Inverting the mass hierarchy of jet quenching with b-jet substructure

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Based on the work arXiv:1801.00008

QCD Evolution, Santa Fe
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The study of jets has been used to test perturbative QCD, to probe proton structure and to search for New Physics
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Where do jets come from, quark, gluon or decaying product of other particles?
Jet substructures

Jet substructures at the LHC

Jet Substructures provide new ways to search for new physics and to probe the Standard Model in extreme regions of phase space.

1-prong jet: Larkoski et al 2014

groomed multi-prong observables Larkoski et al 2017

ratio of 2-prong and 3-prong: Chien et al 2017

See Varun’s talk for Energy-Energy correlator and Kyle’s talk for jet mass for a recent review see arXiv:1709.04464.
Jet in Heavy ion collisions

For example recent measurements

Open angle between the two 2-subjettiness

Measurements of fragmentation functions for jets

The observable we are interested is Jet splitting function
Jet Splitting Function

Defined as a two-prong substructure
Jet Splitting Function

Defined as a two-prong substructure

1 $\rightarrow$ 2 splitting process
Jet Splitting Function

Defined as a two-prong substructure

- An early hard splitting will result in two partons with high transverse momentum.
- Information about these leading partonic components can be obtained by removing the softer wide-angle radiation contributions.
- This is done through the use of jet grooming algorithms that attempt to split a single jet into two subjets, a process referred to as “declustering.”
Jet Splitting Function

Defined as a two-prong substructure

- An early hard splitting will result in two partons with high transverse momentum.
- Information about these leading partonic components can be obtained by removing the softer wide-angle radiation contributions.
- This is done through the use of jet grooming algorithms that attempt to split a single jet into two subjets, a process referred to as “declustering”.

One way to do this is to use Soft-Drop decluttering.
Soft drop decluttering

Original jet with radius $R_0$

Undo last stage of C/A clustering

Define $z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}}$

If $z_g < z_{cut} \left( \frac{\Delta R_{12}}{R_0} \right)^{\beta}$ redefine j to be the harder one, else we have the two-prong subjects

Drop soft divergences systematically

All remaining particles in the jet must be collinear

See Varun, Felix and Kyle’s talks

Larkoski et al 2014
Jet Splitting Function

The QCD splitting function

- Fundamental property of pQCD
- Heart to the collinear universality, DGLAP evolutions
- Most of Parton shower Models are generated by LO splitting functions
Jet Splitting Function

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The jet splitting function or momentum sharing distribution

- Defined as 2-prong jet substructure
- Closely related to the Altarelli-Parisi QCD splitting function, asymptotes to the QCD splitting function in the high-energy limit
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Splitting functions in QCD medium

- Test the in-medium splitting functions.
- Study early stage of the in-medium parton shower evolution.
Splitting functions in medium

The interactions of the outgoing partons with the hot and dense QCD medium, may change the jet splitting functions relative to the simpler proton-proton case.

The modification of $z_g$ distribution in heavy ion collisions has been measured at the LHC and RHIC.

The predictions for the modification of the resumed substructure for light jet

The predictions for the jet substructure of heavy-flavor tagged jet in the vacuum and medium.
Resummation

Why resummation

- Jet splitting function is not IR safe. We have to resum the logs or place a cut on the distance of two subjets.
- Resummation will change the distribution, especially for gluon splitting into massive quarks.

Why heavy flavor

- Predominantly produced in the initial hard scatterings of partons in the incoming nuclei.
- Hard probes to study the full evolution of the medium created by relativistic heavy ion collisions.
- Interaction between the heavy quarks and the medium is sensitive to the medium dynamics.
Vacuum splitting functions

The soft-drop groomed joint distribution is dominant by the first splitting

\[
\frac{dN_{\text{vac}}}{dz_g d\theta_g} \bigg|_j = \frac{\alpha_s}{\pi} \frac{1}{\theta_g} \sum_i P_{j \rightarrow ii}^{\text{vac}}(z_g) , \quad 0 < \theta_g = \frac{\Delta R_{12}}{R_0} < 1
\]

