Nuclear Forces and Currents in Chiral Effective Field Theory

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Polarized light ion physics with EIC
Ghent University, Belgium
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Outline

- Nuclear forces in chiral EFT
  - Introduction to chiral EFT
  - Long & short-range physics
  - Role of pion-nucleon scattering
  - Selective applications and role of 3NF

- Nuclear current in chiral EFT
  - Symmetries for currents
  - Nuclear currents up to N³LO
ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

Existence of light weakly interacting Goldstone bosons

Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

Systematic description of QCD by ChPT in low energy sector
(low momenta and masses \( q, M_\pi < \Lambda \approx 1 \text{ GeV} \) )
From QCD to nuclear physics

- NN interaction is strong: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) $\Rightarrow$ the QM A-body problem

\[
\left[ \sum_{i=1}^{A} -\nabla_i^2 + \mathcal{O}(m_N^{-3}) + V_{2N} + V_{3N} + V_{4N} + \ldots \right] |\Psi\rangle = E |\Psi\rangle
\]

derived within ChPT

Weinberg '91

- unified description of $\pi\pi$, $\pi N$ and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, $\pi$-prod., ...)
- precision physics with/from light nuclei
## Chiral Expansion of the Nuclear Forces

<table>
<thead>
<tr>
<th>Order</th>
<th>Two-nucleon force</th>
<th>Three-nucleon force</th>
<th>Four-nucleon force</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO ($Q^0$)</td>
<td><img src="LO.png" alt="Diagram" /></td>
<td><a href="#">Weinberg '90</a></td>
<td><img src="LO.png" alt="Diagram" /></td>
</tr>
<tr>
<td>NLO ($Q^2$)</td>
<td><img src="NLO.png" alt="Diagram" /></td>
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<td>$N^2$LO ($Q^3$)</td>
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<tr>
<td>$N^3$LO ($Q^4$)</td>
<td><img src="N3LO.png" alt="Diagram" /></td>
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<td><img src="N3LO.png" alt="Diagram" /></td>
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<tr>
<td>$N^4$LO ($Q^5$)</td>
<td><img src="N4LO.png" alt="Diagram" /></td>
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</table>

- **LO ($Q^0$)**: First-order contributions, including two-nucleon forces. 
- **NLO ($Q^2$)**: Contributions to second order, including three-nucleon forces.
- **$N^2$LO ($Q^3$)**: Contributions to third order, including four-nucleon forces.
- **$N^3$LO ($Q^4$)**: Contributions to fourth order, including higher-order interactions.

**References:***

- [Weinberg '90](#)
- [Ordonez, van Kolck '92](#)
- [Kaiser '00 - '02](#)
- [Bernard, Epelbaum, HK, Meißner, '08, '11](#)
- [Epelbaum '06](#)
- [Entem, Kaiser, Machleidt, Nosyk '15](#)
- [Girlanda, Kievsky, Viviani '11](#)
- [HK, Gasparyan, Epelbaum '12, '13](#) *(short-range loop contrib. still missing)*
- [parameter-free](#)
- [still have to be worked out](#)
Couplings of short-range interactions are fixed from NN - data. In the isospin limit we have:

- **LO [Q^0]**: 2 operators (S-waves)
- **NLO [Q^2]**: + 7 operators (S-, P-waves and \( \varepsilon_1 \))
- **N^2LO [Q^3]**: no new terms
- **N^3LO [Q^4]**: + 12 operators (S-, P-, D-waves and \( \varepsilon_1, \varepsilon_2 \))
- **N^4LO [Q^5]**: no new terms

Long range part of the nuclear forces are predictions (chiral symmetry of QCD) once couplings from single-nucleon subprocess are determined.

