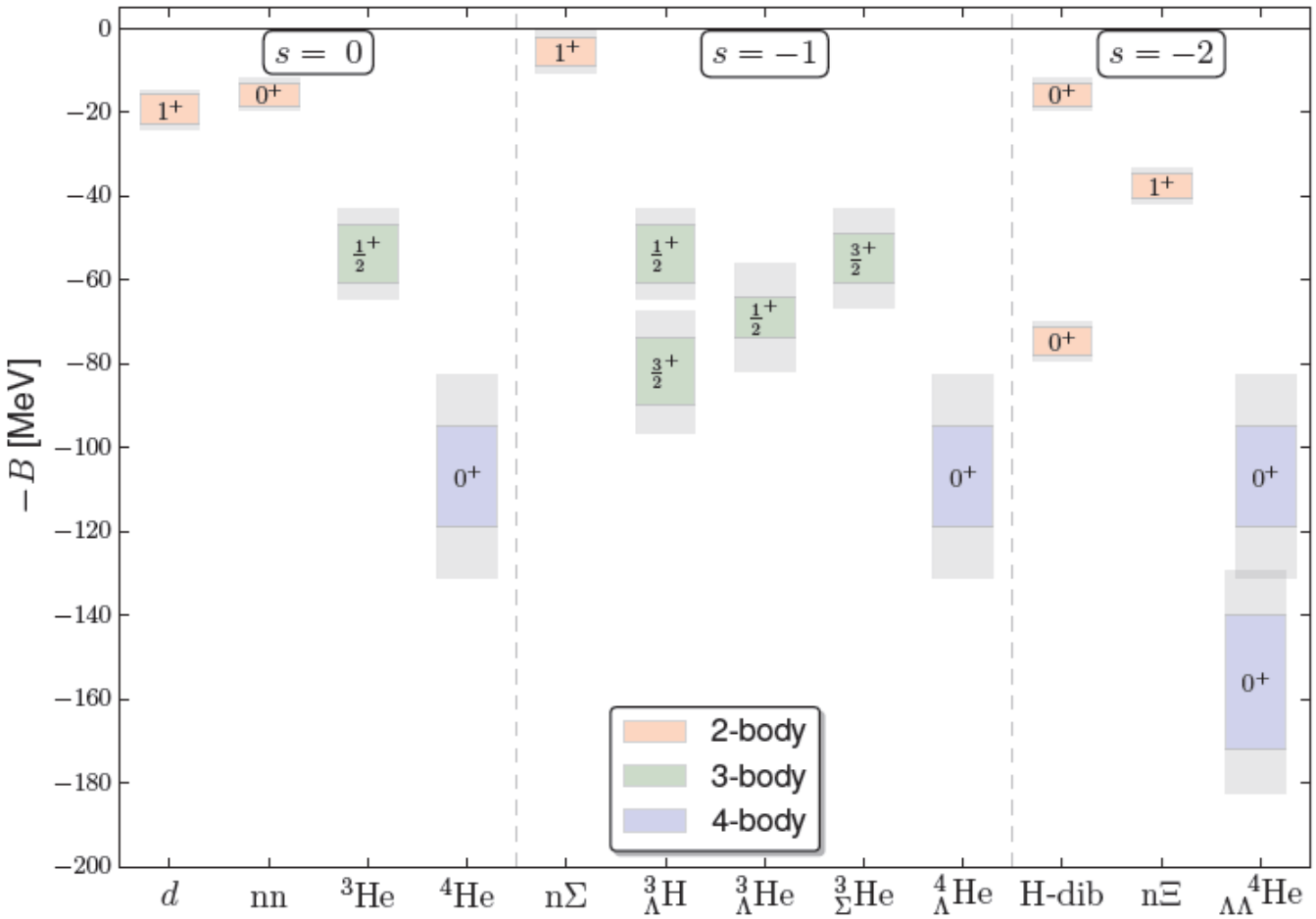
More players since the last Spin

Lighter pion masses

Fundamental Questions

Advanced computing makes it possible!

§ How does QCD bind (hyper)nuclei?



