Kalman Filter Tracking for RTPC12

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Feb 22nd, 2018
TDIS Collaboration Meeting

Outline

1. RTPC12 Design

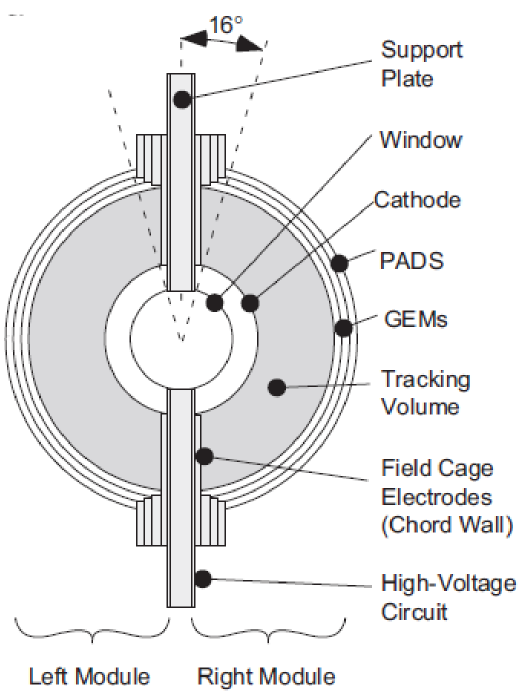
2. Chain Finder

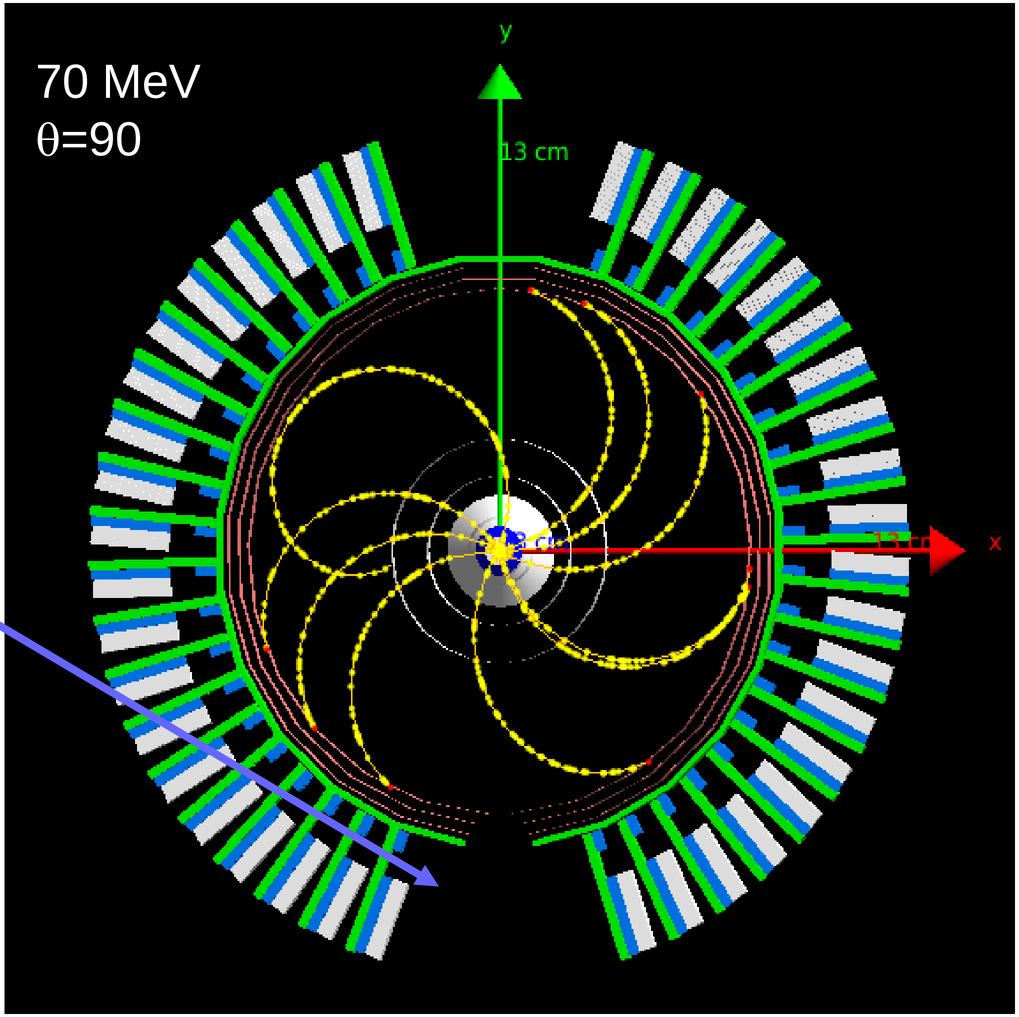
3. Kalman Filter

4. Summary

RTPC12 Design

Target: D2 gas, 293k, 7.5 ATM, 40 cm long
Target Wall: **28** um kapton, 3 mm radius
Drift Region: 3<R<7 cm
Drift Gas: 293k, 1 ATM, Argon/CO_2 (70/30)
Readout pad at R=8 cm
Pad size 2.79 (tran.) x 4 mm (z),
18000 pads in total
phi coverage = 360 degrees,
NO phi acceptance loss here

