Parton distribution functions from the light front parton gas model

Shaoyang Jia

Department of Physics and Astronomy
Iowa State University
Ames, Iowa

Apr. 10, 2019

Parallel Session 8
8th Workshop of the APS Topical Group on Hadronic Physics
Denvor, Colorado

	equal light front time	equal time
quantization condition	$[\phi(x), \partial^+\phi(y)]_{x^+=y^+}$	$[\phi(x), \partial^0\phi(y)]_{x_0=y_0}$
kinetic energy	$(\vec{p}^{\perp 2} + m^2)/p^+$	$\sqrt{p_0^2 - \vec{p}^2 - m^2}$
dynamics	$\mathcal{H}_{\text{eff}} = P^+\mathcal{P}^- - \vec{P}^{\perp 2}$	$\mathcal{L}_{\text{int}}$

Light front coordinates $x^\pm = x^0 \pm x^3$ and momenta $p^\pm = p^0 \pm p^3$, $\vec{p}^\perp = (p^1, p^2)$.

- With light front QFTs, bound state structures are specified by LFWFs

$$|\Psi\rangle = 16\pi^3 P^+ \sum_{N=1}^{+\infty} \int d^{3N}\underline{p}\,\delta\left(\mathbf{P} - \sum_{j=1}^{N}\mathbf{p}_j\right)\psi_N(\mathbf{p})\left[\prod_{k=1}^{N} a_k^\dagger(\mathbf{p})\right]\Big|0\Big\rangle.$$

- Probability interpretation of PDFs $\rightarrow$ statistical description of hadron structures.

Microcanonical molecular dynamics ensemble

An isolated system of classical particles has fixed total momentum $\rightarrow$ microcanonical molecular dynamics ensemble (EVN$\mathbf{P}$ ensemble), whose phase space distribution is given by

$$\rho(E, V, N, \mathbf{P}; \mathbf{q}, \mathbf{p}) = \frac{1}{\Omega(E, V, N, \mathbf{P})} \delta(E - H(\mathbf{q}, \mathbf{p}))\, \delta\left(\mathbf{P} - \sum_{j=1}^{N} \mathbf{p}_j\right),$$

with $\mathbf{q}$ and $\mathbf{p}$ being the coordinates and momenta of the N-body system. The partition function is defined as

$$\Omega(E, V, N, \mathbf{P}) = \frac{1}{N!(2\pi)^{3N}} \int d\mathbf{q}\, d\mathbf{p}\, \rho(E, V, N, \mathbf{P}; \mathbf{q}, \mathbf{p}). \tag{1}$$

Scalar parton gas with light front kinematics

Ergodic time averaging is taken in the sense of light front time: $H(\mathbf{q},\mathbf{p}) \to P^-(\mathbf{x}^\perp,\mathbf{x}^-,\mathbf{p}^\perp,\mathbf{p}^+)$. Let us consider a collection of partons with light front kinematics:

$$P^- = \sum_{j=1}^{N} \frac{\mathbf{p}_j^{\perp 2}+m^2}{p_j^+} = \frac{1}{P^+}\left(\mathbf{P}^{\perp 2} + \sum_{j=1}^{N}\frac{\vec{\kappa}_j^{\perp 2}+m^2}{x_j}\right). \qquad (2)$$

The phase space distribution written in relative momenta $x_j = p_j^+/P^+$ and $\vec{\kappa}_j^\perp = \vec{p}_j^\perp - x_j\vec{P}^\perp$ becomes

$$\rho = \frac{1}{\Omega(E,V,N,\mathbf{P})}\,\delta\left(\left(P^+E-\vec{P}^{\perp 2}\right) - \sum_{j=1}^{N}\frac{\vec{\kappa}_j^{\perp}+m^2}{x_j}\right)\delta\left(1-\sum_{j=1}^{N}x_j\right)\delta\left(\sum_{j=1}^{N}\vec{\kappa}_j^\perp\right), \qquad (3)$$

Define $u = P^+E - \mathbf{P}^{\perp 2}$ as the thermal energy available for the relative motion of partons.

The joint longitudinal momentum fraction distribution

$$\omega(E, V, N, \mathbf{P}; \mathbf{x}) = \frac{(P^+ V)^N}{\Omega(E, V, N, \mathbf{P})} \int d^{2N}\kappa^\perp \, \delta\left(u - \sum_{j=1}^{N} \frac{\vec{\kappa}_j^{\perp 2} + m^2}{x_j} \right) \times \delta\left(1 - \sum_{j=1}^{N} x_j\right) \delta\left(\sum_{j=1}^{N} \vec{\kappa}_j^\perp\right) \quad (4)$$

After the variable transform $\vec{\kappa}_j^\perp = \sqrt{x_j}\,\vec{l}_j^\perp$ one obtains

$$\boxed{\omega(E, V, N, \mathbf{P}; \mathbf{x}) = \frac{1}{\Phi(E, V, N, \mathbf{P})} \left(\prod_{j=1}^{N} x_j\right) \delta\left(1 - \sum_{j=1}^{N} x_j\right) [\tilde{u}(\mathbf{x})]^{N-2} \, \theta(\tilde{u}(\mathbf{x})),}$$

with $\boxed{\tilde{u}(\mathbf{x}) = u - \sum_{i=1}^{N} \frac{m^2}{x_j}.}$ And $\Phi(E, V, N, \mathbf{P})$ ensures the normalization of $\omega(E, V, N, \mathbf{P}; \mathbf{x})$.