At the lowest non-trivial order the splitting functions are

\[
P_{g \rightarrow qg}^{\text{vac}}(z) = C_F \frac{1 + (1 - z)^2}{z} ,
\]

\[
P_{g \rightarrow gg}^{\text{vac}}(z) = 2C_A \left( \frac{1 - z}{z} + \frac{z}{1 - z} + z(1 - z) \right) ,
\]

\[
P_{g \rightarrow \bar{q}q}^{\text{vac}}(z) = T_R \left( z^2 + (1 - z)^2 \right) ,
\]

These splitting functions have been widely used in many applications
Vacuum splitting functions

The soft-drop groomed joint distribution is dominant by the first splitting

\[
\left( \frac{dN^\text{vac}}{dz_g d\theta_g} \right)_j = \frac{\alpha_s}{\pi} \frac{1}{\theta_g} \sum_i P^\text{vac}_{j \rightarrow ii}(z_g) \cdot 0 < \theta_g = \frac{\Delta R_{12}}{R_0} < 1
\]

At the lowest non-trivial order the splitting functions are

\[
P^\text{vac}_{q \rightarrow qg}(z) = C_F \frac{1 + (1 - z)^2}{z},
\]

\[
P^\text{vac}_{g \rightarrow gg}(z) = 2C_A \left( \frac{1 - z}{z} + \frac{z}{1 - z} + z(1 - z) \right),
\]

\[
P^\text{vac}_{g \rightarrow q\bar{q}}(z) = T_R \left( z^2 + (1 - z)^2 \right),
\]

These splitting functions have been widely used in many applications

\[
\left( \frac{dN^\text{vac}}{dzd^2k_\perp} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{2\pi^2} \frac{C_F}{k_\perp^2 + z^2 m^2} \left( \frac{1 + (1 - z)^2}{z} \right) - \frac{2z(1 - z)m^2}{k_\perp^2 + z m^2}
\]

\[
\left( \frac{dN^\text{vac}}{dzd^2k_\perp} \right)_{g \rightarrow Q\bar{Q}} = \frac{\alpha_s}{2\pi^2} \frac{T_R}{k_\perp^2 + m^2} \left( z^2 + (1 - z)^2 + \frac{2z(1 - z)m^2}{k_\perp^2 + m^2} \right)
\]

The dependence on \( z \) and \( k_\perp \) does not factorize.
Medium corrections to splitting functions

The Glauber modes are included using background filed method

\[ \mathcal{L}_{\text{SCET}_G}(\xi_n, A_n, A_G) = \mathcal{L}_{\text{SCET}}(\xi_n, A_n) + \mathcal{L}_G(\xi_n, A_n, A_G) \]

\[ \mathcal{L}_G(\xi_n, A_n, A_G) = \sum_{p,p'} e^{-i(p-p')x} \left( \bar{\xi}_{n,p'} \Gamma_{\text{qqAG}}^{\mu,a} \frac{\not{p}}{2} \xi_{n,p} - i \Gamma_{\text{ggAG}}^{\mu\nu\lambda,abc} (A_{n,p'}^c)_\lambda (A_{n,p}^b)_\nu \right) A_{G,\mu,a}(x) \]

Ovanesyan and Vitev 2011
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This framework was extended by including the finite quark mass

\[ \mathcal{L}_0 = \sum_{\bar{p}, \bar{p}', \bar{q}} e^{-ix \cdot \not{p}} \bar{\xi}_{n,p'} \left[ i m \cdot D + (\not{P}_\perp + g A_{n,q}^\perp) W_n \frac{1}{\not{p}} W_n^\dagger (\not{P}_\perp + g A_{n,q}^\perp) \right] \frac{\not{p}}{2} \xi_{n,p} + \mathcal{L}_m \]

\[ \mathcal{L}_m = \sum_{\bar{p}, \bar{p}', \bar{q}} e^{-ix \cdot \not{p}} \left[ m^2 \bar{\xi}_{n,p'} \left[ (\not{P}_\perp + g A_{n,q}^\perp), W_n \frac{1}{\not{p}} W_n^\dagger \right] \frac{\not{p}}{2} \xi_{n,p} - m^2 \bar{\xi}_{n,p'} W_n \frac{1}{\not{p}} W_n^\dagger \frac{\not{p}}{2} \xi_{n,p} \right] \]
Medium corrections to splitting functions

