The energy-momentum tensor in the NJL model

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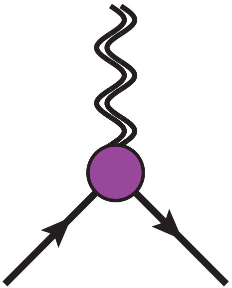
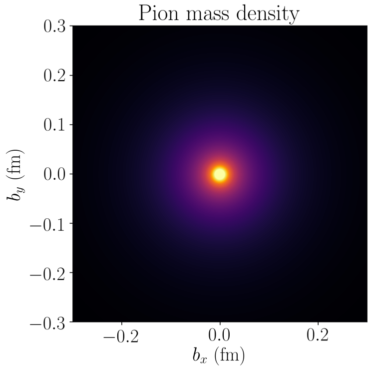
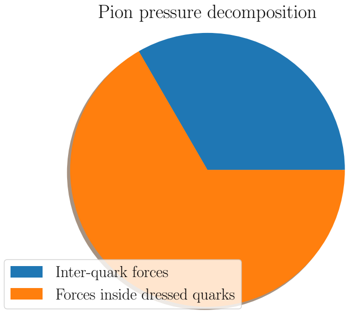
- The **energy-momentum tensor** (EMT) is an operator characterizing the distribution and flow of energy and momentum.
- Matrix elements between hadronic states characterize coveted properties of hadrons:
  - The distribution & decomposition of mass.
  - The distribution & decomposition of angular momentum.
  - The distribution & decomposition of forces, including shear and pressure.







- The **canonical** EMT is obtained by applying Noether's theorem to spacetime translation symmetry:

$$T^{\mu\nu}_{\text{can.}}(x) = \sum_q \left\{ \bar{q}(x) i\gamma^\mu \overleftrightarrow{D}^\nu q(x) - g^{\mu\nu} \bar{q}(x) \left(i \overleftrightarrow{\not{D}} - m_q \right) q(x) \right\} - 2\text{Tr}\left[G^{\mu\lambda} \partial^\nu A_\lambda \right] + \frac{1}{2} g^{\mu\nu} \text{Tr}\left[G^{\lambda\sigma} G_{\lambda\sigma} \right]$$

  - It is conserved in the first index: $\partial_\mu T^{\mu\nu}_{\text{can.}} = 0$.
  - It is not symmetric: $T^{\mu\nu}_{\text{can.}} \neq T^{\nu\mu}_{\text{can.}}$
  - It is not gauge invariant. **Problem!**

- The **gauge invariant kinetic** (gik) EMT is obtained by adding the divergence of a superpotential to the canonical EMT.

See Leader & Lorcé, Phys Rept 541 (2014)

$$T^{\mu\nu}_{\text{gik}}(x) = \sum_q \left\{ \bar{q}(x) i\gamma^\mu \overleftrightarrow{D}^\nu q(x) - g^{\mu\nu}\bar{q}(x)\left(i\overleftrightarrow{\not{D}} - m_q\right)q(x)\right\} - 2\text{Tr}\left[G^{\mu\lambda}G^{\nu}{}_{\lambda}\right] + \frac{1}{2}g^{\mu\nu}\text{Tr}\left[G^{\lambda\sigma}G_{\lambda\sigma}\right]$$

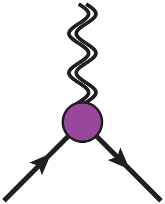
  - It is (still) conserved in the first index: $\partial_\mu T^{\mu\nu}_{\text{gik}} = 0$. . . . but also, $\partial_\nu T^{\mu\nu}_{\text{gik}} = 0$ too.
  - It is (still) not symmetric: $T^{\mu\nu}_{\text{gik}} \neq T^{\nu\mu}_{\text{gik}}$
  - It is gauge invariant.
- The gik EMT is also the source of gravity in Einstein-Cartan theory.
  - Einstein-Cartan theory is a natural extension of general relativity that accommodates spin via spacetime torsion.
  - Also, it's the gauge theory associated with local Poincaré transformations.
  - For a review, see Hehl *et al.*, RMP48 (1976)

- Matrix elements of EMT between hadronic momentum eigenstates give **gravitational form factors** (GFFs).
- For a spin zero hadron:

$$\langle p' \mid T^a_{\mu\nu}(0) \mid p\rangle = 2P_\mu P_\nu A_a(t) + \frac{1}{2}(\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu})C_a(t) + 2m_h^2 g_{\mu\nu}\bar{c}_a(t)$$

where $a = g, q$ is any parton flavor, and m_h is the hadron mass.

