Neutron spin structure from polarized $^3$He

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Example: 12 GeV Experiments @JLab, with $^3\text{He}$

**DIS regime, e.g.**

**Hall A**, [http://hallaweb.jlab.org/12GeV/](http://hallaweb.jlab.org/12GeV/)

**MARATHON Coll. E12-10-103 (Rating A):** MeAsurement of the $F_{2n}/F_{2p}$, d/u Ratios and A=3 EMC Effect in Deep Inelastic Electron Scattering Off the Tritium and Helium Mirror Nuclei

**Hall C**, [https://www.jlab.org/Hall−C/](https://www.jlab.org/Hall−C/)

J. Arrington, et al PR12-10-008 (Rating A−): Detailed studies of the nuclear dependence of $F_2$ in light nuclei

**SIDIS regime, e.g.**

**Hall A**, [http://hallaweb.jlab.org/12GeV/](http://hallaweb.jlab.org/12GeV/)

H. Gao et al, PR12-09-014 (Rating A): Target Single Spin Asymmetry in Semi-Inclusive Deep-Inelastic $(e,e'\pi^{\pm})$ Reaction on a Transversely Polarized $^3\text{He}$ Target

J.P. Chen et al, PR12-11-007 (Rating A): Asymmetries in Semi-Inclusive Deep-Inelastic $(e,e'\pi^{\pm})$ Reactions on a Longitudinally Polarized $^3\text{He}$ Target

**Others? DVCS, spectator tagging...**

In $^3\text{He}$ conventional nuclear effects under control... Exotic ones disentangled
Outline

Selected topics:

- Impulse Approximation and the Spectral Function. 
  DIS $^3\bar{He}(e, e')X$ and extraction of $g_1^n$.

- Beyond the impulse approximation.
  Final state interactions (FSI) and the distorted spectral function.
    * SIDIS $^3\bar{He}(\vec{e}, e'\pi)X$ and the extraction of neutron SSAs;
    * spectator SIDIS $^3\bar{He}(\vec{e}, e'd)X$ and the spin dependent EMC effect.

- Coherent DVCS off $^3He$ (quickly):
  * extraction of neutron GPDs;
  * flavor (isospin) dependence of nuclear effects.

Relevant new calculations to be performed and proper measurements to check theoretical methods, thinking to the EIC, will be addressed.

The neutron information from $^3\text{He}$

$^3\text{He}$ is the ideal target to study the polarized neutron:

$\mu_{^3\text{He}} \approx \mu_n$!

(deuteron: $\mu_{^2\text{H}} \approx \mu_n + \mu_p$)

... But the bound nucleons in $^3\text{He}$ are moving! → theoretical ingredient:

a realistic spin-dependent spectral function for $^3\tilde{\text{He}}, P_{\sigma,\sigma'}(\vec{p}, E)$.

Example: dynamical nuclear effects in inclusive DIS ($^3\tilde{\text{He}}(e, e')X$). The formula

$$A_n \approx \frac{1}{p_nf_n} \left( A_{3\text{exp}}^{\exp} - 2p_pf_p A_p^{\exp} \right), \quad (\text{Ciofi degli Atti et al., PRC}48(1993)R968)$$

($f_p, f_n$ dilution factors, from unpolarized data)

can be safely used → widely adopted by experimental collaborations.

Nuclear effects hidden in the “effective polarizations”, $p_p$ and $p_n$, obtained from the nuclear w.f... But to proof this possibility, the spectral function had to be evaluated.
The spectral function (Impulse Approximation)

\[ P_{M\sigma\sigma}^{N}(\vec{p}, E) = \sum_{f} \left( \begin{array}{c} 3^3\text{He} \\ \vec{p}, E \\ \vec{p}_f, E_f^* \end{array} \right) ^2 \]

\[ = \sum_{f} \delta(E - E_{min} - E_f^*) S_A \langle \Psi_A; J_A M_A \pi_A | \vec{p}, \sigma; \phi_f(E_f^*) \rangle \langle \phi_f(E_f^*); \sigma \vec{p} \pi_A J_A M'; \Psi_A \rangle S_A \]

probability distribution to find a nucleon with given 3-momentum and removal energy \( E \) in the nucleus. It arises in q.e., DIS, SIDIS, DVCS...

In general, if spin is involved, a 2x2 matrix, \( P_{M\sigma\sigma'}^{N}(\vec{p}, E) \), not a density;

the two-body recoiling system can be either the deuteron or a scattering state:

when a deeply bound nucleon, with high \( E = E_{min} + E_f^* \), leaves the nucleus, the recoiling system has high excitation energy \( E_f^* \);

Realistic Spectral Function: 3-body bound state and 2-body final state evaluated within the same Realistic interaction (in our case, Av18, from the Pisa group (Kievsky, Viviani)). Extension to heavier nuclei very difficult
Is the spectral function useful? Does the Impulse Approximation work?

