Two hadron correlations in SIDIS and $e^+e^-$ annihilation reactions

Aram Kotzinian
YerPhI, Armenia & INFN, Torino

Collaborators: H. Matevosyan and A.W. Thomas
University of Adelaide, Australia

Correlations in Partonic and Hadronic Interactions 2018
24-28 September 2018, Yerevan, Armenia

• Introduction
  • accessing TMD PDFs and FFs with electromagnetic probe
  • 1h production in SIDIS (current fragmentation region, CFR) and semi-inclusive $e^+e^-$ annihilation (SIA)

• Two hadron production in CFR of SIDIS and SIA

• Target fragmentation region (TFR) of SIDIS
SIDIS: CFR

\[
\frac{d\sigma}{dx dQ^2 d\phi_s dz d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma}{dQ^2} \otimes D_{q,s'}^{h_1}
\]

\[
D_{q,s'}^{h_1}(z, p_T) = D_1(z, p_T^2) + \frac{p_T \times s'_T}{m_h} H_1(z, p_T^2)
\]

\(H_1\) was measured by BABAR and BELLE to 2 back-to-back jets \(e^+e^- \rightarrow h_1h_2 + X\)
### Twist-2 TMD qDFs

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>Quark Polarization</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1^q (x, k_T^2)$</td>
<td>$e_T^{ij}k_T^j h_{1L}^{q} (x, k_T^2)$</td>
</tr>
<tr>
<td>L</td>
<td>$S_L g_{1L}^q (x, k_T^2)$</td>
<td>$S_L k_T h_{1L}^{q} (x, k_T^2)$</td>
</tr>
<tr>
<td>T</td>
<td>$k_T \times S_T^{q} (x, k^2)$</td>
<td>$k_T \cdot S_T h_{1T}^{q} (x, k_T^2)$ + $k_T (k_T \cdot S_T) h_{1T}^{q} (x, k_T^2)$</td>
</tr>
</tbody>
</table>

All azimuthal dependences are in prefactors. TMDs do not depend on them.
LO cross section in SIDIS CFR

\[
\frac{d\sigma}{dx dQ^2 d\phi_S dz d^2P_T} = \frac{\alpha^2 x}{yQ^2} (1 + (1 - y)^2) \times \left[ F_{UU,T} + D_{nn}(y) F_{UU}^{\cos 2\phi_h} \cos(2\phi_h) + \right.
\]
\[
S_L D_{nn}(y) F_{UL}^{\sin 2\phi_h} \sin(2\phi_h) + \lambda S_L D_{LL}(y) F_{LL} + \left. \right.
\]
\[
S_T \left( F_{UT, T}^{\sin(\phi_h - \phi_S)} \sin(\phi_h - \phi_S) + D_{nn}(y) \left( F_{UT}^{\sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) \right) \right) + \left. \right.
\]
\[
\lambda S_T D_{ll}(y) F_{LT}^{\cos(\phi_h - \phi_S)} \cos(\phi_h - \phi_S) \right] \]

Virtual photon depolarization factors
\[
D_{ll}(y) = \frac{y(2 - y)}{1 + (1 - y)^2}, \quad D_{nn}(y) = \frac{2(1 - y)}{1 + (1 - y)^2}
\]

8 terms out of 18 Structure Functions, 6 azimuthal modulations
4 terms are generated by Collins effect in fragmentation

8 structure functions \( F_{AB}^f(\phi_h, \phi_S) \)

- \( F_{UU,T} \propto f_1^q \otimes D_{1q}^h \)
- \( F_{UU}^{\cos 2\phi_h} \propto h_{1q}^\perp \otimes H_{1q}^\perp \)
- \( F_{UL}^{\sin 2\phi_h} \propto h_{1L}^\perp \otimes H_{1q}^\perp \)
- \( F_{UU}^{\sin(\phi_h - \phi_S)} \propto f_{1T}^q \otimes D_{1q}^h \)
- \( F_{UT}^{\sin(\phi_h + \phi_S)} \propto h_{1q}^\perp \otimes H_{1q}^\perp \)
- \( F_{UT}^{\sin(3\phi_h - \phi_S)} \propto h_{1T}^\perp \otimes H_{1q}^\perp \)
- \( F_{LT}^{\cos(\phi_h - \phi_S)} \propto g_{1T}^q \otimes D_{1q}^h \)
- \( F_{LL} \propto g_1^q \otimes D_{1q}^h \)
Access to $q + \bar{q}$ unpolarized fragmentation functions $D^h_{q+\bar{q}}(z, p^2_\perp)$
Quarks are unpolarized, but their transverse polarization are correlated, inducing an azimuthal correlation of produced hadrons in opposite jets.

