Exploring the Origin of Mass using Pseudoscalar Mesons

Craig Roberts
Only apparent scale in chromodynamics is mass of the quark field

Quark mass is said to be generated by Higgs boson.

In connection with everyday matter, that mass is $1/250^{th}$ of the natural (empirical) scale for strong interactions, viz. more-than two orders-of-magnitude smaller

Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter

**Nuclear physics mass-scale** – 1 GeV – is an *emergent feature of the Standard Model*

– No amount of staring at $L_{\text{QCD}}$ can reveal that scale

Contrast with quantum electrodynamics, *e.g.* spectrum of hydrogen levels measured in units of $m_e$, which appears in $L_{\text{QED}}$
Classically, in a scale invariant theory

the energy-momentum tensor must be traceless: \( T_{\mu\mu} \equiv 0 \)

Regularisation and renormalisation of (ultraviolet) divergences in Quantum Chromodynamics introduces a mass-scale

... *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, \( \zeta \)

\( \alpha \to \alpha(\zeta) \) in QCD’s (massless) Lagrangian density, \( \mathcal{L}(m=0) \)

\[ \partial_\mu D_\mu = \frac{\delta \mathcal{L}}{\delta \sigma} = \alpha \beta(\alpha) \frac{d\mathcal{L}}{d\alpha} = \beta(\alpha) \frac{1}{4} G_{\mu\nu} G^{\mu\nu} = T_{\rho\rho} =: \Theta_0 \]

Quantisation of renormalisable four-dimensional theory
forces nonzero value for trace of energy-momentum tensor
Where is the mass?
Knowing that a trace anomaly exists does not deliver a great deal...
Indicates only that a mass-scale must exist.
Can one compute and/or understand the magnitude of that scale?
One can certainly *measure* the magnitude...

\[
\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu
\]

\[
\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2
\]

\[
= \langle p(P)|\Theta_0|p(P)\rangle
\]

In the chiral limit the entirety of the proton’s mass is produced by the trace anomaly, \(\Theta_0\).

... In QCD, \(\Theta_0\) measures the strength of gluon self-interactions.

... so, from one perspective,

\(m_p\) is (somehow) completely generated by glue.
On the other hand...
In the chiral limit

\[ \langle \pi(q)|T_{\mu\nu}|\pi(q) \rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q)|\Theta_0|\pi(q) \rangle = 0 \]

Does this mean that the scale anomaly vanishes trivially in the pion state, \textit{i.e.} gluons contribute nothing to the pion mass?

Difficult way to obtain “zero”!

Easier to imagine that “zero” owes to cancellations between different operator contributions to the expectation value of \( \Theta_0 \).

Of course, such precise cancellation should not be an accident.

It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.
Whence “1” and yet “0”?  

\[ \langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0 \]

➢ No statement of the question

“How does the mass of the proton arise?”

is complete without the additional clause

“How does the pion remain massless?”

➢ Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems

– Expectation value of \( \Theta_0 \) in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = \( m_p \)
Whence “1” and yet “0”? 

\[ \langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0 \]

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“How does the mass of the proton arise?”

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“How does the pion remain massless?”

➢ Naturally

with

- Expect the entire array

of empirical consequences

of the mechanism responsible

so that the theory can be validated

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\[ \Delta^{-1}_{\mu\nu}(q) = \Delta^{-1} + \frac{1}{2} \Delta^{(a)} + \frac{1}{2} \Delta^{(b)} + \frac{1}{6} \Delta^{(c)} + \frac{1}{2} \Delta^{(d)} + \frac{1}{2} \Delta^{(e)} \]

\[ \Pi_{\mu\nu}(q) = P_{\mu\nu}(q) \Pi(q) \]

\[ P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2 \]
In QCD: Gluons become massive!

Running gluon mass

\[ d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2; \zeta)} \]

\[ \alpha_s(0) \approx \pi \quad m_g^2(0) = (0.46 \text{ GeV})^2 \]

Gluons are **cannibals**
– a particle species whose members become massive by eating each other!