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\[ \mathcal{L}_{\text{SCET}_G} (\xi_n, A_n, A_G) = \mathcal{L}_{\text{SCET}} (\xi_n, A_n) + \mathcal{L}_G (\xi_n, A_n, A_G) \]

\[ \mathcal{L}_G (\xi_n, A_n, A_G) = \sum_{p,p'} e^{-i(p-p')x} \left( \bar{\xi}_{n,p'} \Gamma_{qqA_G}^{\mu,a} \frac{\not p}{2} \xi_{n,p} - i \Gamma_{ggA_G}^{\mu\nu,abc} (A_{n,p'})_{\lambda} (A_{n,p})_{\nu} \right) A_{G,\mu,a}(x) \]

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\[ \mathcal{L}_0 = \sum_{\bar{p}, \bar{p}', \bar{q}} e^{-ix\cdot \bar{p}} \left[ m \bar{\xi}_{n,p'} \left[ (\bar{p}_\perp + g A_{n,q}) W_n \frac{1}{\bar{p}} W_n^\dagger (\bar{p}_\perp + g A_{n,q'}) \right] \frac{\not \bar{q}}{2} \xi_{n,p} + \mathcal{L}_m \right] \]

\[ \mathcal{L}_m = \sum_{\bar{p}, \bar{p}', \bar{q}} e^{-ix\cdot \bar{p}} \left[ m \bar{\xi}_{n,p'} \left[ (\bar{p}_\perp + g A_{n,q}) W_n \frac{1}{\bar{p}} W_n^\dagger \frac{\not \bar{q}}{2} \xi_{n,p} - m^2 \bar{\xi}_{n,p'} W_n \frac{1}{\bar{p}} W_n^\dagger \frac{\not \bar{q}}{2} \xi_{n,p} \right] \right] \]

Feynman Rules are derived directly from the Lagrangian

\[ p \rightarrow p' \quad \quad \quad = \quad i \nu(q_{1\perp}) (b_1)_R (b_1)_T \frac{\not q}{2} \]

\[ p \rightarrow p' \quad \quad \quad = \quad \nu(q_{1\perp}) f^{abc_1} (c_1)_T, g^{\mu\nu} \bar{n} \cdot p \]

Ovanesyan and Vitev 2011

Kang, Ringer and Vitev 2016

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Medium corrections to splitting functions

Calculated in the framework of soft-collinear effective theory with Glauber gluon interactions

\[
\begin{align*}
\frac{dN}{dx} & \sim \begin{cases}
\begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3}
\end{array}
\end{cases} \\
+ 2 \text{Re} \begin{cases}
\text{Diagram 4} \\
\text{Diagram 5} \\
\text{Diagram 6}
\end{cases}
\end{align*}
\]

\[
\left( \frac{dN_{\text{med}}}{dx k_{\perp}} \right)_{q\to gg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1 - x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_\perp \frac{1}{2} \frac{d\sigma_{\text{med}}}{d^2 q_\perp} \left[ \frac{B_1}{B_2^2} \left( \frac{B_1}{B_2^2} - \frac{C_1}{C_2^2} \right) \right] \\
\times (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{C_1}{C_2^2} \left( 2\frac{C_1}{A_2^2} - \frac{A_1}{A_2^2} - \frac{B_1}{B_2^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) \\
+ \left( \frac{B_1}{B_2^2} \cdot \frac{C_1}{C_2^2} \right) (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) + \frac{A_1}{A_2^2} \left( \frac{D_1}{D_2^2} - \frac{A_1}{A_2^2} \right) (1 - \cos[\Omega_4\Delta z]) \\
- \frac{A_1}{A_2^2} \cdot \frac{D_1}{D_2^2} (1 - \cos[\Omega_5\Delta z]) + \frac{1}{N_c} \frac{B_1}{B_2^2} \left( \frac{A_1}{A_2^2} - \frac{B_1}{B_2^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z])
\end{align*}
\]