Obtained $H_{1q}^h(z, p_{\perp}^2)$ FFs are used for transversity $h_1(x, k_T^2)$ extraction from SIDIS data.
2h SIDIS: CFR

\[
d \sigma^{\ell(l, \lambda) + N(P_N, S) \rightarrow \ell(l') + h_1(P_1) + h_2(P_2) + X} \frac{dx dQ^2 d\phi_s dz_1 d^2P_{1T} dz_2 d^2P_{2T}}{dx dQ^2 d\phi_s dz_1 d^2P_{1T} dz_2 d^2P_{2T}} = f_{q,s/N,S} \otimes d \sigma^{\ell(l, \lambda) + q(k,s) \rightarrow \ell(l') + q(k',s')} \otimes D^{h_1,h_2}_{q,s'}
\]

New objects: DiFFs \(D^{h_1,h_2}_{q,s'}\)

\(x_{F,1} > 0, \ x_{F,2} > 0\)
Measured by BELLE: dihadrons production in back-to-back jets in SIA

Access to spin dependent DiFFs $D_{q',s'}^{h_1,h_2}$
Dihadron FFs: pQCD definition

\[ P \equiv P_h = P_1 + P_2, \]
\[ R = \frac{1}{2}(P_1 - P_2), \]
\[ z = z_1 + z_2, \]
\[ \xi = \frac{z_1}{z} = 1 - \frac{z_2}{z} \]
\[ z_i = P_i^- / k^- \]
\[ k_T = -\frac{P_1}{z}, \]
\[ R_T = \frac{z_2 P_{1\perp} - z_1 P_{2\perp}}{z} = (1 - \xi)P_{1\perp} - \xi P_{2\perp}. \]

\[ R_T^2 = \xi(1 - \xi)M_h^2 - M_1^2(1 - \xi) - M_2^2\xi \]

\[ \Delta_{ij}(k; P_1, P_2) = \sum_x \int d^4\xi e^{ik\cdot\xi} \langle 0 | \psi_i(\xi) | P_1 P_2, X \rangle \langle P_1 P_2, X | \psi_j(0) | 0 \rangle. \]

\[ \Delta^F(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) = \frac{1}{4z} \int dk^+ \text{Tr}[\Gamma(\Delta(k, P_1, P_2))]_{k^- = P^-_1 / z}. \]

\[ \Delta[\gamma^{-}] = D_1(z, \xi, k_T^2, R_T^2, k_T \cdot R_T), \]
\[ \Delta[\gamma^{-}\gamma_5] = \frac{\epsilon_{ij}^{\gamma} R_T^j k_T^i}{M_1 M_2} G_1(z, \xi, k_T^2, R_T^2, k_T \cdot R_T), \]
\[ \Delta[i\sigma^{-}\gamma_5] = \frac{\epsilon_{ij}^{\sigma} R_T^j k_T^i}{M_1 + M_2} H_1^\perp(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) \]
\[ + \frac{\epsilon_{ij}^{\sigma} R_T^j k_T^i}{M_1 + M_2} H_1^\perp(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) \]
Number density distribution in quark to 2h fragmentation

<table>
<thead>
<tr>
<th>q pol.</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>DiFF</td>
<td>$D_1$</td>
<td>$G_1^\perp$</td>
<td>$H_1^\times, H_1^\perp$</td>
</tr>
</tbody>
</table>

Unpolarized DiFF

Longitudinal handedness

Interference DiFF (IFF)

Collins-like DiFF

$$F (z, \xi, k_T, R_T; s) = D_1 (z, \xi, k_T^2, R_T^2, \cos (\varphi_{RK}))$$

$$-s_L \frac{R_T k_T \sin (\varphi_{RK})}{M_1 M_2} G_1^\perp (z, \xi, k_T^2, R_T^2, \cos (\varphi_{RK}))$$

$$+s_T \frac{R_T \sin (\varphi_R - \varphi_S)}{M_1 + M_2} H_1^\times (z, \xi, k_T^2, R_T^2, \cos (\varphi_{RK}))$$

$$+s_T \frac{k_T \sin (\varphi_k - \varphi_S)}{M_1 + M_2} H_1^\perp (z, \xi, k_T^2, R_T^2, \cos (\varphi_{RK}))$$

$$\cos (\varphi_{RK}) \equiv \cos (\varphi_R - \varphi_k)$$

$$k_T = -\frac{P_{h^\perp}}{z}$$
Fourier moments of DiFFs

\[ D_1(z, \xi, k_T^2, R_T^2, \cos(\varphi_{KR})) = \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\cos(n \cdot \varphi_{KR})}{1 + \delta_{0,n}} D_1^{[n]}(z, \xi, |k_T|, |R_T|), \]

\[ F^{[n]} = \int d\varphi_{KR} \cos(n\varphi_{KR}) F\left(\cos(\varphi_{KR})\right) \]