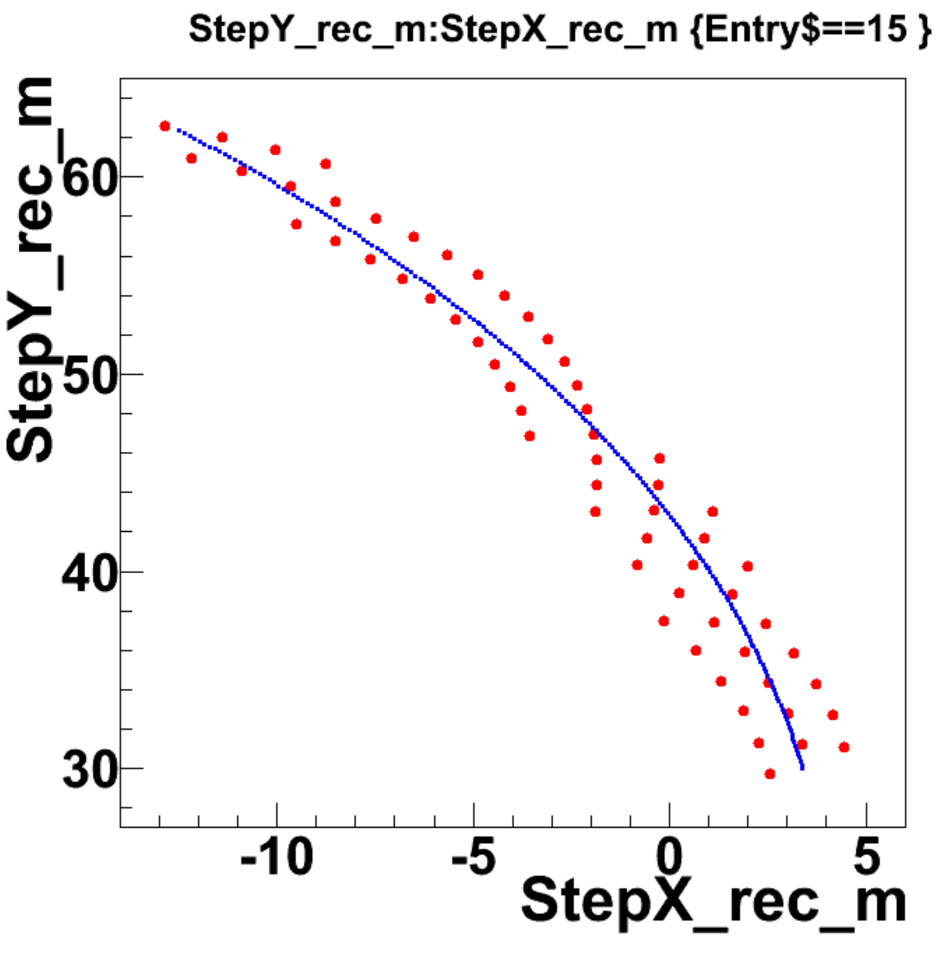




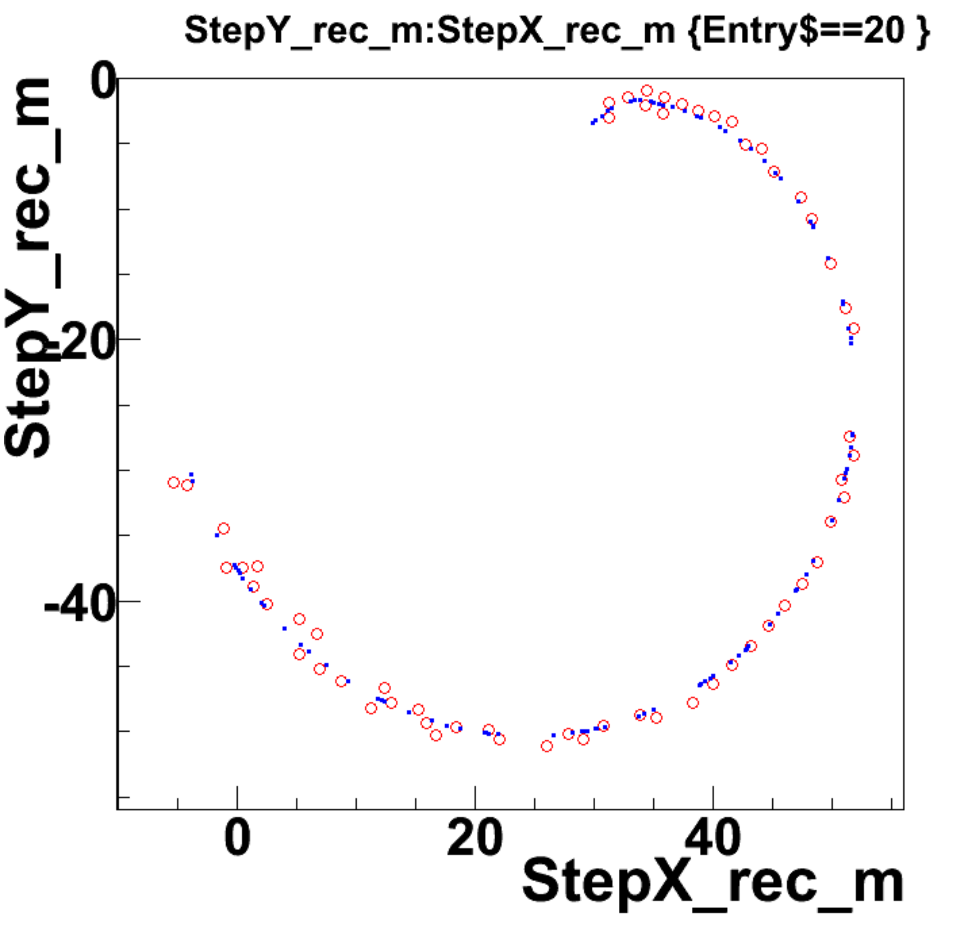
Use CLAS12 Solenoid with -5T field (pointing upstream)

