Radiative transitions between $0^{-+}$ and $1^{--}$ heavy quarkonia on the light front

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LightCone 2018
Thomas Jefferson National Accelerator Facility,
Newport News, VA, May 17th, 2018
Basis Light-Front Quantization (BLFQ): A non-perturbative approach to solve QFT. [J.P. Vary, et al., PRC 81, 2010]

- Light-front (LF) Hamiltonian formalism is a natural framework for tackling relativistic bound-state problems in QCD.
  - The quantum field is quantized on light-front time $x^+ = x^0 + x^3$.
  - LF energy $P^-$, LF 3-momentum $(P^+, P^x, P^y)$, where $P^\pm = P^0 \pm P^3$.
  - Dispersion relation $P^- = (m^2 + \vec{P}_\perp^2)/P^+$

- By solving the eigenvalue equation, it directly produces the invariant masses and the boost invariant wavefunctions:

$$ (P^+ \hat{P}^- - \vec{P}_\perp^2) |\psi_h(P, j, m_j)\rangle = M_h^2 |\psi_h(P, j, m_j)\rangle $$

- Basis representation
  - Basis can encode the analytical approximation to the solution.
  - Optimal basis is the key to numerical efficiency.
The effective Hamiltonian at the $|q\bar{q}\rangle$ Fock sector:

$$H_{\text{eff}} = H_0 + \kappa^4 x(1-x)\bar{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \frac{\partial}{\partial x} \left( x(1-x) \frac{\partial}{\partial x} \right) + V_g,$$

where $x = p_q^+ / P^+$, $\vec{k}_\perp = \vec{p}_q - x\vec{P}$, $\vec{r}_\perp = \vec{r}_q - \vec{r}_{\bar{q}}$.

- Confinement
  - transverse holographic confinement [S.J.Brodsky, et al., PR584, 2015]
  - longitudinal confinement [Y.Li, et al., PLB758, 2016]
- One-gluon exchange with running coupling
  $$V_g = -\frac{4}{3} \frac{4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{\sigma'} \gamma^\mu u_{\sigma} \bar{v}_s \gamma_\mu v_{s'},$$
- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of $H_0$
    $$\phi_{nm}(\vec{k}_\perp / \sqrt{x(1-x)}), \chi_l(x),$$
    with basis truncation: $2n + |m| + 1 \leq N_{\text{max}}, 0 \leq l \leq L_{\text{max}}.$
Heavy Quarkonia [Y.Li, et al., PLB758, 2016; PRD96, 2017]

Light-front wavefunctions (LFWFs): e.g. $\eta_c(1S)$

Mass spectra:
Radiative transitions

The electromagnetic transition between quarkonium states via emission of a photon, offers an insight into the internal structure of quark-antiquark bound states.

\[ \psi_i \rightarrow \psi_f + \gamma \]

Each process is governed by its hadron matrix,

\[ \langle \psi_f | J^\mu(0) | \psi_i \rangle , \]

\( J^\mu(x) \) is the current operator.

Radiative transitions characterized by multipoles. (Figure by Y.Li.)
The M1 transition

\[ \gamma(-q, \lambda) \]

\[ \mathcal{V}(P, m_j) \]

\[ \mathcal{P}(P') \]

**vector(1--)** $\leftrightarrow$ **pseudoscalar (0--)**

spin $J : 1 \leftrightarrow 0$

parity $P : -1 \leftrightarrow -1$

charge conjugation $C : -1 \leftrightarrow +1$

$\mathcal{V} \rightarrow \mathcal{P} + \gamma$ or $\mathcal{P} \rightarrow \mathcal{V} + \gamma$

**Transition form factor:**

\[ I_{m_j}^\mu(P, P') \equiv \langle \mathcal{P}(P') | J^\mu | \mathcal{V}(P, m_j) \rangle = \frac{2V(Q^2)}{m_P + m_V} \epsilon^{\mu\alpha\beta\sigma} P'_\alpha P_\beta e_\sigma (P, m_j) \]

**Decay width:**

\[ \Gamma(\mathcal{V} \rightarrow \mathcal{P} + \gamma) = \frac{(m_V^2 - m_P^2)^3}{(2m_V)^3(m_P + m_V)^2} \frac{|V(0)|^2}{(2J_V + 1)\pi} \]

where $q = P' - P$, $Q^2 \equiv -q^2$ and $e_\sigma (P, m_j)$ is the polarization vector of the vector meson. $J_V = 1$ is the spin of the initial vector meson.
Calculation on the light front

Current components: $J^\mu (\mu = +, -, x, y)$

$J^+: "good current"$ (suppress contributions from pair production/annihilation in vacuum)

$J^-: "bad current"$ (associate with the zero-mode contributions)

$J^\perp (J^R \equiv J^x + iJ^y)$: good/bad?