The answer in the data.

Example: $^3\bar{H}e(e,e')X$ in q.e. kinematics (K. Slifer et al, PRL 101 (2008) 022303)

- Faddeev calc. at low $Q^2$

- PWIA (Av18) calc.
  E. Pace et al, PRC 64 (2001) 055203

Conclusion of the Slifer et al. paper:
“A full three-body Faddeev calculation agrees well with the data but starts to exhibit discrepancies as the energy increases, possibly due to growing relativistic effects. As the momentum transfer increases, the PWIA approach reproduces the data well, but there exists an intermediate range where neither calculation succeeds”

caveat: always check kinematics and set-up (this is q.e., inclusive)
**Status** (Impulse Approximation and beyond)

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|                  | ✓                      | ✓              |
| Non Relativistic | ✓                      | ✓              |
|                  | ✓                      | ✓              |

|                  | Def: ✓                 | Def: ✓         |
| Light-Front      |                        |               |
|                  | Calc: ⬇                 | Calc: ⬇        |
|                  | ✓                      | ✓              |
|                  | ✓                      | ✓              |

Selected contributions from Rome-Perugia:

- Ciofi, Pace, Salmè PRC 21 (1980) 505 ...
- Ciofi, Pace, Salmè PRC 46 (1991) 1591: spin dependence
- Pace, Salmè, S.S., Kievsky PRC 64 (2001) 055203, first Av18 calculation
- Ciofi, Kaptari, PRC 66 (2002) 044004, unpolarized with FSI (q.e.)
- S.S. PRC 70 (2004) 015205, non diagonal SF for DVCS
- Kaptari, Del Dotto, Pace, Salmè, S.S., PRC 89 (2014), spin dependent with FSI
- LF, formal: Del Dotto, Pace, Salmè, SS, PRC 95 (2017) 014001; preliminary calc., S.S., Del Dotto, Kaptari, Pace, Rinaldi, Salmè, Few Body Syst. 56 (2015) 6

February 5th, 2018
# SIDIS off $^3$He and neutron TMDs

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| Non Relativistic      | ✓             | ✓ | ✓ | ✓ |
| Light-Front           | Def: ✓        | Def: ✓ | ![Warning](image) | ![Warning](image) |
|                       | Calc: ![Error](image) | Calc: ![Warning](image) | ![Warning](image) | ![Warning](image) |

Extracting the neutron information from SIDIS off $^3\bar{H}e$.

Basic approach: Impulse Approximation in the Bjorken limit

( S.S., PRD 75 (2007) 054005 )

**Main topic:**

* Evaluation of Final State Interactions (FSI): distorted spectral function SIDIS
  
  (A. Del Dotto, L. Kaptari, E. Pace, G. Salmè, S.S., PRC 96 (2017) 065203 )

* Evaluation of FSI: distorted spectral function and spectator SIDIS
  
  (L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206 )

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Single Spin Asymmetries (SSAs) - 1

\[ \vec{A}(e, e'h)X: \text{Unpolarized beam and T-polarized target} \rightarrow \sigma_{UT} \]

\[ d^6\sigma \equiv \frac{d^6\sigma}{dx dy dz d\phi_S d^2P_{h\perp}} \]

\[ x = \frac{Q^2}{2P \cdot q} \quad y = \frac{P \cdot q}{P \cdot l} \quad z = \frac{P \cdot h}{P \cdot q} \]

\[ \hat{q} = -\hat{e}_z \]

The number of emitted hadrons at a given \( \phi_h \) depends on the orientation of \( \vec{S}_\perp \). In SSAs 2 different mechanisms can be experimentally distinguished.

\[ A_{UT}^{\text{Sivers(Collins)}} = \frac{\int d\phi_S d^2P_{h\perp} \sin(\phi_h - (+)\phi_S) d^6\sigma_{UT}}{\int d\phi_S d^2P_{h\perp} d^6\sigma_{UU}} \]

with

\[ d^6\sigma_{UT} = \frac{1}{2}(d^6\sigma_{U\uparrow} - d^6\sigma_{U\downarrow}) \quad d^6\sigma_{UU} = \frac{1}{2}(d^6\sigma_{U\uparrow} + d^6\sigma_{U\downarrow}) \]
SSAs in terms of parton distributions and fragmentation functions:

\[ A_{UT}^{\text{Sivers}} = N_{\text{Sivers}} / D \]
\[ A_{UT}^{\text{Collins}} = N_{\text{Collins}} / D \]

\[ N_{\text{Sivers}} \propto \sum_q e_q^2 \int d^2 \kappa_T d^2 k_T \delta^2 (k_T + q_T - \kappa_T) \frac{\hat{P}_{h_1} \cdot k_T}{M} f_{1T}^{\perp q}(x, k_T^2) D_{1q,h}^{q,h}(z, (z\kappa_T)^2) \]
\[ N_{\text{Collins}} \propto \sum_q e_q^2 \int d^2 \kappa_T d^2 k_T \delta^2 (k_T + q_T - \kappa_T) \frac{\hat{P}_{h_1} \cdot \kappa_T}{M_h} h_1^{q}(x, k_T^2) H_{1q,h}^{\perp q,h}(z, (z\kappa_T)^2) \]

\[ D \propto \sum_q e_q f_1^q(x) D_{1q,h}^{q,h}(z) \]

