Entanglement entropy at small x

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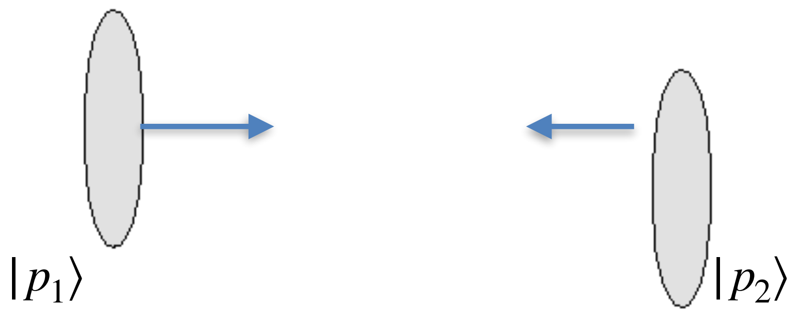
8th Workshop of the APS Topical Group on Hadronic Physics, Denver

Introduction and motivation

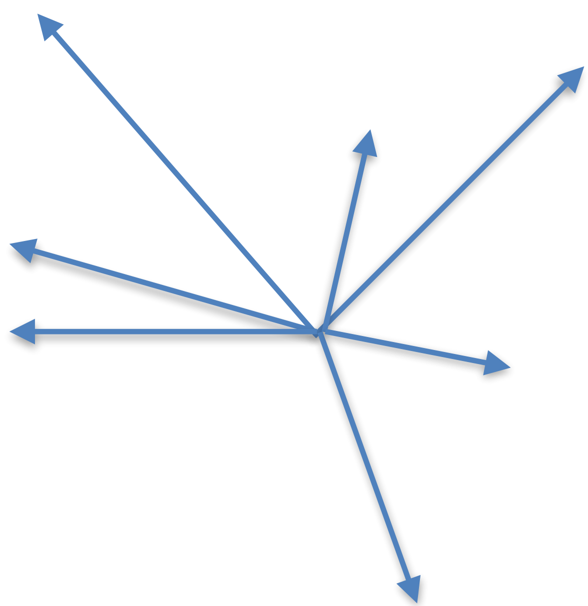
p-p collision



Initial state:

$$|p_1\rangle \otimes |p_2\rangle$$

After the collision



- Correlations
- Thermal-like behavior

Standard procedure

- Density matrix :

$$\rho = |\phi\rangle\langle\phi|$$

- Choose the subsystem of interest(S):

  A local measurement on a spatial region

  A specific energy region (momentum space)

  Exclusive particle production events

- Reduced density matrix :

$$\rho_s = Tr_{\bar{s}}|\phi\rangle\langle\phi|$$

- Von Neumann entropy :

$$S^E = -Tr(\rho_s \ln \rho_s)$$

$S_S^E = S_{\bar{S}}^E$ We can work with either S or $\bar{S}$

Connection to thermodynamics

$$S^E = -Tr(\rho_s \ln \rho_s) \longrightarrow S = -\sum_n p_n \ln p_n$$ (Gibbs entropy)

Q1: $S^E \neq 0?$

Mixing reduced density matrix (not a pure state)

Q2: Maximal $S^E?$

Even distribution in subsystem of interest

- Microcanonical ensemble
- $\bar{S}$(Environment)
- Source of thermodynamical properties?
- Usually true when the number of degrees of freedom is huge