We define Fourier moments of integrated over pair total momentum weighted DiFFs and

\[ D_1^a(z, M_h^2) = z^2 \int d^2k_T \int d\xi \ D_1^{a,[0]}(z, \xi, k_T^2, R_T^2) \]

\[ G_1^{\perp,a,[n]}(z, M_h^2) = z^2 \int d^2k_T \int d\xi \ \left(\frac{k_T^2}{2M_h^2}\right) \frac{|R_T|}{M_h} G_1^{\perp,a,[n]}(z, \xi, k_T^2, R_T^2) \]

\[ H_1^{\perp,a,[n]}(z, M_h^2) = z^2 \int d^2k_T \int d\xi \ \frac{|R_T|}{M_h} H_1^{\perp,a,[n]}(z, \xi, |k_T|, |R_T|) \]

\[ H_1^{\perp,\perp,a,[n]}(z, M_h^2) = z^2 \int d^2k_T \int d\xi \ \frac{|k_T|}{M_h} H_1^{\perp,\perp,a,[n]}(z, \xi, |k_T|, |R_T|) \]
Access to transversity $h_1$ in 2h SIDIS


- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized and Interference*

Dihadron FFs are needed!

$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \sin(\phi_R + \phi_S) \sum_q e_q^2 h_1^q(x) H_1^{\perp q}(z, M_h^2) \sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)$$

$$H_{1,SIDIS}^{\perp}(z, M_h^2) = H_1^{\perp[0]}(z, M_h^2)$$


$$H_{1,SIDIS}^{\perp}(z, M_h^2) = H_1^{\perp[0]}(z, M_h^2) + H_1^{\perp[1]}(z, M_h^2)$$
DiFFs in 2h,2h SIA

\[
\frac{d\sigma(e^+e^- \to (h_1,h_2)(\bar{h}_1,\bar{h}_2)X)}{dq_Tdzd\xi d\phi_T dz d\phi d\phi'} = \sum_{a,a'} c_a^2 \alpha^2 Q^2 z^2 \left[ A(y)F[D^2 D_t^2] + \cos(2\phi_1)B(y)F \left( \frac{2h \cdot k_T h \cdot k_T - k_T^2}{(M_1 + M_2)(M_1 + M_2)} \right) \right] \\
- \sin(2\phi_1)B(y)F \left( \frac{h \cdot k_T h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) \cos(\phi_k + \phi_k' - 2\phi') \\
\times B(y)|R_T||\bar{R}_T|F \left( \frac{h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) + \cos(\phi_k + \phi_k' - 2\phi')B(y)|R_T|F \left( \frac{h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) \\
- \sin(\phi_k + \phi_k' - \phi')B(y)|R_T|F \left( \frac{h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) + \cos(\phi_k + \phi_k' - \phi')B(y)|R_T| \\
\times F \left( \frac{h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) - \sin(\phi_k + \phi_k - \phi')B(y)|R_T|F \left( \frac{h \cdot k_T}{(M_1 + M_2)(M_1 + M_2)} \right) + \Lambda(y)|R_T|R_T| \\
\times \left( \sin(\phi_k - \phi_k' + \phi') \sin(\phi_k + \phi_k + \phi') \cos(\phi_k - \phi_k' + \phi') + \sin(\phi_k - \phi_k + \phi') \cos(\phi_k - \phi_k + \phi') \right) \\
\times F \left( \frac{G_{1,a}^2 G_{1,a}^1}{M_2 M_1 M_1 M_2} \right) + \cos(\phi_k - \phi_k + \phi') \sin(\phi_k - \phi_k + \phi') \cos(\phi_k - \phi_k + \phi') \\
\times F \left( \frac{G_{1,a}^2 G_{1,a}^1}{M_2 M_1 M_1 M_2} \right) + \cos(\phi_k - \phi_k + \phi') \sin(\phi_k - \phi_k + \phi') \cos(\phi_k - \phi_k + \phi') \\
\times (19)
\]
Access to IFF and handedness DiFF in SIA: weighted asymmetries


\[
A^{\cos(\varphi_R+\varphi_R)} \sim \frac{H_1^{\perp}(z, M_h^2) H_1^{\perp}(\bar{z}, M_h^2)}{D_1(z, M_h^2) D_1(\bar{z}, M_h^2)}
\]

\[
A^{\cos(2(\varphi_R-\varphi_R))} \sim \frac{G_1^{\perp}(z, M_h^2) G_1^{\perp}(\bar{z}, M_h^2)}{D_1(z, M_h^2) D_1(\bar{z}, M_h^2)}
\]

\[
H_{1,e^+e^-}^{\perp}(z, M_h^2) = H_1^{\perp[0]}(z, M_h^2) \neq H_{1,SIDIS}^{\perp}(z, M_h^2)
\]

\[
G_{1,e^+e^-}^{\perp}(z, M_h^2) = G_1^{\perp[0]}(z, M_h^2)
\]