NPLQCD, Phys.Rev. D87, 034506 (2013)

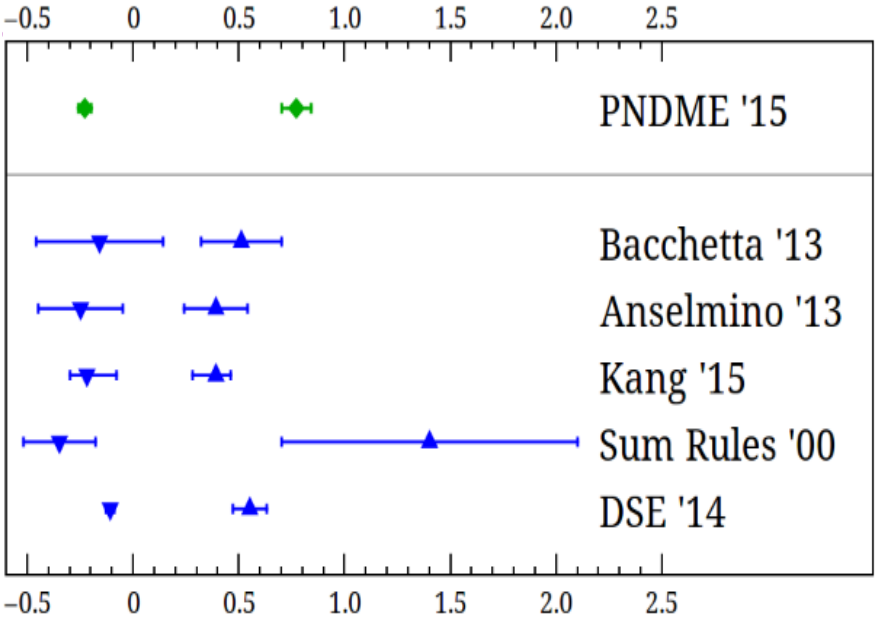
SU(3) flavor-symmetric quark masses

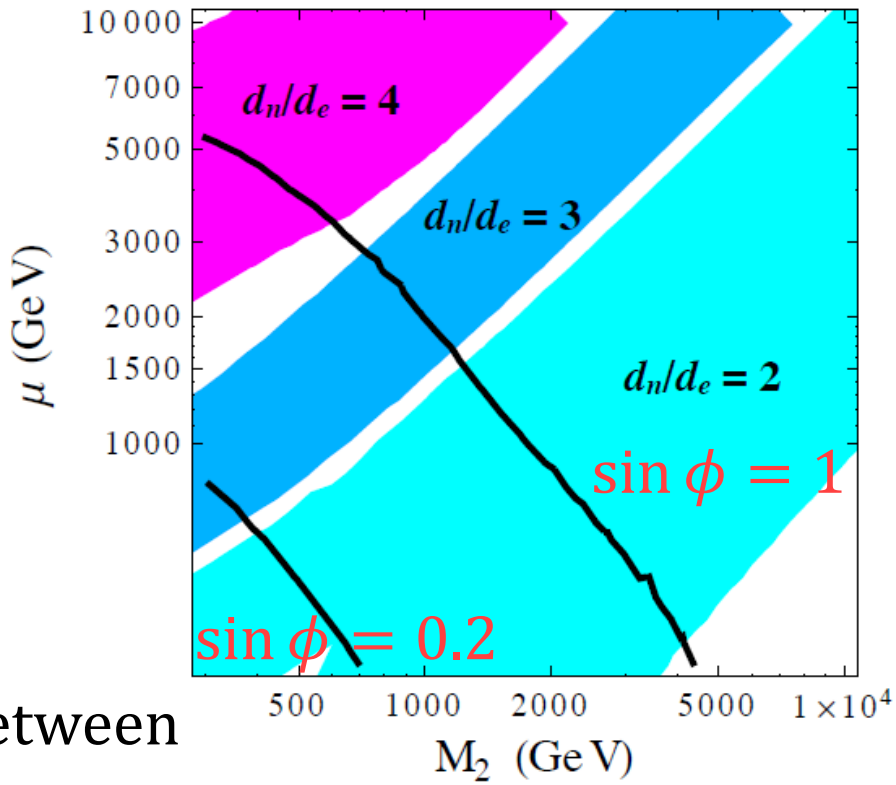
Quark EDM

§ Extrapolate to the physical limit

PNDME, 1506.04196; 1506.06411

$$g_T^d = -0.233(28), g_T^u = 0.774(66), g_T^s = 0.008(9)$$





Observation of a neutron EDM between the current limit and $4 \times 10^{-28} e \cdot \mathrm{cm}$ would falsify the split-SUSY scenario with gaugino mass unification