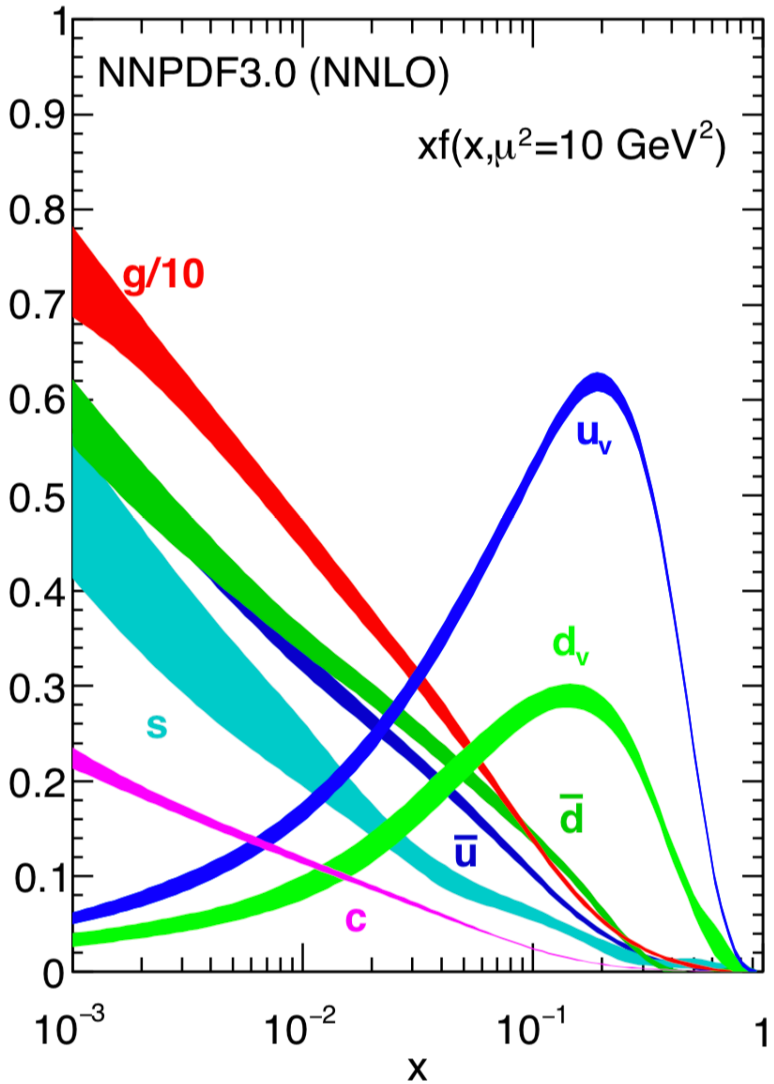
EE Within CGC framework

CGC has well-defined light front wave function

Evolution equation (JIMWLK) with respect to rapidity

The dynamics of EE can also been studied(Scattering)

Parton Distribution at small x



REVIEW OF PARTICLE PHYSICS(2018)

- High Gluon density at small x
  - ➡Gauge field A is large(Classical)
  - ➡Nonlinear
- Degrees of freedom
  - ➡Soft Gluons (small x)
  - ➡Valance quarks(large x)

$$x^+ \sim \frac{1}{k^-} = \frac{k^+}{k_\perp^2}$$

Due to the time dilation, the fast mode (valance quark) can be treated as static source of soft gluon radiation.

Color saturation (Color Glass Condensate)

$$x = \frac{Q^2}{s}$$

DGLAP evolution: Gluon splitting happens in transverse direction.

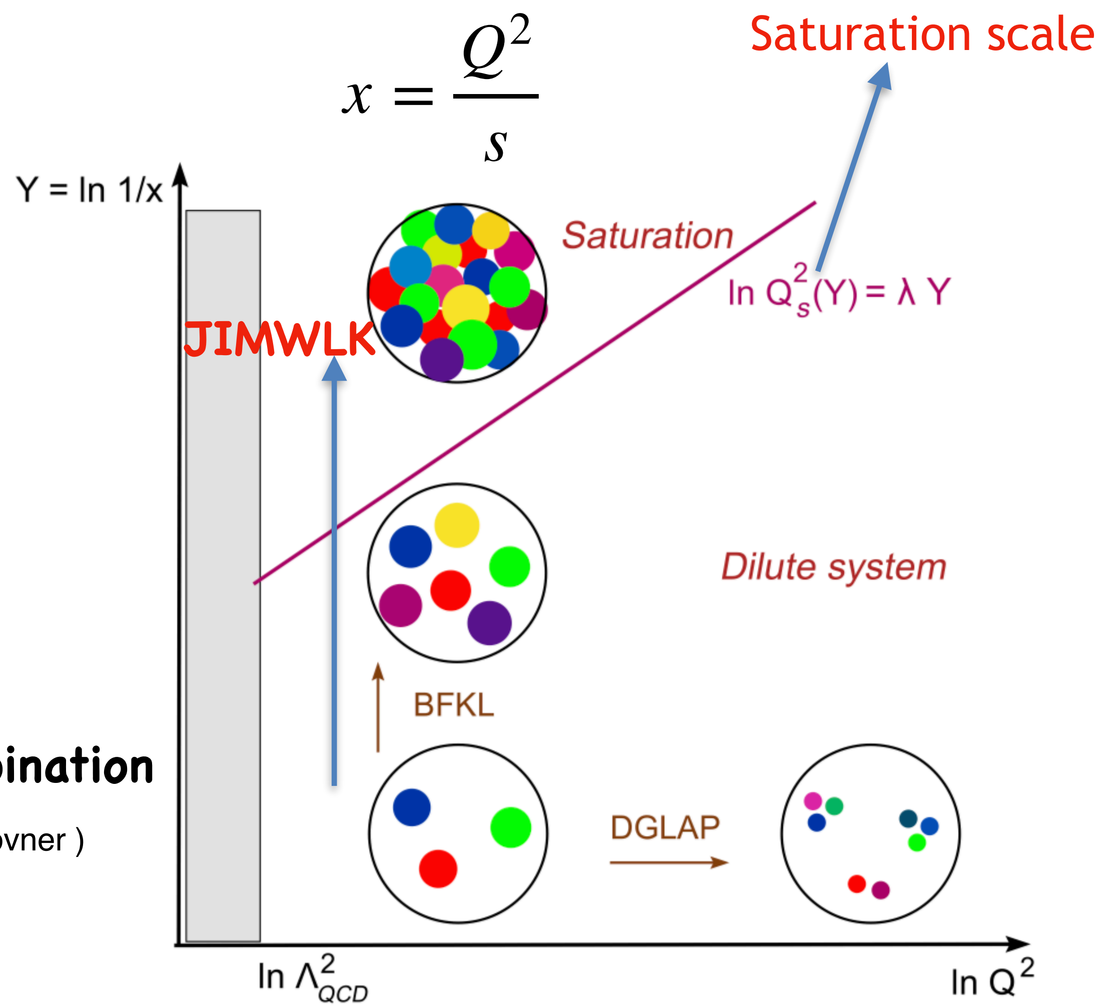
(Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