PRL 107 (2011) 072004 (IFF)
BELLE results
arXiv:1505.08020 (handedness)
Model calculation of FFs and DiFFs

Complete self consistent formalism for spin dependent TMD FFs:
Bentz, AK, Matevosyan, Ninomiya, Thomas, Yazaki: PR D94, 034004 (2016)

MC implementation: Matevosyan, AK, Thomas: One hadron production, PRD 95, 014021 (2017)
Two hadron production: Longitudinally polarized quark, PRD 96, 074010 (2017)
Two hadron production: Transversely polarized quark, PRD 97, 014019 (2018)

All 8 elementary quark-to-quark spin-dependent TMD FFs are taken into account

\[ H_{1,\text{SIDIS}}^\perp \approx 2 H_{1,e^+e^-}^\perp \left( z, M_h^2 \right) \]

\[ G_I^\perp \approx \frac{1}{2} H_{1,e^+e^-}^\perp \left( z, M_h^2 \right) \]
Rederiving dihadron production cross-sections in $e^+e^-$ and SIDIS


Fully differential cross section

\[
\frac{d\sigma(e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X)}{d^2q_Tdzd\xi d\varphi_R dM_h^2 d\bar{z}d\bar{\xi} d\bar{\varphi}_R d\bar{M}_h^2 dy} = \frac{3\alpha^2}{\pi Q^2 z^2 \bar{z}^2} \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \mathcal{F}\left[D_1^a \bar{D}_1^{\bar{a}}\right] + B(y) \mathcal{F}\left[\frac{k_T}{M_h} \frac{\bar{k}_T}{\bar{M}_h} \cos(\varphi_k + \varphi_{\bar{k}}) \bar{H}_1^{\bar{1}a} \bar{H}_1^{1\bar{a}}\right] \\
+ B(y) \mathcal{F}\left[\frac{R_T}{M_h} \frac{\bar{R}_T}{\bar{M}_h} \cos(\varphi_R + \varphi_{\bar{R}}) H_1^{1a} \bar{H}_1^{\bar{1}a}\right] + B(y) \mathcal{F}\left[\frac{k_T}{M_h} \frac{\bar{k}_T}{\bar{M}_h} \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{1a} \bar{H}_1^{\bar{1}a}\right] \\
+ B(y) \mathcal{F}\left[\frac{R_T}{M_h} \frac{\bar{R}_T}{\bar{M}_h} \cos(\varphi_R + \varphi_{\bar{R}}) H_1^{1a} \bar{H}_1^{\bar{1}a}\right] - A(y) \mathcal{F}\left[\frac{R_T}{M_h^2} \frac{\bar{R}_T}{\bar{M}_h^2} \sin(\varphi_k - \varphi_R) \sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{1a} \bar{G}_1^{\bar{1}a}\right]\right\}
\]

\[
\mathcal{F}[w D^a \bar{D}^{\bar{a}}] = \int d^2k_T d^2\bar{k}_T \delta^2(k_T + \bar{k}_T - q_T) w(k_T, \bar{k}_T, R_T, \bar{R}_T) D^a(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) D^{\bar{a}}(\bar{z}, \bar{\xi}, \bar{k}_T^2, \bar{R}_T^2, \bar{k}_T \cdot \bar{R}_T)
\]
IFFs in $e^+e^-$ and SIDIS

- The asymmetry now involves exactly the same integrated IFF as in SIDIS!

$$A^{\cos(\varphi_R + \varphi_{\bar{R}})} = \frac{1}{2} \frac{B(y)}{A(y)} \frac{e_a^2 H_1^{<a}(z, M_h^2) \bar{H}_1^{<\bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_{a,\bar{a}} e_a^2 D_1^{a}(z, M_h^2) \bar{D}_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$

$$D_1(z, M_h^2) \equiv z^2 \int d^2 k_T \int d\xi D_1^{[0]}(z, \xi, |k_T|, |R_T|)$$

$$H_1^{<,e^+e^-}(z, M_h^2) = H_1^{<,[0]} + H_1^{\perp,[1]} \equiv H_1^{<,SIDIS}(z, M_h^2)$$

- All the previous extractions of the transversity are valid!
Handedness DiFF in $e^+e^-$


