Elastic Electron Scattering
for Proton Charge Radius Determination

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for ULQ² (Ultra-Low Q²) Collaboration

Miyazaki Univ. : Y. Maeda
$E_e = 20 - 60 \text{ MeV}$
Short-Lived Exotic Nuclei
world’s highest intensities of exotic beams (2007 ~ )
in-flight fragmentation of U
E = 350 MeV/A
I ~ 1 pμA
size and shape of neutro- and proton-rich nuclei

\[
<r_c^2> = \int r^2 \rho_c(r) \, d\vec{r}
\]

\[
\rho_c(\vec{r}) = \sum_p \psi^*(\vec{r}) \psi(\vec{r})
\]

<table>
<thead>
<tr>
<th></th>
<th>size</th>
<th>shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>isotope shift</td>
<td>electron scattering</td>
</tr>
<tr>
<td>matter</td>
<td>reaction cross section</td>
<td>proton scattering</td>
</tr>
</tbody>
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Electron scattering off short-lived exotic nuclei

H.deVries, C. deJager and C. deVries
Atomic Data and Nuclear Data Tables 36 (987)495

Nuclei targeted so far for electron scattering

Short-lived Exotic Nuclei
Production-hard + Short-lived

Elastic electron scattering

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\text{Mott}}}{d\Omega} |F_c(q)|^2
\]

\[
F_c(q) = \int \rho_c(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d\vec{r}
\]

\[
\rho_c(\vec{r}) = \sum_p \psi_p^*(\vec{r}) \psi_p(\vec{r})
\]

SCRIT (Self-Confining RI ion Target)

\[L \sim 10^{27}/\text{cm}^2/\text{s} \text{ with only } \sim 10^8 \text{ target nuclei}\]

Expected low luminosities

Charge density distribution
Charge radius
SCRIT (Self-Confining RI Ion Target): ion trapping

~10^8 ions are trapped on e-beam (~1 mm^2)
N_t ~10^8 /mm^2 => 10^{10} /cm^2

<table>
<thead>
<tr>
<th>Luminosities</th>
<th>Ee</th>
<th>N_{beam}</th>
<th>\rho \cdot t</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hofstadter’s era (1950s)</td>
<td>150 MeV</td>
<td>~1 nA (~10^9 /s)</td>
<td>~10^{19} /cm^2</td>
<td>~10^{28} /cm^2/s</td>
</tr>
<tr>
<td>JLAB</td>
<td>6 GeV</td>
<td>~100 \mu A (~10^{14} /s)</td>
<td>~10^{24} /cm^2</td>
<td>~10^{38} /cm^2/s</td>
</tr>
<tr>
<td>SCRIT</td>
<td>150 - 300 MeV</td>
<td>~200 mA (~10^{18} /s)</td>
<td>~10^{10} /cm^2</td>
<td>~10^{27} /cm^2/s</td>
</tr>
</tbody>
</table>
SCRIT facility in RIKEN/RI Beam Factory

world’s first electron scattering facility for exotic nuclei
The SCRT Electron Scattering Facility at RIKEN: The World's First Electron Femtoscope for Short-Lived Unstable Nuclei

A. Enokizono, T. Ohnishi & K. Tsukada


To link to this article: https://doi.org/10.1080/10619127.2018.1427951
Proton Charge Radius
by
Elastic Electron Scattering
Why is the proton radius a hot topics?

1) the radius is one of the basic properties of the nucleon

2) the radius is strongly correlated to the Rydberg constant

\[ \Delta E = R_{\text{Rydberg}} \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \]

\[ \Delta E = \alpha \cdot R_{\text{Rydberg}} + \beta \cdot <r^2> \]

\[ R_\infty = 10973 \ 731.568 \ 539 \pm 0.000 \ 055 \ \text{m}^{-1} \]

3) possible new physics beyond Standard Model (??)

Lepton Universality (e <-> \mu) ??

4) the neutron-skin thickness of neutron-rich nuclei

=> EOS of neutron matter
Charge radius and charge density

\[
<r^2> = \int r^2 \rho(r) \, dr = 4\pi \int r^4 \rho(r) \, dr
\]

Proton \(\sim 0.8 \text{ fm}\)

\[
\int_0^\infty r^2 \rho(r) \, dr = \text{Proton Charge integration}
\]

\[
\int_0^\infty \frac{r^2 \rho(r) \, dr}{\int_0^\infty r^2 \rho(r) \, dr}
\]

Mainz (2014)

\text{Ee} \geq 180 \text{ MeV}

PRC 90 (2014) 015206.

\[
G_E(Q^2) \sim 1 - \frac{<r^2>^{1/2}}{6} Q^2 + \frac{<r^4>^{1/2}}{120} Q^4 - \ldots
\]

\[
<r^2> \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}
\]

Proportion = 98%
Absolute $G_E(Q^2)$ at lower $Q^2$ region

1) no absolute $G_E(Q^2)$
2) $\chi^2$ is quite similar ("floating")

$$<r^2> = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

Figure 5. (a) Padé fit (solid) together with the "standard" fit having $\chi^2 \lesssim 0.88$ fm. (b) Density corresponding to the Padé fit.

This educational example demonstrates that it is important to examine the density implied by the parameterized $G(q)$. In $r$-space, the outrageous behavior of the Padé fit is immediately visible (see Figure 5). The peculiar nature of the fit results from the correlation between $a_1$ and $b_1$, which, when assuming large values, can generate the behavior shown in Figure 5.