Tracking of RTPC12

1. Track finder
2. fitter: global helix fitter and Kalman Filter



Step limit = 0.6 mm
Forward track

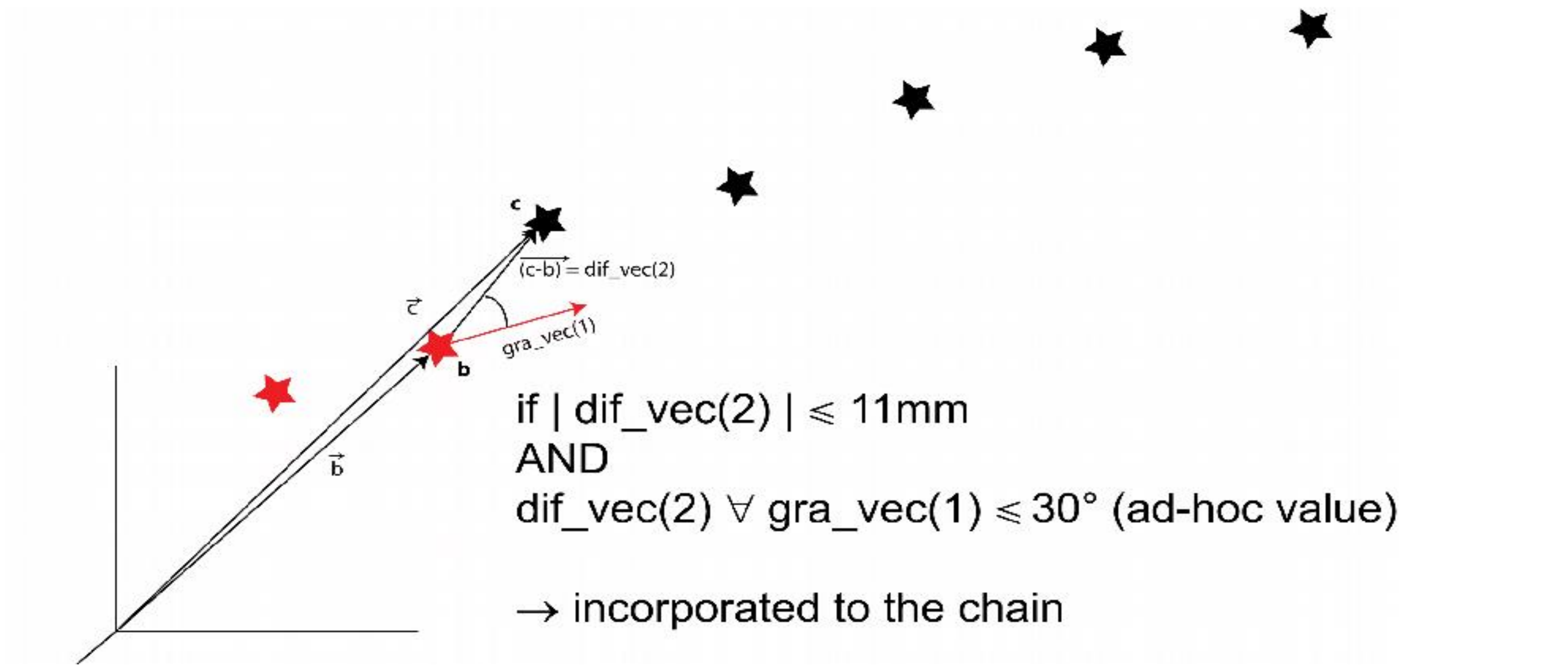


No step limit,
Curve-back track

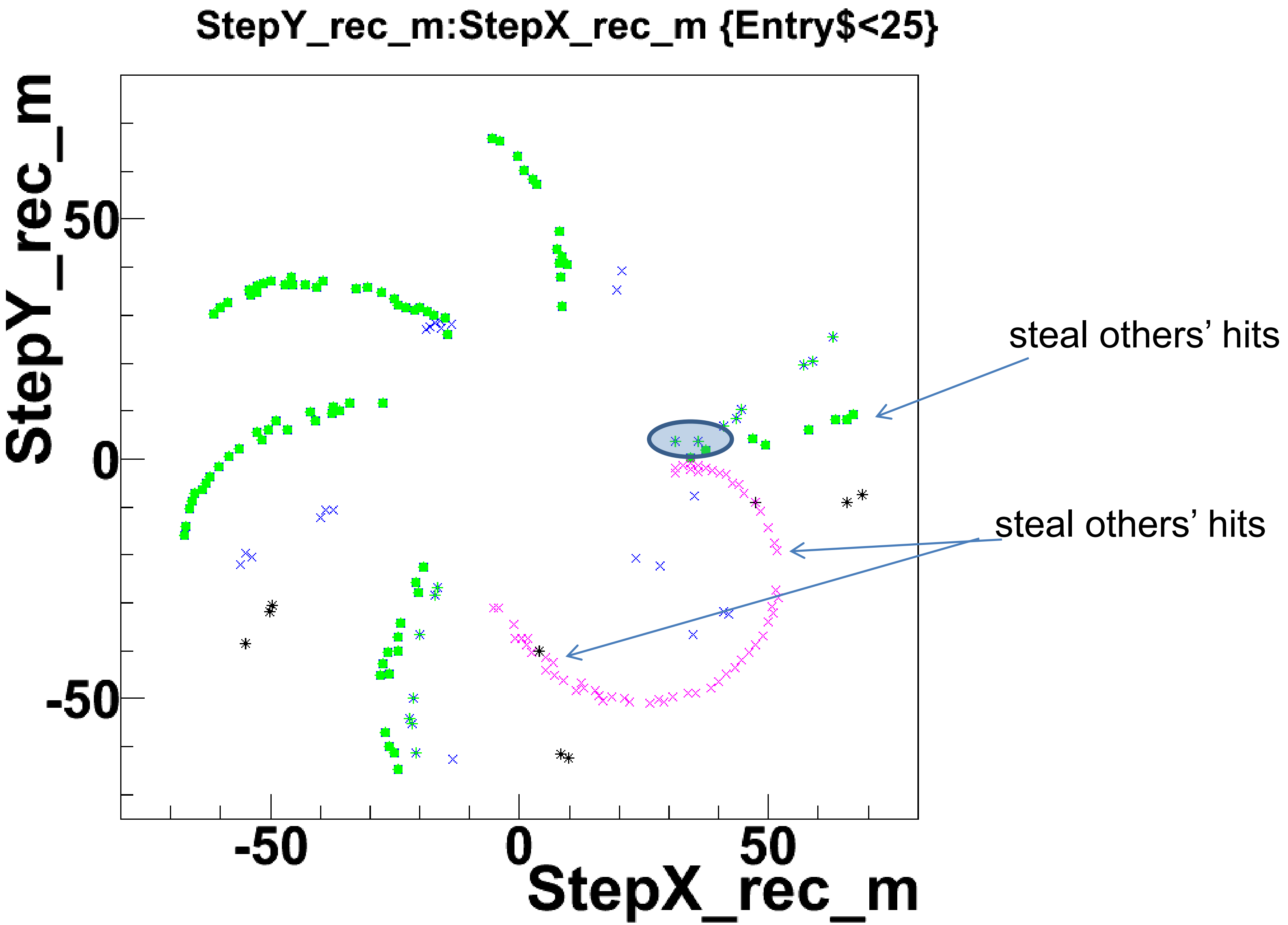
How ChainFinder Works

1) Start from a seed, add itself and nearby hits (requiring distance < 15mm) into the chain. All hits added should be marked as 'USED'.
2) Move to next hit in the chain, loop over the whole hit pool to get all nearby hits, requiring:
A) Distance < 19mm and deflection angle < 30deg, or
B) Distance < 4mm and deflection angle < 40deg;
Add all found hits into the chain and mark them as 'USED'.
3) repeat step 2) till all hits in the chain finish.

4) Choose next hit in the pool as the seed for next chain, repeat step 1) 2) 3) till all hits in the pool been marked as 'USED'.