Expression of trace anomaly:
Massless glue becomes massive gluon mass-squared function

Class A: Combining DSE, lQCD and pQCD analyses of QCD’s gauge sector

Power-law suppressed in ultraviolet, so invisible in perturbation theory


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This is where we live

What’s happening out here?!

QCD’s Running Coupling
Modern continuum & lattice methods for analysing gauge sector enable
“Gell-Mann – Low” running charge to be defined in QCD

Combined continuum and lattice analysis of QCD’s gauge sector yields a parameter-free prediction

N.B. Qualitative change in $\hat{\alpha}_{\text{PI}}(k)$ at $k \approx \frac{1}{2} m_p$
QCD Effective Charge

- $\hat{\alpha}_{PL}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, \textit{i.e.} completely determined by the gauge-boson two-point function.

- $\hat{\alpha}_{PL}$ is
  - process-independent
  - appears in every one of QCD’s dynamical equations of motion
  - known to unify a vast array of observables

- $\hat{\alpha}_{PL}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD

- QCD is IR finite, owing to dynamical generation of gluon mass-scale

- Asymptotic freedom $\Rightarrow$ QCD is well-defined at UV momenta

- \textit{QCD is therefore unique amongst known 4D quantum field theories}
  - Potentially, defined \& internally consistent at all momenta
QCD Effective Charge

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- QCD is therefore unique amongst known 4D quantum field theories
  - Potentially, defined & internally consistent at all momenta.

Conceivably, therefore, QCD can serve as a basis for theories that take physics beyond the Standard Model.
Enigma of Mass
Pion’s Goldberger-Treiman relation

➢ Pion’s Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

\[ \Gamma_{\pi^j}(k; P) = \tau_{\pi^j} \gamma_5 \left[ iE_\pi(k; P) + \gamma \cdot PF_\pi(k; P) \right. \]

\[ \left. + \gamma \cdot k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right] \]

➢ Dressed-quark propagator

\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]

➢ Axial-vector Ward-Takahashi identity entails

\[ f_\pi E_\pi(k; P = 0) = B(k^2) \]

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known

This means that \( \pi \) necessarily has dressed-quark \( L=0 \) & \( L=1 \) components in any frame Twist-3 on light-front
The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

This emphasises that Goldstone's theorem has a pointwise expression in QCD

Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Thus, enigmatically, the properties of the massless pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

This algebraic identity is why QCD’s pion is massless in the chiral limit
\[ \langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0 \]

**Whence “0”?**
\[ \langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0 \]

**Whence “0”?**

The answer is algebraic
Obtain a coupled set of gap- and Bethe-Salpeter equations

- Bethe-Salpeter Kernel:
  - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
  - Interactions of arbitrary but enumerable complexity involving these “basis vectors”

- Chiral limit:
  - Algebraic proof
    - at any & each finite order in symmetry-preserving construction of kernels for
      » the gap (quark dressing)
      » and Bethe-Salpeter (bound-state) equations,
    - there is a precise cancellation between
      » mass-generating effect of dressing the valence-quarks
      » and attraction introduced by the scattering events
  - Cancellation guarantees that
    - simple system, which began massless,
    - becomes a complex system, with
      » a nontrivial bound-state wave function
      » attached to a pole in the scattering matrix, which remains at $P^2=0$ ...
  - Interacting, bound system remains massless!
Obtain a coupled set of gap- and Bethe-Salpeter equations

Quantum field theory statement:
In the pseudoscalar channel, the dynamically
generated mass of the two fermions is
precisely cancelled by the attractive
interactions between them – iff –

\[ f_\pi E_\pi(p^2) = B(p^2) \]

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Revealing Mass
Consequences ... 1

➢ Mass is dynamically generated in QCD: Scale $\sim \Lambda_{\text{QCD}}$
  – Empirically $\Lambda_{\text{QCD}} \approx 0.2$ GeV ... Standard Model can’t predict this value.

➢ Gluon self-interactions make $\Lambda_{\text{QCD}} \approx 0.2$ GeV possible. They do not guarantee it.

