Possibilities for learning about the nucleon spin structure with positrons and electrons

Charlotte Van Hulse
University of the Basque Country
Structure of the nucleon
Structure of the nucleon
Structure of the nucleon

Wigner distributions  \( W(x, \vec{k}_T, \vec{b}_\perp) \)
Structure of the nucleon

\[
\int d^2 \vec{b}_\perp
\]

Wigner distributions \( W(x, \vec{k}_T, \vec{b}_\perp) \)

transverse-momentum dependent PDFs (TMDs)
Structure of the nucleon

Wigner distributions \( W(x, \vec{k}_T, \vec{b}_\perp) \)

\[
\int d^2 \vec{b}_\perp
\]

transverse-momentum dependent PDFs (TMDs)

impact-parameter distributions

\[
\int d^2 \vec{k}_T
\]

Fourier transform

generalized parton distributions (GPDs)
Structure of the nucleon

Wigner distributions $W(x, \vec{k}_T, \vec{b}_\perp)$

$\int d^2 \vec{b}_\perp$

transverse-momentum dependent PDFs (TMDs)

impact-parameter distributions

$\int d^2 \vec{k}_T$

Fourier transform

generalized parton distributions (GPDs)

$\int d^2 \vec{k}_T$

PDFs

$t = 0, \xi = 0$
Structure of the nucleon

Wigner distributions \( W(x, \vec{k}_T, \vec{b}_\perp) \)

- Transverse-momentum dependent PDFs (TMDs)

- Impact-parameter distributions

- Fourier transform

- Generalized parton distributions (GPDs)

- Semi-inclusive deep-inelastic scattering (DIS)
Structure of the nucleon

Wigner distributions
\[ W(x, \vec{k}_T, \vec{b}_\perp) \]

\[ \int d^2 \vec{b}_\perp \]

transverse-momentum dependent PDFs (TMDs)

\[ \int d^2 \vec{k}_T \]

impact-parameter distributions

Fourier transform

generalized parton distributions (GPDs)

t = 0, \xi = 0

semi-inclusive deep-inelastic scattering (DIS)

hard exclusive reactions

PDFs

TMD PDF

TMD FF

GPDs
Deeply virtual Compton scattering and beam-charge asymmetries
GPDs and DVCS

\[ x + \xi \quad x - \xi \]

\[ \text{GPD} \]

\[ p \quad t \quad p \]

- \( x \) = average longitudinal momentum fraction
- \( 2\xi \) = average longitudinal momentum transfer
- \( t \) = four-momentum transfer squared

\[ \tilde{H}(x, \xi, t) \quad \tilde{H}_T(x, \xi, t) \]

\[ \tilde{E}(x, \xi, t) \quad \tilde{E}_T(x, \xi, t) \]

<table>
<thead>
<tr>
<th>quark-helicity conserving twist-2 GPDs</th>
<th>quark-helicity flip twist-2 GPDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(x, \xi, t) )</td>
<td>( H_T(x, \xi, t) )</td>
</tr>
<tr>
<td>( E(x, \xi, t) )</td>
<td>( E_T(x, \xi, t) )</td>
</tr>
<tr>
<td>spin independent</td>
<td></td>
</tr>
<tr>
<td>( \tilde{H}(x, \xi, t) )</td>
<td>( \tilde{H}_T(x, \xi, t) )</td>
</tr>
<tr>
<td>( \tilde{E}(x, \xi, t) )</td>
<td>( \tilde{E}_T(x, \xi, t) )</td>
</tr>
<tr>
<td>spin dependent</td>
<td></td>
</tr>
<tr>
<td>nucleon helicity conservation</td>
<td>nucleon helicity flip</td>
</tr>
</tbody>
</table>

\[ J = \lim_{t \to 0} \frac{1}{2} \int_{-1}^{1} dx \ x \left[ H(x, \xi, t) + E(x, \xi, t) \right] \]

GPDs and DVCS

\[ \gamma^* \gamma \]

\[ x + \xi \rightarrow x - \xi \]

\[ p \rightarrow p \]

\[ t \]

DVCS
GPDs and DVCS

\[ e \rightarrow e^* \rightarrow x + \xi \rightarrow x - \xi \rightarrow p + p \rightarrow t \]

GPD

DVCS
GPDs and DVCS

\[ x + \xi \quad \rightarrow \quad x - \xi \]

GPD

DVCS

Bethe-Heitler
GPDs and DVCS

\[ d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{DVCS}^* \tau_{BH} + \tau_{DVCS}^* \tau_{BH} \]
DVCS cross section

\[ d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{DVCS}\tau_{BH}^* + \tau_{DVCS}^*\tau_{BH} \]

Unpolarized nucleon
Longitudinally polarized lepton beam
DVCS cross section

\[ d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{DVCS}\tau_{BH}^* + \tau_{DVCS}^*\tau_{BH} \]

**Unpolarized nucleon**

**Longitudinally polarized lepton beam**

\[ |\tau_{BH}|^2 = \frac{K_{BH}}{P_1(\phi) P_2(\phi)} \left\{ \sum_{n=0}^{2} c_{BH}^n \cos(n\phi) \right\} \]

calculable with knowledge Pauli & Dirac form factors

\[ |\tau_{DVCS}|^2 = \frac{1}{Q^2} \left\{ \sum_{n=0}^{2} c_{DVCS}^n \cos(n\phi) + \lambda s_{DVCS}^2 \sin(\phi) \right\} \]

coefficients: bilinear in GPDs

\[ \mathcal{I} = \frac{-e_l K_{\mathcal{I}}}{P_1(\phi) P_2(\phi)} \left\{ \sum_{n=0}^{3} c_{\mathcal{I}}^n \cos(n\phi) + \lambda \sum_{n=1}^{2} s_{\mathcal{I}}^n \sin(n\phi) \right\} \]

coefficients: linear in GPDs
DVCS cross section

\[ d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{DVCS} \tau_{BH}^* + \tau_{DVCS}^* \tau_{BH} \]