Massless partons: Ovanesyan and Vitev 2011
Medium corrections to splitting functions

Calculated in the framework of soft-collinear effective theory with Glauber gluon interactions

\[
\frac{dN}{dx} \sim \left( \frac{dN_{\text{med}}}{dx d^2 k_\perp} \right)_{\nu \rightarrow qg}^2 \times \left( \frac{dN_{\text{med}}}{dx d^2 k_\perp} \right)_{q \rightarrow gg} \times \frac{\alpha_s}{2\pi^2} \frac{C_F}{x} \left( 1 + \frac{(1 - x)^2}{x} \right) \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_\perp \frac{d\sigma_{\text{el}}}{\sigma_{\text{el}}} \frac{dN_{\text{med}}}{d^2 q_\perp} \left[ \frac{B_\perp}{B_\perp^2} \left( \frac{B_\perp}{C_\perp} - \frac{C_\perp}{B_\perp} \right) \right] \times \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) + \frac{C_\perp}{C_\perp^2} \cdot \left( \frac{2 C_\perp}{C_\perp^2} - \frac{A_\perp}{A_\perp^2} - \frac{B_\perp}{B_\perp^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_3)\Delta z] \right) + \frac{B_\perp}{B_\perp^2} \cdot \frac{C_\perp}{C_\perp^2} \left( 1 - \cos[(\Omega_2 - \Omega_3)\Delta z] \right) + \frac{A_\perp}{A_\perp^2} \cdot \left( \frac{D_\perp}{D_\perp^2} - \frac{A_\perp}{A_\perp^2} \right) \left( 1 - \cos[\Omega_4 \Delta z] \right)
\]

\[
\frac{dN_{\text{med}}}{dx d^2 k_\perp} \mid _{Q \rightarrow Qg} \mid = \frac{\alpha_s}{2\pi^2} \frac{C_F}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_\perp \frac{d\sigma_{\text{el}}}{\sigma_{\text{el}}} \left\{ \left( 1 + \frac{(1 - x)^2}{x} \right) \right\} \left[ \frac{B_\perp}{B_\perp^2 + \nu^2} \right.
\]
\[
\times \left( \frac{B_\perp}{B_\perp^2 + \nu^2} - \frac{C_\perp}{C_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) + \frac{C_\perp}{C_\perp^2 + \nu^2} \cdot \left( \frac{2 C_\perp}{C_\perp^2 + \nu^2} - \frac{A_\perp}{A_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_2 - \Omega_3)\Delta z] \right) + \frac{A_\perp}{A_\perp^2 + \nu^2} \cdot \left( \frac{D_\perp}{D_\perp^2 + \nu^2} - \frac{A_\perp}{A_\perp^2 + \nu^2} \right) \left( 1 - \cos[\Omega_4 \Delta z] \right)
\]

\[
\times \left( \frac{B_\perp}{B_\perp^2 + \nu^2} - \frac{C_\perp}{C_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_3)\Delta z] \right) + \frac{C_\perp}{C_\perp^2 + \nu^2} \cdot \left( \frac{2 C_\perp}{C_\perp^2 + \nu^2} - \frac{A_\perp}{A_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_2 - \Omega_3)\Delta z] \right) + \frac{A_\perp}{A_\perp^2 + \nu^2} \cdot \left( \frac{D_\perp}{D_\perp^2 + \nu^2} - \frac{A_\perp}{A_\perp^2 + \nu^2} \right) \left( 1 - \cos[\Omega_4 \Delta z] \right)
\]

\[
\times \frac{1}{N_c} \frac{B_\perp}{B_\perp^2 + \nu^2} \cdot \left( \frac{A_\perp}{A_\perp^2 + \nu^2} - \frac{B_\perp}{B_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) + \frac{1}{N_c} \frac{B_\perp}{B_\perp^2 + \nu^2} \cdot \left( \frac{A_\perp}{A_\perp^2 + \nu^2} - \frac{B_\perp}{B_\perp^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) + \ldots
\]