➢ Understanding of observables (almost always) depends on frame of reference and scale of probe
  – gluons and quarks $\rightarrow$ dressed quasiparticles:
    • massless in perturbation theory
    • possess mass functions which are large at infrared momenta $\leq m_g \approx 2 \Lambda_{\text{QCD}}$
  – at hadronic scale: wave functions, cross-sections, etc. are most readily understood using evolving quasiparticle operators for dressed-g, -q
    • Each contains a (distinct) countable infinity of partons

$\Rightarrow$ All bound-states have GeV-scale masses

$\Rightarrow$ Except Nambu-Goldstone modes

✓ DCSB: whilst constituents are massive, NG-modes are (nearly) massless
Consequences ... 2

➢ QCD’s unique Gell-Mann–Low effective coupling
  ✓ Infrared finite ... \( \alpha(\sim 0) \approx \pi \)
  ✓ Landau pole of perturbation theory is eliminated by emergence of gluon mass
  ✓ Cross-sections are free of infrared divergences

➢ PDAs of ground-state S-wave mesons and baryons are broad, concave functions
  – Numerous empirical consequences ⇒ empirically verifiable
    • Hadron elastic and transition form factors

➢ Emergent vs Explicit (Higgs) mass generation
  – s-quark defines a boundary:
    • emergent mass generation dominates for \( m < m_s \)
    • but explicit (Higgs) mass is most important for \( m > m_s \)
  – s-quark/u-quark comparisons in parton distributions are a sensitive probe of emergent mass and its distribution
Consequences ... 3

➢ Existence of nonpointlike scalar and axial-vector diquarks in nucleon.
  – Axial-vector correlations are essential

➢ Empirically verifiable consequences
  – Example ... proton’s tensor charges:
    • $\delta_Td \neq 0 \Rightarrow$ rules-out scalar-diquark-only nucleon
    • $\delta_Tu \approx 4 | \delta_Td |$ ... understood via highly-correlated proton wave function

➢ Baryon resonances:
  – Relative strengths of 0+ and 1+ change & new diquarks appear (0- & 1-)
  – Vast array of new predictions whose testing is crucial to validating the emergent mass paradigm

➢ Hybrid Mesons
  – Just like diquark correlations exist in baryons ...
  – $q_g = g+q$ and $\bar{q}_g = g+\bar{q}$ correlations very probably also exist in systems with valence glue.
  – Hybrid mesons may be understood as highly-correlated $q_g\bar{q} \leftrightarrow q\bar{q}_g$ bound-states
Observing Mass
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π & K Valence-quark Distribution Functions
Continuum QCD prediction of π valence-quark distributions

➢ Owing to absence of pion targets, the pion’s valence-quark distribution functions are measured via the Drell-Yan process:

\[ \pi p \rightarrow \mu^+ \mu^- X \]

➢ Consider a theory in which quarks scatter via a vector-boson exchange interaction whose \( k^2 \gg m_G^2 \) behaviour is \( (1/k^2)^\beta \),

➢ Then at a resolving scale \( Q_0 \)

\[ u_\pi(x;Q_0) \sim (1-x)^{2\beta} \]

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

➢ In QCD, \( \beta=1 \) and hence

\[ QCD \ u_\pi(x;Q_0) \sim (1-x)^2 \]

QCD: \( Q>Q_0 \Rightarrow 2 \rightarrow 2+\gamma, \gamma>0 \)
Empirical status of the Pion’s valence-quark distributions

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\[ \pi p \rightarrow \mu^+ \mu^- X \]

➢ Three experiments:

➢ None more recent

➢ Conway et al.

  – Leading-order analysis of the Drell-Yan data: \( Q^2 \approx 27 \text{ GeV}^2 \)
  – \( \sim 400 \) citations
Empirical status of the Pion’s valence-quark distributions

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➢ Three experiments:
  - CERN (1983 & 1985)

➢ None more recent

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  – Leading-order analysis of the Drell-Yan data: \( Q^2 \approx 27 \text{ GeV}^2 \)

➢ Controversial!