LARGE \( A_{UT}^{\text{Sivers}} \) measured in \( p(e, e'\pi)x \) HERMES PRL 94, 012002 (2005)

SMALL \( A_{UT}^{\text{Sivers}} \) measured in \( D(e, e'\pi)x \); COMPASS PRL 94, 202002 (2005)

A strong flavor dependence

Importance of the neutron for flavor decomposition!
Is the extraction procedure tested in DIS valid also for the SSAs in SIDIS?

In a first paper on this subject, (S.Scopetta, PRD 75 (2007) 054005) the process $^3\bar{He}(e, e'\pi)X$ has been evaluated:

* in the Bjorken limit
* in IA → no FSI between the measured fast, ultrarelativistic $\pi$ the remnant and the two nucleon recoiling system

$E_\pi \simeq 2.4 \text{ GeV}$ in JLAB exp at 6 GeV - Qian et al., PRL 107 (2011) 072003

SSAs involve convolutions of the spin-dependent nuclear spectral function, $\vec{P}(\vec{p}, E)$, with parton distributions and fragmentation functions

$$A \simeq \int d\vec{p} dE ... \vec{P}(\vec{p}, E) \ f_{1T}^{\perp q} \left( \frac{Q^2}{2p \cdot q}, k_T^2 \right) D_{1}^{q, h} \left( \frac{p \cdot h}{p \cdot q}, \left( \frac{p \cdot h}{p \cdot q} \kappa_T \right)^2 \right)$$

Specific nuclear effects, new with respect to the DIS case, can arise and have to be studied carefully.
The IA @ JLab kinematics: a few words more

The convolution formulae for a generic structure function can be cast in the form

\[
\mathcal{F}^A(x_{Bj}, Q^2, \ldots) = \sum_N \int_{x_{Bj}}^A f_N^A(\alpha, Q^2, \ldots) \mathcal{F}^N(x_{Bj}/\alpha, Q^2, \ldots) \, d\alpha
\]

with the light-cone momentum distribution:

\[
f_N^A(\alpha, Q^2, \ldots) = \int dE \int_{p_{min}(\alpha, Q^2, \ldots)}^{p_{max}(\alpha, Q^2, \ldots)} P_N^A(p, E) \delta \left( \alpha - \frac{pq}{m\nu} \right) \theta \left( W_x^2 - (M_N + M_\pi)^2 \right) \, d^3p
\]

- Bjorken limit:
  - \( p_{min, max} \) not dependent on \( Q^2, x \):
  - \( f_N^A(\alpha) \) depends on \( \alpha \) only,
  - \( 0 \leq \alpha \leq A \)

- @ JLab kinematics,
  - \((E = 8.8 \text{ GeV}, E' \simeq 2 \div 3 \text{ GeV}, \theta_e \simeq 30^\circ) q \neq \nu \) and \( \alpha_{min} \neq 0 \)
Calculation within the \textbf{Av18} interaction:

- weak depolarization of the neutron, \( p_n = \int d\alpha f_n^{3\text{He}}(\alpha) = 0.878 \)

- strong depolarization of the protons, \( p_p = \int d\alpha f_p^{3\text{He}}(\alpha) = -0.023 \)
  (cancellation between contributions in the 2-body and 3-body channels)
Results: $\vec{n}$ from $^3\vec{He}$: $A_{UT}^{Sivers}$, @ JLab, in IA

**FULL:** Neutron asymmetry (model: from parameterizations or models of TMDs and FFs)

**DOTS:** Neutron asymmetry extracted from $^3\text{He}$ (calculation) neglecting the contribution of the proton polarization

$$\bar{A}_n \simeq \frac{1}{f_n} A_{3}^{calc}$$

**DASHED:** Neutron asymmetry extracted from $^3\text{He}$ (calculation) taking into account nuclear structure effects through the formula:

$$A_n \simeq \frac{1}{p_n f_n} \left( A_{3}^{calc} - 2p_p f_p A_{p}^{model} \right)$$
Results: $\vec{n}$ from $^3\vec{He}$: $A_{UT}^{Collins}$, @ JLab

In the Bjorken limit the extraction procedure successful in DIS works also in SiDIS, for both the Collins and the Sivers SSAs!

What about FSI effects?