- GFFs characterize different aspects of gravitational structure.
- $A_a(t)$ encode mass distributions,
- $C_a(t)$ & $\bar{c}_a(t)$ encode force distributions,
- $\bar{c}_a(t)$ encode force balancing between quarks & gluons: $\bar{c}_q(t) = -\bar{c}_g(t)$.
- See Polyakov & Schweitzer for details



$p = P - \frac{\Delta}{2}$ $\quad$ $p' = P + \frac{\Delta}{2}$

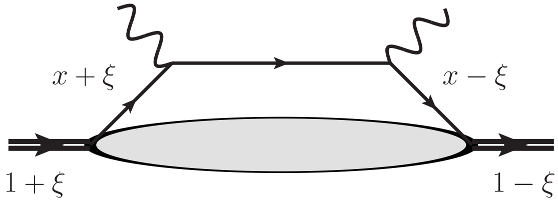
So, can we measure the structure of pions and protons with gravitational waves?

So, can we measure the structure of pions and protons with gravitational waves?

No.

- **Hard exclusive reactions** are used to measure GFFs—not gravitational experiments.
  - Deeply virtual Compton scattering (DVCS) to probe quark structure.
  - Deeply virtual meson production (DVMP), *e.g.*, J/ψ or Υ to probe gluon structure.
  - ... and more!
- Related to GPDs—spin-zero example:

$$\int_{-1}^{1} \mathrm{d}x\, x H_a(x,\xi,t) = A_a(t) + \xi^2 C_a(t)$$



- The gik EMT is the current that a graviton "sees" in Einstein-Cartan theory.
- Of course, gravity is too weak for us to do graviton-exchange experiments.
- But using graviton exchange as a *purely theoretical means of calculation* is still helpful—can think in field theory terms.
  - Ward-Takahashi identities
  - Dyson-Schwinger equations
  - Feynman diagrams
- Matrix elements of the EMT related to graviton vertex:

$$\langle p' \,|\, T^{\mu\nu} \,|\, p \rangle = \langle p' \,|\, \Gamma^{\mu\nu}_{G} \,|\, p \rangle$$

Gravitational Ward-Takahashi Identities

- Quark-graviton vertex satisfies a simple Ward-Takahashi identity (WTI):

$$\Delta_\mu \Gamma^{\mu\nu}_{qG}(p',p) = p^\nu S^{-1}(p') - p'^\nu S^{-1}(p)$$

  - Identical to WTI for canonical EMT.
  - Applies to anything made of only quarks.
- WTI for gluon-graviton vertex more complicated:

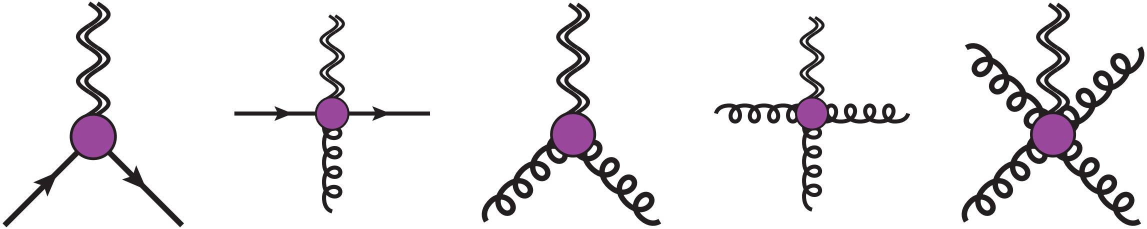
$$\Delta_\mu \Gamma^{\mu\nu}_{gG}(p',p) = p^\nu S^{-1}(p') - p'^\nu S^{-1}(p) + \frac{1}{2i}\Delta_\mu \left[S^{-1}(p')\Sigma^{\mu\nu} - \Sigma^{\mu\nu} S^{-1}(p)\right]$$

where $\Sigma^{\mu\nu}$ is the generator of Lorentz transforms.

  - Identical to WTI for Belinfante EMT [proved by DeWitt, PR162 (1967)]

Dyson-Schwinger Equations

- **Equivalence principle**: all energy gravitates the same way.
- This includes potential energy, encoded in EMT via $-g^{\mu\nu}\mathcal{L}$.
- Graviton vertex diagrams (excluding ghosts—also necessary for a covariant gauge!):



- Every one of these must be dressed by **Dyson-Schwinger equations**.
- These equations are coupled, and there are an infinite tower of them.
- A simpler model of QCD would be a nice starting point.

