Jefferson Lab’s Contribution to the Characterization of the Proton Form Factors

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First, what are form factors

For a spin-$\frac{1}{2}$ nucleon, and assuming that $ep$ elastic scattering proceeds by the exchange of a single virtual photon:

the finite size of the proton requires 2 form factors to describe the hadron current:

one related to the electric charge and Dirac magnetic moment:

$$F_1$$ (named Dirac form factor),

the other for the anomalous part of the proton magnetic moment,

$$\kappa_p = \mu_p - 1$$, $$F_2$$ (named Pauli form factor).
R. Hofstadter (1961 Nobel) introduced a form factor for the $ep \rightarrow e'p'$ cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{\text{Mott}} \left| \int_{\text{volume}} \rho(r) \exp(iq \cdot r) dr \right|^2 = \sigma_{\text{Mott}} \left| F(q) \right|^2$$

$$F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr$$

$F(q)$ Fourier transform of charge distribution; $(d\sigma/d\Omega)_{\text{Mott}}$ for massless $e^-$ and point-like target, all spin $\frac{1}{2}$.

In lowest order, one virtual photon exchange, or OPEX, amplitude is product of leptonic and hadronic currents:

$$\Gamma^\mu = F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu}q_\nu}{2M}$$

Both $F_1$ (Dirac) and $F_2$ (Pauli) depending on $q$, momentum transfer, only.
Cross Section: the Rosenbluth method

Better to write the Cross sections in terms of the Sachs form factors:

Electric $G_{E_p} = F_1 - \tau F_2$ and

Magnetic $G_{M_p} = F_1 + F_2$ with $\tau = Q^2/4m_p^2$; then:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{\text{Mott}} \left\{ G_{M_p}^2 + \left( \frac{\epsilon}{\tau} \right) G_{E_p}^2 \right\} \tau / \epsilon (1+\tau)$$

with $\epsilon = [1+2(1+\tau)\tan^2(\theta_e/2)]^{-1}$

and $\frac{d\sigma}{d\Omega}_{\text{Mott}} = \frac{\alpha^2}{4E_e^2} \frac{\cos^2\theta}{\sin^4\frac{\theta}{2}}$

$$\sigma_R = \epsilon(1+\tau) \frac{\sigma}{\sigma_{\text{Mott}}} = \epsilon G_{E_p}^2 + \tau G_{M_p}^2$$
By the 1990's $G_{Ep}$ and $G_{Mp}$ showed stability, both $G_{Ep}/G_D$ and $G_{Mp}/\mu_p G_D \approx Q^2$-independent, with $G_D=(1+Q^2/0.71)^{-2}$ the dipole FF.
Recoil Polarization Observables in OPEX


\[ P_t = -hP_e 2\sqrt{\tau(1 + \tau)} \frac{G_{Ep}G_{Mp}}{G_{Ep}^2 + \frac{\tau}{e}G_{Mp}^2} \tan \frac{\vartheta e}{2} \]

\[ P_l = hP_e \frac{E_e + E_{el}}{M} \sqrt{\tau(1 + \tau)} \frac{G_{Mp}^2}{G_{Ep}^2 + \frac{\tau}{e}G_{Mp}^2} \tan^2 \frac{\theta_e}{2} \]

\[ P_n = 0 \]

\[ \frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_l} \frac{E_e + E_{el}}{2M} \tan \frac{\theta_e}{2} \]

\( h = + \text{ or } -1 \) is beam helicity, \( P_e \) beam polarization

most importantly, \( G_E \) and \( G_M \) are measured, not \( G_E^2 \) and \( G_M^2 \)

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In a first proposal (Perdrisat and Punjabi, 1989), we aimed at getting $P_t$ and $P_\ell$, then calculate $P_t/P_\ell$ and use the good $G_{Mp}$ data base to get $G_{Ep}$. In a second submission (1993) we proposed to obtain $G_{Ep}/G_{Mp}$ directly from measured $P_t/P_\ell$, for smaller uncertainties, statistical and systematic. Best way!

Measuring the polarization ratio cancels the radiative corrections, requires no knowledge of the analyzing power of the reaction, and cancels many other fluctuating effects like beam polarization, target density, detector efficiencies… Little did we expect to get $G_{Ep}$ at $Q^2$ of 8.5 GeV$^2$, so we now know that it contributes less than one 2 parts in 1000 to the cross section at that $Q^2$!
\( \theta \) and \( \phi \) polar- and azimuthal angles after re-scattering in analyzer. If \( \epsilon(\theta, \phi) \) efficiency, and \( A_y(\theta) \) analyzing power, then proton detection probability for both beam helicities, \( f^\pm \):

\[
f^\pm(\theta, \phi) = \frac{\epsilon(\theta, \phi)}{2\pi} \{ 1 \pm A_y(\theta)(P_t^{fpp}\cos \phi - P_n^{fpp}\sin \phi) \}
\]
Obtain azimuthal asymmetry difference by flipping beam helicity

\[ f^+ - f^- = \frac{1}{\Delta \phi} \left[ \frac{N^+(\phi)}{N_{in}^+(\phi)} - \frac{N^-(\phi)}{N_{in}^-(\phi)} \right] = A_y(\phi) \left[ P_t^{fpp} \cos \phi - P_n^{fpp} \sin \phi \right] \]