**NN parameter free predictions**

\[
\begin{align*}
\bar{p}_\mu & \rightarrow W^- \\
\mu^- & \rightarrow ...
\end{align*}
\]
Pion-Nucleon Scattering

**Effective chiral Lagrangian:**

\[ \mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots \]

\[ \mathcal{L}_{\pi N} = \bar{N} \left( i \gamma^\mu D_\mu [\pi] - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu [\pi] \right) N + \sum_i c_i \bar{N} \hat{O}_i^{(2)} [\pi] N + \sum_i d_i \bar{N} \hat{O}_i^{(3)} [\pi] N + \sum_i e_i \bar{N} \hat{O}_i^{(4)} [\pi] N + \ldots \]

**Pion-nucleon scattering is calculated up to Q^4 in heavy-baryon ChPT**

*Fettes, Meißner '00; HK, Gasparyan, Epelbaum '12*

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**Order Q:**

- \[ \mathcal{L}_\pi^{(1)} \]

**Order Q^2:**

- \( c_i \)

**Order Q^3:**

- \( d_i \)

**Order Q^4:**

- \( e_i \)
Dispersive analysis of $\pi N$ scattering

- Roy-Steiner equations for $\pi N$ scattering
  
  *Hoferichter et al., Phys. Rept. 625 (16) 1*

  Partial Wave Decomposition of Hyperbolic dispersion relations
  $\pi N \rightarrow \pi N \& \pi \pi \rightarrow \bar{N}N$ channels

**Input:**
- S- and P-waves above $s_m = (1.38 \text{ GeV})^2$
- Higher partial waves for all $s$
- Inelasticities for $s < s_m$ and scattering lengths

**Output:**
- S- and P-waves with error bands, $\sigma$-term,
- Subthreshold coefficients $\bar{X} = \sum_{m,n} x_{mn} \nu^{2m+kn}$, $X = \{A^\pm, B^\pm\}$

- $c_i, d_i, e_i$ are fixed from subthreshold coefficients (within Mandelstam triangle where one expects best convergence of chiral expansion)

- Subthreshold point is closer to kinematical region of NN force than the physical region of $\pi N$ scattering
NN Data Used in the Fits

Reinert, HK, Epelbaum ’17

- From 1950 on around 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured
- Not all of these data are compatible. Rejections are required to get a reasonable fit
- Granada 2013 base used: Navarro Perez et al. ’13 rejection by $3\sigma$-criterion
  - 31% of np + 11% of pp data have been rejected

Resulting data base consists of 2697 np + 2158 pp data for $E_{\text{lab}}$$=0$-300 MeV
Chiral Expansion of np Phase Shifts

Reinert, HK, Epelbaum '17

- Good convergence of chiral expansion & excellent agreement with NPWA data
- Chiral potential match in precision phenomenological potentials (CD Bonn, Av18, …) with around 40% less parameter
Uncertainty Estimate

*Epelbaum, HK, Meißner ’15*

- Uncertainties in the experimental data
- Uncertainties in the estimation of $\pi N$ LECs
- Uncertainties in the determination of contact interaction LECs
- Uncertainties of the fits due to the choice of $E_{\text{max}}$

- Systematic uncertainty due to truncation of the chiral expansion at a given order

*Estimate the uncertainty via expected size of higher-order corrections*

For a $N^4$LO prediction of an observable $X^{N^4\text{LO}}$ we get an uncertainty

$$
\Delta X^{N^4\text{LO}}(p) = \max \left( Q \times |X^{N^3\text{LO}}(p) - X^{N^4\text{LO}}(p)|, Q^2 \times |X^{N^2\text{LO}}(p) - X^{N^3\text{LO}}(p)|, 
Q^3 \times |X^{N\text{LO}}(p) - X^{N^2\text{LO}}(p)|, Q^4 \times |X^{\text{LO}}(p) - X^{N\text{LO}}(p)|, Q^6 \times |X^{\text{LO}}(p)| \right)
$$

with chiral expansion parameter

$$
Q = \max \left( \frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right)
$$

For $\sigma_{\text{tot}}$ errors $\rightarrow$ 68% degree-of-belief intervals (Bayesian analysis): *Furnstahl et al. ’15*
Uncertainty Quantification

Reinert, HK, Epelbaum ’17

Effective range, deuteron properties and phase-shift with quantified uncertainty

Example: deuteron asymptotic normalization

\[ A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1) \text{ fm}^{-1/2} \]
\[ \eta = 0.0255^{(+1)}_{(-1)}(1)(4)(1) \]

\[ \eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1) \]

Exp: \( A_S = 0.8781(44) \text{ fm}^{-1/2}, \quad \eta = 0.0256(4) \)