(thinking to E12-09-018, A.G. Cates et al., approved with rate A @JLab 12)
Relative energy between $A - 1$ and the remnants: a few GeV

$\rightarrow$ eikonal approximation.

$$d\sigma \simeq \epsilon_{\mu\nu} W^A_{\mu\nu}(S_A)$$

$$W^A_{\mu\nu}(S_A) \simeq \sum_{S_{A-1}, S_X} J^A_{\mu} J^A_{\nu} \quad J^A_{\mu} \simeq \langle S_A P | \hat{J}^A_\mu(0) | S_X, S_{A-1}, P_{A-1} E_{A-1}^f \rangle$$
Relevant part of the (GEA-distorted) spin dependent spectral function:

\[ \mathcal{P}^{IA(FSI)}_\parallel = \mathcal{O}^{IA(FSI)}_{\frac{1}{2} \frac{1}{2}} - \mathcal{O}^{IA(FSI)}_{-\frac{1}{2} -\frac{1}{2}}; \]

with:

\[ \mathcal{O}^{IA(FSI)}_{\lambda \lambda'}(p_N, E) = \sum_{\epsilon_{A-1}} \rho(\epsilon_{A-1}^*) \langle S_A, P_A | (\hat{S}_{Gl})\{\Phi_{\epsilon_{A-1}^*}, \lambda', p_N\} \times \langle (\hat{S}_{Gl})\{\Phi_{\epsilon_{A-1}^*}, \lambda, p_N\} | S_A, P_A \rangle \delta(E - B_A - \epsilon_{A-1}^*) \].

Glauber operator: \( \hat{S}_{Gl}(r_1, r_2, r_3) = \prod_{i=2,3} [1 - \theta(z_i - z_1) \Gamma(b_i - b_i, z_i)] \)

(generalized) profile function: \( \Gamma(b_{1i}, z_{1i}) = \frac{(1 - i \alpha) \sigma_{eff}(z_{1i})}{4 \pi b_0^2} \exp \left[ -\frac{b_{1i}^2}{2 b_0^2} \right] \),

GEA (\( \Gamma \) depends also on the longitudinal distance between the debris and the scattering centers \( z_{1i} \)) very successful in q.e. semi-inclusive and exclusive processes off \(^3\)He see, e.g., Alvioli, Ciofi & Kaptari PRC 81 (2010) 02100 and references there in

A hadronization model is necessary to define \( \sigma_{eff}(z_{1i}) \)
FSI: the hadronization model

Hadronization model (Kopeliovich et al., NPA 2004) + $\sigma_{eff}$ model for SIDIS (Ciofi & Kopeliovich, EPJA 2003)

GEA + hadronization model successfully applied to unpolarized SIDIS $^2H(e, e'p)X$ (Ciofi & Kaptari PRC 2011).

\[ Q^2 \]

\[ N \]

\[ N_1 \]

\[ \pi_1 \]

\[ \pi_2 \]

\[ \pi_n \]

\[ 0 \]

\[ z_0 \]

\[ z_1 \]

\[ z_{n-1} \]

\[ X \]

\[ 0 \]

\[ \infty \]

The hadronization model is phenomenological: parameters are chosen to describe the scenario of JLab experiments (e.g., $\sigma_{tot}^{NN} = 40$ mb, $\sigma_{tot}^{\pi N} = 25$ mb, $\alpha = -0.5$ for both $NN$ and $\pi N ...$).

According to high energy $N - N$ scattering data, $\sigma_{eff}(z)$ is taken spin-independent (see, e.g., Alekseev et al., PRD 79 (2009) 094014 )

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FSI: *distorted* spin-dependent spectral function of $^3$He

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206

While $P^{IA}$ is “static”, i.e. depends on ground state properties, $P^{FSI}$ is dynamical ($\propto \sigma_{eff}$) and process dependent;

For each experimental point (given $x, Q^2$...), a different spectral function has to be evaluated!

Quantization axis (w.r.t. which polarizations are fixed) and eikonal direction (fixing the “longitudinal” propagation) are different)... States have to be rotated...

$P^{FSI}$: a really cumbersome quantity, a very demanding evaluation

$(\approx 1$ Mega CPU*hours @ “Zefiro” PC-farm, PISA, INFN “gruppo 4”).

The convolution formulae for a generic structure function can be cast in the form

$$F^A(x_{Bj}, Q^2, ...) = \sum_N \int_{x_{Bj}}^A f_N^A(\alpha, Q^2, ...) \int_{x_{Bj}}^A F^N(x_{Bj}/\alpha, Q^2, ...) d\alpha$$

with the distorted light-cone momentum distribution:

$$f_N^A(\alpha, Q^2, ...) = \int dE \int_{p_m(\alpha, Q^2, \ldots)}^{p_M(\alpha, Q^2, \ldots)} P^{A, FSI}_N(p, E, \ldots) \delta \left( \alpha - \frac{pq}{m\nu} \right) \theta \left( W^2_{x} - (M_N + M_\pi)^2 \right) d^3p$$
light-cone momentum distributions with FSI:
Del Dotto, Kaptari, Pace, Salmè, S.S., PRC 96 (2017) 065203

PROTON @ $E_i = 8.8$ GeV

NEUTRON @ $E_i = 8.8$ GeV

Effective polarizations change...
Does the strong FSI effect hinder the neutron extraction?