Unpolarized nucleon
Longitudinally polarized lepton beam

\[ |\tau_{BH}|^2 = \frac{K_{BH}}{\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^{2} c_{n}^{BH} \cos(n\phi) \right\} \]

calculable with knowledge Pauli & Dirac form factors

\[ |\tau_{DVCS}|^2 = \frac{1}{Q^2} \left\{ \sum_{n=0}^{2} c_{n}^{DVCS} \cos(n\phi) + \lambda s_{1}^{DVCS} \sin(\phi) \right\} \]

coefficients: bilinear in GPDs

\[ \mathcal{I} = \frac{-e_{l} K_{I}}{\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^{3} c_{n}^{I} \cos(n\phi) + \lambda \sum_{n=0}^{2} s_{n}^{I} \sin(n\phi) \right\} \]

coefficients: linear in GPDs

beam polarization
DVCS cross section

\[ d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{DVCS}\tau_{BH}^* + \tau_{DVCS}^*\tau_{BH} \]

Unpolarized nucleon
Longitudinally polarized lepton beam

\[ |\tau_{BH}|^2 = \frac{K_{BH}}{\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) \right\} \]
calculable with knowledge Pauli & Dirac form factors

\[ |\tau_{DVCS}|^2 = \frac{1}{Q^2} \left\{ \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) + \lambda s_1^{DVCS} \sin(\phi) \right\} \]
coefficients: bilinear in GPDs

\[ I = \frac{-e_1 K_I}{\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^{3} c_n^I \cos(n\phi) + \lambda \sum_{n=1}^{2} s_n^I \sin(n\phi) \right\} \]
coefficients: linear in GPDs

beam charge
beam polarization
DVCS cross section

\[ I = \frac{-e_{l} K_{T}}{P_{1}(\phi) P_{2}(\phi)} \left\{ \sum_{n=0}^{3} c_{n}^{T} \cos(n\phi) + \lambda \sum_{n=1}^{2} s_{n}^{T} \sin(n\phi) \right\} \]

\[ c_{1}^{T} \propto \Re M^{1,1}, \quad s_{1}^{T} \propto \Im M^{1,1} \]

\[ M^{1,1} = F_{1}(t) H(\xi, t) + \frac{x_{B}}{2 - x_{B}} (F_{1}(t) + F_{2}(t)) \tilde{H}(\xi, t) - \frac{t}{4M_{p}^{2}} F_{2}(t) E(\xi, t) \]

\( \text{CFF } H, \tilde{H}, E = \text{convolution GPD x hard scattering amplitude} \)

At LO: \( \Im \) direct access to GPDs at \( x = \pm \xi \)
\( \Re \) convolution integral over \( x \)
+ access to D-term
Beam-charge asymmetry

\[ A_C(\phi) \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{-\frac{K_I}{P_1(\phi)P_2(\phi)} \sum_{n=0}^{3} c_n^I \cos(n\phi)}{\frac{K_{BH}}{P_1(\phi)P_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)} \]
Beam-charge asymmetry

\[ A_C(\phi) \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{-\frac{K_I}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{3} c_n^I \cos(n\phi)}{\frac{K_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)} \]
Beam-charge asymmetry

\[ A_C(\phi) \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{K_{BH}}{p_1(\phi)p_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) \]

- \( c_n^{DVCS} \) expected small at HERMES
- suppressed as \( 1/Q^2 \)
Beam-charge asymmetry

\[ \mathcal{A}_C(\phi) \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{K_{BH}}{p_1(\phi)p_2(\phi)} \sum_{n=0}^{2} c^B_n \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c^\text{DVCS}_n \cos(n\phi) \]

- \[ c^B_n \] are the terms calculable.
- \[ c^\text{DVCS}_n \] expected small at HERMES.
- \[ c^\text{DVCS}_n \] suppressed as \( 1/Q^2 \).
Beam-charge asymmetry

Figure 4: The $\cos(n\phi)$ amplitude of the beam-charge asymmetry extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of $-t$, $x_B$, and $Q^2$. The error bars (bands) represent the statistical (systematic) uncertainties. The theoretical calculations are based on the models that are unable to describe the data in Fig. 2. For the VGG model the parameter settings $b_{\text{val}}=\infty$ and $b_{\text{sea}}=1$ are used and the contribution from the D–term is set to zero. The bottom row shows the fractional contribution of associated BH production as obtained from a MC simulation.

The theoretical calculations shown in Fig. 4 are based on either the Dual-GT or the VGG model. For the VGG model the parameter settings $b_{\text{val}}=\infty$ and $b_{\text{sea}}=1$ are used and the contribution from the D–term is set to zero, as only this set of parameters yields a good description of the BC data [24,25]. Note that the same set, in particular the setting $b_{\text{sea}}=1$, leads to an amplitude with the largest magnitude represented in the bands in the top row of Fig. 2, i.e., it clearly does not describe the data related to the imaginary part of the DVCS amplitude. It appears that additional degrees of freedom are needed.
Beam-charge asymmetry

**Figure 4:** The $\cos(n\phi)$ amplitude ($n = 0 – 3$) of the beam-charge asymmetry extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of $-t$, $x_B$, and $Q^2$. The error bars (bands) represent the statistical (systematic) uncertainties. The theoretical calculations are based on the models that are unable to describe the data in Fig. 2. For the VGG model the parameter settings $b_{val} = \infty$ and $b_{sea} = 1$ are used and the contribution from the D–term is set to zero. The bottom row shows the fractional contribution of associated BH production as obtained from a MC simulation.

No striking additional features are observed in Fig. 5 where the $\cos(n\phi)$ amplitude shows a function of $-t$ for three distinct $x_B$ ranges. The theoretical calculations shown in Fig. 4 are based either the Dual-GT or the VGG model. For the VGG model the parameter settings $b_{val} = \infty$ and $b_{sea} = 1$ are used and the contribution from the D–term is set to zero, as only this set of parameters yields a good description of the BC data [24,25]. Note that this same setting, in particular the setting $b_{sea} = 1$, leads to an amplitude with the largest magnitude represented in the bands in the top row of Fig. 2, i.e., it clearly does not describe the data related to the imaginary part of the DVCS amplitude. It appears that additional degrees of freedom are needed for a proper description of the data.
Beam-charge asymmetry