Borbely et al. ’85

Rodning, Knutson ’90

Nijmegen PWA [errors are „educated guesses“] \( A_S = 0.8845(8) \text{ fm}^{-1/2}, \quad \eta = 0.0256(4) \)

Stoks et al. ’95

Granada PWA [errors purely statistical] \( A_S = 0.8829(4) \text{ fm}^{-1/2}, \quad \eta = 0.0249(1) \)

Navarro Perez et al. ’13
Role of the 3NFs

NN-force from Epelbaum, HK, Meißner '15

Total cross section for Nd scattering: without 3NF

LENPIC collaboration, Binder et al. '15

Significant discrepancy between experiment and theory

The discrepancy at 10 MeV is much lower than at other energies

Cross section at low energy is governed by S-wave spin-doublet and spin-quartet Nd scattering lengths: $^4a > > ^2a$ (one order of magnitude) and $^4a$ is much less sensitive to 3NF (Pauli principle)
Role of the 3NFs for $A > 3$

*NN-force from Epelbaum, HK, Meißner ’15*

Selected observables for $^4$He & $^6$Li

*LENPIC collaboration, Binder et al. ’15*

- Results for $^4$He are obtained by solving Faddeev-Yakubovski eq. and No-Core Shell Model (NCSM) which agree within estimated uncertainties.
- Results for $^6$Li are obtained by NCSM with Similarity Renormalization Group (SRG) evolution (induced 3NF’s are taken into account).
Summary

Chiral Nucleon-Nucleon Force

- Chiral nuclear NN forces are calculated up to N^4LO
- Phase-shifts, deuteron properties, … with uncertainty quantification
- Chiral NN force match in precision phenomenological potentials (CD Bonn, Av18, …) with around 40% less parameter
- Clear evidence for missing 3NF for A > 2
Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism.

Chiral EFT Hamiltonian depends on external sources.

\[ H[a, v, s, p] \]

- Axial-vector source
- Pseudoscalar source
- Vector source
- Scalar source
Siegert theorem + $N^4$LO

Skibinski et al. PRC93 (2016) no. 6, 064002

Generate longitudinal component of NN current by continuity equation

$$\left[ H_{\text{strong}}, \rho \right] = \vec{k} \cdot \vec{J} \quad \text{regularized longitudinal current (Siegert theorem)}$$

Deuteron photo-disintegration

$$\gamma + d \rightarrow p + n$$

- consistent regularization via cont. eq.
- improvement by 1N+Sievert
- implementation of transverse part & exchange currents work in progress

Nucleon-deuteron radiative capture: $p(n) + d \rightarrow ^3H( ^3He) + \gamma$

Data for different energies and angles are plotted, showing the predicted and experimental cross sections. The graphs illustrate the comparison between different models and the effects of energy on the reaction probabilities.

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**Figure Details:***
- **Figure 1**: Total cross section for the reaction $\gamma + d \rightarrow p + n$.
- **Figure 7**: Truncation errors at different orders of the chiral expansion, showing convergence of the predictions at different energies and orders.

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**Equations and Expressions:**
- $H_{\text{strong}}$ as the strong Hamiltonian,
- $\rho$ the density operator,
- $\vec{k} \cdot \vec{J}$ the regularized longitudinal current.

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**Theoretical and Experimental Considerations:**
- Comparisons are made between experimental data and predictions from different models.
- The truncation errors are displayed to show the convergence of the predictions at different energies and orders of the chiral expansion.

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**Analysis:**
- The Siegert theorem is applied to obtain very similar predictions, practically indistinguishable at photon energies $E_\gamma$.
- The predictions agree well with experimental data at different photon energies $E_\gamma$ and angles $\Theta_{\gamma d}$.

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**Footnotes:**
- Siegert, $N^4$LO, 1N, AV18, SNC, SNC+Siegert, and SNC+MEC models are compared.
- The implementation of transverse parts and exchange currents is under progress.
1 Introduction

In 2013 we published the final result of the MuCap experiment, which is a precision determination of $g_P$, the weak-pseudoscalar coupling of the proton. The hydrogen time projection chamber (TPC) technique employed is relatively immune to the poorly known molecular physics complications that plagued previous efforts. The result, $g_P = 8.06 \pm 0.55$, settles a long-standing experimental challenge. It provides a sensitive test of QCD symmetries and finally confirms a fundamental prediction of chiral perturbation theory.