Actually, one should also consider the effect on dilution factors $f_N$

\[
A_3^{\exp} \approx \frac{\Delta \sigma_{3}^{\exp}}{\sigma_{\text{unpol.}}^{\exp}} \Rightarrow \frac{\langle \tilde{s}_n \rangle \Delta \bar{\sigma}(n) + 2 \langle \tilde{s}_p \rangle \Delta \bar{\sigma}(p)}{\langle N_n \rangle \sigma_{\text{unpol.}}(n) + 2 \langle N_p \rangle \sigma_{\text{unpol.}}(p)} = \langle \tilde{s}_n \rangle f_n A_n + 2 \langle \tilde{s}_p \rangle f_p A_p
\]

PWIA:
\[
\langle \tilde{s}_{n(p)} \rangle = \int dE \int d^3 p P_{\parallel}(E, p) = p_{n(p)};
\]
\[
\langle N \rangle = \int dE \int d^3 p P_{\text{unpol.}}(E, p) = 1.
\]

FSI:
\[
\langle \tilde{s}_{n(p)}^{\text{FSI}} \rangle = \int dE \int d^3 p P_{\parallel}^{\text{FSI}}(E, p) = p_{n(p)}^{\text{FSI}};
\]
\[
\langle N \rangle = \int dE \int d^3 p P_{\text{unpol.}}^{\text{FSI}}(E, p) < 1.
\]

\[
A_n \approx \frac{1}{p_n^{\text{FSI}} f_n^{\text{FSI}}} \left( A_3^{\exp} - 2 p_p^{\text{FSI}} f_p^{\text{FSI}} A_p^{\exp} \right) \approx \frac{1}{p_n f_n} \left( A_3^{\exp} - 2 p_p f_p A_p^{\exp} \right)
\]
Good news from GEA studies of FSI!

Effects of GEA-FSI (shown at $E_i = 8.8$ GeV) in the dilution factors and in the effective polarizations compensate each other to a large extent: the usual extraction is safe!

$$A_n \approx \frac{1}{p_n^{FSI} f_n^{FSI}} \left( A_3^{exp} - 2 p_p^{FSI} f_p^{FSI} A_p^{exp} \right) \approx \frac{1}{p_n f_n} \left( A_3^{exp} - 2 p_p f_p A_p^{exp} \right)$$

A. Del Dotto, L. Kaptari, E. Pace, G. Salmè, S.S., PRC 96 (2017) 065203

February 5th, 2018
Now: **spectator SIDIS**...

We studied the process $A(e, e'(A - 1))X$ many years ago

In this process, in IA, no convolution!

$$d^2 \sigma_A \propto F^N_2(x)$$

for the deuteron: Simula PLB 1997;

Melnitchouk, Sargsian, Strikman ZPA 1997; BONUS@JLab

Example: through $^3\text{He}(e,e'd)X$, $F^p_2$, check of the reaction mechanism (EMC effect); measuring $^3\text{H}(e,e'd)X$, direct access to the neutron $F^n_2$!

new perspectives: loi to the JLab PAC, already in November 2010; now: approved experiments at JLab!

ALERT coll., arXiv:1708.00891 [nucl-ex], for $^4\text{He}$...
Spectator SIDIS $^{3}\text{He}(e', e\,^{2}\text{H})X \rightarrow g_{1}^{p}$ for a bound proton

Kaptari, Del Dotto, Pace, Salmè, Scopetta PRC 89, 035206 (2014)

The distorted spin-dependent spectral function with the Glauber operator $\hat{G}$ can be applied to the "spectator SIDIS" process, where a slow deuteron is detected. Goal $\rightarrow g_{1}^{N}(x_{N} = \frac{Q^{2}}{2p_{N}q})$ of a bound nucleon. $A_{LL}$ of electrons with opposite helicities scattered off a longitudinally polarized $^{3}\text{He}$ for parallel kinematics ($p_{N} = -p_{mis} \equiv -P_{A-1} \parallel \hat{z}$, with $\hat{z} \equiv \hat{q}$)

$$\frac{\Delta \sigma \hat{S}_{A}}{d\varphi_{e} \, dx \, dy \, dP_{D}} \equiv \frac{d\sigma \hat{S}_{A}(h_{e} = 1) - d\sigma \hat{S}_{A}(h_{e} = -1)}{d\varphi_{e} \, dx \, dy \, dP_{D}} =$$

$$\approx 4 \frac{\alpha_{em}^{2}}{Q^{2}z_{N}\mathcal{E}} \frac{m_{N}}{E_{N}} \frac{g_{1}^{p}}{g_{1}^{p}} \left( \frac{x}{z} \right) \mathcal{P}_{\parallel}^{1/2}(p_{mis}) \mathcal{E}(2 - y) \left[ 1 - \frac{|p_{mis}|}{m_{N}} \right] \quad \text{Bjorken limit}$$