Figure 4: The \( \cos(n\phi) \) amplitude\((n = 0 – 3)\) of the beam-charge asymmetry extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of \(-t\), \(x_B\), and \(Q^2\). The error bars (bands) represent the statistical (systematic) uncertainties. The theoretical calculations are based on the models that are unable to describe the data in Fig. 2. For the VGG model the parameter settings \(b_{\text{val}} = \infty\) and \(b_{\text{sea}} = 1\) are used and the contribution from the D–term is set to zero. The bottom row shows the fractional contribution of associated BH production as obtained from a MC simulation.
Beam-charge asymmetry

\[ c_0^T \propto -k c_1^T \]

\[ \Re M^{1,1} \]
twist-2 GPDs

\[ \Re M^{0,\pm 1} : \]
twist-3 GPDs
Beam-charge asymmetry

\[ C \cos (\phi - 0.1) \]

\[ \langle M_0, \pm 1 : \text{twist-2 GPDs} \rangle \]

\[ \mathcal{R} M^{1,1} \]

\[ \mathcal{R} M^{0, \pm 1} : \text{twist-3 GPDs} \]

\[ \mathcal{R} M^{\mp 1, \pm 1} : \text{twist-2 gluon helicity-flip GPDs} \]
Beam-charge asymmetry

Figure 4: The $\cos(n\phi)\text{amplitude}(n=0–3)$ of the beam-charge asymmetry extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of $-t$, $x_B$, and $Q^2$. The error bars (bands) represent the statistical (systematic) uncertainties. The theoretical calculations are based on the models that are unable to describe the data in Fig. 2. For the VGG model the parameter settings $b_{val} = \infty$ and $b_{sea} = 1$ are used and the contribution from the D–term is set to zero. The bottom row shows the fractional contribution of associated BH production as obtained from a MC simulation. The error of the helicity-flip GPDs, is found to be consistent with zero. No striking additional features are observed in Fig. 5 where the $\cos(n\phi)\text{amplitude}$ are shown as a function of $-t$ for three distinct $x_B$ ranges. The theoretical calculations shown in Fig. 4 are based on either the Dual-GT or the VGG model. For the VGG model the parameter settings $b_{val} = \infty$ and $b_{sea} = 1$ are used and the contribution from the D–term is set to zero, as only this set of parameters yields a good description of the BC data [24,25]. Note that the same set, in particular the setting $b_{sea} = 1$, leads to an amplitude with the largest magnitude among those represented in the bands in the top row of Fig. 2, i.e., it clearly does not describe the data related to the imaginary part of the DVCS amplitude. It appears that additional degrees of freedom –14–

$\mathcal{M}^{1,1}$
twist-2 GPDs

$\mathcal{M}^{0,\pm 1}$:
twist-3 GPDs

$\mathcal{M}^{\mp 1,\pm 1}$:
twist-2 gluon helicity-flip GPDs

$e p \rightarrow e \gamma N \pi$
Beam-charge asymmetry

Figure 5: The \( \cos(n\phi) \) amplitude for beam-Charge asymmetry extracted from the 1996–2005 hydrogen data as a function of \(-t\) for three \(x_B\) ranges. The error bars (bands) represent the statistical (systematic) uncertainties.

Previously unmeasured charge-difference and charge-averaged beam-helicity asymmetries in hard electroproduction of real photons from an unpolarized proton target are extracted. The twist-3 and twist-2 GPDs provide a description of the data. The twist-3 GPDs provide a description of the data.

\( c_0^T \propto -k_1^T \)

\( \mathcal{R}M_{1,1} \):

\( \mathcal{R}M_{0,\pm 1} \):

\( \mathcal{R}M_{\mp 1,\pm 1} \): twist-2 gluon helicity-flip GPDs
Beam-helicity asymmetry
Beam-helicity asymmetry

\[ |\tau_{BH}|^2 = \frac{K_{BH}}{P_1(\phi) P_2(\phi)} \left\{ \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) \right\} \]

Calculable with knowledge Pauli & Dirac form factors

\[ |\tau_{DVCS}|^2 = \frac{1}{Q^2} \left\{ \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) + \lambda s_1^{DVCS} \sin(\phi) \right\} \]

coefficients: bilinear in GPDs

\[ I = \frac{-e_l K_I}{P_1(\phi) P_2(\phi)} \left\{ \sum_{n=0}^{3} c_n^{I} \cos(n\phi) + \lambda \sum_{n=1}^{2} s_n^{I} \sin(n\phi) \right\} \]

coefficients: linear in GPDs

Unpolarized nucleon
Longitudinally polarized lepton beam

beam charge
beam polarization
Beam-helicity asymmetry

\[ A_{LU}(\phi, e_\ell) \equiv \frac{d\sigma^\rightarrow - d\sigma^\leftarrow}{d\sigma^\rightarrow + d\sigma^\leftarrow} \]

\[
= \frac{1}{P_1(\phi)P_2(\phi)} \left[ K_{BH} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) - e_\ell K_I \sum_{n=0}^{3} c_n^{I} \cos(n\phi) \right] + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) + \frac{1}{Q^2} s_1^{DVCS} \sin \phi
\]
Beam-helicity asymmetry

\[ A_{LU}(\phi, e_{\ell}) \equiv \frac{d\sigma^{\rightarrow} - d\sigma^{\leftarrow}}{d\sigma^{\rightarrow} + d\sigma^{\leftarrow}} \]

\[ = \frac{1}{p_{1}(\phi)p_{2}(\phi)} \left[ K_{BH} \sum_{n=0}^{2} c_{n}^{BH} \cos(n\phi) - e_{\ell} K_{I} \sum_{n=0}^{3} c_{n}^{I} \cos(n\phi) \right] + \frac{1}{Q^{2}} s_{1}^{DVCS} \sin\phi + \frac{1}{Q^{2}} \sum_{n=0}^{2} c_{n}^{DVCS} \cos(n\phi) \]
Beam-helicity asymmetry

\[
\mathcal{A}_{LU}(\phi, e_\ell) \equiv \frac{d\sigma^{-} - d\sigma^{\leftarrow}}{d\sigma^{+} + d\sigma^{\leftarrow}}
\]

\[
= \frac{1}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left[ K_{BH} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) - e_\ell K_{I} \sum_{n=0}^{3} c_n^{I} \cos(n\phi) \right] + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)
\]

- \( s_1^{DVCS} \) twist-3
- suppressed as \( 1/Q^2 \)
Beam-helicity asymmetry

\[ A_{LU}(\phi, e_\ell) \equiv \frac{d\sigma^- - d\sigma^\leftarrow}{d\sigma^+ + d\sigma^\leftarrow} \]

\[ = \frac{1}{P_1(\phi)P_2(\phi)} \left[ K_{BH} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) - e_\ell K_I \sum_{n=0}^{3} c_n^I \cos(n\phi) \right] + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) + \frac{1}{Q^2 s_1^{DVCS}} \sin \phi \]