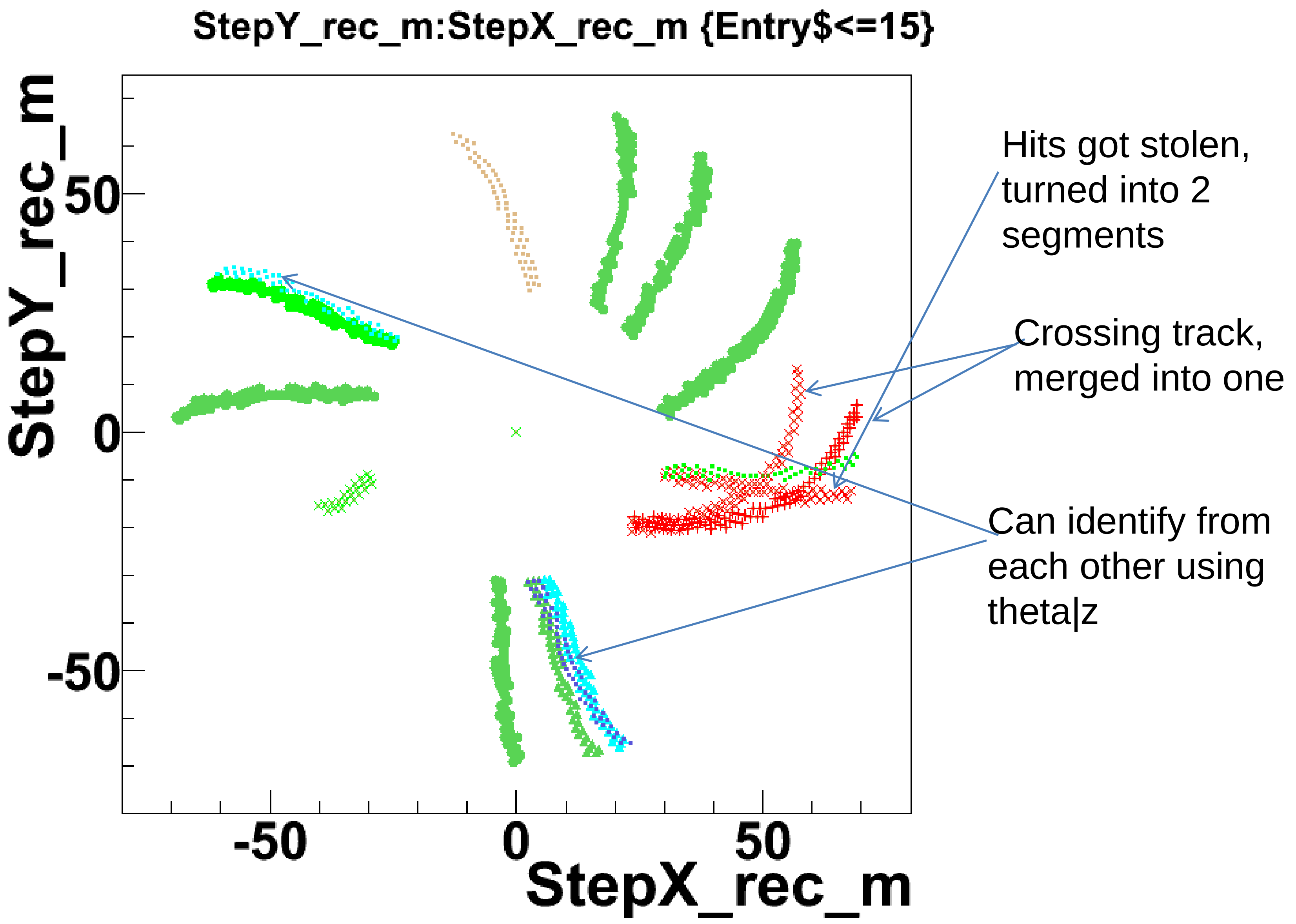


How The Chain-Finder Works?



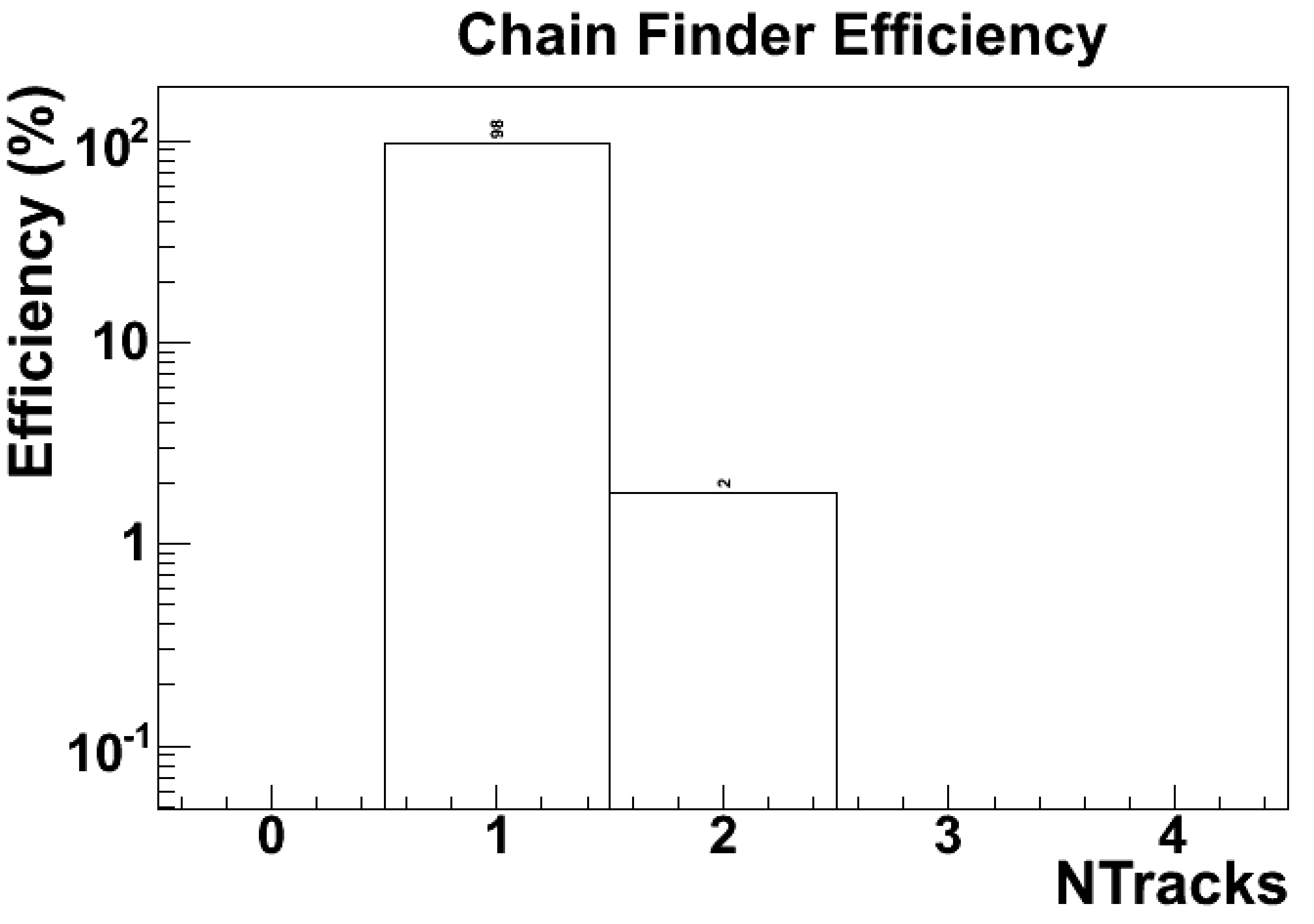
Normal Geant4 simulation, no step length interference

When Does Chain-Finder Fail?



Abnormal Geant4 simulation, step limit = 0.6 mm

ChainFinder Efficiency @ No Noise



The input GEMC root file contains 1 track per event(maximum step limit unknown).
<0.5% that a particle enter into the drift region twice.
CF efficiency is > 98%