Medium corrections to splitting functions

Calculated in the framework of soft-collinear effective theory with Glauber gluon interactions

\[ \frac{dN}{dx} \sim \left( \begin{array}{c}
\text{\footnotesize \includegraphics{diagram1.png}}
+ \text{\footnotesize \includegraphics{diagram2.png}}
+ \text{\footnotesize \includegraphics{diagram3.png}}
\end{array} \right)^2 \]

\[ \left( \frac{dN_{\text{med}}}{dx d^2 k_{\perp}} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi^2} C_F \left( 1 + \frac{(1-x)^2}{x} \right) \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_{\perp} \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{med}}}{d^2 q_{\perp}} \left[ \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right)^2 \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \right] \times \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \left( 1 - \cos[(\Omega_3 - \Omega_4)\Delta z] \right) \frac{A_1}{A_1^2 + \nu^2} \left( 1 - \cos[(\Omega_5 - \Omega_6)\Delta z] \right) \]

\[ \frac{dN_{\text{med}}}{dx d^2 k_{\perp}} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_{\perp} \frac{1}{\sigma_{\text{el}}} \frac{d\sigma_{\text{med}}}{d^2 q_{\perp}} \left\{ \left( 1 + \frac{(1-x)^2}{x} \right) \left[ \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right)^2 \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \right] \}
\]

\[ \begin{align*}
&\times \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} - \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) + \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} - \frac{A_1}{A_1^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_3 - \Omega_4)\Delta z] \right) \\
&+ \frac{A_1}{A_1^2 + \nu^2} \left( \frac{D_{\perp}}{D_{\perp}^2 + \nu^2} - \frac{A_1}{A_1^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_5 - \Omega_6)\Delta z] \right) \\
&+ \frac{1}{N_c^2 \nu^2} \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} - \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) \\
&+ x^3 m^2 \left( \frac{1}{B_{\perp}^2 + \nu^2} - \frac{1}{C_{\perp}^2 + \nu^2} \right) \left( 1 - \cos[(\Omega_1 - \Omega_2)\Delta z] \right) \right\} \\
&\times \left( 1 - \cos[(\Omega_3 - \Omega_4)\Delta z] \right) \frac{A_1}{A_1^2 + \nu^2} \left( 1 - \cos[(\Omega_5 - \Omega_6)\Delta z] \right)
\end{align*} \]


See Matt Sievert’s talk on Sunday for the all opacity results
Applications
Applications

Modification of fragmentation functions for gluon and quark

Kang et al 2014
Applications

Modification of fragmentation functions for gluon and quark
Kang et al 2014

Modification of jet shape
Chien et al 2015

Modification of fragmentation functions for gluon and quark
Kang et al 2014
Applications

Modification of fragmentation functions for gluon and quark

Chien et al 2015

Nuclear modification factor $R_{AA}$ for $D^0$ meson (massive)

Kang et al 2016

Kang et al 2014

Modification of jet shape

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Applications

Modification of fragmentation functions for gluon and quark
Kang et al 2014

Modification of jet shape
Chien et al 2015

Nuclear modification factor $R_{AA}$ for D0 meson (massive)
Nuclear modification factor $R_{AA}$ for heavy-ion collisions at
Kang et al 2017

The nuclear modification factor $R_{AA}$ for heavy-ion collisions at...
Resummation formalism