**The relevant terms involving $G_{1}^\perp$:**

$$d\sigma_L \sim \mathcal{F}\left[\frac{(R_T \times k_T)_3}{M_h^2} \frac{(\bar{R}_T \times \bar{k}_T)_3}{\bar{M}_h^2} G_{1}^{\perp a} (R_T \cdot k_T) \bar{G}_{1}^{\perp \bar{a}} (\bar{R}_T \cdot \bar{k}_T)\right]$$

Weighting break-up the convolution

$$\langle T \rangle \equiv \int d\xi \int d\bar{\xi} \int d\varphi_R \int d\varphi_{\bar{R}} \int d^2 q_T \int d^2 q_{\bar{T}} \frac{d\sigma(e^+e^- \rightarrow (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{d^2 qTd\xi d\bar{\xi} d\varphi_R d\varphi_{\bar{R}} dM_h^2 d\bar{M}_h^2 dy}$$

$$\langle f(\varphi_R, \varphi_{\bar{R}}) \rangle_L = 0$$

**The old asymmetry by Boer et. al. exactly vanishes!**

**Explains the BELLE results.**

$$A \Rightarrow = \frac{\langle \cos(2(\varphi_R - \varphi_{\bar{R}})) \rangle}{\langle 1 \rangle} = 0!$$

C PHI-2018, 23-Sep-18, Yerevan, Armenia

Aram Kotzinian
New weight to access handedness DiFF in $e^+e^-$


\[
q_T^2 \left( 3 \sin(q_R) \sin(q_\overline{R}) + \cos(q_R) \cos(q_\overline{R}) \right) / M_h \bar{M}_h
\]

\[
= \frac{12\alpha^2 A(y)}{\pi Q^2} \sum_{a,\bar{a}} e_a^2 \left( G_1^{a,[0]} - G_1^{\bar{a},[2]} \right) \left( \bar{G}_1^{\bar{a},[0]} - G_1^{\bar{a},[2]} \right)
\]

\[
A^{e^+e^-}_R (z, \bar{z}, M_h^2, \bar{M}_h^2) = 4 \frac{\sum_{a,\bar{a}} G_1^{a} (z, M_h^2) G_1^{\bar{a}} (\bar{z}, \bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^a (z, M_h^2) D_1^{\bar{a}} (\bar{z}, \bar{M}_h^2)}
\]

\[
G_1^{\perp a} (z, M_h^2) \equiv G_1^{\perp a,[0]} (z, M_h^2) - G_1^{\perp a,[2]} (z, M_h^2)
\]
Our MC model results

$G_1^\perp$ naturally smaller than $H_1^\perp$, but should be measurable!

$G_1^\perp (z, M_h^2) \equiv G_1^{\perp a,[0]}(z, M_h^2) - G_1^{\perp a,[2]}(z, M_h^2)$

$z_{1,2} \geq 0.1$

$u \rightarrow \pi^+ \pi^-, N_L = 6$
Fracture function $M$ is a Conditional Probability Distribution Function (CPDF) to observe the hadron $h$ produced in nucleon flight direction when hard probe interacts with parton carrying fraction $x$ of nucleon momentum.
Quark transverse spin in hard $l$-$q$ scattering

Nucleon and initial quark spin

$S_T$ \hspace{2cm} $\gamma^* N$ CMS

$\phi_s$ \hspace{2cm} $\phi_{s'}$

Final quark spin $S'_T$ \hspace{2cm} CFR

QED: $lq \rightarrow l'q' \Rightarrow s'_T = D_{nn}(y)s_T$, $D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$, $\phi_{s'} = \pi - \phi_s$

$[s'_T \times p_T] \propto \sin(\phi_h - \phi_{s'}) = -\sin(\phi_h + \phi_s)$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.

→ No access to quark transverse polarization dependent fracture functions.

No Collins like modulation.
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda) + N(P_N, S) \rightarrow \ell(l') + h(P) + X}}{dx dQ^2 d\phi_S d\zeta d^2 P_T} = M_{q,s/N,S}^h \otimes \frac{d\sigma^{\ell(l,\lambda) + q(k,s) \rightarrow \ell(l') + q(k',s')}}{dQ^2}$$

$$\zeta \approx \frac{P^-}{P_N} \approx x_F (1 - x)$$
Karl Linney: plants classification

Plants were divided by it into 24 classes and 116 groups on the basis of features of a structure of their reproductive organs.