- \( s_1^{DVCS} \) twist-3
- suppressed as \( 1/Q^2 \)

\( c_n^{DVCS} \) expected small at HERMES
- suppressed as \( 1/Q^2 \)
Beam-helicity asymmetry

\[
A_{LU}(\phi, e_\ell) \equiv \frac{d\sigma^\to - d\sigma^\leftarrow}{d\sigma^\to + d\sigma^\leftarrow}
\]

\[
= \frac{1}{p_1(\phi)p_2(\phi)} \left[ K_{BH} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) \right] - e_\ell K_I \sum_{n=0}^{3} c_n^I \cos(n\phi)
+ \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)
\]

- \text{calculable}

- twist-2 \( c_0^I, c_1^I \neq 0 \)

- \( s_1^{DVCS} \) twist-3

- suppressed as \( 1/Q^2 \)

- \( c_n^{DVCS} \) expected small at HERMES

- suppressed as \( 1/Q^2 \)
Beam-helicity asymmetries

Charge-difference beam-helicity asymmetry

\[ A_{LU}^{I}(\phi) \equiv \frac{(d\sigma^{\to\to} - d\sigma^{\to\leftarrow}) - (d\sigma^{\leftarrow\to} - d\sigma^{\leftarrow\leftarrow})}{(d\sigma^{\to\to} + d\sigma^{\to\leftarrow}) + (d\sigma^{\leftarrow\to} + d\sigma^{\leftarrow\leftarrow})} \]

\[ = \frac{-K_{I}}{T_{1}(\phi)T_{2}(\phi)} \sum_{n=1}^{2} s_{n}^{I} \sin(n\phi) \]

\[ - \frac{K_{BH}}{T_{1}(\phi)T_{2}(\phi)} \sum_{n=0}^{2} c_{n}^{BH} \cos(n\phi) + \frac{1}{Q^{2}} \sum_{n=0}^{2} c_{n}^{DVCS} \cos(n\phi) \]
Charge-difference beam-helicity asymmetry

\[ A^{LU}_{I}(\phi) \equiv \frac{(d\sigma^{+--} - d\sigma^{+-+-}) - (d\sigma^{--} - d\sigma^{+-})}{(d\sigma^{+--} + d\sigma^{+-+-}) + (d\sigma^{--} + d\sigma^{+-})} \]

\[ = \frac{K_B}{P_1(\phi)P_2(\phi)} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) \]
Beam-helicity asymmetries

**Charge-difference beam-helicity asymmetry**

\[
A^I_{LU}(\phi) \equiv \frac{(d\sigma^{+\rightarrow} - d\sigma^{+\leftarrow}) - (d\sigma^{-\rightarrow} - d\sigma^{-\leftarrow})}{(d\sigma^{+\rightarrow} + d\sigma^{+\leftarrow}) + (d\sigma^{-\rightarrow} + d\sigma^{-\leftarrow})}
\]

\[
= -\frac{K_I}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=1}^{2} s_n^I \sin(n\phi)
\]

\[
\frac{\mathcal{K}_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)
\]

**Charge-averaged beam-helicity asymmetry**

\[
A^{DVCS}_{LU}(\phi) \equiv \frac{(d\sigma^{+\rightarrow} - d\sigma^{+\leftarrow}) + (d\sigma^{-\rightarrow} - d\sigma^{-\leftarrow})}{(d\sigma^{+\rightarrow} + d\sigma^{+\leftarrow}) + (d\sigma^{-\rightarrow} + d\sigma^{-\leftarrow})}
\]

\[
= \frac{1}{Q^2} s_1^{DVCS} \sin \phi
\]

\[
= \frac{\mathcal{K}_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi)
\]
Beam-helicity asymmetries

Charge-difference beam-helicity asymmetry

\[ A_{LU}^I(\phi) \equiv \frac{(d\sigma^{+-} - d\sigma^{-+}) - (d\sigma^{--} - d\sigma^{++})}{(d\sigma^{+-} + d\sigma^{-+}) + (d\sigma^{--} + d\sigma^{++})} \]

\[ = \frac{K_I}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left[ \sum_{n=1}^{2} s_n^I \sin(n\phi) \right] \]

\[ = \frac{K_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) \]

Charge-averaged beam-helicity asymmetry

\[ A_{LU}^{DVCS}(\phi) \equiv \frac{(d\sigma^{+-} - d\sigma^{-+}) + (d\sigma^{--} - d\sigma^{++})}{(d\sigma^{+-} + d\sigma^{-+}) + (d\sigma^{--} + d\sigma^{++})} \]

\[ = \frac{1}{Q^2} s_{DVCS}^1 \sin \phi \]

\[ = \frac{K_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{2} c_n^{BH} \cos(n\phi) + \frac{1}{Q^2} \sum_{n=0}^{2} c_n^{DVCS} \cos(n\phi) \]
Figure 2: The first (second) row shows the $\sin \phi$ amplitude of the beam-helicity asymmetry $A_{LU,I}$, which is sensitive to the interference term (squared DVCS term), extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of $-t$, $x_B$, and $Q^2$. The third row shows the $\sin 2\phi$ amplitude of $A_{LU,I}$. The error bars (bands) represent the statistical (systematic) uncertainties. Not included is a 2.8% scale uncertainty due to the beam polarization measurement. The calculations are based on the recently corrected minimal implementation [33, 34] of a dual-parameterization GPD model (Dual–GT) and on a GPD model [30, 38] based on double–distributions (VGG). Both models use a Regge–motivated $t$-dependence. The band for the VGG model results from varying the parameters $b_{\text{val}}$ and $b_{\text{sea}}$ between unity and infinity. The bottom row shows the fractional contribution of associated BH product as obtained from a MC simulation.
Charge-difference and charge-average beam-helicity asymmetry

Figure 2: The first (second) row shows the $\sin \phi$ amplitude of the beam-helicity asymmetry $A_{LU,I}$, which is sensitive to the interference term (squared DVCS term), extracted from the 1996–2005 hydrogen data in the entire experimental acceptance, and as a function of $-t$, $x_B$, and $Q^2$. The third row shows the $\sin 2\phi$ amplitude of $A_{LU,I}$. The error bars (bands) represent the statistical (systematic) uncertainties. Not included is a 2.8% scale uncertainty due to the beam polarization measurement. The calculations are based on the recently corrected minimal implementation [33, 34] of a dual-parameterization GPD model (Dual–GT) and on a GPD model [30, 38] based on double–distributions (VGG). Both models use a Regge–motivated $t$-dependence. The band for the VGG model results from varying the parameters $b_{\text{val}}$ and $b_{\text{sea}}$ between unity and infinity. The bottom row shows the fractional contribution of associated BH product as obtained from a MC simulation.