Resummed splitting kernels in the vacuum

Larkoski et al 2015
Resummation formalism

Resummed splitting kernels in the vacuum

Larkoski et al 2015

\[
\frac{dN^{FO}_j}{dz_g d\theta_g} \quad \text{is divergent when} \quad \theta_g \to 0
\]

Collinear singularities

\[
\frac{dN^F_j}{dz_g} \quad \text{is not well-defined at any fixed perturbative order}
\]
Resummation formalism

Resummed splitting kernels in the vacuum \textit{Larkoski et al 2015}

\[
\frac{dN_{j}^{FO}}{dz_g d\theta_g} \quad \text{is divergent when} \quad \theta_g \to 0
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\frac{dN_{j}^{F}}{dz_g} \quad \text{is not well-defined at any fixed perturbative order}
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\text{Collinear singularities}

\text{but is well defined if we resum logs to all order}
Resummation formalism

Resummed splitting kernels in the vacuum

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but is well defined if we resum logs to all order

Collinear singularities

The MLL resummation for light jet to modified leading-logarithmic (MLL) accuracy,

\[
\frac{dN_{j}^{\text{vac,MLL}}}{dz_g d\theta_g} = \sum_i \left( \frac{dN_{j}^{\text{vac}}}{dz_g d\theta_g} \right)_{j \rightarrow i \bar{i}} \exp \left[ - \int_{\theta_g}^{1} d\theta \int_{z_{\text{cut}}}^{1/2} dz \sum_i \left( \frac{dN_{j}^{\text{vac}}}{dz d\theta} \right)_{j \rightarrow i \bar{i}} \right]
\]

Sudakov Factor

Larkoski et al 2015
Resummation formalism

Resummed splitting kernels in the vacuum

Larkoski et al 2015

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\frac{dN_{j}^{FO}}{dz_{g}d\theta_{g}} \quad \text{is divergent when} \quad \theta_{g} \rightarrow 0
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\]

Sudakov Factor

MLL includes running coupling effects and subleading terms in the splitting functions compared to LL resummation.
Theoretical formalism

Resummed splitting kernels for heavy flavors

Suppose that we can distinguish the splitting process involving heavy flavor

For \( b \rightarrow b g \) the formula is the similar with massless quark

\[
\frac{dN_{j}^{\text{vac, MLL}}}{dz_g d\theta_g} = \sum_i \left( \frac{dN_{j}^{\text{vac}}}{dz_g d\theta_g} \right)_{j \rightarrow i \bar{i}} \exp \left[ - \int_{\theta_g}^{1} d\theta \int_{z_{\text{cut}}}^{1/2} dz \sum_i \left( \frac{dN_{j}^{\text{vac}}}{dz d\theta} \right)_{j \rightarrow i \bar{i}} \right]
\]

For \( g \rightarrow b \bar{b} \) the resumed distribution is

\[
p(\theta_g, z_g)|_{g \rightarrow Q \bar{Q}} = \frac{\left( \frac{dN_{j}^{\text{vac}}}{dz_g d\theta_g} \right)_{g \rightarrow Q \bar{Q}} \sum g(\theta_g)}{\int_{0}^{1} d\theta \int_{z_{\text{cut}}}^{1/2} dz \left( \frac{dN_{j}^{\text{vac}}}{dz d\theta} \right)_{g \rightarrow Q \bar{Q}} \sum g(\theta)}
\]
Theoretical formalism

Resummed splitting kernels for heavy flavors

Suppose that we can distinguish the splitting process involving heavy flavor $dN_{\text{vac}}$, $\text{MLL}_j dz_g d\theta_g = \sum_i \left( \frac{dN_{\text{vac}}}{dz_g d\theta_g} \right)_{j \rightarrow \tilde{i}} \exp \left[ - \int_{\theta_g}^{1} d\theta \int_{z_{\text{cut}}}^{1/2} dz \sum_i \left( \frac{dN_{\text{vac}}}{dz d\theta} \right)_{j \rightarrow \tilde{i}} \right]$

For $b \rightarrow bg$ $c \rightarrow cg$ formula is the similar with massless quark