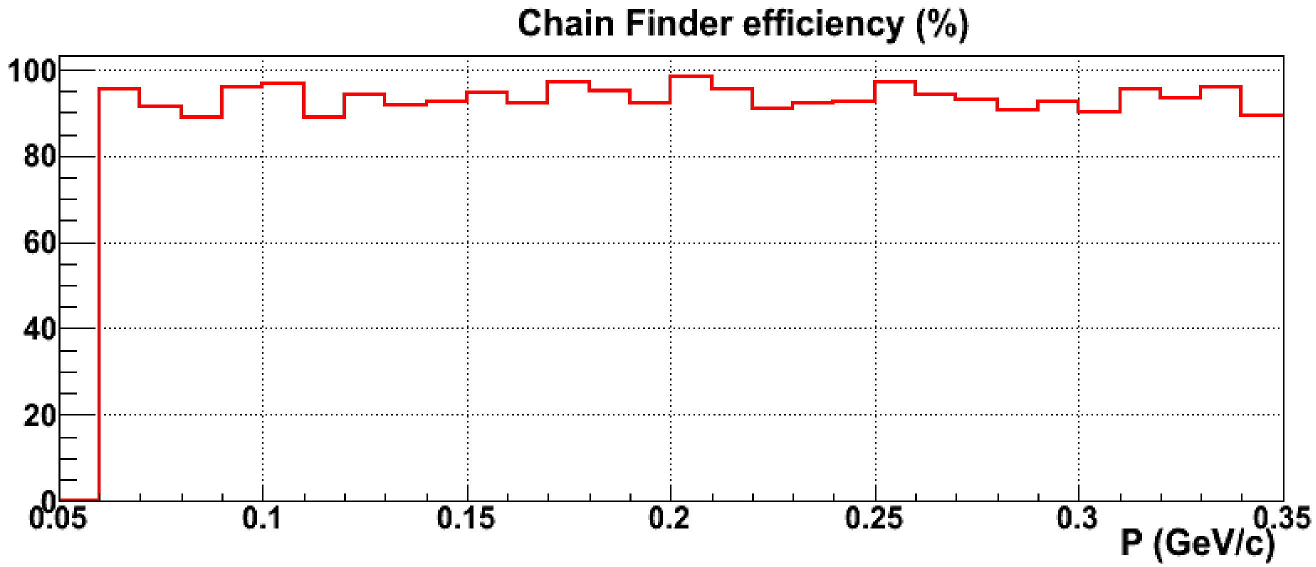
ChainFinder Efficiency @ With Noise

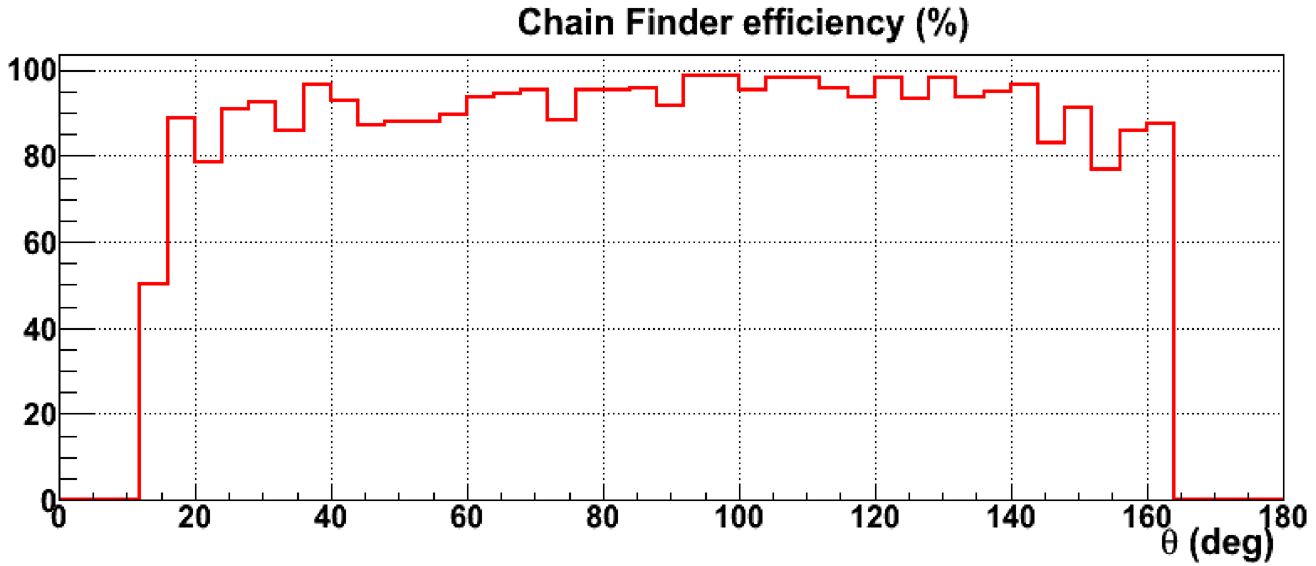
51 tracks per event, only one track is on time, the other 50 tracks is randomly shifted by time with +/- 7 us.

Using Jixie's Geant4 RTPC12 simulation root file, all tracks are forced to have a maximum step length of 1mm.

To be a valid track, none of the following is true:

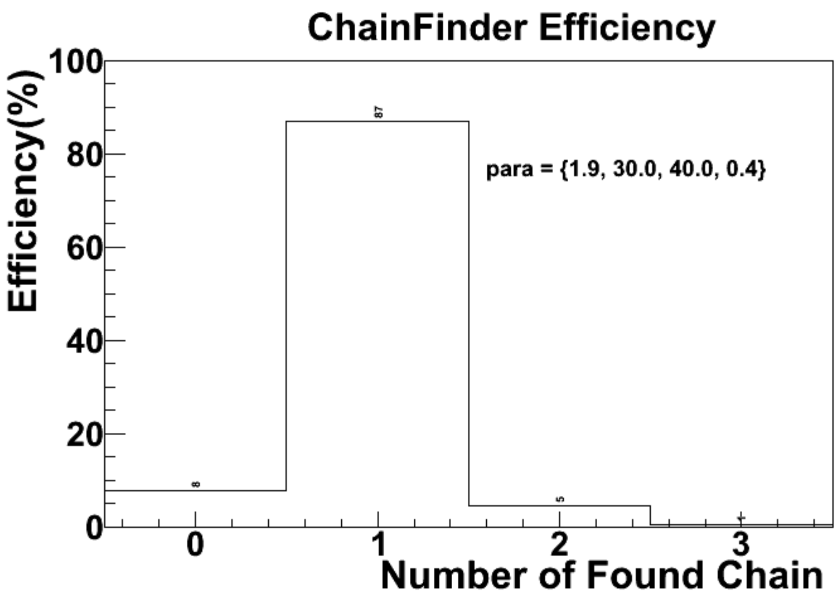
a) number of hits < 5, or
b) Smin > 5 cm, or
c) Smax < 5 cm, or
d) Smax-Smin<2cm.