There are other examples in the literature that emphasize the importance of considering $r(r)$ at the same time. Bernauer et al. [63], for instance, make an inverse polynomial fit to their data ($q_{\text{max}} \lesssim 5$ fm).

The resulting values for $\chi^2$ as a function of the order of the polynomial are plotted in Figure 6. The jump of $\chi^2$ at order 10 (not used for the determination of $\chi^2$) results from a pole of $G(q)$, which happens close to the $q_{\text{max}}$ of the data. Such a form factor with a pole corresponds to a density that shows large-amplitude oscillations out to very large values of $r$ [87], which of course affect $\chi^2$.

A look at the density would have immediately revealed the unphysical nature of the form factor fit.

Figure 6. Charge and magnetic rms-radius from the inverse polynomial fit, together with the $c^2$ per degree of freedom (right-hand scale) [63].

The lesson from the above examples is that it is important to check on the behavior of the density implied by the chosen $G(q)$. The most important corollary is that it is very dangerous to employ $<r^2> \rightarrow 6 dG_E(Q^2)/dQ^2 |_{Q^2 \to 0}$ as a toy model for larger radius.
The goal of this project

\( G_E(Q^2) \) measurements at \( 0.0003 \leq Q^2 \leq 0.008 \,(\text{GeV}/c)^2 \)

Absolute cross section measurement with \( 10^{-3} \) precision

Rosenbluth separated \( G_E(Q^2), G_M(Q^2) \)

Exp. @ Tohoku Low-Energy Electron Linac (\( E_e = 20 - 60 \,\text{MeV} \))

只駄 ~ 2%
ULQ\(^2\) collaboration (Ultra-Low Q\(^2\))

Tohoku Univ.
Sendai
1.3 GeV Booster Ring
- tagged photons (~1 GeV)
- meson photoproduction, hypernucleus

60 MeV electron linac
- ~10 kW electron beam (150 uA)
- Radioactive Isotope photo-production
e-scattering off proton at ultra-low $Q^2$ region

Goal of our experiment

$G_E(Q^2)$ measurements in $0.0003 \leq Q^2 \leq 0.008$ (GeV/c)$^2$

Our experiments

Low-energy electron scattering
Absolute cross section measurement
Rosenbluth separation ($G_E(Q^2)$, $G_M(Q^2)$)

accelerator, instruments

Tohoku low-energy electron linac + experimental hall

$20 \leq E_e \leq 60$ MeV
$30 \leq \theta \leq 150^\circ$
$\Delta p/p \sim 10^{-3}$

new beam line + double-arm spectrometer

Challenges

Absolute cross section ($G_E(Q^2)$) with $10^{-3}$ accuracy

experimental challenges for measurement
theoretical challenges for interpretation
Proton charge radius by e-scattering

One Photon Exchange Approx.

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \]

\[ \tau = \frac{Q^2}{4m_p^2} \]

\[ \epsilon = \frac{1}{1 + 2(1 + \tau)\tan^2\frac{\theta}{2}} \]

\[ \omega = e - e' \]

\[ Y = e^2 q^2 \sin^2(\theta/2) \]

\[ Q^2 = q^2 - \omega^2 \]

\[ \vec{q} = \vec{e} - \vec{e}' \]

momentum transfer
energy transfer
4 momentum transfer

Beam Energy
- 60MeV
- 50MeV
- 40MeV
- 30MeV
- 20MeV

Proton charge radius by e-scattering

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \]

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\[ Q^2 = q^2 - \omega^2 \]

\[ \vec{q} = \vec{e} - \vec{e}' \]
Absolute cross section ($G_E(Q^2)$) with $10^{-3}$ accuracy

- relative measurement to well-known (established) cross section Moeller cross section: PRAD@JLAB
- large scattering angle coverage for GE/GM separation

$^{12}\text{CH}_2(e,e')$ cross section ULQ$^2$@Tohoku

Low energy electron detection with high resolution
- no tracking, frequent spectrometer setting changes

Ultra Relativistic Limit: $m_e \rightarrow 0$?
- finite effects: up to a few % depending on kinematics

Coulomb distortion effects
- not negligible ($\sim 0.2\%$ level)
New beam line and spectrometers

new beam line
+ new spectrometer(s)
Electron spectrometer (P = 20 - 60 MeV/c)

Low energy: Ee = 20 - 60 MeV
high-resolution without tracking

→ “old-fashioned” spectrometer

**Electron spectrometer**

- **radius**: 500 mm
- **bending angle**: 90°
- **max. B**: 0.4T@60MeV
- **gap**: 70 mm
- **dispersion**: 850 mm
- **Δp/p**: 8×10^{-4}
- **momentum bite**: 10%
- **Δθ**: 5 mrad
- **solid angle**: 10 mSr
Summary

1) elastic e+p scattering at ultra-low $Q^2$ region
2) $G_E(Q^2)$ at $0.0003 \leq Q^2 \leq 0.008 \text{ (GeV/c)}^2$
3) $G_E$ is extracted by Rosenbluth separation
4) Absolute cross section measurement
   relative to $^{12}\text{C}(e,e)^{12}\text{C}$ : sys. err. $\sim 3 \times 10^{-3}$
5) $E_e = 20 - 60 \text{ MeV}, \theta = 30 - 150^\circ$
6) the new beam line, and spectrometer are under construction
7) the experiments will start in 2019