For $g \rightarrow b\bar{b}$ $g \rightarrow c\bar{c}$ the resumed distribution is

$$p(\theta_g, z_g)_{g \rightarrow QQ} = \frac{\left( \frac{dN_{\text{vac}}}{dz_g d\theta_g} \right)_{g \rightarrow QQ}}{\int_0^1 d\theta \int_{z_{\text{cut}}}^{1/2} dz \left( \frac{dN_{\text{vac}}}{dz d\theta} \right)_{g \rightarrow QQ} \sum_g(\theta)}$$

Exponentiate all the possible contributions for gluon evolution

Resummation changes the distribution a lot compared to LO results
Results for light jet

In pp collisions uncertainties are generated by varying scales.

In heavy-ion collisions uncertainties are generated by varying scales and coupling (between medium and jet) independently.

Tripathee, et al 2017
Results for light jet

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Results for light jet

increase jet $P_T$

The splitting function in the medium becomes steeper

MLL changes the modification by a few percent

The modification is larger for small jet $P_T$

The theoretical predictions are consistent with the measurements
In general the path for recoil jet in the medium is longer than the one for trigger jet. To compare with data this effect is included in our splitting functions.
Results for heavy flavor tagged jet

In order to compare with the predictions from PYTHIA

A recent study for charm and beauty quarks at colliders using Monte Carlo event generators

see the work for details: Ilten et al 2017
Results for heavy flavor tagged jet

In order to compare with the predictions from PYTHIA

Label two subjets \((n_1^c, n_2^c) (n_1^b, n_2^b)\)

If there is no b-quark or b-hadron

\[
(n_1^c, n_2^c) = \begin{cases} 
(1,0) \text{ or } (0,1) & c \to cg \\
(1,1) & g \to c\bar{c}
\end{cases}
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\]

The other cases are ignored in the analysis during comparing with Pythia
Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged jet

The splitting kernel $\frac{C_F}{\pi^2} \frac{\alpha_s}{k_T^2 + x^2 m^2}$ is zero after integration when $k_T$ is zero.
Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged jet

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LO and MLL predictions for b-tagged jet

Non-perturbative

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Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged subjets

- Huge Sudakov suppression in the small angle region
- Wide-angle gluon splittings
Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged subjets

Non-perturbative corrections

- Huge Sudakov suppression in the small angle region
- Wide-angle gluon splittings
CMS is preparing to measure the double-b-tagged gluon splittings

Figures from Slides by Kurt Jung at Quark matter 2018
CMS is preparing to measure the double-b-tagged gluon splittings.

\[ \sqrt{s_{NN}} = 5.02 \text{ TeV} \quad pp \]

ant-\( k_T \) \( R = 0.4 \) \( \eta_{\text{jet}} \) \(< 1.3 \)

140 < \( p_{T,\text{jet}} \) < 160 GeV

Soft-Drop \( \beta = 0 \) \( z_{\text{cut}} = 0.1 \)

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Results for heavy flavor tagged jet

\[ \sqrt{s_{NN}} = 5.02 \text{ TeV} \]

- \( c \rightarrow gc \)
  - MLL
  - \( 140 < P_{T,j} < 160 \text{ GeV} \)
  - \( 250 < P_{T,j} < 300 \text{ GeV} \)

- \( b \rightarrow bg \)
  - SD: \( z_b = 0.1 \)
  - \( \Delta R_{12} > 0.1 \)
  - \( g = 1.9 \pm 0.1 \)

- \( g \rightarrow cc \)
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- \( g \rightarrow bb \)
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Results for heavy flavor tagged jet

When the jet energy is high the mass effect is small. The heavy flavor tagged jet behaves similar to the light jet in the medium.
Results for heavy flavor tagged jet

\[ \sqrt{s_{NN}}=200 \text{ GeV} \]
\[ \text{ant-}k_t, R=0.4 \]
\[ g=2\pm0.1 \text{ MLL} \]
\[ c \rightarrow cg \]
\[ b \rightarrow bg \]
\[ 10<P_{Tj}<30 \text{ GeV} \]
\[ \text{Soft-Drop} \quad \beta=0 \quad z_{cut}=0.1 \]
\[ g \rightarrow c\bar{c} \]
\[ g \rightarrow b\bar{b} \]
Results for heavy flavor tagged jet