Beam-charge asymmetry on nuclear targets

- Nuclear DVCS: (anti-)shadowing, EMC effect
- New nuclear effect, absent in forward DIS amplitude?
- Coherent scattering: mesonic degrees of freedom:
  - non-linear $A$ dependence of first moment of D-term
  - at HERMES, beam-charge asymmetry grows with increasing $A$: $\tau_{DVCS}$ increases.
    Absence of mesons: asymmetry independent of $A$.
- Incoherent scattering: similar to proton GPDs
Beam-charge asymmetry on nuclear targets
Beam-charge asymmetry on nuclear targets

No dependence on $A$ observed
(TMD) PDFs
Semi-inclusive DIS production
Semi-inclusive DIS cross section

\[
\sigma^h(\phi, \phi_S) = \sigma_{UU}^h \left \{ 1 + 2\langle \cos(\phi) \rangle_{UU}^h \cos(\phi) + 2\langle \cos(2\phi) \rangle_{UU}^h \cos(2\phi) \\
+ \lambda_i \langle \sin(\phi) \rangle_{LU}^h \sin(\phi) \\
+ S_L \left [ 2\langle \sin(\phi) \rangle_{UL}^h \sin(\phi) + 2\langle \sin(2\phi) \rangle_{UL}^h \sin(2\phi) \\
+ \lambda_i \left ( 2\langle \cos(0\phi) \rangle_{LL}^h \cos(0\phi) + 2\langle \cos(\phi) \rangle_{LL}^h \cos(\phi) \right ) \right ] \\
+ S_T \left [ 2\langle \sin(\phi - \phi_S) \rangle_{UT}^h \sin(\phi - \phi_S) + 2\langle \sin(\phi + \phi_S) \rangle_{UT}^h \sin(\phi + \phi_S) \\
+ 2\langle \sin(3\phi - \phi_S) \rangle_{UT}^h \sin(3\phi - \phi_S) + 2\langle \sin(\phi_S) \rangle_{UT}^h \sin(\phi_S) \\
+ 2\langle \sin(2\phi - \phi_S) \rangle_{UT}^h \sin(2\phi - \phi_S) \\
+ \lambda_i \left [ 2\langle \cos(\phi - \phi_S) \rangle_{LT}^h \cos(\phi - \phi_S) \\
+ 2\langle \cos(\phi_S) \rangle_{LT}^h \cos(\phi_S) + 2\langle \cos(2\phi - \phi_S) \rangle_{LT}^h \cos(2\phi - \phi_S) \right ] \right \} \right \}
\]
Semi-inclusive DIS cross section

\[
\sigma^h(\phi, \phi_S) = \sigma_{UU}^h \left\{ 1 + 2\langle \cos(\phi) \rangle_{UU}^h \cos(\phi) + 2\langle \cos(2\phi) \rangle_{UU}^h \cos(2\phi) \\
+ \lambda_I 2\langle \sin(\phi) \rangle_{LU}^h \sin(\phi) \\
+ S_L \left[ 2\langle \sin(\phi) \rangle_{UL}^h \sin(\phi) + 2\langle \sin(2\phi) \rangle_{UL}^h \sin(2\phi) \\
+ \lambda_I \left( 2\langle \cos(0\phi) \rangle_{LL}^h \cos(0\phi) + 2\langle \cos(\phi) \rangle_{LL}^h \cos(\phi) \right) \right] \\
+ S_T \left[ 2\langle \sin(\phi - \phi_S) \rangle_{UT}^h \sin(\phi - \phi_S) + 2\langle \sin(\phi + \phi_S) \rangle_{UT}^h \sin(\phi + \phi_S) \\
+ 2\langle \sin(3\phi - \phi_S) \rangle_{UT}^h \sin(3\phi - \phi_S) + 2\langle \sin(\phi_S) \rangle_{UT}^h \sin(\phi_S) \\
+ 2\langle \sin(2\phi - \phi_S) \rangle_{UT}^h \sin(2\phi - \phi_S) \\
+ \lambda_I \left( 2\langle \cos(\phi - \phi_S) \rangle_{LT}^h \cos(\phi - \phi_S) \\
+ 2\langle \cos(\phi_S) \rangle_{LT}^h \cos(\phi_S) + 2\langle \cos(2\phi - \phi_S) \rangle_{LT}^h \cos(2\phi - \phi_S) \right) \right] \right\}
\]
Semi-inclusive DIS cross section

\[
\sigma^h(\phi, \phi_S) = \sigma_{UU}^h \left( 1 + 2\langle \cos(\phi) \rangle_{UU}^h \cos(\phi) + 2\langle \cos(2\phi) \rangle_{UU}^h \cos(2\phi) \\
+ \lambda_l 2\langle \sin(\phi) \rangle_{LU}^h \sin(\phi) \\
+ S_L \left[ 2\langle \sin(\phi) \rangle_{UL}^h \sin(\phi) + 2\langle \sin(2\phi) \rangle_{UL}^h \sin(2\phi) \\
+ \lambda_l \left( 2\langle \cos(0\phi) \rangle_{LL}^h \cos(0\phi) + 2\langle \cos(\phi) \rangle_{LL}^h \cos(\phi) \right) \right] \\
+ S_T \left[ 2\langle \sin(\phi - \phi_S) \rangle_{UT}^h \sin(\phi - \phi_S) + 2\langle \sin(\phi + \phi_S) \rangle_{UT}^h \sin(\phi + \phi_S) \\
+ 2\langle \sin(3\phi - \phi_S) \rangle_{UT}^h \sin(3\phi - \phi_S) + 2\langle \sin(\phi_S) \rangle_{UT}^h \sin(\phi_S) \\
+ 2\langle \sin(2\phi - \phi_S) \rangle_{UT}^h \sin(2\phi - \phi_S) \\
+ \lambda_l \left( 2\langle \cos(\phi - \phi_S) \rangle_{LT}^h \cos(\phi - \phi_S) \\
+ 2\langle \cos(\phi_S) \rangle_{LT}^h \cos(\phi_S) + 2\langle \cos(2\phi - \phi_S) \rangle_{LT}^h \cos(2\phi - \phi_S) \right) \right] \right) \right}\]
\]
Semi-inclusive DIS cross section