For STMD Fracture Functions I was expecting 32 (Trentadue) independent structures. Fortunately for unpolarized hadron production we end up with only 16 of them at twist-two.
At LO 16 independent STMD fracture functions. Probabilistic interpretation at LO: the conditional probabilities to find an unpolarized, a longitudinally polarized or a transversely polarized quark with longitudinal momentum fraction $x_B$ and transverse momentum $k_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction $\zeta$ of the nucleon longitudinal momentum and a transverse momentum $P_{h\perp}$. 

$$\mathcal{M}^{[\Gamma]}(x_B, \vec{k}_\perp, \zeta, \vec{P}_{h\perp}) = \frac{1}{4\zeta} \int \frac{d^2 \zeta}{(2\pi)^6} e^{i(x_B P^+ - \vec{k}_\perp \cdot \vec{\zeta})} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times \langle P, S | \bar{\psi}(0) \Gamma | P_h, S_h; X \rangle \langle P_h, S_h; X | \psi(\zeta^+, 0, \vec{\zeta}_\perp) | P, S \rangle$$

$$\Gamma = \gamma^-, \gamma^- \gamma_5, \ i\sigma^i \gamma_5$$
### STMD Fracture Functions for spinless hadron production

**Quark polarization**

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$\hat{U}_1$</td>
<td>$\frac{k_T \times P_T}{m_N m_h} \hat{i}^h_1$</td>
<td>$\epsilon^{ij}_T P_T^j \frac{\hat{i}^h_1}{m_h} + \epsilon^{ij}_T k_T^j \frac{\hat{i}^\perp_1}{m_N}$</td>
</tr>
<tr>
<td>L</td>
<td>$S_L \frac{(k_T \times P_T)}{m_N m_h} \hat{u}^\perp_{1L}$</td>
<td>$S_L \hat{T}$</td>
<td>$S_L \frac{P_T}{m_h} \hat{i}^h_1 + S_L \frac{k_T}{m_N} \hat{i}^\perp_1$</td>
</tr>
<tr>
<td>T</td>
<td>$\frac{P_T \times S_T}{m_h} \hat{u}^h_{1T}$</td>
<td>$\frac{P_T \cdot S_T}{m_h} \hat{i}^h_{1T}$</td>
<td>$S_T \hat{i}^h_{1T}$ + $\frac{P_T (P_T \cdot S_T)}{m_h^2} \hat{i}^{hh}<em>{1T}$ + $\frac{k_T (k_T \cdot S_T)}{m_N^2} \hat{i}^{\perp\perp}</em>{1T}$ + $\frac{P_T (k_T \cdot S_T) - k_T \cdot (P_T \cdot S_T)}{m_N m_h} \hat{i}^{\perp h}_{1T}$</td>
</tr>
</tbody>
</table>

STMD fracture functions depend on $x, k_T^2, \zeta, P_T^2, k_T \cdot P_T$

$k_T \cdot P_T = k_T P_T \cos(\phi_h - \phi_q)$

azimuthal dependence in fracture functions
LO cross-section for single hadron production in TFR

\[
\frac{d\sigma}{dxdQ^2 d\phi_S d\zeta d^2P_T} (x_F < 0) = \frac{\alpha^2 x}{yQ^4} \left(1 + (1-y)^2\right) \sum q e_q^2 \times
\]

\[
\left[ \tilde{u}_1(x, \zeta, P_T^2) - \frac{P_T}{m_h} \tilde{u}_{1T}^h(x, \zeta, P_T^2) \sin(\phi_h - \phi_S) +
\right]
\]

\[
\times \lambda y(2-y) \left( S_L \tilde{l}_{1L}(x, \zeta, P_T^2) + S_T \frac{P_T}{m_h} \tilde{l}_{1T}^h(x, \zeta, P_T^2) \cos(\phi_h - \phi_S) \right)
\]

At LO only 4 terms out of 18 Structure Functions,
Only 2 azimuthal modulations

No Collins-like \(\sin(\phi_h+\phi_S)\) modulation

No access to quark transverse polarization
Double hadron production in DIS (DSIDIS): TFR & CFR

\[ x_{F2} < 0, \quad x_{F1} > 0 \]

\[ \frac{d\sigma}{dx dQ^2 d\phi_s dz d^2 P_{T1} d\zeta d^2 P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma}{dQ^2} \otimes D^{h_1}_{q,s'} \]

\[ D^{h_i}_{q,s'}(z, p_T) = D_1(z, P_T^2) + \frac{p_T \times s'}{m_h} H_1(z, P_T^2) \]
Unintegrated DSIDIS cross-section: accessing quark polarization

\[
\frac{d\sigma_{\ell(l', \lambda)+N(P_N, S)\rightarrow \ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx dQ^2 d\phi_S dz d^2P_{T1} d\zeta d^2P_{T2}} =
\]

\[
= \frac{\alpha^2 x}{Q^4 y} \left(1 + (1 - y)^2\right) \left(\hat{u}^{h_2} \otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{t}^{h_2} \otimes D_1^{h_1} \right)
\]

\[
+ \hat{t}^{h_2} \otimes \frac{p_T \times s'_T}{m_{h_1}} H_1^{h_1}
\]

\[
= \frac{\alpha^2 x}{Q^4 y} \left(1 + (1 - y)^2\right) \left(\sigma_{UU} + S_L \sigma_{UL} + S_T \sigma_{UT} + \lambda D_{ll} \left(\sigma_{LU} + S_L \sigma_{LL} + S_T \sigma_{LT}\right)\right)
\]