PNDME

Precision Neutron-Decay Matrix Elements

https://sites.google.com/site/pndmelqcd/

Tanmoy Bhattacharya

Rajan Gupta

HWL

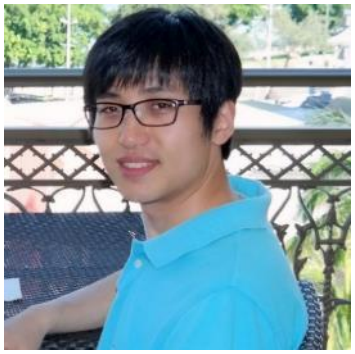
+

Vincenzo Cirigliano

Saul Cohen

Anosh Joseph

Yong-Chull Jang



Boram Yoon

The Trouble with Nucleons

Nucleons are more complicated than mesons because…

§ Noise issue

- Signal diminishe

§ Excited-state cor

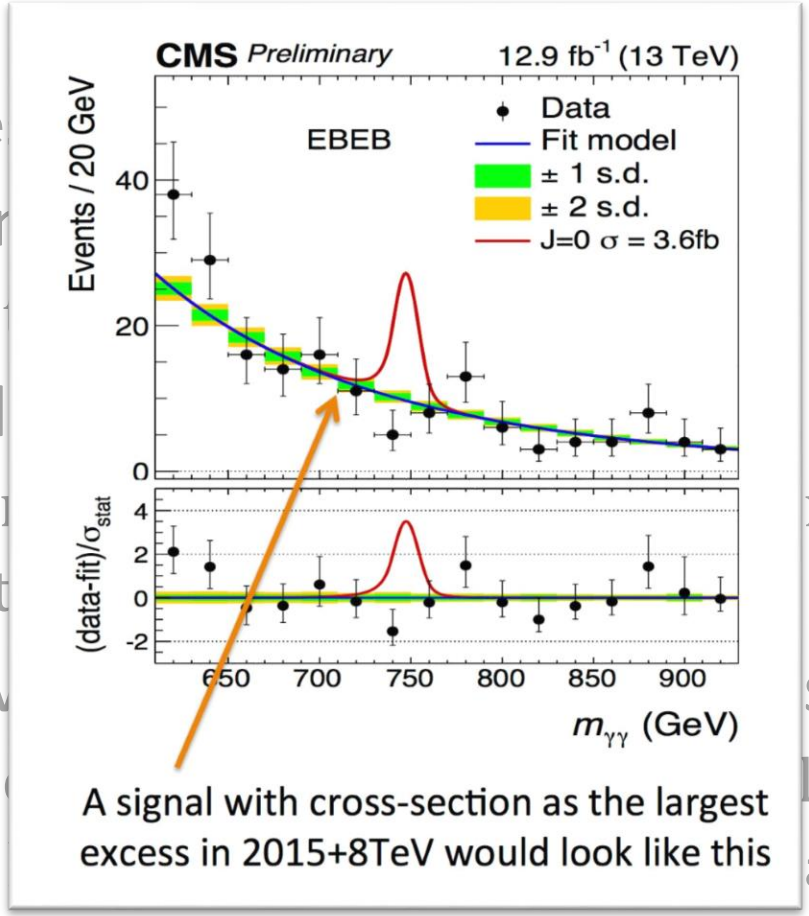
- Nearby excited s

§ Hard to extrapol

- Δ resonance nea r convergence…
- Less an issue in t

§ Requires larger v s

- Ensembles are n leons in mind
- **High-statistics:** amples



A signal with cross-section as the largest excess in 2015+8TeV would look like this

The disappearance of X(750)