BFKL: Gluon splitting in longitudinal direction

(Balitsky-Fadin-Kuraev-Lipatov)

JIMWLK: Gluon splitting + recombination

(Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov- Kovner)



F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan 2010

Light front wave function

$$|p\rangle = \sum_a |v, \rho_a\rangle |s, \rho_a, \phi_a\rangle$$

Stochastic sampling of the configuration

$$\langle v, \rho_a | v, \rho_a \rangle = \exp\left\{ -\int_{k_\perp} \frac{\rho_a(k_\perp)\rho_a^*(k_\perp)}{2\mu^2(k_\perp)} \right\} \longrightarrow W[\rho_a] \text{ in JIMWLK}$$

Classical gluon field can be written as coherent state

$$|s, \rho_a, \phi_a\rangle = \exp\left\{ i \int_{k_\perp} b_a^i(k_\perp)\phi_a^i(k_\perp) \right\} |0\rangle$$

With

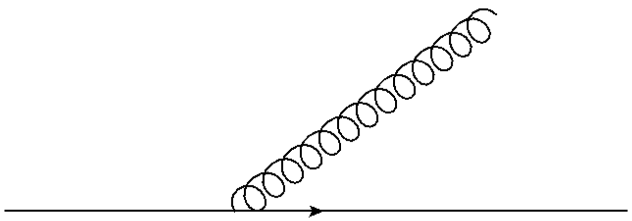
Gluon field operator: $\phi_a^i(k_\perp) = a_a^{+i}(k_\perp) + a_a^i(-k_\perp)$

Coherent vacuum : $\langle \phi | 0 \rangle = N \exp\left\{ -\frac{\pi\phi_i^2}{2} \right\}$

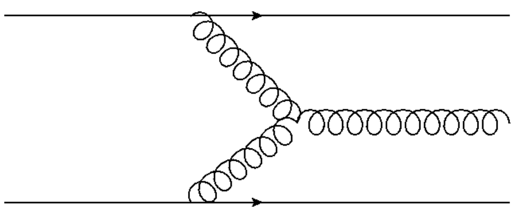
The classical gluon field:

Valance color charge density

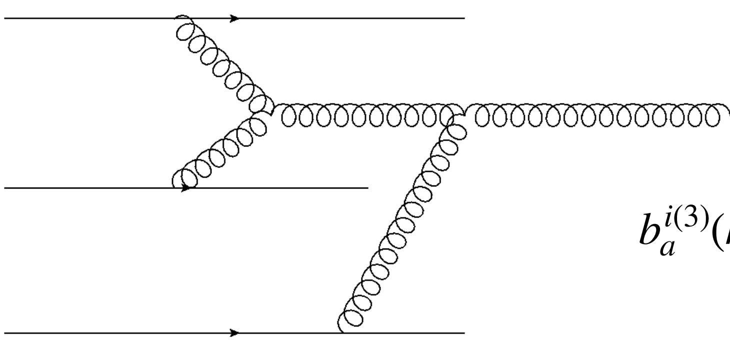
$$b^i(x_\perp) = -\frac{1}{ig}U(x_\perp)\partial^i U(x_\perp)^\dagger \qquad \partial^i b_a^i(x_\perp) = g\rho_a(x_\perp)$$