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π valence-quark distributions
20 Years of Evolution

  - Leading-order analysis of Drell-Yan data: $Q^2 \approx 27 \text{ GeV}^2$
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   – QCD-connected model prediction
π valence-quark distributions
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   – Partial NLO analysis of E615 data
   – Large-\( x \) power-law \( \rightarrow 1.54 \pm 0.08 \)
\(\pi\) valence-quark distributions
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- **2010/02 ... Controversy highlighted:** Holt & Roberts, Rev. Mod. Phys. **82** (2010) 2991-3044
π valence-quark distributions
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- Consistent next-to-leading-order analysis
- Large-$x$ power-law $\rightarrow 2.6(1)$
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π valence-quark distributions
20 Years of Evolution

➢ 2010/09 ... Reconsideration of data:
  - Consistent next-to-leading-order analysis

➢ 2019/04 ... Ding, et al.
  - Continuum QCD prediction, using most sophisticated bound-state kernels currently available
π valence-quark distributions
20 Years of Evolution

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➢ 2019/01 ... Sufian, et al.
  - 1st exploratory lattice-QCD calculation,
  - using lattice-calculable matrix element obtained through spatially-separated current-current correlations in coordinate space
  - \( m_\pi^2 = 9 m_\pi^2\)-physical

Large-\(x\) exponent \( \zeta = 5.2\text{GeV} \) and momentum \( \zeta = 2\text{GeV} \)
Continuum ... 2.3(1) & \( <2x> = 0.52\pm0.04 \)
Lattice ... 2.2(1) & \( <2x> = 0.42\pm0.05 \)
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Modellers still ignoring QCD & violating symmetries
Phenomenologists question analysis of Aicher et al.


**π & K PDFs**

➢ **Urgent need for Newer Data**
   - Persistent controversy regarding the Bjorken-\(x \approx 1\) behaviour of the pion’s valence-quark PDF
   - Single modest-quality measurement of \(u^K(x)/u^\pi(x)\) (1980) cannot be considered definitive.

➢ **Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question**

➢ **Similar technique might also serve for the kaon ... experiment approved**

➢ **Future:**
   - New mesonic Drell-Yan measurements at modern facilities (COMPASS at LHC) could yield valuable information on \(\pi\) and \(K\) PDFs
     - Discussed extensively in “Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER)”
   - EIC would be capable of providing access to \(\pi\) and \(K\) PDFs through measurements of forward nucleon structure functions.
Kaon’s gluon content

\[ \langle x \rangle_{g}^{K}(\zeta_H) = 0.05 \pm 0.05 \]
\[ \Rightarrow \text{Valence quarks carry 95\% of kaon’s momentum at } \zeta_H \]

\[ \text{DGLAP-evolved to } \zeta_2 \]

<table>
<thead>
<tr>
<th>( q )</th>
<th>( \langle x \rangle_{q}^{K} )</th>
<th>( \langle x^2 \rangle_{q}^{K} )</th>
<th>( \langle x^3 \rangle_{q}^{K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>0.28</td>
<td>0.11</td>
<td>0.048</td>
</tr>
<tr>
<td>( \bar{s} )</td>
<td>0.36</td>
<td>0.17</td>
<td>0.092</td>
</tr>
</tbody>
</table>

Valence-quarks carry \( \frac{2}{3} \) of kaon’s light-front momentum

*Cf. Only \( \frac{1}{2} \) for the pion*
Marked differences between $\pi$ & $K$ gluon content

- $\zeta_H$:
  - Whilst $\frac{1}{3} \sim \frac{1}{5}$ of pion’s light-front momentum carried by glue
  - *Only* $\frac{1}{20}$ of the kaon’s light-front momentum lies with glue

- $\zeta_2^2 = 4 \text{ GeV}^2$
  - Glue carries $\frac{1}{2}$ of pion’s momentum and $\frac{1}{3}$ of kaon’s momentum

- Evident in differences between large-$x$ behaviour of valence-quark distributions in these two mesons

Signal of Nambu-Goldstone boson character of $\pi$

- Nearly complete cancellation between one-particle dressing and binding attraction in this almost-massless pseudoscalar system

$$2 \text{ Mass}_Q + U_g \approx 0$$
Understanding the emergence and character of Nambu-Goldstone modes in the Standard Model is critical

- Nambu-Goldstone modes are nonpointlike!
- Intimately connected with origin of mass!
- Possibly/Probably(?) inseparable from expression of confinement!