Left, \( Q^2 = 2.5 \text{ GeV}^2 \), M. Meziane et al, P.R.L. 106, 132501 (2011)
Right, \( Q^2 = 8.6 \text{ GeV}^2 \), A. Puckett et al, P.R.L. 104, 242301 (2010)
Following GEp(1), GEp(2), GEp(3) and GEp(2ϒ), form factor ratios from Jlab differ drastically from most recent Rosenbluth data: Andivahis, Christy and Qattan

Rosenbluth cross sections corrected for radiative effects

Double polarization results:
Blue, red, black and magenta:
GEp(1 to 3 and 2ϒ).

No significant radiative corrections for polarization ratio required at the ~% level.
Vector Meson Dominance (VMD) models describe all four nucleon FF’s well (Lomon, Iachello, Bijker...)

Relativistic Constituent Quark Models show importance of relativistic dynamics; can separate dynamical from nucleon structure effects (Chung & Coester, Miller, Gross, de Melo ....).

Dyson-Schwinger equations, continuum approach to QCD (Roberts et al.)

Generalized Parton Distribution (GPD) related to Form Factors (see Guidal...)
Simultaneous fits of all 4 form factors in Fourier transf. including relativistic effects due to use of Breit frame.

In infinite momentum frame transverse charge density $\rho(b)$ is a relativistic invariant, 2-dim. Fourier transform of $F_1$:

$$
\rho(b) = \sum_q e_q \int \text{d}x q(x, b) = \int \frac{d^2q}{(2\pi)^2} F_1(Q^2 = q^2)e^{i q \cdot b}.
$$


GPDs and FF

First moments of Generalized Parton Distributions (GPD) from deeply virtual Compton scattering are related to Form Factors (Radyushkin, 96, Ji, 97) at valence quark level.

\[ \int_{-1}^{+1} dx H^q(x, \xi, Q^2) = F_1^q(Q^2), \]
\[ \int_{-1}^{+1} dx E^q(x, \xi, Q^2) = F_2^q(Q^2), \]

- \( x-\xi \) initial, \( x+\xi \) final momentum fraction of valence quark struck.

- \( H^q \) and \( E^q \): quark correlation functions; emission, re-absorption of quark in non-perturbative realm.

Curves shown from Regge parametrization for \( H \) and \( E \) (from Guidal e. a., PR D72 (2005), 054013)
Virtual photon absorbed on 1 of the 3 leading quarks; momentum of that quark must be shared equally among the 3 quarks by exchange of 2 gluons, each with virtuality $\alpha \frac{1}{Q^2}$:

$$Q^2F_2/F_1 \approx \text{constant.}$$

Data do not agree with expectation.
Elastic $ep$ scattering in $1-10$ GeV$^2$ $Q^2$ range is in domain of non-perturbative QCD. Dressed quarks from Dynamical Chiral Symmetry Breaking: (C. Roberts e.a. PRL 111, 101803 (2013)). Described by Dyson-Schwinger equations.

The quark-partons of QCD acquire in infra-red region a momentum-dependent mass 2 orders of magnitude larger than current-quark mass; from cloud of gluons surrounding a low-momentum quark.
Can both data be physically true?

Cross sections only (Rosenbluth)  Double polarization only
Is there explanation for “dramatic” difference between results of the 2 types of experiments: Rosenbluth and Polarization? is this difference real?

Results of both obtained within OPEX approximation. But, in 2003 Guichon and Vanderhaeghen proposed two-photon exchange as origin.

Must be evaluated, or measured. Difficult to calculate: virtual nucleon can be any excited baryon.

Many model calculations since: Afanasev ea., Arrington, Kondratyuk ea., Bystritskiy ea., Vanderhaeghen ea., Blunden, Carlson and Vanderhaeghen)....

Direct way to measure TPE from the cross section ratio for electrons and positrons on the proton. (VEEP-3, Jlab HallB, Olympus).
Hall B $e^+p$ results (Rinal et al, PR C95 (2017)): Rosenbluth and polarization compatible up to $Q^2=1.77$ GeV$^2$; extend these results to 8.5 GeV$^2$! Main culprit 1994 Andivahis data? reanalyzed by Tomasi-Gustafsson, and Arrington and others: better agreement with polarization results possible, but not proven!
At Jefferson Lab $G_{Ep}(2\gamma)$ experiment measured $P_t/P_\ell$ and obtained $\mu G_E/G_M$ at $Q^2=2.5$ GeV$^2$ for 3 values of $\epsilon$, with small error bars.