\[
\sigma^h(\phi, \phi_S) = \sigma^h_{UU} \left\{ 1 + 2\langle \cos(\phi) \rangle^h_{UU} \cos(\phi) + 2\langle \cos(2\phi) \rangle^h_{UU} \cos(2\phi) \right. \\
+ \lambda_l 2\langle \sin(\phi) \rangle^h_{LU} \sin(\phi) \\
+ \left. S_L \left[ 2\langle \sin(\phi) \rangle^h_{UL} \sin(\phi) + 2\langle \sin(2\phi) \rangle^h_{UL} \sin(2\phi) \right] \\
+ \lambda_l \left( 2\langle \cos(0\phi) \rangle^h_{LL} \cos(0\phi) + 2\langle \cos(\phi) \rangle^h_{LL} \cos(\phi) \right) \right\] \\
+ \left[ S_T \left[ 2\langle \sin(\phi - \phi_S) \rangle^h_{UT} \sin(\phi - \phi_S) + 2\langle \sin(\phi + \phi_S) \rangle^h_{UT} \sin(\phi + \phi_S) \right] \\
+ 2\langle \sin(3\phi - \phi_S) \rangle^h_{UT} \sin(3\phi - \phi_S) + 2\langle \sin(\phi_S) \rangle^h_{UT} \sin(\phi_S) \right] \\
+ 2\langle \sin(2\phi - \phi_S) \rangle^h_{UT} \sin(2\phi - \phi_S) \\
+ \lambda_l \left( 2\langle \cos(\phi - \phi_S) \rangle^h_{LT} \cos(\phi - \phi_S) \right) \\
+ \left. 2\langle \cos(\phi_S) \rangle^h_{LT} \cos(\phi_S) + 2\langle \cos(2\phi - \phi_S) \rangle^h_{LT} \cos(2\phi - \phi_S) \right) \right\} 
\]
Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi+\phi_S)}$$
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle^h_{UT} = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$$F_{XY} \propto C [\text{TMD PDF}(x, k_{\perp}) \times \text{TMD FF} (z, p_{\perp})]$$
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$F_{XY} \propto C [\text{TMD PDF}(x, k_{\perp}) \times \text{TMD FF}(z, p_{\perp})]$
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2 \langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$$F_{XY} \propto C \left[ \text{TMD PDF}(x, k_\perp) \times \text{TMD FF}(z, p_\perp) \right]$$

<table>
<thead>
<tr>
<th>quark polarization</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1$</td>
<td></td>
<td>$h_1^T$</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>$g_{1L}$</td>
<td>$h_{1L}$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^T$</td>
<td></td>
<td>$h_{1T} h_{1T}^T$</td>
</tr>
</tbody>
</table>

Note that our goal is to study the applicability of Bessel weighting to experimental data, for which we explicitly need $k_\perp$ and $p_\perp$ dependences in the Monte Carlo generator. Alongside with this goal it is interesting to investigate how well the approximations of the simple parton model are justified in the current relatively low energy experimental set-up. One would expect that if approximations that lead to the parton model expressions for structure functions are justified, then the generalized parton model expression would not spoil this approximation numerically. On the other hand if the generalized parton model gives notably different results with respect to a naive parton model, one would expect that kinematics of the experiment does not allow a certain type of approximations and the approximations we have made, which are consistent with a generalized parton model framework, enable us to implement a Monte Carlo that incorporates the correct phase space momentum constraints and satisfies the requirements we outlined in this section. Thus, our Monte Carlo simulation allows us to take the factorized form of the generalized parton model cross section Eq. (3.2) as a basis and then to impose four-momentum conservation for the partons according to Fig. 1, assuming the initial quark is on-shell with non-zero mass. We also take a non-zero target mass into account. This procedure does not necessarily lead to a more accurate description of the underlying physics, because it still rests on the simplified picture of the generalized parton model and involves the approximation of an on-shell quark. Nonetheless, implementing these modifications can give us an indication for the magnitude of the uncertainties resulting from the aforementioned kinematic approximations in the parton model.
Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$$F_{XY} \propto \mathcal{C} \left[ \text{TMD PDF}(x, k_\perp) \times \text{TMD FF} (z, p_\perp) \right]$$

<table>
<thead>
<tr>
<th>nucleon polarization</th>
<th>quark polarization</th>
<th>hadron polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U</td>
<td>L</td>
</tr>
<tr>
<td>U</td>
<td>$f_1$</td>
<td>$h_1^\perp$</td>
</tr>
<tr>
<td>L</td>
<td>$g_{1L}$</td>
<td>$h_{1L}^\perp$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^\perp$</td>
<td>$g_{1T}^\perp$</td>
</tr>
</tbody>
</table>

Figure 1. Kinematics of the process. $q$ is the virtual photon, $k$ and $k_0$ are the initial and struck quarks, $k_\parallel$ is the quark transverse component. $P_h$ is the final hadron with a $p_\parallel$ component, transverse with respect to the fragmenting quark $k_0$ direction.

As the beam energy becomes, the more serious the inaccuracies of the parton model have to be taken. On the other hand, the "fully differential" cross section Eq. (3.2) of the generalized parton model allows us to include in our Monte Carlo both transverse momentum and the physical energy and momentum phase space constraints. We used the widely accepted parton model approximation of setting the initial parton on-shell (assumption that virtual photon interacts with an on-mass shell quark). But it is important to emphasize that the approximations we have made, which are consistent with a generalized parton model framework, enable us to implement a Monte Carlo that incorporates the correct phase space momentum constraints and satisfies the requirements we outlined in this section. Thus, our Monte Carlo simulation allows us to take the factorized form of the generalized parton model cross section Eq. (3.2) as a basis and then to impose four-momentum conservation for the partons according to Fig. 1, assuming the initial quark is on-shell with non-zero mass. We also take a non-zero target mass into account. This procedure does not necessarily lead to a more accurate description of the underlying physics, because it still rests on the simplified picture of the generalized parton model and involves the approximation of an on-shell quark. Nonetheless, implementing these modifications can give us an indication for the magnitude of the uncertainties resulting from the aforementioned kinematic approximations in the parton model.