$x = \frac{Q^{2}}{2m_{N}\nu}$, $y = (\mathcal{E} - \mathcal{E}')/\mathcal{E}$, $z = (p_{N} \cdot q)/m_{N}\nu$

$\mathcal{P}_{\parallel}^{1/2}(p_{mis}) = \mathcal{O}_{\frac{1}{2} \frac{1}{2}}^{\frac{1}{2} \frac{1}{2}} - \mathcal{O}_{\frac{1}{2} \frac{1}{2}}^{-\frac{1}{2} - \frac{1}{2}}$ parallel component of the spectral function

$\mathcal{O}_{\lambda \lambda'}^{M M'}(FSI) (P_{D}, E_{2bbu}) = \left\langle \hat{G} \{ \Psi_{P_{D}}, \lambda, P_{N} \} | \Psi_{A}^{M} \right\rangle_{\hat{q}} \left\langle \Psi_{A}^{M'} | \hat{G} \{ \Psi_{P_{D}}, \lambda', P_{N} \} \right\rangle_{\hat{q}}$

Using $^{3}\text{H}$ one would get the neutron!
Spectator SIDIS $^3\text{He}(\vec{e}, e' \, ^2\text{H})X \rightarrow g_1^p$ for a bound proton

Kaptari, Del Dotto, Pace, Salme', Scopetta PRC 89, 035206 (2014)

The kinematical variables upon which $g_1^N(x_N)$ depends can be changed independently from the ones of the nuclear-structure $P_1^{1/2}(p_{mis})$. This allows to single out a kinematical region where the final-state effects are minimized: $|p_{mis} \equiv P_D| \approx 1 \text{fm}^{-1}$

Possible direct access to $g_1^N(x_N)$.

At JLab, $E = 12 \text{ GeV}$, $-p_{mis} \parallel q$:

![Graph showing the dependence of $|P_{1/2}(p_{mis})|$ on $|p_{mis}|$]
Spectator SIDIS $^3\text{He}(\vec{e}, e' \, ^2\text{H})X \rightarrow g_1^P$ for a bound proton

Even if $^3\text{H}$ is not available, relevant information is at hand:

- $g_1^P$ for the bound proton: comparison with the free case, polarized EMC effect and possible evidence of off-shell effects, beyond the IA;
- these effects are neglected in the extraction formula used in DIS for $g_1^n$

$$g_1^n \simeq \frac{1}{p_n} \left( g_1^3 - 2p_p g_1^P \right),$$

Nonetheless it was noted long time ago (L. Frankfurt, V. Guzey, M. Strikman PLB 381 (1996) 379) that, if the IA is used to describe $g_1^3\text{He}$ and $g_1^3\text{H}$, the Bjorken Sum Rule for the trinucleon system is violated by 4%: possible evidence of explicit $\Delta$ isobar degrees of freedom in the nuclear wave function.

- the Bjorken Sum Rule (in the nucleon case) is obtained considering also $^3\text{He}$ data; used even to constrain $\alpha_s(Q^2)$ (see, e.g., A. Deur et al, Prog.Part.Nucl.Phys. 90 (2016) 1)
- FSI under control in specific regions; they can be evaluated elsewhere

I understand that at JLab the use of a recoil detector in a polarized set-up is difficult, and the use of $^3\text{H}$ targets complicated...

Actually none of these problems occur at the EIC

February 5th, 2018
DVCS off $^3$He and neutron GPDs (a fast look)

**Question:** Which of these transverse sections is more similar to that of a nucleus?

To answer, we should perform a *tomography*...

We can! M. Burkardt, PRD 62 (2000) 07153

**Answer:** Deeply Virtual Compton Scattering & Generalized Parton Distributions (GPDs)

For $^4$He, recent data (M. Hattawy et al., PRL 119, (2017) 20204) and new experiments approved (ALERT coll., e-Print: arXiv:1708.00888 [nucl-ex])

February 5$^{th}$, 2018
GPDs: Definition (X. Ji PRL 78 (97) 610)

For a $J = \frac{1}{2}$ target, in a hard-exclusive process, (handbag approximation) such as (coherent) DVCS:

\[
\begin{align*}
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi} q(-\lambda n/2) | P \rangle & = H_q(x, \xi, \Delta^2) \tilde{U}(P') \gamma^\mu U(P) \\
+ E_q(x, \xi, \Delta^2) \tilde{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \ldots
\end{align*}
\]

