Electron Scattering from Deeply Bound Nucleon on the Light-Front

Frank Vera

FIU

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In collaboration with M. Sargsian

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Why a description of electromagnetic scattering from deeply bound nucleons?

Experiments at quasi-elastic kinematics established signatures that deeply bound nucleons emerge from short range nucleon-nucleon correlations.

Study the structure of nuclei at short distance.

Use of nucleons as the main degrees of freedom

Study high momentum components of the bound nucleon

\[ \frac{m_N}{2} \lesssim p_i \simeq m_N \quad \rightarrow \quad Q^2 \sim \text{a few GeV} \quad \rightarrow \quad p_f \sim \text{a few GeV} \]
Introduction

Simplest case: Exclusive Electro-Disintegration of Deuteron

\[ e + d \rightarrow e' + N_f + N_r \]  \hspace{1cm} (1)

\( N_f \): ejected nucleon
\( N_r \): recoil nucleon

\( q \): virtual photon momentum, \( q^2 < 0 \)

Within the Plane Wave Impulse Approx. (PWIA):

Easy and general enough to extend to bigger nuclei
Feynman amplitude:

\[ M = \langle \lambda_f | j^\nu_e | \lambda_i \rangle \frac{e^2 g_{\nu \mu}}{q^2} \langle s_f, s_r | A_\mu^o | s_d \rangle, \]  

(2)

with, \( \gamma^* d \rightarrow NN \) scattering amplitude:

\[ A_\mu^o = \langle s_f, s_r | A_\mu^o | s_d \rangle = -\bar{u}(p_f, s_f) \Gamma_{\gamma^*}^{\mu} \frac{p_i^\mu + m_N}{p_i^2 - m_N^2} \bar{u}(p_r, s_r) \Gamma_D \chi^{s^d} \]  

(3)

Extracting electro-bound nucleon Cross Sect.  \implies \ Time ordered amplitude:

\[ p_i \gtrsim m_N/2 \ (Q \gtrsim 2m_N) \rightarrow \text{"Z-graph" can not be ignored} \]
Set Up

Light Front $\tau$-time ordered scattering amplitude:

LF coordinates: $(x^+, x^-, x, y)$, $x^\pm = t \pm z$, $x^+ = \tau$

LF momenta: $(p^+, p^-, p_x, p_y)$, $p^\pm = E \pm p_z$

No "Z-Graph" in a collinear-frame where $\hat{z}$ is opposite to $q$
Nuclear Amplitude

LFPT diagrammatic rules produces two contributions (Kogut and Soper 1970) (Lepage and Brodsky 1980)

Propagating:

\[ A_{\text{prop}}^\mu = -\bar{u}(p_f, s_f)\Gamma_{\gamma^*N}^\mu \left( \frac{1}{p_i^+} \frac{(p_i^+ + m_N)_{on}}{(p_d^- - p_r^- - p_{i, on}^-)} \right) \bar{u}(p_r, s_r) \Gamma_{DNN} \chi^{sd} \]  

(4)

Instantaneous:

\[ A_{\text{inst}}^\mu = -\bar{u}(p_f, s_f)\Gamma_{\gamma^*N}^\mu \left( \frac{1}{2 \gamma^+} \right) \bar{u}(p_r, s_r) \Gamma_{DNN} \chi^{sd} \]  

(5)

Factorisation of the amplitude:

\[ A^\mu = A_{\text{prop}}^\mu + A_{\text{inst}}^\mu = \sum_{s_i} J_N^\mu (p_f s_f, p_i s_i) \frac{\psi_{s_i s_r s_d}^{s_i s_r s_d}(\alpha, p_T)}{\alpha} \sqrt{2 (2\pi)^3} \]
\[ J_N^\mu = J_{(\text{prop})}^\mu + J_{(\text{inst})}^\mu \] (6)

where,

\[ J_{(\text{prop})}^\mu = \bar{u}(p_f, s_f) \left( \gamma^\mu F_1 + i\sigma^{\mu\nu} q_\nu F_2 \frac{\kappa}{2m_N} \right) u(p_i, s_i) \] (7)

\[ J_{(\text{inst})}^\mu = \bar{u}(p_f, s_f) \Gamma_{\gamma^*N}^{(\text{inst})\mu} u(p_i, s_i) \] (8)

\[ \Gamma_{\gamma^*N}^{(\text{inst})\mu} = \left( \gamma^\mu F_1 + i\sigma^{\mu\nu} q_\nu F_2 \frac{\kappa}{2m_N} \right) \frac{\Delta p_i^\mu}{2m_N} - F_1 \frac{q^\mu}{q^2} q \left( 1 + \frac{\Delta p_i}{2m_N} \right) \] (9)

dynamic off-shell factor: \[ \Delta p_i^\mu = p^\mu_d - p^\mu_r - p^\mu_{i,on} = p^\mu_i - p^\mu_{i,on} \]
\[ 2\Delta p_i^\mu = \gamma^+ (p_i^- - p_i^-_{i,on}) \]

\( J_{(\text{prop})}^\mu \) not an on-shell current, \( q^\mu \neq p^\mu_f - p^\mu_{i,on} \)
Structure Functions