Inverting the mass hierarchy in jet quenching effects
Results for heavy flavor tagged jet

Inverting the mass hierarchy in jet quenching effects

Splitting function in the vacuum

\[
\left( \frac{dN_{\text{vac}}^\gamma}{dz dk_{\perp}} \right)_{g \rightarrow Q\bar{Q}} = \frac{\alpha_s}{2\pi^2} \frac{C_F}{k_{\perp}^2 + z^2m^2} \left( \frac{1 + (1 - z)^2}{z} - \frac{2z(1 - z)m^2}{k_{\perp}^2 + z^2m^2} \right)
\]

\[
\left( \frac{dN_{\text{vac}}^\gamma}{dz dk_{\perp}} \right)_{c \rightarrow cg} = \frac{\alpha_s}{2\pi^2} \frac{1}{k_{\perp}^2 + m^2} \left( z^2 + (1 - z)^2 + \frac{2z(1 - z)m^2}{k_{\perp}^2 + m^2} \right)
\]

\[
\left( \frac{dN_{\text{vac}}^\gamma}{dz dk_{\perp}} \right)_{b \rightarrow bg} = \frac{\alpha_s}{2\pi^2} \frac{T_R}{k_{\perp}^2 + 2m^2} \left( z^2 + (1 - z)^2 + \frac{2z(1 - z)m^2}{k_{\perp}^2 + m^2} \right)
\]
Results for heavy flavor tagged jet

Inverting the mass hierarchy in jet quenching effects

Splitting function in the vacuum

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\]

Corrections in QCD medium contain terms such as

\[
\left( \frac{1}{k_\perp^2 + z^2 m^2} \right)^2 \times f(k_\perp, z) \quad \text{for} \quad Q \rightarrow Qg
\]

\[
\left( \frac{1}{k_\perp^2 + m^2} \right)^2 \times f'(k_\perp, z) \quad \text{for} \quad g \rightarrow Q\bar{Q}
\]
Conclusions

- Presented the resummation formula for jet splitting function in vacuum and QCD medium
- Compared the MLL predictions with Pythia8 at pp collider
- Compared the MLL modifications with measurements from CMS and STAR and found a good agreement within all the uncertainties
- Presented the modifications of the momentum sharing distributions for heavy flavor tagged jet

Heavy flavor tagged-jet may be better probes of the QGP properties than light jet.
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Thank you
Back up
Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged jet

\( \sqrt{s_{NN}} = 5.02 \text{ TeV} \) pp
\( \text{ant-}k_T R = 0.4 \ |\eta| < 1.3 \)
\( 140 < p_{T,jet} < 160 \text{ GeV} \)
Soft-Drop \( \beta = 0 \), \( z_{cut} = 0.1 \)

K factor
$\sqrt{s_{NN}} = 5.02$ TeV \( pp \)

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Soft-Drop \( \beta = 0, z_{\text{cut}} = 0.1 \)

CMS 2010 Open Data

- Theory (MLL)
- Pythia 8.219
- Herwig 7.0.3
- Sherpa 2.2.1

- $p_T > 1.0$ GeV
- AK5; $|\eta| < 2.4$
- $p_T > 150$ GeV

Soft-Drop: \( \beta = 0, z_{\text{cut}} = 0.1 \)
Results for heavy flavor tagged jet

LO and MLL predictions for b-tagged jet

\[ \sqrt{s_{NN}} = 5.02 \text{ TeV} \]

pp

ant-\( k_t \), \( R = 0.4 \), \( l < 1.3 \)

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\( g \rightarrow b \bar{b} \)

\[ K \text{ factor} \]
Back up