- \( \Delta = P' - P \), \( q^\mu = (q_0, \vec{q}) \), and \( \bar{P} = (P + P')^\mu / 2 \)
- \( x = k^+ / P^+ \); \( \xi = \text{"skewness"} = -\Delta^+ / (2\bar{P}^+) \)
- \( x \leq -\xi \rightarrow \text{GPDs describe antiquarks} \)
- \( -\xi \leq x \leq \xi \rightarrow \text{GPDs describe } q\bar{q} \text{ pairs} \)
- \( x \geq \xi \rightarrow \text{GPDs describe quarks} \)

Neutron spin structure from polarized $^3$He – p.28/37

February 5\textsuperscript{th}, 2018
GPDs: constraints

when $P' = P$, i.e., $\Delta^2 = \xi = 0$, one recovers the usual PDFs:

$$H_q(x, \xi, \Delta^2) \implies H_q(x, 0, 0) = q(x); \quad E_q(x, 0, 0) \text{ unknown}$$

the $x$–integration yields the q-contribution to the Form Factors (ffs)

$$\int dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q (-\lambda n/2) \gamma^\mu \psi_q (\lambda n/2) | P \rangle =$$

$$\int dx H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + \int dx E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{\sigma^{\mu\nu} \Delta_{\nu}}{2M} U(P) + \ldots$$

$$\implies \int dx H_q(x, \xi, \Delta^2) = F_1^q(\Delta^2) \quad \int dx E_q(x, \xi, \Delta^2) = F_2^q(\Delta^2)$$

$$\implies \text{Defining } \tilde{G}_M^q = H_q + E_q \quad \text{one has } \int dx \tilde{G}_M^q(x, \xi, \Delta^2) = G_M^q(\Delta^2)$$
GPDs: a unique tool...

not only 3D structure, at parton level; many other aspects, e.g., contribution to the solution to the “Spin Crisis” (J.Ashman et al., EMC collaboration, PLB 206, 364 (1988)), yielding parton total angular momentum...

... but also an experimental challenge:

Hard exclusive process $\rightarrow$ small $\sigma$;

Difficult extraction:

$$T_{DVCS} \propto CFF \propto \int_{-1}^{1} dx \frac{H_q(x, \xi, \Delta^2)}{x - \xi + i\epsilon} + \ldots,$$

Competition with the BH process! ($\sigma$ asymmetries measured).

$$d\sigma \propto |T_{DVCS}|^2 + |T_{BH}|^2 + 2 \Re\{T_{DVCS}T_{BH}^*\}$$

Nevertheless, for the proton, we have results:

(Guidal et al., Rep. Prog. Phys. 2013...

February 5th, 2018
Nuclei and DVCS tomography

In impact parameter space, GPDs are densities:

\[
\rho_q(x, \vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, 0, \Delta^2)
\]

Coherent DVCS: nuclear tomography

Incoherent DVCS: tomography of bound nucleons, realization of the EMC effect

\(^3\)He is a unique target for GPDs studies. Examples:

* access to the neutron information in coherent processes
* heavier targets do not allow refined theoretical treatments. Test of the theory
* Between \(^2\)H (“not a nucleus”) and \(^4\)He (a true one). Not isoscalar!
Extracting GPDs: $^3\text{He} \simeq p$

One measures asymmetries: $A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$

- Polarized beam, unpolarized target:
  $$\Delta\sigma_{LU} \simeq \sin \phi \left[ F_1 \mathcal{H} + \xi (F_1 + F_2) \tilde{\mathcal{H}} + \left( \Delta^2 F_2 / M^2 \right) \mathcal{E} / 4 \right] d\phi \implies \mathcal{H}$$

- Unpolarized beam, longitudinally polarized target:
  $$\Delta\sigma_{UL} \simeq \sin \phi \left\{ F_1 \tilde{\mathcal{H}} + \xi (F_1 + F_2) \left[ \mathcal{H} + \xi / (1 + \xi) \mathcal{E} \right] \right\} d\phi \implies \tilde{\mathcal{H}}$$

- Unpolarized beam, transversely polarized target:
  $$\Delta\sigma_{UT} \simeq \cos \phi \sin (\phi_S - \phi) \left[ \Delta^2 (F_2 \mathcal{H} - F_1 \mathcal{E}) / M^2 \right] d\phi \implies \mathcal{E}$$

To evaluate cross sections, e.g. for experiments planning, one needs $\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}$

This is what we have calculated for $^3\text{He}$. $\mathcal{H}$ alone, already very interesting.
GPDs of $^3$He in IA