The Trouble with Nucleons

Nucleons are more complicated than mesons because…

§ Noise iss

- Signal di

§ Excited-s

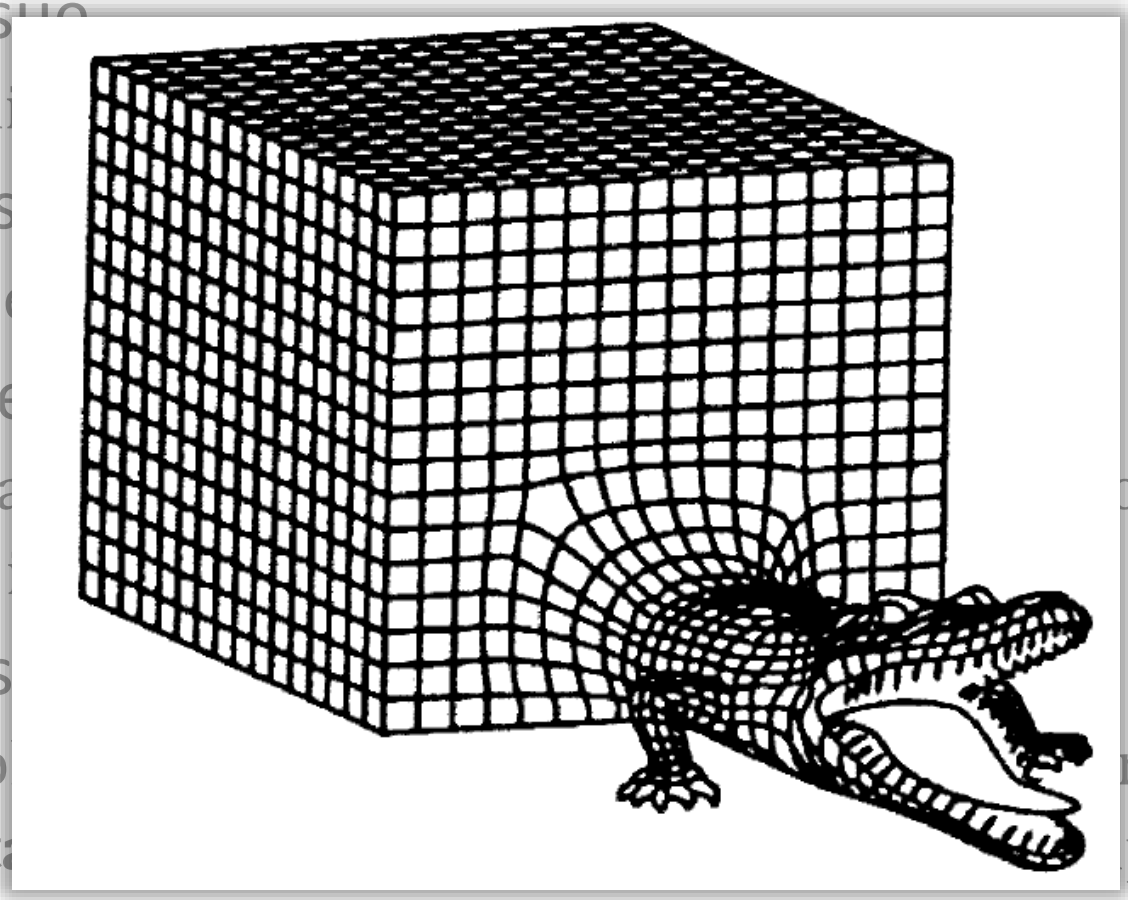
- Nearby

§ Hard to e

- Δ resona onvergence…
- Less an

§ Requires

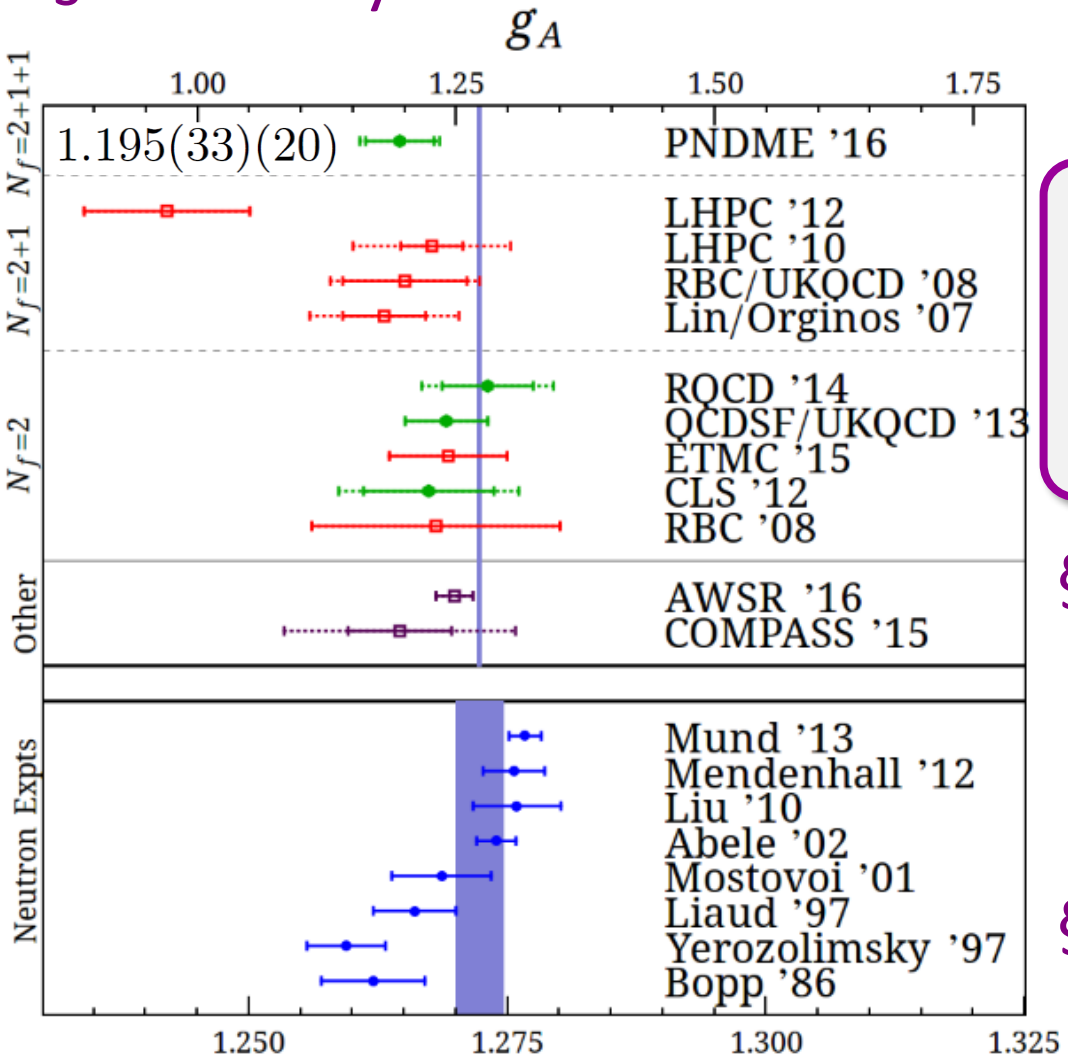
- Ensemb ns in mind
- **High-sta** ples



PROCEED WITH CAUTION

Nucleon Axial Charge

§ Summary



§ Implications?

- 2σ might go away with greater statistics

Lattice 2016 Prelim.

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.8(4)

§ New physics?

