E97-110
The Generalized GDH Sum Rule for Neutron and $^3$He at Low $Q^2$

Chao Peng (Duke University)
For the Hall A and E97-110 Collaborations
January 24, 2018
Hall A Collaboration Meeting
Outline

• Generalized GDH Sum Rule

• Experiment E97-110

• Progress Report
GDH Sum Rule

• Gerasimov-Drell-Hearn (GDH) Sum Rule

\[ I^{GDH} = \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} (\sigma_P(\nu) - \sigma_A(\nu)) = 4\pi^2 \alpha \frac{\kappa^2}{M^2} S, \]

• Spin \( S \), anomalous magnetic moment \( \kappa \), target mass \( M \), and virtual photon energy \( \nu \)

• Relates the helicity-dependent photoabsorption cross sections to static properties

• Derived from general principles
Generalized GDH sum rule

- Generalized for virtual photon via unsubtracted dispersion relation

\[
I_{TT}(Q^2) = \frac{M^2}{4\pi^2\alpha} \int_{\nu_{th}}^{\infty} \frac{K(\nu, Q^2)\sigma_{TT}(\nu, Q^2)}{\nu^2} d\nu
\]

\[
= \frac{2M^2}{Q^2} \int_{0}^{x_{th}} \left[ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right] dx.
\]

\[
I_1(Q^2) = \frac{2M^2}{Q^2} \int_{0}^{x_{th}} g_1(x, Q^2) dx
\]

\[
= \frac{M^2}{4\pi^2\alpha} \int_{\nu_{th}}^{\infty} \frac{K(\nu, Q^2)}{\nu^2 + Q^2} \left[ \sigma_{TT}(\nu, Q^2) + \frac{Q}{\nu} \sigma_{LT}(\nu, Q^2) \right] d\nu.
\]
First Moment of $g_1$

• First Moment of $g_1$

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2) dx$$

• Connects to the total spin carried by the quarks
• $\approx I_1(Q^2)$ at large $Q^2$

• Bjorken Sum Rule

$$\Gamma_1^P(Q^2) - \Gamma_1^N(Q^2) = \frac{g_A}{6} + O(\alpha_s(Q^2)) + O(\frac{1}{Q^2})$$

• $g_A$, nucleon axial charge
• Consistent with experimental result within 10%
Importance of Generalized GDH Sum Rule

- Recovers the GDH sum rule for real photons ($Q^2 = 0$)
- Connects with Bjorken sum rule ($Q^2 = \infty$)
- Relates the moments of the spin dependent structure functions to virtual Compton amplitude ($Q^2 > 0$)
- Tests the theoretical calculations for the Compton amplitudes at very low $Q^2$
  - Baryon Chiral Perturbation Theory (HBChPT, IRBChPT, RBChPT)
  - Lattice QCD
- Studies the non-perturbative region and the transition region
E94-010 Results

Neutron

Helium-3


E94-010 Results

Heavy Baryon $\chi$PT Calculation
Kao, Spitzenberg, Vanderhaeghen

Relativistic Baryon $\chi$PT
Bernard, Hemmert, Meissner

\[
\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right]
\]

\[
\delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 + g_2 \right]
\]

Need further tests at low $Q^2$
Theoretical developments

\[ \gamma_0^p \left( 10^{-4} \text{fm}^4 \right) \]

\[ Q^2 \left( \text{GeV}^2 \right) \]

\[ \gamma_0^n \left( 10^{-4} \text{fm}^4 \right) \]

\[ Q^2 \left( \text{GeV}^2 \right) \]

Curves:
- MAID (empir.)
- LO-HBChPT
- NLO-HBChPT
- NLO-IRBChPT
- LO-BChPT
- NLO-BChPT
- NLO-BChPT