$H_q^A$ can be obtained in terms of $H_q^N$ (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} \sum_S \sum_s P_{SS,ss}^N(\vec{p}, \vec{p}', E) \xi \frac{\xi'}{\xi} H_q^N(x', \Delta^2, \xi') ,$$

and $\tilde{G}_{M}^{3,q}$ in terms of $\tilde{G}_{M}^{N,q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_{M}^{3,q}(x, \Delta^2, \xi) = \sum_N \int dE \int d\vec{p} \left[ P_{+-,+-}^N - P_{+-,--}^N \right] (\vec{p}, \vec{p}', E) \xi \frac{\xi'}{\xi} \tilde{G}_{M}^{N,q}(x', \Delta^2, \xi') ,$$

where $P_{SS,ss}^N(\vec{p}, \vec{p}', E)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon $N$ in the nucleus,

$$P_{SS',ss'}^N(\vec{p}, \vec{p}', E) = \frac{1}{(2\pi)^6} \frac{M \sqrt{ME}}{2} \int d\Omega_t \sum_{st} \langle \vec{P}'S' | \vec{p}' s', \vec{t}_s \rangle_N \langle \vec{p}s, \vec{t}_s | \vec{P}S \rangle_N ,$$

evaluated by means of a realistic treatment based on Av18 wave functions ("CHH" method in A. Kievsky et al NPA 577, 511 (1994); Av18 + UIX overlaps in E. Pace et. al, PRC 64, 055203 (2001)).

Nucleon GPDs given by an old version of the VGG model (VGG 1999, $x-$ and $\Delta^2-$ dependencies factorized)
Nuclear effects - flavor dependence

Nuclear effects are bigger for the $d$ flavor rather than for the $u$ flavor:

\[ R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)} \]

\[ H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2) \]

$R_q^{(0)}(x, \xi, \Delta^2)$ would be one if there were no nuclear effects;

This is a typical conventional, IA effect (spectral functions are different for $p$ and $n$ in $^3$He, not isoscalar!); if (not) found, clear indication on the reaction mechanism of DIS off nuclei. Not seen in $^2$H, $^4$He.

February 5\(^{th}\) , 2018
Nuclear effects - flavor dependence

The \(d\) and \(u\) distributions follow the pattern of the neutron and proton light-cone momentum distributions, respectively:

\[
R_q(x_3, 0, 0) x_3 \quad d \quad u
\]

\[
R_q(x_3, \xi_3 = 0.2, \Delta^2 = -0.25 \text{ GeV}^2) x_3 \quad d \quad u
\]

\[
f_N(z) \quad n \quad p
\]

\[
h_N(z, \xi_3 = 0.2, \Delta^2 = -0.25 \text{ GeV}^2) \quad n \quad p
\]

How to perform a flavor separation? Take the triton \(^3\text{H}\)!

Possible (see MARATHON@JLab). Possible for DVCS (ALERT).


\[
H_t, H_H \rightarrow H_u^H \simeq H_d^t, H_d^H \simeq H_u^t
\]

in the valence region...

February 5th, 2018
Calculations of $^3\text{He}$ GPDs: summary

Our results, for $^3\text{He}$: (S.S. PRC 2004, 2009; M. Rinaldi and S.S., PRC 2012, 2013)

* I.A. calculation of $H_3$, $E_3$, $\tilde{H}_3$, within AV18;
* Interesting predictions: strong sensitivity to details of nuclear dynamics;
* extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;

Coherent DVCS off $^3\text{He}$ would be:

* a test of IA; relevance of non-nucleonic degrees of freedom;
* a test of the $A$-dependence of nuclear effects;
* complementary to incoherent DVCS off the deuteron in extracting the neutron information (with polarized targets).

No data; no proposals at JLAB... difficult to detect slow recoils using a polarized target... But even unpolarized, $^3\text{He}$ would be interesting!
Together with $^3\text{H}$, nice possibilities (flavor separation of nuclear effects, test of IA)

at the EIC, beams of polarized light nuclei will operate. $^3\tilde{H}e$ can be used.

Our codes available to interested colleagues.
Conclusions

Further calculations to support DIS EIC measurements:

1. Calculations with a Light-Front spectral function (in IA);
   LF, formal: Del Dotto, Pace, Salmè, SS, PRC 95 (2017) 014001. Relativistic FSI?
2. Extension to low $x$ kinematics (coherent effects)
3. Towards 4-body systems: DVCS off $^4$He (in collaboration with M. Viviani, Pisa), non diagonal spectral function. Upgrade, using realistic dynamics, of old results for (unp.) spectator tagging. (also for JLab, ALERT collaboration)

Contribution of EIC to validate underlying assumptions

1. Provide also absolute cross sections, not only asymmetries!
2. Spectator tagging (any kind of): off-shell effects, (polarized) EMC effect, validity of the impulse approximation
3. SIDIS $^3H e(\vec{e}, e'd)X$ measurements and polarized EMC effect
4. Even better with $^3H$ beams: direct access to $g_1^n$; check of IA and extraction formulae (even the inclusive measurement is sufficient for this)
5. Even unpolarized $^3H$ very interesting (DVCS) for isospin dependent nuclear effects and the possible breaking of IA
6. DVCS off $^3$He: onset of nuclear effects, great theoretical control

February 5$^{th}$, 2018