$$b_a^{i(1)}(k_\perp) = g\rho_a(k_\perp)\frac{ik_\perp^i}{k_\perp^2}$$



$$b_a^{i(2)}(k) = \frac{if_{abc}g^3}{2}(\delta_{ij} + \frac{k_ik_j}{k^2})\int \frac{d^2p}{(2\pi)^2}\frac{p^j}{p^2(p-k)^2}\rho_b(p)\rho_c(k-p)$$



$$b_a^{i(3)}(k) = \frac{if_{abc}g^3}{6}(\delta_{ij} + \frac{k_ik_j}{k^2})\int \frac{d^2p}{(2\pi)^2}\frac{1}{g(p-k)^2}b_b^{j(2)}(p)\rho_c(k-p)$$

The full density matrix

$$\hat{\rho} = \sum_{ab} |v, \rho_a\rangle\, |s, \rho_a, \phi_a\rangle\langle s, \rho_b, \phi_b|\, \langle v, \rho_b|$$

The reduced density matrix for soft modes

$$\hat{\rho}_r = \sum_a \int D[\rho_a] W[\rho_a] |s, \rho_a, \phi_a\rangle\langle s, \rho_a, \phi_a|$$

One can evolve this weighting functional with respect to rapidity

We want Von Neumann entropy:

$$S^E = -Tr(\hat{\rho}_r \ln \hat{\rho}_r)$$

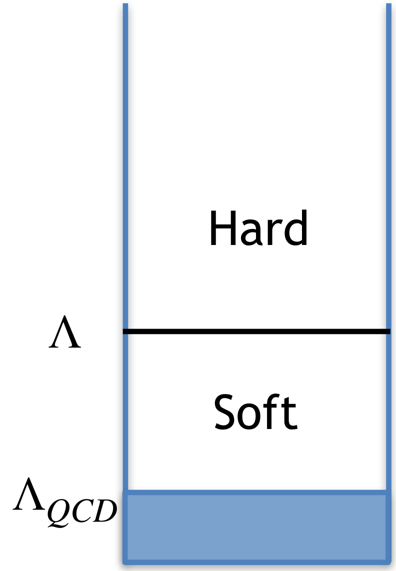
Infinite dimensional & impossible to diagonalize!

$$\ln \rho = \lim_{\epsilon \to 0} \frac{1}{\epsilon}(\rho^\epsilon - 1) \longrightarrow S^E = -\lim_{N \to 1} \left(\frac{\rho^N - \rho}{N - 1} \right)$$

Some results & future work

LO (Alex Kovner and Michael Lublinsky(2015))

$$S^E \simeq \frac{SQ_s^2}{4\pi g^2}(N_c^2-1)\Big[\overbrace{\ln^2\big(\frac{g^2\Lambda^2}{Q_s^2}\big)+\ln\big(\frac{g^2\Lambda^2}{Q_s^2}\big)}^{UV}+\overbrace{\frac{3}{2}}^{IR}\Big]$$



Consider NLO correction

At $Tr(\hat{\rho}_r^2)$ level, it's the coherence between $b^{(1)}$and $b^{(3)}$

The correction is $\#g^8\phi^4$

JIMWLK evolution of $W[\rho_a]$

Increase the color charge density to the order of $\frac{1}{g^2}$ (Saturation effect)

Applications of EE

EE

Thermal model of particle production

K.Kutak, arXiv 1103.3654
B.Muller, A.Schaefer, arXiv:1712.03567
X.Feal, C.Pajares, R.A.Vazquez, arXiv 1809.04409

Final state correlation

Duff Neill and Wouter Waalewijn, arXiv 1811.01021

Conformal duality?!
A.H. Mueller arXiv:1804.07249
D. M. Hofman and J. Maldacena arXiv:0803.1467
Y.Hatta arXiv:0810.0889

Initial state correlation

A. Kovner and M. Lublinsky, arXiv1506.05394
A.Kovner, M.Lublinsky,M.Serino, arXiv:1806.01089
N. Armesto, et. al. arXiv:1901.08080

Final state thermal-like properties in hadronic collisions

Infrared information

D.Carney, L. Chaurette, D.Neuenfeld, Gordon Semenoff(2017 PRL, arXiv 1803.02370)

Eigenstate thermalization hypothesis

J. M. Deutsch, Phys. Rev. A 43, 2046 (1991)
M. Srednicki, Phys. Rev. E 50, 888 (1994).