Data points:
K. Slifer, J.-P. Chen, S. Kuhn, et al [Jefferson Lab spin program]

see talk by H. Krebs
Theoretical developments

Curves:
- MAID (empir.)
- LO-HBChPT
- NLO-HBChPT
- NLO-IRBChPT
  [Bernard et al (2006)]
- LO-BChPT
- NLO-BChPT
  [Lensky, Alarcon & V.P., PRC (2014)]
- NLO-BChPT
  [Bernard et al (2013)]
  see talk by H. Krebs

Data points:
K. Slifer, J.-P. Chen, S. Kuhn, et al
[Jefferson Lab spin program]
E97-110 at Jefferson Lab

- Inclusive measurement, $^3\text{He}(e, e')X$

  - Scattering angles: $6^\circ$ and $9^\circ$
  - Polarized electron beam, $P_{\text{beam}} = 75\%$
  - Polarized $^3\text{He}$ target, $P_{\text{target}} = 40\%$
  - Septum magnet for detection of forward scattering angles

- Measured the differences of polarized cross sections

  - Parallel (anti-parallel)
  - Perpendicular

Spokespersons: J.-P. Chen, A. Deur, F. Garibaldi
Graduate students: J. Singh, V. Sulkosky, J. Yuan, C. Peng, N. Ton
Kinematic Coverage

- **First Period** – lowest $Q^2$
- **Second Period** – higher $Q^2$
Progress Update (V. Sulkosky)

- **Current Students:**
  - Chao Peng (Duke): analysis on the $^3$He moments
  - Nguyen Ton (UVA): first period analysis
- **Finalized** acceptance (V. Sulkosky)
- **Finalized** radiative corrections
  - Preliminary work done by J. Singh
  - Work continued by M. Meziane (Duke) and C. Peng
- **Finalized** estimation of QE contribution to neutron results (V. Sulkosky)
- Constant $Q^2$ interpolation and extraction of neutron moments **in progress**
First Period Analysis (N. Ton)

- Finished optic study
- Analyzed $^3$He elastic cross sections
- Study of the saturation effect in the septum magnet for 2.2 GeV

<table>
<thead>
<tr>
<th>1.1 GeV</th>
<th>$\sigma_{sim}$ [nb]</th>
<th>$\sigma_{data}$ [nb]</th>
<th>Rel. Diff. [%]</th>
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<td>Dp = -2%</td>
<td>187954</td>
<td>191766</td>
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<tr>
<td>Dp = 0%</td>
<td>187138</td>
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<tr>
<td>Dp = 2%</td>
<td>180331</td>
<td>173123</td>
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<table>
<thead>
<tr>
<th>Ebeam (all dp =0%)</th>
<th>$\sigma_{sim}$ (nb)</th>
<th>$\sigma_{data}$ (nb)</th>
<th>Rel. Diff. [%]</th>
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<td>78774</td>
<td>72836</td>
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<td>2.2 GeV</td>
<td>12412</td>
<td>8819</td>
<td>29</td>
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Quasi-elastic subtraction (V. Sulkosky)
Preliminary Neutron Result (V. Sulkosky)

- First moment of $g_1$

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**Graphical Content**

The graphs show data points and curves for different experiments, including.

- **JLab E97110 Preliminary**
- **JLab E94010**
- **SLAC E143**
- **JLab CLAS EG1a**
- **EG1b data+extr.**

Curves for comparison include:

- **GDH slope**
- **Ji et al., HBXpt**
- **Bernard et al., Xpt**
- **Lensky et al., Xpt**

The data is plotted on a log-log scale for $Q^2(\text{GeV}^2)$ against the $g_1$ value.
Neutron Spin Polarizabilities (V. Sulkosky)
Second Period $^3$He Analysis

- **Finished**
  - Elastic tail subtraction
  - Radiative correction
  - Interpolation on constant $Q^2$
  - Associated systematics

- **Ongoing**
  - Generalized GDH integral
  - Parameterization for the unmeasured high energy contribution
Structure functions at constant beam energy