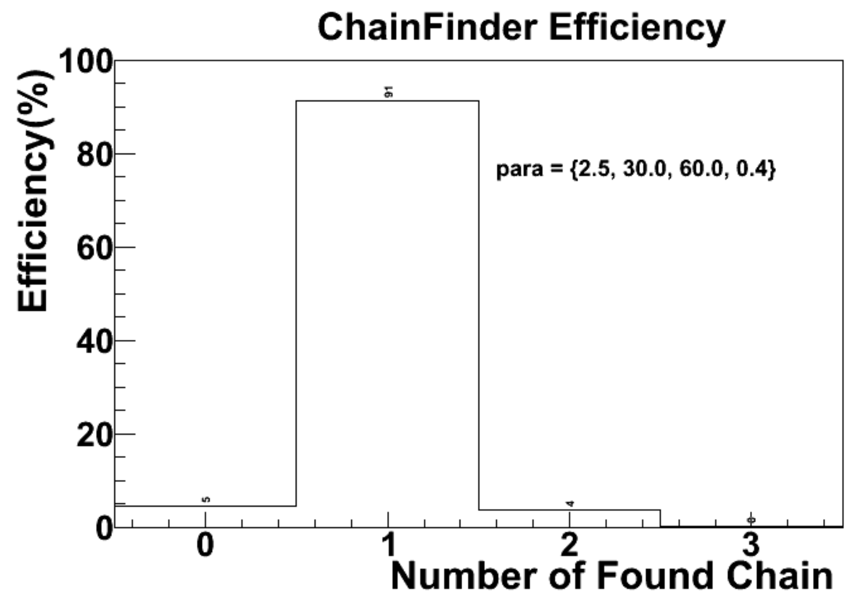


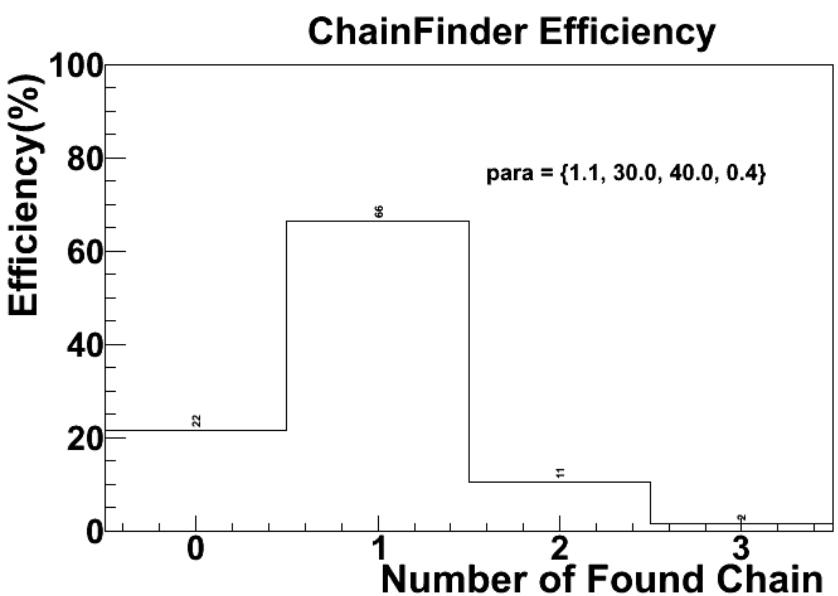


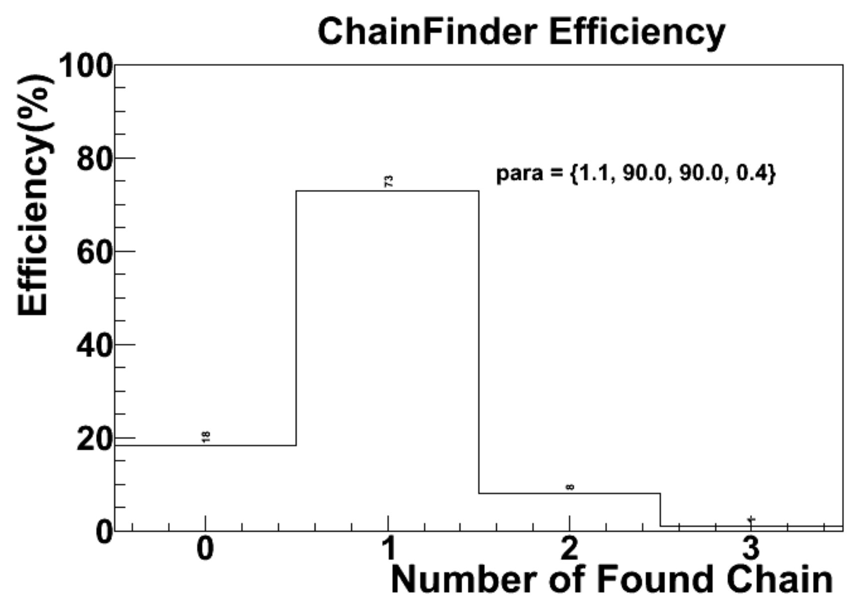
2000 such events prepared flatly in P, Theta, Phi and Z, but only 1848 events whose true track is a 'valid track', 1742 true tracks found. Overall CF efficiency is **1742/1848 ~= 93%**

Chain-Finder Efficiency









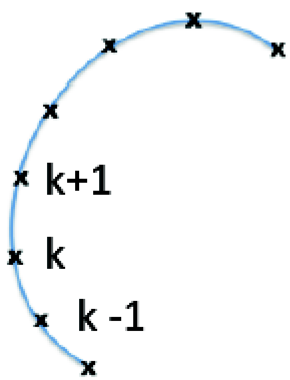
Optimize the parameter can improve efficiency.

Using Jixie’s Geant4 simulated events. One track per event, no step limit.

Introduction of Kalman Filter (I)

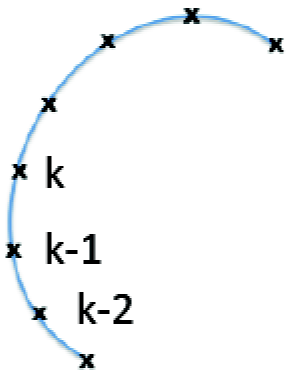
The Kalman filter is an algorithm that uses a series of measurements observed which contain noise (random variations i.e. MS) and other errors.

It produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone and it produces a statistically optimal estimate of the underlying system state.



1. A vector describing the particle at a particular location *(e.g k-1)* in the tracker (i.e. measurement site): $\mathbf{a}_{k-1}$
2. Equation of motion describing the evolution of state from one site to the next: f($\mathbf{a}_{k-1}$) = $\mathbf{a}_k$
   – deterministic function ignores process noise ($\mathbf{w}_{k-1}$) with covariance matrix ($Q_{k-1} = < \mathbf{w}_{k-1}\, \mathbf{w}^T_{k-1} >$), incorporated in state covariance matrix
3. A measurement vector: observables at a given site (e.g. midplane wire position)
   – assumes measurement error ($\boldsymbol{\varepsilon}_{k-1}$) is unbiased; covariance matrix ($V_{k-1} = < \boldsymbol{\varepsilon}_{k-1}\, \boldsymbol{\varepsilon}^T_{k-1} >$)
4. A function mapping the prediction at a given site onto the measurement: h($\mathbf{a}_{k-1}$)

Introduction of Kalman Filter (II)



- Goal to update the predicted state vector at site k: $\boldsymbol{a}_k^{k-1}$, using all measurements *up to site k* → $\boldsymbol{a}_k^*$

  - Let $(\chi^2)_{k-1}^{k-1}$ be the χ^2 up to site k-1
  - The χ^2 including the measurement and covariance matrix propagation to site k is: $(\chi^2)_k^k = (\chi^2)_{k-1}^{k-1} + \chi^2_+$
  - The optimal value of the state at site k is obtained by minimizing the χ^2 increment: $\frac{\partial \chi^2_+}{\partial \boldsymbol{a}_k^*} = 0$
  - where: $$\chi^2_+ = (\boldsymbol{a}_k^* - \boldsymbol{a}_k^{k-1})^T \left(\boldsymbol{C}_k^{k-1}\right)^{-1} (\boldsymbol{a}_k^* - \boldsymbol{a}_k^{k-1}) + (\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^*))^T \boldsymbol{G} (\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^*))$$

    (G = inverse measurement error matrix)
  - yields ***filtered*** state:

    Kalman Gain

    $$\boldsymbol{a}_k = \boldsymbol{a}_k^{k-1} + \left[\left(\boldsymbol{C}_k^{k-1}\right)^{-1} + \boldsymbol{H}_k^T \boldsymbol{G}_k \boldsymbol{H}_k\right]^{-1} \boldsymbol{H}_k^T \boldsymbol{G}_k \left(\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^{k-1})\right)$$

Ref.*Extended*Kalman*Filter*Keisuke*Fujii*[The*ACFA@Sim@J*Group]***

Introduction of Kalman Filter (III)

state propagator

$$a_k^{k-1} = f_{k-1}(a_{k-1}^{k-1}) = f_{k-1}(a_{k-1}).$$

Predict what the state at site k is based on information up to site k-1

- A site is a location in the detector where a measurement is taken (e.g. plane)
- Energy loss correction in state propagation