The result was recognized as an Editor's Suggestion and described in an American Physical Society synopsis and in several press releases.

![Figure 2: Recent theoretical and experimental results on muon capture rate](image)

The good agreement between the MuCap result and theory demonstrates that all parameters entering the one-nucleon weak amplitudes are well under control. This allows the MuSun experiment to extend this program with a precise determination of the strength of the weak interaction in the two-nucleon system, using the process

$$\mu^- + d \rightarrow v_\mu + n + n$$

(1)

MuSun will determine the sole unknown low-energy constant involved in modern – QCD-based – effective field theory (EFT) calculations of weak nuclear reactions. The anticipated precision is 5 times greater than presently available from the 2N system and will be essential for calibrating these reactions in a model-independent way. This will provide a benchmark for extending the EFT method to more complicated few-body processes. Regarding the 2N system, muon capture will provide unique constraints on electro-weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:

- triton half life, $fT_{1/2} = 1129.6 \pm 3.0$ s, and the muon capture rate on $^3$He, $\Lambda_0 = 1496 \pm 4$ s$^{-1}$ → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:
  - $p + p \rightarrow d + e^+ + v_e$
  - $p + p + e^- \rightarrow d + v_e$
  - $p + ^3$He $\rightarrow ^4$He + $e^+ + v_e$
  - $^7$Be + $e^- \rightarrow ^7$Li + $v_e$
  - $^8$B $\rightarrow ^8$Be* + $e^+ + v_e$
Historical remarks

Meson-exchange theory, Skyrme model, phenomenology, …

Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubodera, Riska, Sauer, Friar, Gari, …

First derivation within chiral EFT to leading 1-loop order using TOPT


— only for the threshold kinematics
— pion-pole diagrams ignored
— box-type diagrams neglected
— renormalization incomplete

Leading one-loop expressions using TOPT for general kinematics (still incomplete, e.g. no 1/m corrections)

Pastore, Girlanda, Schiavilla, Goity, Viviani, Wiringa;

Baroni, Girlanda, Pastore, Schiavilla, Viviani;

Complete derivation to leading one-loop order using the method of UT

Kölling, Epelbaum, HK, Meißner;
PRC80 (2009) 045502; PRC84 (2011) 054008

# Vector currents in chiral EFT

Chiral expansion of the electromagnetic current and charge operators

<table>
<thead>
<tr>
<th>Q⁻³</th>
<th>single-nucleon</th>
<th>two-nucleon</th>
<th>three-nucleon</th>
</tr>
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<tbody>
<tr>
<td><img src="Q%E2%81%BB%C2%B3.png" alt="Diagram" /></td>
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<tbody>
<tr>
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</table>

*Up to order Q only single-nucleon current operator does depend on energy-transfer $k_0$*

![Diagram](Q¹.png)

Needed for verification of continuity equation for OPE part

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Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)
Axial vector operators in chiral EFT

Chiral expansion of the axial vector current and charge operators

<table>
<thead>
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<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
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</table>

| Q^{-1} | | | |
|--------|--------|--------|
| ![Diagram](image5) | ![Diagram](image6) | ![Diagram](image7) |

<table>
<thead>
<tr>
<th>Q^0</th>
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<tr>
<th>Q^1</th>
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<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
<td><img src="image13" alt="Diagram" /></td>
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</tbody>
</table>

1/m corrections

Parameter-free static two-pion exchange

Ando, Park, Kubodera, Myhrer (2002)

Bernard, Kaiser, Meißner (HBCHPT)

Parameter-free

depend on \(d_i, d_5, d_6, d_{15}, d_{18}, d_{23}\), no 1/m corrections...

depend on \(C_T\)

depend on \(z_1, z_2, z_3, z_4\); no loop corrections

Park, Min, Rho (earlier works, incomplete, TOPT)
Baroni et al. (TOPT), HK, Epelbaum, Meißner (UT)
Summary

Nuclear Currents  
*Forthcoming review HK, EPJA*

- Vector & axial-vector currents are calculated up to $N^3$LO
- Numerical implementation require symmetry-respecting regularization
  (work in progress)