$E = 1147 \text{ MeV}$

$g_1$: Solid circle and red
$g_2$: Open circle and blue

Preliminary
Interpolation to constant $Q^2$

\[ Q^2 = 0.032 \sim 0.23 \text{ GeV}^2 \text{ (Second Period)} \]
Structure functions at constant $Q^2$

$g_1$

$g_2$

$Q^2 = 0.032 \text{ GeV}^2$

Preliminary

$v$ (MeV)
Summary

• First period analysis
  • Progress on the optic study
  • Cross section analysis is ongoing
  • Publication forseen in 2018-2019

• Second period analysis
  • Neutron result, finalizing systematics
  • $^3$He result, obtained spin structure functions, getting generalized GDH sum rule and $g_1$, $g_2$ moments
  • Neutron data soon to be published (several weeks)
  • $^3$He data to be published in Spring of 2018
Thank you

The work is supported in part by US DOE grant DE-FG02-03ER41231
Second Period Systematics (V. Sulkosky)

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<th>Source</th>
<th>$\sigma_{syst}$ [%]</th>
<th>$\sigma_{syst}$ [%]</th>
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<td></td>
<td>Preliminary</td>
<td>Final</td>
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<td>Target density</td>
<td>2</td>
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<td>VDC Multi-tracks</td>
<td>2.5 – 3</td>
<td>&lt; 1</td>
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<td>Yield Stability $\nu$-dependent</td>
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<td>Acceptance</td>
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<td>2 – 3</td>
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<td>Beam polarization</td>
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<tr>
<td>Target Polarization</td>
<td>7.5</td>
<td>3 – 5.2</td>
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Second Period Systematics (V. Sulkosky)

**Other systematics:**

1. Radiative corrections, including elastic tail subtraction.
3. Interpolation and extrapolation to constant $Q^2$.
4. Neutron extraction from $^3$He: 5% to 10%.
## Experimental progress

<table>
<thead>
<tr>
<th>Observable</th>
<th>H target</th>
<th>D target</th>
<th>$^3$He target</th>
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<tbody>
<tr>
<td>$g_1, g_2, \Gamma_1 &amp; \Gamma_2$ at high $Q^2$</td>
<td>SLAC</td>
<td>SLAC</td>
<td>SLAC</td>
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<tr>
<td></td>
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<td>JLAB E06-014</td>
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<td>$g_1 &amp; \Gamma_1$ at high $Q^2$</td>
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<td>SMC</td>
<td>HERMES</td>
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<td>$\Gamma_1 &amp; \Gamma_2$ at low $Q^2$</td>
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<td>JLab E94-010</td>
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<td>JLab E97-103</td>
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<tr>
<td>$\Gamma_1$ at low $Q^2$</td>
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<td>$\Gamma_2, Q^2 &lt;&lt; 1 \text{ GeV}^2$</td>
<td>JLab E08-027</td>
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<td>JLab E97-110</td>
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GDH Measurements

- Proton, verified: Mainz, Bonn, LEGS
- Neutron, in progress: Mainz, Bonn, LEGS, HIGS
- Measurements on Deuteron and $^3\text{He}$

<table>
<thead>
<tr>
<th></th>
<th>$M,[\text{GeV}]$</th>
<th>Spin</th>
<th>$\kappa$</th>
<th>$I_{\text{GDH}},[\mu \text{b}]$</th>
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<tbody>
<tr>
<td>Proton</td>
<td>0.938</td>
<td>$\frac{1}{2}$</td>
<td>1.79</td>
<td>−204.8</td>
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<tr>
<td>Neutron</td>
<td>0.940</td>
<td>$\frac{1}{2}$</td>
<td>−1.91</td>
<td>−233.2</td>
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<td>Deuteron</td>
<td>1.876</td>
<td>1</td>
<td>−0.14</td>
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<tr>
<td>Helium-3</td>
<td>2.809</td>
<td>$\frac{1}{2}$</td>
<td>−8.38</td>
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