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms, similarly to 1h SIDIS cross section.
Twist-2 $A_{LU}$ asymmetry in DSIDIS

AK @ DIS2011,
Anselmino, Barone and AK, PLB 713 (2012) 317

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2m_N} F_{k1}^{\hat{i}_{1h} \cdot D_1} \sin(\phi_1 - \phi_2)$$

$F^{\hat{u} \cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(P_{T1} \cdot P_{T2})$

$P_{T1} \cdot P_{T2} = P_{T1}P_{T2} \cos(\Delta \phi)$, with $\Delta \phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta \phi$ and $\phi_2$ ($\phi_1 = \Delta \phi + \phi_2$)

Integrating $\sigma_{UU}$ and $\sigma_{iU}$ over $\phi_2$ we obtain

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = -\frac{P_{T1}P_{T2}}{m_2m_N} F_{k1}^{\hat{i}_{1h} \cdot D_1} \left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta \phi)\right) \sin(\Delta \phi)$$

$$\frac{\int d\phi_2 \sigma_{UU}}{\int d\phi_2 \sigma_{UU}} = F_0^{\hat{u} \cdot D_1} \left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta \phi)\right)$$
Presence of higher harmonics indicate that $\sigma_{LU}(\Delta \Phi) \neq \sigma_{UU}(\Delta \Phi)$

Courtesy of S. Pisano & H. Avakian (unpublished 😔)
Conclusions

• Azimuthal correlations in dihadron production in SIDIS and SIA provide a new way to study nucleon structure and hadronization process

• In our recent work
  • The inconsistency between IFF definitions in SIDIS and SIA was resolved
  • The BELLE zero result in quark handedness TMD FF study was explained
  • New weighted asymmetries are proposed for measurement of this FFs both in SIDIS and SIA

• To describe TFR of SIDIS 16 LO spin-dependent TMD fracture functions

• For one hadron in TFR SIDIS SSA contains only a Sivers-type modulation.
  • Observation of Collins-type SSA will indicate that LO factorized approach fails
    • Indication of long range correlation between the struck quark polarization and $P_T$ of produced in TFR hadron might be important
  • Preliminary data from JLab show nonzero $A_{LU}$

• We expect more news for JLab 12 and EIC
adds
Collinear Frac.Func.: application to HERA data, 1

\[
\frac{d^3 \sigma_{\text{target}}^p}{d \beta d Q^2 dx_p} = \frac{4 \pi \alpha^2}{\beta Q^4} \left( 1 - \frac{y^2}{2} \right) M_p^b(\beta, Q^2, x_p), \quad \beta = \frac{x}{1 - \zeta}, \quad \zeta = \frac{p_h^+}{p_N^+}
\]

\[
x_M^{b/p}(\beta, Q_0^2, x_p) = N_s \beta^{a_s}(1 - \beta)^{b_s} \left\{ C_p \beta x_p^{\alpha_p} + C_{LP}(1 - \beta)^{\gamma_{LP}} \right\} [1 + a_{LP}(1 - x_p)^{\beta_{LP}}]
\]

**FIG. 2.** H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

**FIG. 8.** ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).
Collinear Frac.Func.: application to HERA data, 2

Shoeibi et al., Neutron fracture functions. PRD 95, 074011 (2017)
\[ \sigma_{UU} = F_0^{\Delta D} - D_{nm} = \left( \frac{P^2_{T1}}{m_1 m_N} F_{k_1 p_1}^{\Delta H_1} \cos(2\phi) \right) - \left( \frac{P^2_{T1}}{m_1 m_2} F_{k_{p1}}^{\Delta H_1} \cos(\phi_1 + \phi_2) \right) + \left( \frac{P^2_{T2}}{m_1 m_N} F_{k_{p2}}^{\Delta H_1} + \frac{P^2_{T2}}{m_1 m_2} F_{p_2}^{\Delta H_1} \right) \cos(2\phi_2) \]

\[ D_{nm}(y) = \frac{2(1-y)}{1 + (1-y)^2} \]

\[ F_k^{\Delta D} = C \left[ \frac{\hat{M} \cdot D \left( (P_{T1} \cdot P_{T2})(P_{T2} \cdot k) - (P_{T1} \cdot k)P^2_{T2} \right)}{(P_{T1} \cdot P_{T2})^2 - P^2_{T1} P^2_{T2}} \right] \]

\[ C[\hat{M} \cdot Dw] = \sum_a e^2_a \int d^2k_T d^2p_T \delta^{(2)}(z k_T + p_T - P_{T1}) \hat{M}_a(x, \zeta, k^2_T, P^2_{T2}, k_T \cdot P_{T2}) D_a(z, p^2_T) \]