Difference between gluon content of $\pi$ & $K$ is measurable ... using well-designed EIC

Write a definitive new chapter in future textbooks on the Standard Model
Epilogue
Challenge: Explain and Understand
the Origin and Distribution of the Vast Bulk of Visible Mass

Current Paradigm: Quantum Chromodynamics

QCD is plausibly a mathematically well-defined quantum field theory,
*The only one we’ve ever produced*
– Consequently, it is a worthwhile paradigm for developing Beyond-SM theories

Challenge is to reveal the content of strong-QCD

**Tough Problem**

*Progress* and *Insights*
being delivered by amalgam of
– Experiment
– Phenomenology
– Theory

Must continue into eras of
\[ T_{\mu\nu} = \frac{1}{4} \beta(\alpha(\zeta)) C_{\mu\nu} C^{\mu\nu} \]

**Trace Anomaly**

- Knowing that a trace anomaly exists does not deliver a great deal... indicates only that a mass-scale must exist.
- Can one compute and/or understand the magnitude of that scale?
- One can certainly measure the magnitude... consider proton:
  \[ \langle p(P)|T_{\mu\nu}|p(P)\rangle = -p_{\mu}p_{\nu} \]
  \[ \langle p(P)|T_{\mu\nu}|p(P)\rangle = -m_{p}^{2} = \rho_{\mu}p_{\mu} \]

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, \( \rho_{\mu} \).
- In QCD, \( \rho_{\mu} \) measures the strength of gluon self-interactions... so, not from one perspective, \( m_{p} \) is (somehow) completely generated by glue.

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**Process-Independent effective-charge in QCD**

- Modern continuum & lattice methods for analyzing gauge sector enable... Self-interaction - low running charge to be defined in QCD.
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction.
- N.B. Qualitative change in \( g_{A}(k) \) at \( k = m_{p} \).

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**Pion’s Goldberger – Treiman relation**

- Solution of the Bethe-Salpeter equation
  \[ T_{\mu\nu}(k; p) = 2(2\pi)^{3} \int E_{\pi}(k; p) + \gamma \cdot P_{\pi}(k; p) \]
  \[ + (1 - \gamma \cdot k) P_{\pi}(k; p) + \rho_{\mu} p_{\nu} - \rho_{\nu} p_{\mu} \]

- Dressed-quark propagator
  \[ S(p) = \mathcal{Z}(p) = \left( \frac{1}{p^{2} - m^{2} + i\epsilon} \right) \]

- Axial-vector Ward-Takahashi identity entails
  \[ f_{p_{\pi}}(k; P = 0) = 0 \]

**Miracle - two body problem solved, almost completely, once solution of one body problem is known**

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**Pion masslessness**

- Obtain a coupled set of gap and Bethe-Salpeter equations.
- Quantum field theory statement:
  In the pseudoscalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them.

\[ \int d^{3}k E_{\pi}(p^{2}) = B(p^{2}) \]

- Cancellation guarantees:
  - simple system, without masses.
  - Becomes a complex system, with.
  - A non-trivialisation state wave function.
  - Attached to a pole in the scattering matrix, which remains at \( P^{2} = \ldots \)

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**\( \pi \) & K PDFs**

- Marked differences between \( \pi \) & K gluon content.
  - \( \xi_{G} \)
  - When \( 1/3 \) of \( \pi \)’s light-front momentum carried by glue.
  - Only \( 1/3 \) of kaon’s light-front momentum zero with glue.
  - \( \xi_{G}^{2} = 4 \text{GeV}^{2} \)
  - Glue carries \( 1/3 \) of pion’s momentum and \( 2/3 \) of kaon’s momentum.
  - Evident in differences between large-x behaviour of valence-quark distributions in these two mesons.
  - Signal of Nambu-Goldstone boson character of \( \pi \)
  - Nearly complete cancellation between oneparticle dressing and binding attraction in this almost-massless pseudoscalar system.

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