- $\lambda = g_A / g_V \, f_{NP}$

$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

§ Stay tuned…

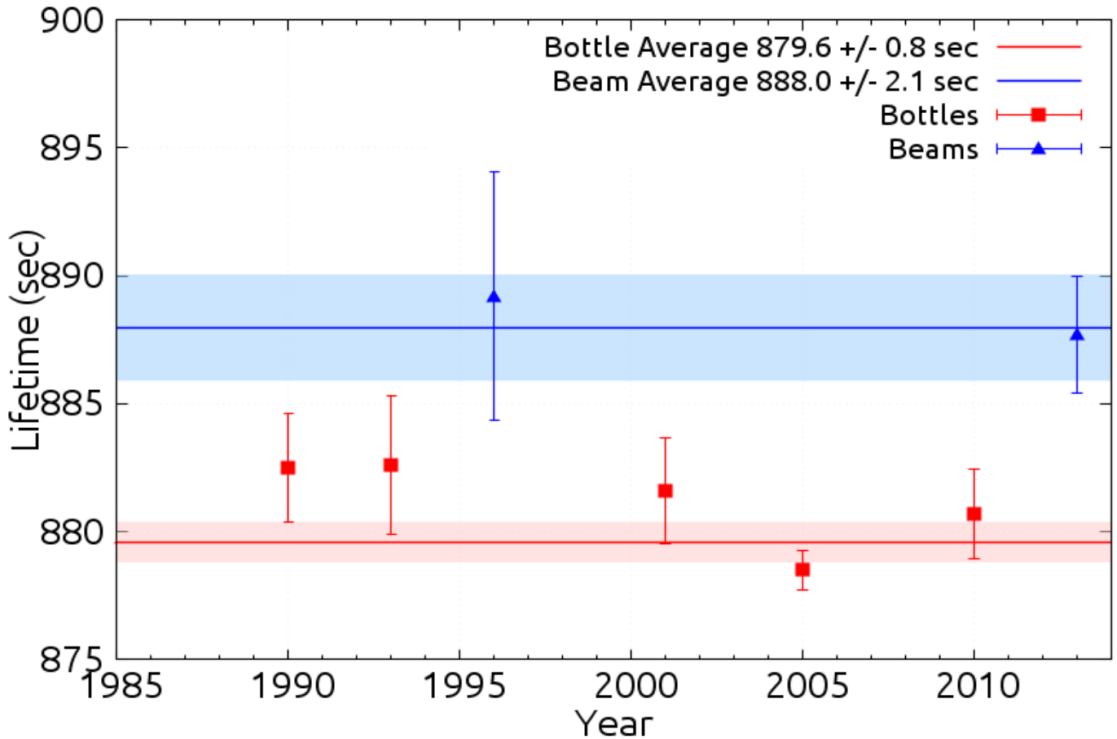
n-Lifetime Discrepancy

§ Neutron lifetime discrepancy?

- $\tau_n = 980(50)$ s Using lattice g_A and V_{ud}

The Situation…Today

Slide by Geoff Greene



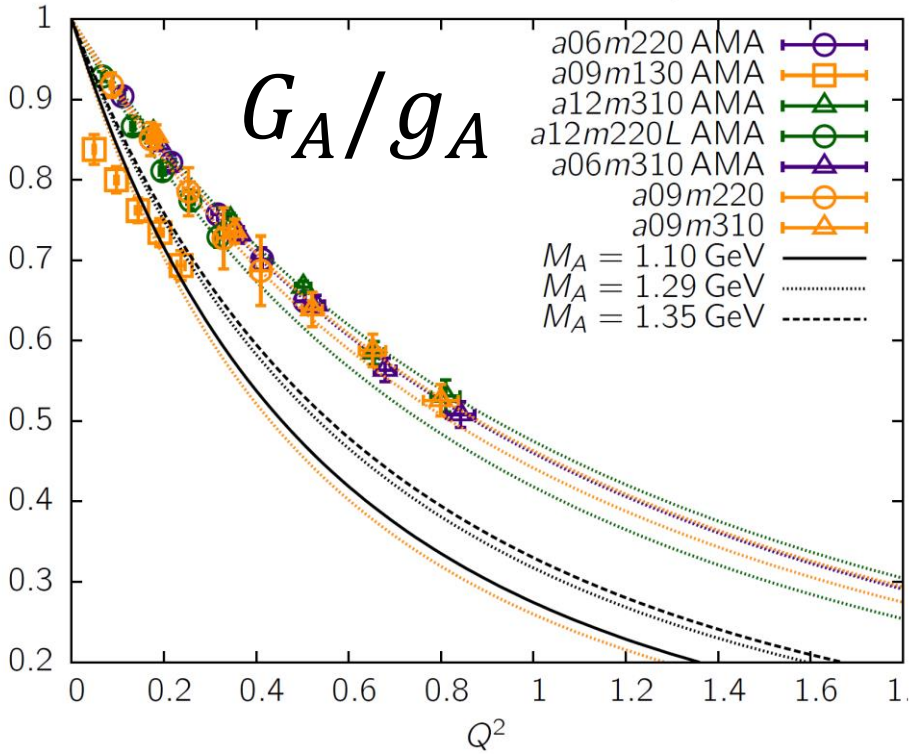
A. Yue, et al, Phys Rev Lett, 111, 222501 (2013)