Let us model mesons in the **Nambu–Jona-Lasinio** (NJL) model of QCD.

- Low-energy effective field theory.
- Models QCD with gluons integrated out. **Four-fermi contact interaction**.

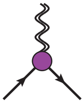
$$\mathcal{L} = \overline{\psi}(i\overleftrightarrow{\not\partial} - \hat{m})\psi + \frac{1}{2}G_\pi[(\overline{\psi}\psi)^2 - (\overline{\psi}\gamma_5\boldsymbol{\tau}\psi)^2 + (\overline{\psi}\boldsymbol{\tau}\psi)^2 - (\overline{\psi}\gamma_5\psi)^2] \\ - \frac{1}{2}G_\omega(\overline{\psi}\gamma_\mu\psi)^2 - \frac{1}{2}G_\rho[(\overline{\psi}\gamma_\mu\boldsymbol{\tau}\psi)^2 + (\overline{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi)^2] - \frac{1}{2}G_f(\overline{\psi}\gamma_\mu\gamma_5\psi)^2]$$

- Reproduces **dynamical chiral symmetry breaking** (DCSB).
- Gap equation:

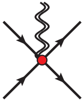
$$M = m + 8iG_\pi(2N_c)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{M}{k^2 - M^2 + i0}$$

- Mesons appear as poles in T-matrix after solving a **Bethe-Salpeter equation** (BSE).

- NJL model has three- and five-point graviton vertices.

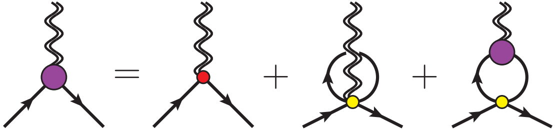


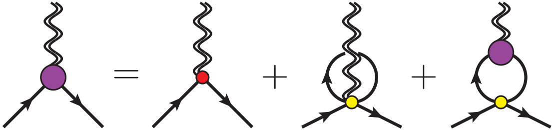
$$= \gamma^\mu k^\nu - g^{\mu\nu}(\not{k} - M) + \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{4M} C_Q(t) + \frac{i\epsilon^{\mu\nu\Delta\sigma}\gamma_\sigma\gamma_5}{4} D'_Q(t)$$



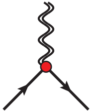
$$= -g^{\mu\nu} \sum_\Omega 2G_\Omega \Omega \otimes \Omega \qquad \text{(sum over contact interactions)}$$

- Five-point vertex comes from **equivalence principle**.
  - Contact interaction is a potential energy
  - All energy gravitates the same way
  - Formally, are introduced to EMT through $-g^{\mu\nu}\mathcal{L}$ term
- Three-point satisfies a Bethe-Salpeter equation:



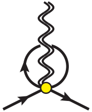


- Graviton vertex BSE driven by elementary interaction:



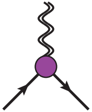
$$= \gamma^\mu k^\nu - g^{\mu\nu}(\not{k} - m)$$

- Vaccuum condensate turns bare into dressed mass:



$$= 8iG_\pi(2N_c)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{M}{k^2 - M^2 + i0}g^{\mu\nu} = (M-m)g^{\mu\nu}$$

- When added, these two terms obey WTI. Last term must be transverse.



$$= \gamma^\mu k^\nu - g^{\mu\nu}(\not{k} - M) + \frac{\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu}}{4M} C_Q(t) + \frac{i\epsilon^{\mu\nu\Delta\sigma}\gamma_\sigma\gamma_5}{4} D'_Q(t)$$

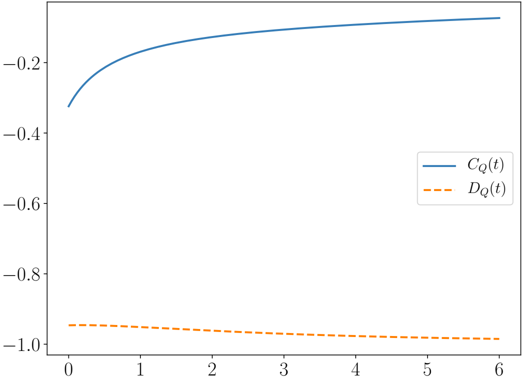
NJL model dressed quarks:

$$A(t) = 1 \qquad C(t) = C_Q(t)$$
$$B(t) = 0 \qquad D(t) = -1 + D'_Q(t)$$

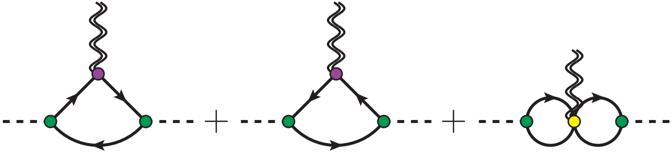
Elementary quarks:

$$A(t) = 1 \qquad C(t) = 0$$
$$B(t) = 0 \qquad D(t) = -1$$

Dressed quarks look bare at high $-t$.
arXiv:1903.09222 (AF & Ian Cloët)



- Sum **three diagrams** to get meson EMT.



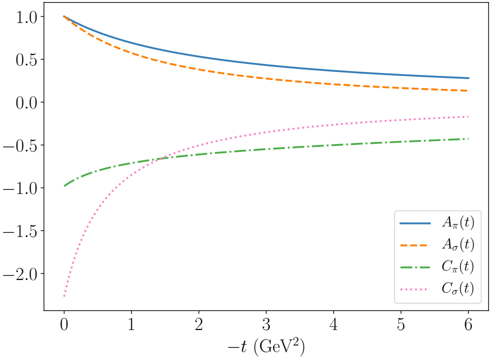
- First two are typical triangle diagrams (appear in EM current, axial current, *etc.*)
- Third **bicycle diagram** is new to EMT (is NJL-model specific)
  - But there are (more complicated) analogues in QCD
  - **Equivalence principle**: all forms of energy look the same to gravity
  - Vertices look like potential energy
  - Graviton can couple to **any vertex** in the Lagrangian
- All three are needed for energy/momentum conservation!

Spin-zero mesons

$$\langle p' \mid T^a_{\mu\nu}(0) \mid p \rangle = 2P_\mu P_\nu A_a(t) + \frac{1}{2}(\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}) C_a(t) + 2m_h^2 \bar{c}_a(t) g_{\mu\nu}$$

Let's look at π and σ mesons (summed over all quarks)...

- $A(t)$ encodes spatial distribution of energy *on the light cone* [via 2D Fourier transform]
- $C(t)$ encodes spatial distribution of forces *on the light cone* [via 2D Fourier transform]
- $\bar{c}(t) = 0$—required by energy conservation



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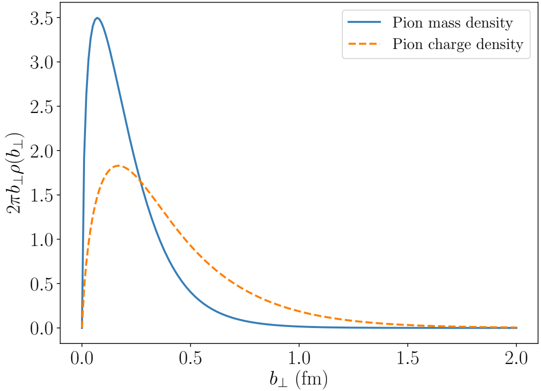
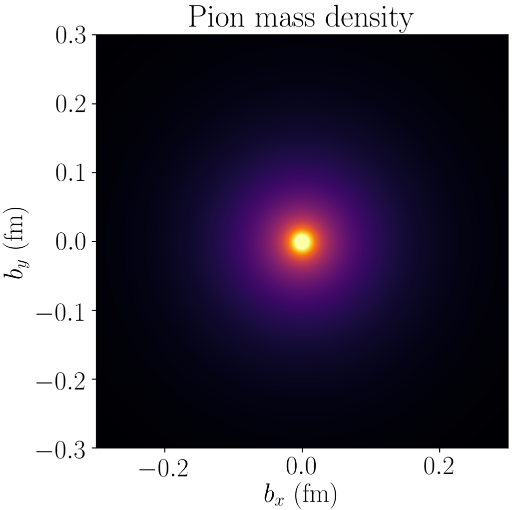
Pion: light cone mass density





- Mass is more centrally concentrated than charge.
- Suggests inhomogeneous distribution of charge.
- NJL model **dressed quarks** have extended charge density, but pointlike mass density.

Predict 0.27 fm for pion light cone mass radius.