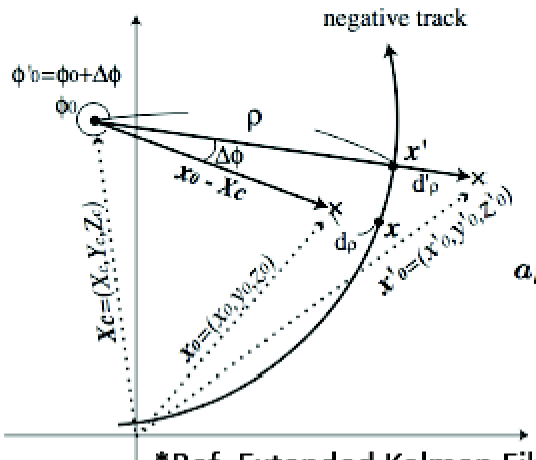
propagator matrix / **process noise (Multiple Scattering)**

$$C_k^{k-1} = F_{k-1} C_{k-1} F_{k-1}^T + Q_{k-1},$$

$$F_{k-1} \equiv \left(\frac{\partial f_{k-1}}{\partial a_{k-1}}\right) = \begin{pmatrix} \frac{\partial d'_\rho}{\partial a} \\ \frac{\partial \phi'_0}{\partial a} \\ \frac{\partial \kappa'}{\partial a} \\ \frac{\partial d'_z}{\partial a} \\ \frac{\partial \tan\lambda'}{\partial a} \end{pmatrix}$$

Propagate the Covariance matrix to site k is based on information up to site k-1

* e.g. for a helical track *(i.e track fitting in ~ constant B-field)*



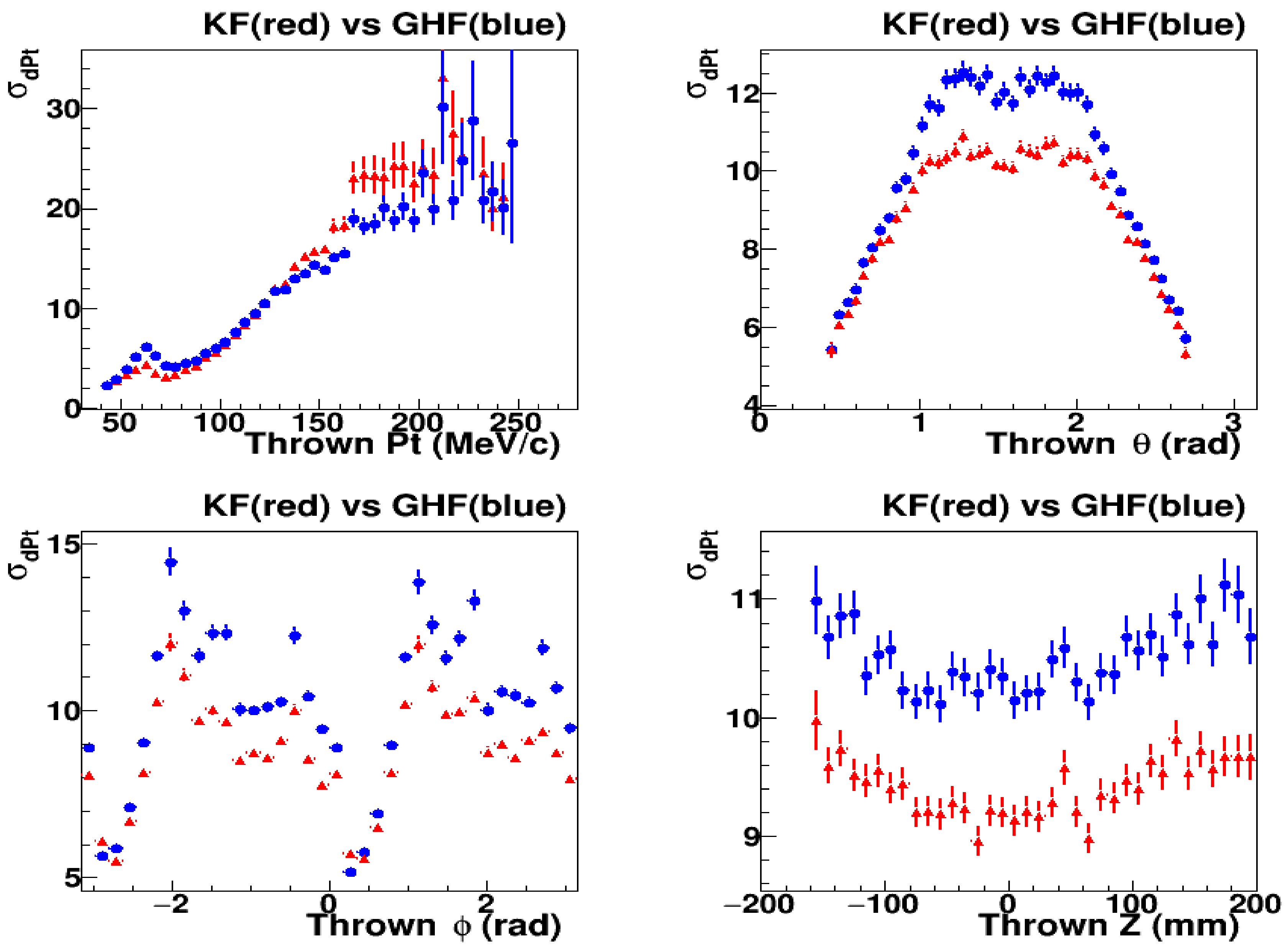
$$a' \equiv a_k^{k-1} = (d'_\rho, \phi'_0, \kappa', d'_z, \tan\lambda')^T = f_{k-1}(a_{k-1}) \begin{cases} d'_\rho = (X_c - x'_0)\cos\phi'_0 + (Y_c - y'_0)\sin\phi'_0 - \frac{\alpha}{\kappa} \\ \phi'_0 = \begin{cases} \tan^{-1}\left(\frac{Y_c - y'_0}{X_c - x'_0}\right) & (\kappa > 0) \\ \tan^{-1}\left(\frac{y'_0 - Y_c}{x'_0 - X_c}\right) & (\kappa < 0) \end{cases} \\ \kappa' = \kappa \\ d'_z = z_0 - z'_0 + d_z - \left(\frac{\alpha}{\kappa}\right)(\phi'_0 - \phi_0)\tan\lambda \\ \tan\lambda' = \tan\lambda, \end{cases}$$