Structure functions \( F_{\Delta D} \) depend on \( x, z, \zeta, P^2_{T1}, P^2_{T2} \) and \( P_{T1} \cdot P_{T2} \)

\[ P_{T1} \cdot P_{T2} = P^2_{T1} P^2_{T2} \cos(\Delta \phi), \text{ with } \Delta \phi = \phi_1 - \phi_2 \]
\[ \sigma_{LU} = - \frac{P_{T1}P_{T2}}{m_2m_N} F_{k1}^{i_{1T}} \cdot D_1 \sin(\phi_1 - \phi_2) \]

\[ \sigma_{LL} = F_0^{i_{1}} \cdot D_1 \]

\[ \sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{i_{1T}} \cdot D_1 \cos(\phi_1 - \phi_S) \]

\[ + \left( \frac{P_{T2}}{m_2} F_0^{i_{1T}} \cdot D_1 + \frac{P_{T2}}{m_N} F_{k2}^{i_{1T}} \cdot D_1 \right) \cos(\phi_2 - \phi_S) \]
\[ \sigma_{UL} = - \frac{P_T^1 P_T^2}{m_2 m_N} F_{k_1}^{\hat{L} h} \cdot D_{1} \sin(\phi_1 - \phi_2) + D_{nn} \begin{pmatrix} \frac{P_T^2}{m_1 m_N} F_{k_p 1}^{\hat{L} h} \cdot H_1 \sin(2\phi_1) + \frac{P_T^2}{m_1 m_2} F_{p_1}^{\hat{L} h} \cdot H_1 \sin(\phi_1 + \phi_2) + \left( \frac{P_T^2}{m_1 m_N} F_{k_p 2}^{\hat{L} h} \cdot H_1 + \frac{P_T^2}{m_1 m_2} F_{p_2}^{\hat{L} h} \cdot H_1 \right) \sin(2\phi_2) \end{pmatrix} \]
\[ \sigma_{UT} = -\frac{P_{T1}}{m_N} F_{k1}^{\phi_{PT}} \sin(\phi_1 - \phi_2) \]

\[- \left( \frac{P_{T2}}{m_2} F_{0}^{\phi_{PT}} + \frac{P_{T2}}{m_3} F_{k2}^{\phi_{PT}} \right) \sin(\phi_2 - \phi_3) \]

\[ \left( \frac{P_{T1}^2}{m_1^2} F_{p1}^{\phi_{PT}} + \frac{P_{T1}^2}{2m_2^2} F_{p1}^{\phi_{PT}} - \frac{P_{T1}^2 P_{T2}}{2m_2 m_N} F_{kp3}^{\phi_{PT}} + \frac{P_{T1}^2 P_{T2}^2}{m_2 m_N} F_{kp3}^{\phi_{PT}} \right) \sin(\phi_1 + \phi_3) \]

\[ + \frac{P_{T1}^2}{2m_2 m_N} F_{kp1}^{\phi_{PT}} \sin(3\phi_1 - \phi_3) \]

\[ + D_{\sin}(y) \]

\[ \left( \frac{P_{T2}^3}{2m_2 m_N} F_{p2}^{\phi_{PT}} + \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp2}^{\phi_{PT}} + \frac{P_{T1}^2}{2m_2 m_N} F_{kp2}^{\phi_{PT}} + \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp2}^{\phi_{PT}} + \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp2}^{\phi_{PT}} \right) \sin(3\phi_2 - \phi_3) \]

\[ + \left( \frac{P_{T1}^2 P_{T2}}{2m_2 m_N} F_{pli}^{\phi_{PT}} + \frac{P_{T1}^2 P_{T2}}{2m_2 m_N} F_{kpli}^{\phi_{PT}} \right) \sin(3\phi_2 - \phi_3) \]

\[ + \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp4}^{\phi_{PT}} \sin(2\phi_2 - \phi_3) \]

\[ - \frac{P_{T1}^2 P_{T2}}{2m_2 m_N} F_{kp4}^{\phi_{PT}} \sin(2\phi_1 + \phi_3) \]

\[ - \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp3}^{\phi_{PT}} \sin(\phi_2 - 2\phi_3 - \phi_5) \]

\[ + \frac{P_{T1}^2 P_{T2}^2}{2m_2 m_N} F_{kp4}^{\phi_{PT}} \sin(2\phi_1 + \phi_3 - \phi_5) \]