Electro Nucleon Cross Section in terms of Structure Functions

$$\sigma_{eN} = \frac{1}{2p_dp_i} \sigma_{Mott} \frac{k_i}{E_f} \left( \eta_L V_L^N + \eta_T L V_T^N \cos \phi + \eta_T V_T^N + \eta_T T V_{TT}^N \cos(2\phi) \right)$$

Nucleonic EM tensor in LF components

$$V_L^N = \frac{q_V^4}{4} \left( H_N^{++} \frac{1}{q^+ q^+} + \frac{2}{Q^2} H_N^{+-} + \frac{q^+ q^+}{Q^4} H_N^{--} \right)$$

$$V_{TL}^N = \frac{q_V}{Q^2} \left( H_N^{||} q^- - H_N^{\perp\perp} q^+ \right)$$

$$V_T^N = H_N^{||\perp\perp}$$

$$V_{TT}^N = H_N^{||\perp\perp} - H_N^{\perp\perp\perp}$$
Structure Functions

\[ V_{L \text{ prop}}^N = q^2 \left[ F_1^2 \tau^{-1} \left( 1 + \frac{p_T^2}{m_N^2} + \tau \eta_i (\eta_i + \eta_q) \right) - F_1 F_2 \kappa \left( 2 + \eta_q \right) + F_2^2 \kappa^2 \left( \frac{p_T^2}{m_N^2} + \tau (1 + \eta_q) \right) \right] \]

\[ V_{L \text{ inst}}^N = q^2 \left[ F_1^2 \eta_i \left( \tau \eta_i (1 + \eta_q) - 2 - \eta_q \right) + F_1 F_2 \kappa \left( \tau \eta_i (2 - 2 \eta_i - \eta_q) + \eta_q \right) + F_2^2 \kappa^2 \tau \left( \tau \eta_i (\eta_i + \eta_q) - \eta_q \right) \right] \]

\[ V_{TL \text{ prop}}^N = 2 |q| p_T \left( F_1^2 + F_2^2 \kappa^2 \tau \right) \left[ 2 \eta_q + 2 \eta_i + \eta_q \right] \]

\[ V_{TL \text{ inst}}^N = 2 |q| p_T \left( F_1^2 + F_2^2 \kappa^2 \tau \right) (1 - \tau \eta_i) \eta_q \]

\[ V_{T \text{ prop}}^N = 4m_N^2 \left[ F_1^2 \left( \frac{p_T^2}{m_N^2} + 2 \tau (1 + \eta_q) \right) + 2 F_1 F_2 \kappa \tau \left( 2 + \eta_q \right) + F_2^2 \kappa^2 \tau \left( 2 + \frac{p_T^2}{m_N^2} + 2 \tau \eta_i (\eta_i + \eta_q) \right) \right] \]

\[ V_{T \text{ inst}}^N = 2 Q^2 \left[ F_1^2 \left( \tau \eta_i (\eta_i + \eta_q) - \eta_q \right) + F_1 F_2 \kappa \left( \tau \eta_i (2 \eta_i + \eta_q - 2) - \eta_q \right) + F_2^2 \kappa^2 \tau \eta_i \left( \tau \eta_i (1 + \eta_q) - 2 - \eta_q \right) \right] \]

\[ V_{TT \text{ prop}}^N = 4p_T^2 \left( F_1^2 + F_2^2 \kappa^2 \tau \right) \]

\[ V_{TT \text{ inst}}^N = 0 \]

with,

\[ \eta = \frac{1}{Q^2} \left( 4 \frac{(m_N^2 + p_T^2)}{\alpha (2 - \alpha)} - m_d^2 \right), \text{ a universal parameter controlling off-shell effects} \]

and,

\[ \tau = Q^2 / (4m_N^2), \quad \eta_i = \eta \alpha_N / 2, \quad \eta_q = \eta \alpha_q / 2 \]
Different off-shell Scattering Cross Sections lead to significantly different results (specially for deeply bound nucleons).

Comparison with the widely used de Forest off-shell extrapolations:

**Figure 1**: Angular distribution. \[ R = \frac{\sigma_{eN}}{\sigma_{eN}^{\text{free}}} \]
Results

Comparison with the widely used de Forest off-shell extrapolations:

![Graph showing $Q^2$ distribution](image)

**Figure 2:** $Q^2$ distribution. \( R = \frac{\sigma_{eN}}{\sigma_{eN}^{free}} \)

Increasing the photon’s virtuality reduces the off-shell effects.
Results

Kinematics for E01-020 (Boeglin et al., arXiv:1410.6770)

Figure 3: $p_i$ distribution. $R = \frac{\sigma_{eN}}{\sigma_{eN}^{free}}$
For wide range of kinematics, $\eta < 0.1 \implies \text{off-shellness} \lesssim 5\%$

![Graphs showing the distribution of $|R - 1|$ for protons and neutrons with different momenta and light-front parameters.]

**Figure 4:** $\eta = \frac{1}{Q^2} \left(4 \frac{m_N^2 + p_T^2}{\alpha (2 - \alpha)} - m_d^2\right)$

Effective method for controlling the uncertainties in the reaction mechanism.
Summary

Electron–bound-nucleon cross section calculation based on Light Front Perturbation Theory.

Identification of parameter ($\eta$) that universally characterizes the off-shell extend of the electromagnetic current.

$\eta$ can be used by experimentalists to suppress or isolate the off shell effects for dedicated studies.

Results are more general than just PWIA: The EM current is applicable to FSI within eikonal approximation.
Thank You