$$a_{k-1} \equiv a = (d_\rho, \phi_0, \kappa, d_z, \tan\lambda)^T$$

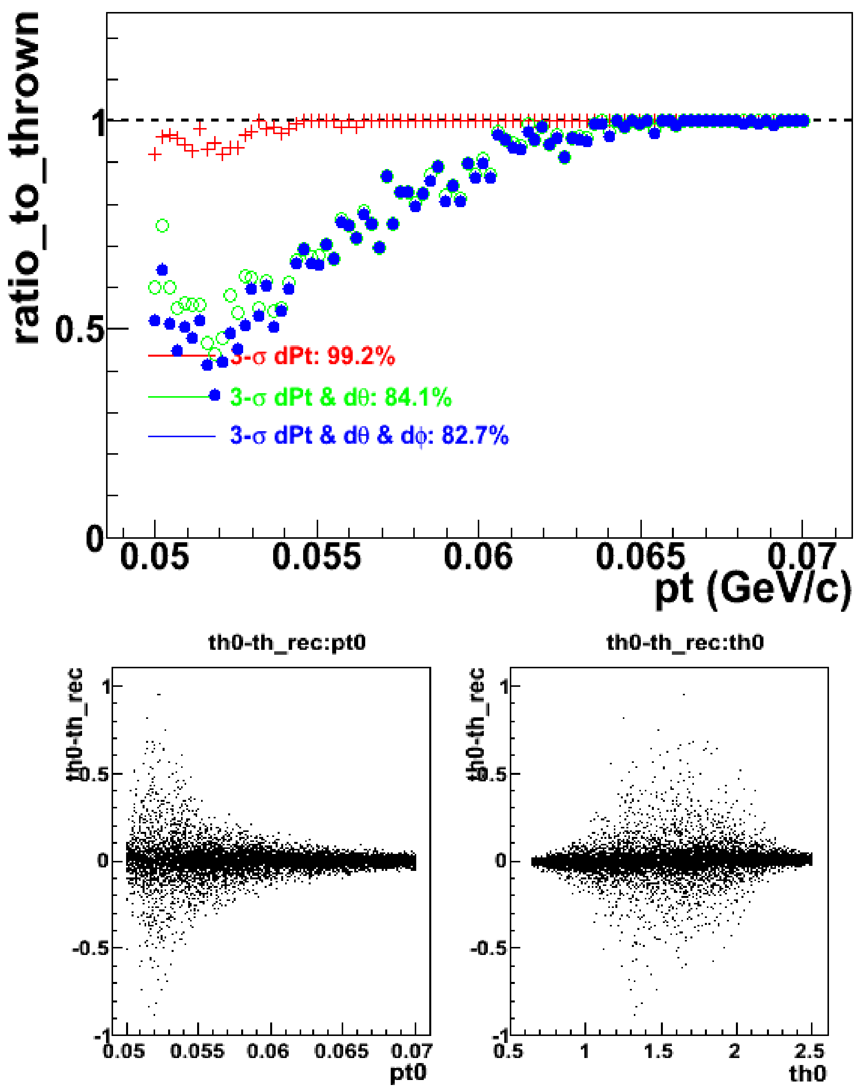
$$\begin{cases} X_c \equiv x_0 + \left(d\rho + \frac{\alpha}{\kappa}\right)\cos\phi_0 \\ Y_c \equiv y_0 + \left(d\rho + \frac{\alpha}{\kappa}\right)\sin\phi_0 . \end{cases}$$

*Ref. Extended Kalman Filter Keisuke Fujii [The ACFA-Sim-J Group]

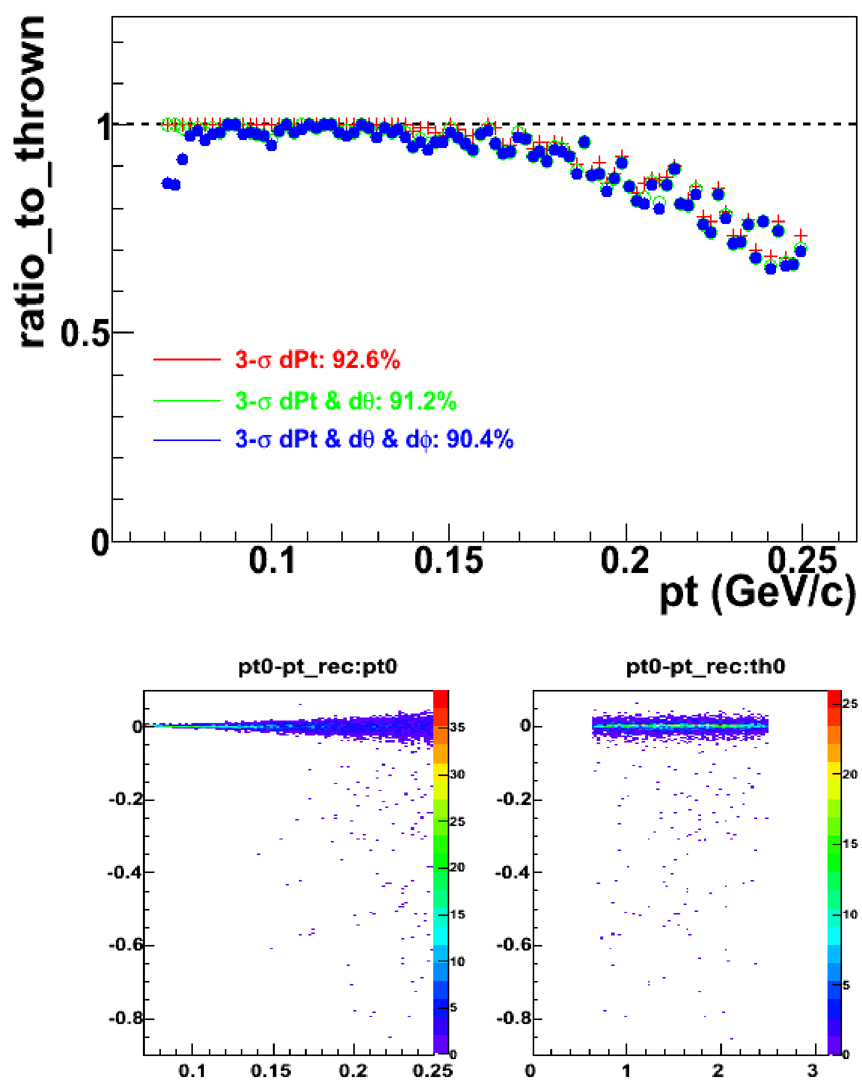
Kalman Filter Resolution: Pt and Theta



Kalman Filter Performance



50<Pt<70, -0.8<CosTh<0.8



70<Pt<250, -0.8<CosTh<0.8

50<Pt<70: 82.7% valid reconstruction, 15% lost due to Theta reconstruction at small Pt region (Pt<0.63)
70<Pt<250: 90.4% valid reconstruction, 7.4% lost due to Pt reconstruction in large Pt region (Pt>0.17)

Conclusion

1.Tracking software for RTPC12 is ready to use.
2. Global helix fitter has been improved.
3. Detail study have been done to Kalman Filter of RTPC12. It is sensitive to given initial values. To first order, reconstructed Pt and Phi are sensitive to initial R and Phi), while reconstructed Theta is sensitive to initial Theta. If there are offsets in these initial values, these offsets are hard to be removed.
4. For none-curve-back tracks, both global helix fitter or 1Iter-KF works well. 1Iter-KF is a little bit better.
5. For curve-back tracks, 1-Iter-KF will not reconstruct phi well. 2-Iter-KF (with the first iteration going backward then smooth back to the last site) will fit phi well.

For more details, check the following:
https://userweb.jlab.org/~jixie/bonus12/Jixie_KalmanFilter_Tracking_RTPC12.pdf
https://userweb.jlab.org/~jixie/bonus12/Jixie_RTPC_Tracking_v0.97_ReleaseNote.pdf
https://userweb.jlab.org/~jixie/bonus12/Jixie_ChainFinder_Efficiency_08032017.pdf