Average $\mu G_{Ep}/G_{Mp}=0.688+/-.004$ ($\approx 0.6\%$)

Lack of an $\epsilon$-dependence confirms form factor ratio results unaffected by TPEX contribution at this $Q^2$.


$Y$-axis is in fact $\mu G_{Ep}/G_{Mp}$ in the absence of a 2-photon effect.
Is there a physical reason for the “difference”, or are we seeing the limit of sensitivity to $G_{Ep}$ of cross section data?

$$\frac{\sigma}{\sigma_{Mott}} = \frac{\tau}{\varepsilon (1+\tau)} G_{Ep}^2 (1 + \frac{\varepsilon G_E^2}{\tau G_M^2})$$

Then plot $\frac{\varepsilon G_E^2}{\tau G_M^2}$ versus $Q^2$.

Polar. transfer

Rosenbluth

together
Quark Flavor separation

If assume charge symmetry for hadron current at the dressed quark level: \( <p|e_u \bar{u} \gamma_\mu u + e_d \bar{d} \gamma_\mu d|p> \), with \( e_u \) and \( e_d \) the charge of the up and down quarks, then: the dressed quark Dirac and Pauli form factors obey:

\[
  f^u_{1(2)p} = f^d_{1(2)n}, \quad f^d_{1(2)p} = f^u_{1(2)n}
\]

and the Dirac and Pauli dressed quark ff's can be obtained from the nucleon Dirac and Pauli proton FF's, \( F_{1(2)p} \) and \( F_{1(2)n} \):

\[
  f^u_{1(2)p} = 2F_{1(2)p} + F_{1(2)n} \quad \text{and} \quad f^d_{1(2)p} = F_{1(2)p} + 2F_{1(2)n}
\]

Quark Flavor separation and di-quark

Solid lines for polynomial fits to nucleon FF

Interference of axial-vector and scalar di-quark generates zero in $f_{1p}^d$


Dashed lines for Dynamical Chiral Sym. Breaking
Next experiment, GEp(5), with upgraded Jlab accelerator, SuperBigbiteSpectrometer in Hall A, and significant others improvements
CONCLUSION

Even though this was a drastically shortened presentation of the field of elastic electron/nucleon scattering, as it evolved towards increasing $Q^2$ after our start of double polarization experiments at Jefferson Lab,

I hope to have given you a sense of the magnitude of the changes in understanding of the proton structure resulting from the use of the polarization ratio method to obtain the proton form factors.


Also A.J.R. Puckett et al. PR C96 (2017) 055203.
The END
The Jefferson Lab physics results I have shown here were obtained by a large collaboration, lasting from the late nineteen-eighties to this day. I mention particularly Prof. Vina Punjabi, who presented the first proposals, and got them first conditionally (1989), and then fully approved in 1993.

Mark Jones, Ed Brash, L. Pentchev and F. Wesselmann were crucially involved in this work. I also mention the graduate students of these experiments: G. Quemener, O. Gayou, A. Puckett, M. Meziane and W. Luo.

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The polarimeter in HRS in Hall A: $G_{Ep}(1)$ and $G_{Ep}(2)$

The double polarimeter in HMS in Hall C: $G_{Ep}(3)$ and $G_{Ep}(2\gamma)$
Some consequences of Polarization results

1. Vector Dominance (VMD) and Constituent Quark models (CQM) revisited, made relativistic (E. Lomon, G. Miller)

2. Concept that $F_1$ is Fourier transf. of charge densities lost: $F_1$ Fourier transf. of 2-dimens. transverse density in IMF (G. Miller)

3. Proton in ground state not necessarily spherically symmetric, may show a typical multipole shape (G. Miller)

4. Elastic $ep$ scat. in 1–10 GeV² in non-perturbative QCD realm; Dynam. Chiral Symm. Breaking has visible effects. (C. Roberts)

5. Di-quark structure of the nucleon has observable consequences (I. Cloet and C. Roberts e.a)

6. Obtain flavor separated dressed quark form factors: different for $u$ and $d$ quarks.
“Best” knowledge of $G_{Ep}$ and $G_{Mp}$

Recoil polarization

Rosenbluth

![Graph of $G_{Ep}$ vs $Q^2$ (GeV^2)](image)

![Graph of $G_{Mp}$ vs $Q^2$ (GeV^2)](image)
“Patience et longueur de temps font plus que force ni que rage”

dans la fable du Lion et du Rat,

de Jean de la Fontaine, 1621-1695

Patience and length of time achieve more than force or rage
Sachs FF: $F_{1p,n}$ and $F_{2p,n}$ from Kelly-like polynomial fits to $G_{Ep}$, $G_{Mp}$ and $G_{En}$, $G_{Mn}$, as in Punjabi et al EPJA (2015) 51: 79

All 4 form factors have a smooth behavior, and the data are internally consistent.