Note that our goal is to study the applicability of Bessel weighting to experimental data, for which we explicitly need $k_\parallel$ and $p_\parallel$ dependences in the Monte Carlo generator. Alongside with this goal it is interesting to investigate how well the approximations of the simple parton model are justified in the current relatively low energy experimental set-up. One would expect that if approximations that lead to the parton model expressions for structure functions are justified, then the generalized parton model expression would not spoil this approximation numerically. On the other hand if the generalized parton model gives notably different results with respect to a naive parton model, one would expect that kinematics of the experiment does not allow a certain type of approximations and the approximations we have made, which are consistent with a generalized parton model framework, enable us to implement a Monte Carlo that incorporates the correct phase space momentum constraints and satisfies the requirements we outlined in this section. Thus, our Monte Carlo simulation allows us to take the factorized form of the generalized parton model cross section Eq. (3.2) as a basis and then to impose four-momentum conservation for the partons according to Fig. 1, assuming the initial quark is on-shell with non-zero mass. We also take a non-zero target mass into account. This procedure does not necessarily lead to a more accurate description of the underlying physics, because it still rests on the simplified picture of the generalized parton model and involves the approximation of an on-shell quark. Nonetheless, implementing these modifications can give us an indication for the magnitude of the uncertainties resulting from the aforementioned kinematic approximations in the parton model.

Note that our goal is to study the applicability of Bessel weighting to experimental data, for which we explicitly need $k_\parallel$ and $p_\parallel$ dependences in the Monte Carlo generator. Alongside with this goal it is interesting to investigate how well the approximations of the simple parton model are justified in the current relatively low energy experimental set-up. One would expect that if approximations that lead to the parton model expressions for structure functions are justified, then the generalized parton model expression would not spoil this approximation numerically. On the other hand if the generalized parton model gives notably different results with respect to a naive parton model, one would expect that kinematics of the experiment does not allow a certain type of approximations and the approximations we have made, which are consistent with a generalized parton model framework, enable us to implement a Monte Carlo that incorporates the correct phase space momentum constraints and satisfies the requirements we outlined in this section. Thus, our Monte Carlo simulation allows us to take the factorized form of the generalized parton model cross section Eq. (3.2) as a basis and then to impose four-momentum conservation for the partons according to Fig. 1, assuming the initial quark is on-shell with non-zero mass. We also take a non-zero target mass into account. This procedure does not necessarily lead to a more accurate description of the underlying physics, because it still rests on the simplified picture of the generalized parton model and involves the approximation of an on-shell quark. Nonetheless, implementing these modifications can give us an indication for the magnitude of the uncertainties resulting from the aforementioned kinematic approximations in the parton model.
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$$F_{XY} \propto C \left[ \text{TMD PDF}(x, k_{\perp}) \times \text{TMD FF} (z, p_{\perp}) \right]$$
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2 \langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi+\phi_S)}$$

$$F_{XY} \propto C \left[ \text{TMD PDF}(x, k_\perp) \times \text{TMD FF} \left( z, p_\perp \right) \right]$$

<table>
<thead>
<tr>
<th>nucleon polarization</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1$</td>
<td></td>
<td>$h_1^T$</td>
</tr>
<tr>
<td>L</td>
<td>$g_{1L}$</td>
<td>$h_{1L}^T$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}$</td>
<td>$g_{1T}$</td>
<td>$h_{1T}^T$ $h_{1T}^L$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>quark polarization</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$D_1$</td>
<td></td>
<td>$H_1^T$</td>
</tr>
</tbody>
</table>

Note that our goal is to study the applicability of Bessel weighting to experimental data, for which we explicitly need $k_\perp$ and $p_\perp$ dependences in the Monte Carlo generator. Alongside with this goal it is interesting to investigate how well the approximations of the simple parton model are justified in the current relatively low energy experimental set-up. One would expect that if approximations that lead to the parton model expressions for structure functions are justified, then the generalized parton model expression would not spoil this approximation numerically. On the other hand if the generalized parton model gives notably different results with respect to a naive parton model, one would expect that kinematics of the experiment does not allow a certain type of approximations and the

3 The confined quark has a non-zero virtuality. Such effects in Monte Carlo generators have been studied in Ref. [62].
TMD PDFs and fragmentation functions (FFs)

Azimuthal amplitudes related to structure functions $F_{XY}$:

$$2\langle \sin(\phi + \phi_S) \rangle_{UT}^h = \epsilon F_{UT}^{\sin(\phi + \phi_S)}$$

$$F_{XY} \propto C [\text{TMD PDF}(x, k_\perp) \times \text{TMD FF}(z, p_\perp)]$$

Survive integration over transverse momentum $h_{1T}$:
- convolution via single-hadron semi-inclusive DIS
- direct production via
  - two-hadron production
  - $\langle \sin(\phi_S) \rangle = \text{twist-3}$

$f_1, g_{1L}$: via (semi-)inclusive DIS
$h_{1T}$: via semi-inclusive DIS
Sivers amplitudes

\[ C[f_{1T}^{q} \times D_{1}^{q}] \]

- Sivers function:
  - requires non-zero orbital angular momentum (model)
  - naive T-odd
  - final-state-interactions \(\rightarrow\) azimuthal asymmetries
Sivers amplitudes

\[ C[f_{1T}^q \times D_1^q] \]

- Sivers function:
  - requires non-zero orbital angular momentum (model)
  - naive T-odd
  - final-state-interactions → azimuthal asymmetries

\[ \pi^+ : \]
- positive → non-zero orbital angular momentum
- amplitude dominated by u-quark scattering:
  \[
  \approx - \frac{C f_{1T}^{u \to \pi^+} \times D_1^{u \to \pi^+}}{C f_{1T}^{u \to \pi^+} \times D_1^{u \to \pi^+}} \quad \rightarrow f_{1T}^{u < 0}
  \]

\[ \pi^- : \]
- consistent with zero
- u and d quark cancelation → \( f_{1T}^{u,d} > 0 \)

\[ K^+ : \]
- larger amplitude than for \( \pi^+ \)
Accessing collinear helicity PDFs: complementary probes to inclusive $\gamma^*\text{DIS}$