Calculation on the light front

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\( J^\perp (J^R \equiv J^x + iJ^y) \): good/bad?


\[
I^+_{m_j} = \frac{2V(Q^2)}{m_\rho + m_\nu} \begin{cases} 
0, & m_j = 0 \\
\frac{i}{\sqrt{2}} \frac{P^+ \Delta^R}{}, & m_j = 1 \\
-\frac{i}{\sqrt{2}} P^+ \Delta^L, & m_j = -1 
\end{cases}
\]

\[
I^R_{m_j} = \frac{2V(Q^2)}{m_\rho + m_\nu} \begin{cases} 
-\frac{im_\nu \Delta^R}{}, & m_j = 0 \\
\frac{i}{\sqrt{2}} \frac{P^R \Delta^R}{}, & m_j = 1 \\
\frac{i}{\sqrt{2z}} (z^2 m_\nu^2 - m_\rho^2 - P'^R \Delta^L), & m_j = -1 
\end{cases}
\]

Boost invariants:
\( z \equiv P'^+ / P^+ \)
\( \tilde{\Delta}_\perp \equiv \vec{P}'_\perp - z\vec{P}_\perp \).

Complex forms:
\( k^{R/L} \equiv k^x \pm ik^y \).
Light-front wavefunction representation in $|q\bar{q}\rangle$

Impulse approximation: $V(Q^2) = 2eQ_f \hat{V}(Q^2)$, $Q_f$ is the quark charge.

The hadron matrix element in the Drell-Yan frame ($q^+ = 0$) is,

$$
\langle \mathcal{P}(P') | J_\mu^q(0) | \mathcal{V}(P, m_j) \rangle
= \sum_{s, \bar{s}, s', \bar{s}'} \int_0^1 \frac{dx}{2x^2(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_{s\bar{s}/\mathcal{V}}^{(m_j)}(k_\perp, x) \psi^*_{s'\bar{s}',\mathcal{P}}(k_\perp + (1-x)q_\perp, x) \nonumber
\bar{u}_{s'}(xP^+, k_\perp + xP_\perp + \bar{q_\perp}) \gamma^\mu u_s(xP^+, k_\perp + xP_\perp).$$

Convolutions of LFWFs: $\psi_{s\bar{s}/\mathcal{V}}^{(m_j)}, \psi_{s'\bar{s}',\mathcal{P}}.$
Transition form factor at $|q\bar{q}\rangle$

$\hat{V}(Q^2)$ as convolutions of LFWFs: $\psi_{s\bar{s}/V}^{(m_j)}(\vec{k}_\perp, x)\psi^*_{s'\bar{s}'/P}(\vec{k}_\perp + (1 - x)\vec{q}_\perp, x)$.

- $J^+$ and $m_j = 1$
  - $m_j = 0$ is not available, $m_j = -1$ is equivalent to $m_j = 1$.

$$\hat{V}_{m_j=1}(Q^2) = \frac{\sqrt{2}(m_P + m_V)}{iqR} \int_0^1 \frac{dx}{2x(1 - x)} \int \frac{d^2\vec{k}_\perp}{(2\pi)^3} \left[ \psi^{(m_j=1)}_{\uparrow\uparrow/V} \psi^*_{\uparrow\uparrow/P} + \psi^{(m_j=1)}_{\uparrow\downarrow-\downarrow/\chi} \psi^*_{\uparrow\uparrow\downarrow/\chi} + \psi^{(m_j=1)}_{\uparrow\downarrow\uparrow/\chi} \psi^*_{\uparrow\downarrow\uparrow/P} + \psi^{(m_j=1)}_{\downarrow\downarrow/V} \psi^*_{\downarrow\downarrow/P} \right]$$

- $J^R$ and $m_j = 0$
  - $m_j = \pm 1$ can be related to that in $J^+$ through a transverse Lorentz boost.

$$\hat{V}_{m_j=0}(Q^2) = \frac{i(m_P + m_V)}{m_V} \int_0^1 \frac{dx}{2x^2(1 - x)} \int \frac{d^2\vec{k}_\perp}{(2\pi)^3} \left[ -\frac{1}{2} \psi^{(m_j=0)}_{\uparrow\downarrow\uparrow\downarrow/\chi} \psi^*_{\uparrow\downarrow\uparrow/\chi} + \psi^{(m_j=0)}_{\downarrow/\chi} \psi^*_{\downarrow/\chi/P} \right]$$

Dominant spin components: exist in the nonrelativistic limit
Subdominant spin components: relativistic origin
Transition form factor at $|q\bar{q}\rangle$

For the less relativistic heavy systems:

$\hat{V}_{m_j=0}(Q^2)$: depends on dominant components $\rightarrow$ more robust! ($\hat{V}(Q^2)$)

$\hat{V}_{m_j=1}(Q^2)$: relies on subdominant components

dominant components and subdominant components of charmonium and bottomonium:

![Diagrams](a) $\mathcal{P}(nS)$
(b) $\mathcal{V}^{(m_j=0)}(nS)$
(c) $\mathcal{V}^{(m_j=1)}(nS)$
Transition form factor 1

The allowed transition: initial and final states have the same radial or angular quantum numbers.

\[ nS \rightarrow nS + \gamma \]

Nonrelativistic limit: \( \hat{V}(0) \rightarrow 2. \)

Shaded area: numerical uncertainty from basis truncation.
Transition form factor II

The *hindered* transition: initial and final states have different radial or angular quantum numbers.

\[ nS \rightarrow n'S + \gamma(n \neq n'), \quad nD \rightarrow n'S + \gamma \]

Nonrelativistic limit: \( \hat{V}(0) \rightarrow 0 \).

Shaded area: numerical uncertainty from basis truncation.
Transition form factor III

The *hindered* transition: initial and final states have different radial or angular quantum numbers.

\[ nS \rightarrow n'S + \gamma(n \neq n'), \quad nD \rightarrow n'S + \gamma \]

Nonrelativistic limit: \( \hat{V}(0) \rightarrow 0 \).

Shaded area: numerical uncertainty from basis truncation.
Transitions between vector and pseudoscalar mesons for charmonia and bottomonia below their open flavor threshold. (#21)

The diagram shows various transitions involving mesons and their decay channels. The PDG, Lattice, relativistic quark model (rQM), and Godfrey-Isgur model (GI model) are referenced.

PDG: [C. Patrignani, et al., CPC40, 2016]
relativistic quark model (rQM): [D. Ebert, et al., PRD67, 2003]
Decay constant of the vector meson $f_V$

\[ \langle 0 | J^\mu(0) | V(P, m_j) \rangle = e^\mu(P, m_j) M_V f_V \]

Integrals of LFWFs:

\[ f_V \mid_{J^+, J_R, m_j=0} : \psi^{(m_j=0)} \uparrow\downarrow+\downarrow\uparrow/V. \]

\[ f_V \mid_{J_R, m_j=1} : \psi^{(m_j=1)} \uparrow\uparrow/V, \psi^{(m_j=1)} \uparrow\downarrow+\uparrow\downarrow/V, \psi^{(m_j=1)} \uparrow\downarrow-\downarrow\uparrow/V. \]

Dominant spin components for the S-wave states.
Decay constant of the vector meson $f_V$

$$\langle 0| J^\mu(0) | V(P, m_j) \rangle = e^\mu(P, m_j) M_V f_V$$

Integrals of LFWFs:

$$f_V|_{J^+/JR, m_j=0} : \psi^{(m_j=0)}_{\uparrow\downarrow+\downarrow\uparrow/V} \cdot \quad f_V|_{JR, m_j=1} : \psi^{(m_j=1)}_{\uparrow\uparrow/V}, \psi^{(m_j=1)}_{\downarrow\downarrow+\uparrow/V}, \psi^{(m_j=1)}_{\downarrow\uparrow-\downarrow\uparrow/V}.$$  

Dominant spin components for the S-wave states.

- Lorentz symmetry is reasonably preserved.
- $\hat{V}(Q^2)_{m_j=0}$ using the dominant components is reliable.
Summary and outlook

- The radiative transitions between $0^{-+} (\mathcal{P})$ and $1^{--+} (\mathcal{V})$ heavy quarkonia is calculated from the LFWFs, providing predictions.
- Comparison of different components of current operator ($J^+$ and $J^\perp$) provides insights on light-front dynamics.
  - $J^\perp$ is preferred to $J^+$ for the M1 transition in heavy systems
  - $J^+$ and $J^\perp$ agree on the decay constants of the vector mesons.
- Other radiative transitions: $\mathcal{V} \to S + \gamma$, $\mathcal{V} \to A + \gamma$
- EM Dalitz decay: $\mathcal{V} \to \mathcal{P} + \gamma^* \to \mathcal{P} + e^+ + e^-$.

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- $J^\perp$ is preferred to $J^+$ for the M1 transition in heavy systems
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Thank you very much!
Acknowledgements

- Co-authors
  Yang Li, Pieter Maris, James P. Vary (my supervisor).

- People from our group
  Shaoyang Jia, Wenyang Qian, Shuo Tang, Anji Yu, Lekha Adhikari.

- Others
  Guangyao Chen, Xingbo Zhao, V.A. Karmanov, Sofia Leitão, Wayne N. Polyzou.