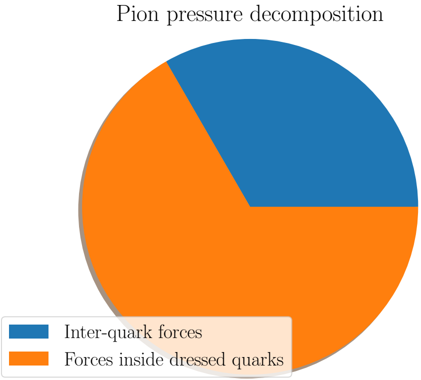
Pion: pressure

In the **chiral limit**:

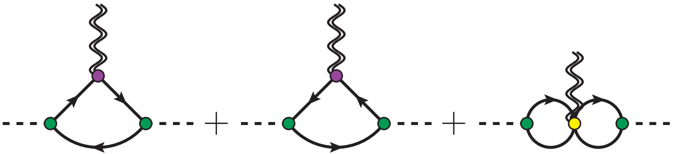
$$\langle \pi(p) \mid T_{\mu\nu}(0) \mid \pi(p) \rangle = 2P_\mu P_\nu - \frac{1}{2}(\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu})$$

i.e., $C_\pi(0) \xrightarrow[m_\pi \to 0]{} -1$. This is a **low energy pion theorem.**

See Voloshin & Zakharov, PRL45 (1980), Novikov & Shifman, Z. Phys. C8 (1981)



- **NJL model does satisfy this theorem.**
- Two-thirds majority of $C_\pi(0)$ comes from dressing term in BSE.
- It is necessary to self-consistently solve **all** non-perturbative dynamical equations.
- For GPD calculations involving constituent quarks, **the bilocal operator must also be dressed** (or one will get $C_\pi(0)$ wrong by a factor 3).
- *n.b.*, no low-energy sigma theorem. We get $C_\sigma(0) = -2.27$.



- $\bar{c}(t) = 0$ is satisfied by the NJL model.
- This requires the bicycle diagram.

$$2m_\pi^2 \bar{c}_\pi(t) = Z_\pi \Pi_{PP}(m_\pi^2) \left[1 + 2G_\pi \Pi_{PP}(m_\pi^2)\right] = 0$$
$$2m_\sigma^2 \bar{c}_\sigma(t) = Z_\sigma \Pi_{SS}(m_\pi^2) \left[1 - 2G_\sigma \Pi_{SS}(m_\sigma^2)\right] = 0$$

  - Green terms are from bicycle diagram
  - The overall vanishing is identical to **mass shell condition**
- Necessity of bicycle diagram: energy conservation is for sum of **kinetic energy** (three-point vertex) and **potential energy** (five-point vertex)

Conclusions & outlook

- The NJL model can be used to compute **gravitational structure** of hadrons that **respects expected non-perturbative dynamics**.
- Gravitons couple to every vertex in the Lagrangian—a consequence of the **equivalence principle**.
- These couplings are **necessary** to observe energy conservation.
- This talk was mostly on the pion (with a little sigma), but we have rho & proton results (see backup slides).
- We predict a pion light cone mass radius of 0.27 fm.

. . . and, most importantly:

Thanks for your time and attention!

(Backup slides follow)

Spin-one mesons

New work with Cedric Lorce, Wim Cosyn, & Sabrina Cotogno, arXiv:1903.00408

$$
\begin{aligned}
\langle p', \lambda' \mid T^a_{\mu\nu}(0) \mid p, \lambda\rangle = -2P_\mu P_\nu & \left[(\epsilon'^* \epsilon)\mathcal{G}^a_1(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2m_\rho^2}\mathcal{G}^a_2(t) \right] \\
& - \frac{1}{2}(\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}) \left[(\epsilon'^* \epsilon)\mathcal{G}^a_3(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2m_\rho^2}\mathcal{G}^a_4(t) \right] + P_{\{\mu} \left(\epsilon'^*_{\nu\}}(\Delta\epsilon) - \epsilon_{\nu\}}(\Delta\epsilon'^*) \right) \mathcal{G}^a_5(t) \\
& + \frac{1}{2} \left[\Delta_{\{\mu} \left(\epsilon'^*_{\nu\}}(\Delta\epsilon) + \epsilon_{\nu\}}(\Delta\epsilon'^*) \right) - \epsilon'^*_{\{\mu}\epsilon_{\nu\}}\Delta^2 - g_{\mu\nu}(\Delta\epsilon'^*)(\Delta\epsilon) \right] \mathcal{G}^a_6(t) \\
& + \epsilon'^*_{\{\mu}\epsilon_{\nu\}} m_\rho^2 \mathcal{G}^a_7(t) + g_{\mu\nu} m_\rho^2 (\epsilon'^*\epsilon)\mathcal{G}^a_8(t) + \frac{1}{2} g_{\mu\nu}(\Delta\epsilon'^*)(\Delta\epsilon)\mathcal{G}^a_9(t) \\
& + P_{[\mu} \left(\epsilon'^*_{\nu]}(\Delta\epsilon) - \epsilon_{\nu]}(\Delta\epsilon'^*) \right) \mathcal{G}^a_{10}(t) + \Delta_{[\mu} \left(\epsilon'^*_{\nu]}(\Delta\epsilon) + \epsilon_{\nu]}(\Delta\epsilon'^*) \right) \mathcal{G}^a_{11}(t)
\end{aligned}
$$