Single-particle longitudinal momentum fraction distribution

The single-particle x-distribution is defined as

$$\omega_x(E, V, N, \mathbf{P}; x_1) = \left(\prod_{j=2}^{N} \int dx_j \right) \omega(E, V, N, \mathbf{P}; \mathbf{x}). \qquad (5)$$

For $N = 2$, we have

$$\omega_x(E, V, 2, \mathbf{P}; x) = \frac{6\, x(1-x)\, \theta\left(u - 4m^2\right)}{\left(1 + \frac{2m^2}{u}\right)\sqrt{1 - \frac{4m^2}{u}}}\, \theta\left(x - \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{m^2}{u}}\right) \times \theta\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{m^2}{u}} - x\right). \qquad (6)$$

Theta functions ensure the positive definiteness of the Hamiltonian.

For $N = 3$ in the units of $m = 1$, we have

$$\omega_x(x) = \frac{[(1-x)(x_+ - x)(x - x_-)]^{3/2}}{\phi(u)\sqrt{ux-1}}\theta(u-9)\theta(x-x_-)\theta(x_+ - x) \quad (7)$$

with

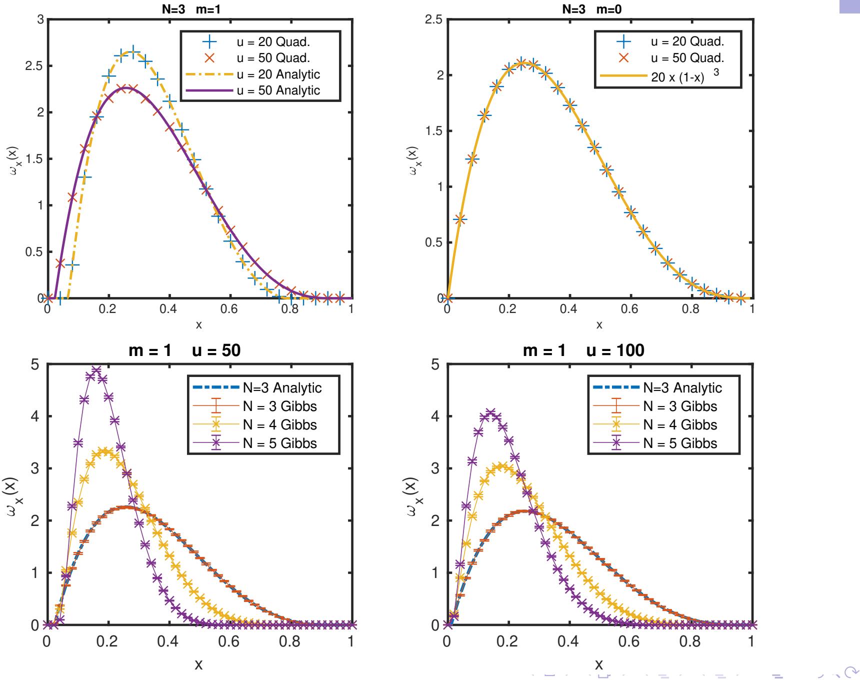
$$x_\pm = \frac{u - 3 \pm \sqrt{(u-9)(u-1)}}{2u}. \quad (8)$$

The normalization $\phi(u)$ is defined as

$$\phi(u) = \int_{x_-}^{x^+} dx\, \frac{[(1-x)(x_+ - x)(x - x_-)]^{3/2}}{\sqrt{ux-1}}. \quad (9)$$

In scenarios where all partons are massless, the single-parton x-distribution is given by

$$\omega_x(E, V, N, \mathbf{P}; x) = (2N-2)(2N-1)\, x\,(1-x)^{2N-3}\, \theta\left(P^+ E - \overrightarrow{P}^{\perp 2}\right). \quad (10)$$


N=3 m=1
u = 20 Quad.
u = 50 Quad.
u = 20 Analytic
u = 50 Analytic
ω_x(x)
x
N=3 m=0
u = 20 Quad.
u = 50 Quad.
20 x (1-x) 3
ω_x(x)
x
m = 1 u = 50
N=3 Analytic
N = 3 Gibbs
N = 4 Gibbs
N = 5 Gibbs
ω_x(x)
x
m = 1 u = 100
N=3 Analytic
N = 3 Gibbs
N = 4 Gibbs
N = 5 Gibbs
ω_x(x)
x

Summary and outlook

- The statistical light front parton gas model has been introduced for scalar partons.
    1. The joint phase space distribution of partons are given by the microcanonical molecular dynamics ensemble.
    2. With the light front kinetic energy in the Hamiltonian, the exact expression for the join parton longitudinal momentum fraction distribution has been derived.
    3. This joint distribution has been verified using the quadrature marginalization of the phase space distribution.
    4. The sampling of this joint distribution is available through a Gibbs sampler with a fixed number of partons.

  SJ and J. P. Vary, arxiv:1812.09340

- Further developments:
    1. introducing color, flavor, and spin to the partons,
    2. allowing particle creation and annihilation.