This work was supported in part by the US Department of Energy (DOE) under Grant Nos. DE-FG02-87ER40371, DE-SC0018223 (SciDAC-4/NUCLEI), DE-SC0015376 (DOE Topical Collaboration in Nuclear Theory for Double-Beta Decay and Fundamental Symmetries) and DE-FG02-04ER41302.
The transition form factor $\hat{V}(Q^2)$ extracted from the quark current is related to $V(Q^2)$ as,

$$V(Q^2) = 2eQ_f \hat{V}(Q^2)$$

Quark charge: $Q_f = Q_c = +2/3$, $Q_f = Q_b = -1/3$. 
Backup: More on the spin components

dominant components and subdominant components of D-wave states:

\[
\psi(\pi) \quad \Upsilon(\pi) \quad \Upsilon(\pi)
\]

\[
\psi(\pi) \quad \Upsilon(\pi) \quad \Upsilon(\pi)
\]

\[
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\]

\[
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\]

Dominant components: exist in the nonrelativistic limit
Subdominant components: relativistic origin
Backup: Decay constant of the vector meson

The decay constant $f_V$ is defined from the vacuum-to-hadron matrix elements:

$$\langle 0 | J^\mu(0) | \mathcal{V}(P, m_j) \rangle = \epsilon^\mu(P, m_j) M_\mathcal{V} f_\mathcal{V}$$

- Same current operator as for the transition form factor.
- Help check Lorentz symmetry.

$$f_\mathcal{V}(m_j = 0): f_\mathcal{V}^+(m_j = 0) = f_\mathcal{V}^\perp(m_j = 0)$$

$$f_\mathcal{V}(m_j = \pm 1): f_\mathcal{V}^+(m_j = \pm 1)\text{ (not available), } f_\mathcal{V}^\perp(m_j = \pm 1)$$

The light-front wavefunctions representation,

$$f_\mathcal{V}(m_j = 0) = \sqrt{2N_c} \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_{\uparrow\downarrow+\uparrow\uparrow/\mathcal{V}}(\mathbf{k}_\perp, x) \cdot$$

$$f_\mathcal{V}(m_j = 1) = \frac{\sqrt{N_c}}{2m_\mathcal{V}} \int_0^1 \frac{dx}{[x(1-x)]^{3/2}} \int \frac{d^2k_\perp}{(2\pi)^3} [k^L (1 - 2x) \psi_{\uparrow\downarrow+\uparrow\uparrow/\mathcal{V}}(\mathbf{k}_\perp, x)$$

$$- k^L \psi_{\downarrow\uparrow-\downarrow\uparrow/\mathcal{V}}(\mathbf{k}_\perp, x) + \sqrt{2m_q} \psi_{\uparrow\uparrow/\mathcal{V}}(\mathbf{k}_\perp, x)] \cdot$$

Dominant spin configurations for S-wave states.
The two sets of hadron matrix elements \( \langle \mathcal{P}(P') | J^+ | \mathcal{V}(P, m_j) \rangle \) and \( \langle \mathcal{P}(P') | \vec{J}_\perp | \mathcal{V}(P, m_j) \rangle \) can be related through the transverse Lorentz boost specified by the velocity vector \( \vec{\beta}_\perp \),

\[
v^+ \rightarrow v^+, \quad \vec{v}_\perp \rightarrow \vec{v}_\perp + v^+ \vec{\beta}_\perp.
\]

That is,

\[
\langle \mathcal{P}(P'^+', \vec{P}'_\perp + P'^+ \vec{\beta}_\perp) | \vec{J}_\perp | \mathcal{V}(P^+, \vec{P}_\perp + P^+ \vec{\beta}_\perp, m_j) \rangle \\
= \langle \mathcal{P}(P'^+, \vec{P}'_\perp) | \vec{J}_\perp | \mathcal{V}(P^+, \vec{P}_\perp, m_j) \rangle + \vec{\beta}_\perp \langle \mathcal{P}(P'^+, \vec{P}'_\perp) | J^+ | \mathcal{V}(P^+, \vec{P}_\perp, m_j) \rangle.
\]

In consequence, \( V(Q^2) \) obtained using \( J^+ \) and \( \vec{J}_\perp \) with the same \( m_j \) should be the same.
Backup: $\hat{V}(0)$

Ratio of $\hat{V}_{m_j=1}(0)$ (using $J^+$) to $\hat{V}_{m_j=0}(0)$ (using $J^R$).

▲ Allowed transition: the transition with the same radial or angular quantum numbers (e.g. $nS \rightarrow n' S + \gamma$)

◇ Hindered transition: the transition between states with different radial or angular excitations. (e.g. $nS \rightarrow n' S + \gamma$ and $nD \rightarrow n' S + \gamma$)