Well, it's a bit complicated...

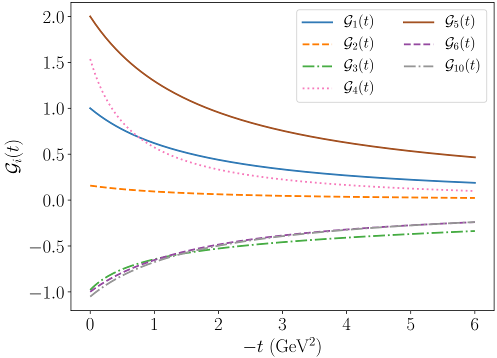
Let's remove the stuff that's zero in the NJL model (from energy/momentum conservation)

$$\langle p', \lambda' | T^a_{\mu\nu}(0) | p, \lambda \rangle = -2P_\mu P_\nu \left[(\epsilon'^* \epsilon)\mathcal{G}^a_1(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2m_\rho^2}\mathcal{G}^a_2(t) \right] - \frac{1}{2}(\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}) \left[(\epsilon'^* \epsilon)\mathcal{G}^a_3(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2m_\rho^2}\mathcal{G}^a_4(t) \right]$$
$$+ \frac{1}{2}\left[\Delta_{\{\mu} \left(\epsilon'^*_{\nu\}}(\Delta\epsilon) + \epsilon_{\nu\}}(\Delta\epsilon'^*) \right) - \epsilon'^*_{\{\mu}\epsilon_{\nu\}}\Delta^2 - g_{\mu\nu}(\Delta\epsilon'^*)(\Delta\epsilon) \right] \mathcal{G}^a_6(t)$$
$$+ P_{\{\mu} \left(\epsilon'^*_{\nu\}}(\Delta\epsilon) - \epsilon_{\nu\}}(\Delta\epsilon'^*) \right) \mathcal{G}^a_5(t) + P_{[\mu} \left(\epsilon'^*_{\nu]}(\Delta\epsilon) - \epsilon_{\nu]}(\Delta\epsilon'^*) \right) \mathcal{G}^a_{10}(t)$$

Look at ρ meson

- $\mathcal{G}_1(0) = 1$ from momentum conservation
- $\mathcal{G}_3(0) \approx -1$, but no low-energy theorem for rho
- $\mathcal{G}_{1,2,6}(t)$ encode spatial distribution of energy
- $\mathcal{G}_{3,4,6}(t)$ encode spatial distribution of forces (pressure, shear, surface tension)
- $\mathcal{G}_6(t)$ related to tensor polarization mode
- $\mathcal{G}_5(t)$ encodes spatial distribution of *total* angular momentum
- $\mathcal{G}_{10}(t)$ encodes spatial distribution of parton *intrinsic* spin

Still a lot... **unpacking in a future talk!**



The proton

$$\langle p', \lambda' \mid T^a_{\mu\nu}(0) \mid p, \lambda \rangle = \bar{u}(p', \lambda') \Big[\frac{P_\mu P_\nu}{M} A_a(t) + \frac{iP_{\{\mu}\sigma_{\nu\}\Delta}}{2M} [A_a(t) + B_a(t)] + \frac{\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}}{M} C_a(t) + M g_{\mu\nu} \bar{c}_a(t) + \frac{iP_{[\mu}\sigma_{\nu]\Delta}}{2M} D_a(t) \Big] u(p, \lambda)$$

We have partial proton results ...

- This is in a quark-diquark model.
- Scalar and axial-vector diquarks included.
- We get $B(0) = 0$ exactly.
- **Light front basis is not necessary to respect this identity.**
- $\bar{c}(t) = 0$ not yet proved.
- Have not yet calculated $D(t)$, from antisymmetric part of EMT.