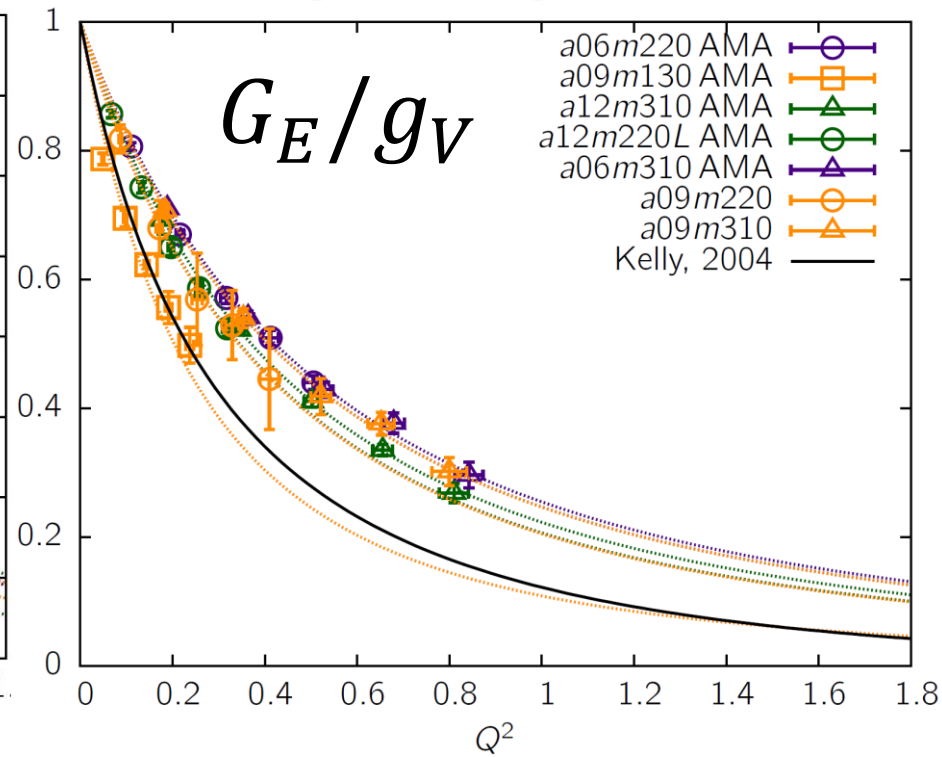
plot courtesy K. Grammer

Others' Results

§ Isovector form factors

Plots by Yong-Chull Jang





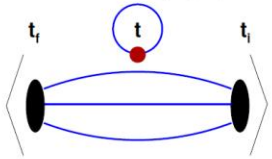
§ Flavor-dependent couplings, 1st moments of PDFs, …

- qEDM by Cirigliano (this afternoon)

§ Updates from Lattice 2016

Quark and gluon momentum fraction

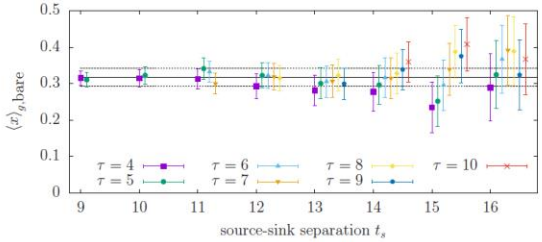
First moment of q/g parton distribution function: $\langle x \rangle_{q/g} = \int dx\, x\, F_{q/g}(x)$.



Connected insertion: u, d.
Disconnected insertion: u, d, s, g

ETMC: [Vaquero,Thu,17:30] $N_f = 2$ twisted mass with clover term,

$m_\pi = 131$ MeV, $Lm_\pi = 3$,
$a = 0.093$ fm.
Stout smearing to reduce noise.
Approx: 2000 (cfgs) $\times$ 100 (sources)



Sara Collins, Lattice 2016

Renormalisation: mixing between $\sum_q \langle x \rangle_q$ and $\langle x \rangle_g$: 1-loop to $\overline{\mathrm{MS}}$ at 2 GeV.

$\langle x \rangle_g^{bare} = 0.318(24) \rightarrow \langle x \rangle_g^{\overline{\mathrm{MS}}} = 0.320(24)$, $\quad (\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s)^{\overline{\mathrm{MS}}} = 0.72(11)$

Momentum sum satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1.04(11)$

Consistent with χQCD quenched calculation [Deka,1312.4816].

Also computed: $g_A^{u,d}$, $g_T^{u,d}$, $g_S^{u,d}$.

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