- **Semi-inclusive DIS**
  - Fragmentation function $\rightarrow$ flavour tagging, but additional non-perturbative object
  - Reconstruction kinematic variables: initial/scattered lepton
  - Longitudinally polarized beam and polarized target needed

- **Charged-current DIS**
  - No fragmentation functions
  - Probe combinations of flavours different from inclusive $\gamma^*\text{DIS}$
  - Reconstruction kinematic variables:
    - initial lepton and detected final-state particles
    - \[ y_{JB} = \frac{\sum_i (E_i - p_{z,i})}{2E_e} \]
    - \[ Q^2_{JB} = \frac{|\sum_i \vec{p}_{T,i}|^2}{1 - y_{JB}} \]
  - Longitudinally polarized target needed
  - Need high enough $Q^2$!
Accessing collinear helicity PDFs: complementary probes to inclusive $\gamma^{*}\text{DIS}$

**Semi-inclusive DIS**

**Charged-current DIS**

Inclusive DIS, independent of beam charge:

$$g_{1,p}(x) = \frac{1}{2} \sum_{q} e_{q}^{2} \Delta q(x)$$

$$g_{1,p}^{W^{-}}(x) = \Delta u(x) + \Delta \bar{d}(x) + \Delta c(x) + \Delta \bar{s}(x)$$

$$g_{5,p}^{W^{-}}(x) = -\Delta u(x) + \Delta \bar{d}(x) - \Delta c(x) + \Delta \bar{s}(x)$$

$$g_{1,p}^{W^{+}}(x) = \Delta \bar{u}(x) + \Delta d(x) + \Delta \bar{c}(x) + \Delta s(x)$$

$$g_{5,p}^{W^{+}}(x) = \Delta \bar{u}(x) - \Delta d(x) + \Delta \bar{c}(x) - \Delta s(x)$$
Inclusive DIS production
Longitudinal target single-spin asymmetries

\[ A_p^{W^-} = \frac{2b g_{1p}^{W^-} - a g_{5p}^{W^-}}{a F_{1p}^{W^-} + b F_{3p}^{W^-}} \]

\[ a, b = f(y_{JB}) \]

- Charged currents @ EIC:
  - PRD 88 (2013) 114025
  - DSSV analysis @ NLO

Charged currents @ EIC:

E. Aschenauer et al., PRD 88 (2013) 114025

Detector effects, radiative corrections, charged currents @ EIC:

PRD 88 (2013) 114025

DSSV+ analysis:

DSSV+ analysis @ NLO:

Longitudinal target single-spin asymmetries

\[ A_p^{W^-} = \frac{2b g_{1p}^{W^-} - a g_{5p}^{W^-}}{a F_{1p}^{W^-} + b F_{3p}^{W^-}} \]

\[ a, b = f(y_{JB}) \]
Structure function $g_2$

\[ \frac{d^3 \sigma_{LT}}{dx \, dy \, d\phi'} \propto -\lambda \left[ \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right] \cos(\phi') \]

\[ A_{LT}(x, Q^2, \phi') = \lambda \frac{\sigma_{LT}(x, Q^2, \phi')}{\sigma_{UU}(x, Q^2, \phi')} = -A_T \cos(\phi') \]

Count # events with target spin up - target-spin down

- transversely pol. target
- longitudinal spin: beam/boson
Structure function $g_2$

- transversely pol. target
- longitudinal spin: beam/boson

\[
\frac{d^3 \sigma_{LT}}{dx \, dy \, d\phi'} \propto -\lambda \left[ \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right] \cos(\phi')
\]

\[
A_{LT}(x, Q^2, \phi') = \lambda \frac{\sigma_{LT}(x, Q^2, \phi')}{\sigma_{UU}(x, Q^2, \phi')} = -A_T \cos(\phi')
\]

Count # events with target spin up - target-spin down

\[
g_2(x) = g_{2WW} + \bar{g}_2(x) \quad d_2 = 3 \int_0^1 dx \, x^2 \, \bar{g}_2(x)
\]
Structure function $g_2$

- $\frac{d^3\sigma_{LT}}{dx \, dy \, d\phi'} \propto -\lambda \left[ \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right] \cos(\phi')$

- $A_{LT}(x, Q^2, \phi') = \lambda \frac{\sigma_{LT}(x, Q^2, \phi')}{\sigma_{UU}(x, Q^2, \phi')} = -A_T \cos(\phi')$

- Count # events with target spin up - target-spin down

- $g_2(x) = g_2^{WW} + \bar{g}_2(x) \quad d_2 = 3 \int_0^1 dx \, x^2 \, \bar{g}_2(x)$

- Sivers effect

Force on struck quark at $t=0 \propto -d_2$

M. Burkardt arXiv:0810.3589

FSI from $t=0 \to \infty$
A₂ and g₂

HERMES:  \( d₂ = 0.0148 \pm 0.0096 \text{(stat)} \pm 0.0048 \text{(syst)} \)

E143+155:  \( d₂ = 0.0032 \pm 0.0017 \)
Furthermore...
Spin-dependent fragmentation functions

Probe with longitudinal spin: polarization transfer to quark → fragmentation to Λ

Parity-violating weak decay of Λ:
in Λ rest frame, proton preferably emitted along Λ spin direction

Spin-dependent fragmentation functions $G_1, H_1, \tilde{G}_T$
Two-photon exchange in inclusive DIS

* transversely pol. target
* unpolarized beam

\[
\begin{array}{l}
\langle Q^2 \rangle [\text{GeV}^2] \\
\langle X_B \rangle
\end{array}
\]


\[
\begin{array}{l}
e^- p^\uparrow \rightarrow e^- X \\
e^+ p^\uparrow \rightarrow e^+ X
\end{array}
\]
Summary

Usage of $e^+$ and $e^-$ beam offers advantages to access:

- GPDs in DVCS measurements: beam charge but also spin asymmetries
- Helicity distributions (collinear)
- $g_2$ structure function
- Spin-dependent, collinear fragmentation functions
- Two-photon exchange in elastic and deep-inelastic scattering
Summary

Usage of $e^+$ and $e^-$ beam offers advantages to access

- GPDs in DVCS measurements: beam charge but also spin asymmetries
- helicity distributions (collinear)
- $g_2$ structure function
- spin-dependent, collinear fragmentation functions
- two-photon exchange in elastic and deep-inelastic scattering

Thank you