Visualising data in Longitudinal Phase Space (Van Hove Analysis)

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The latest generation of hadron physics experiments are providing high quality, large statistics datasets.
These will enable non-perturbative QCD to be probed to new depths.
Many facilities, many types of beam, but still much common ground.
Regardless of the beam type or detector system used, modern experiments share many features:

- Large acceptance coverage
- Charged and neutral particle identification across a wide momentum range
- Multi-particle final states
- Current experiments are already adept at detecting multi-particle final states
- Calorimetry, tracking, timing, triggering all combine to enable selection of such events
Even assuming perfect particle identification, different processes can result in the same final state.

This is particularly true as the number of particles in the final state increases.

For example, in the reaction $\gamma p \rightarrow pK^+K^-$, the final state particles could arise from the production of an intermediate meson ($\phi \rightarrow K^+K^-$), or baryon ($\Lambda \rightarrow pK^-$).
Separating these signals via conventional analysis cuts never completely removes the other process.
These competing processes, however, have very different reaction kinematics.

In the centre of mass frame, the two processes CAN be separated.
This isn’t a new idea, it was first proposed and studied fifty years ago by Leon Van Hove.

His basic premise is that at sufficiently high centre-of-mass energy, phase space is dominated by longitudinal components of particle momenta.

Transverse components can be neglected, reducing dimensionality of phase space.
This gave rise to the Longitudinal Phase Space plot, a way of visualising reaction kinematics of an n-particle final state in an n-1 dimensional plane.
Consider the process

$$A + B \rightarrow C_1 C_2 C_3$$

Let $W$ be the total CM energy, and $q_i, r_i$ be the longitudinal and transverse momentum components of particle $C_i$, i.e. $p_i = q_i + r_i$

We can also assume

$$\sum_{i=1}^{N} E_i = W, \quad \sum_{i=1}^{N} q_i = 0, \quad \sum_{i=1}^{N} r_i = 0$$
Van Hove Plots

- At sufficiently high centre-of-mass energy, the $r_i$s can be neglected
- The result is a reduction in the dimensionality of phase space
- For example, in a three particle final state, the longitudinal phase space can be represented on a two dimensional plane

- We can define the following relations
  
  \[ q_1 = \sqrt{\frac{2}{3}} q \sin(\omega) \]
  \[ q_2 = \sqrt{\frac{2}{3}} q \sin(\omega + \frac{2}{3} \pi) \]
  \[ q_3 = \sqrt{\frac{2}{3}} q \sin(\omega + \frac{4}{3} \pi) \]
  \[ q = (q_1^2 + q_2^2 + q_3^2)^{\frac{1}{2}} \]
Van Hove Plots (continued)

\[ q_1 = \sqrt{\frac{2}{3}} q \sin(\omega) \]
\[ q_2 = \sqrt{\frac{2}{3}} q \sin(\omega + \frac{2}{3} \pi) \]
\[ q_3 = \sqrt{\frac{2}{3}} q \sin(\omega + \frac{4}{3} \pi) \]
\[ q = (q_1^2 + q_2^2 + q_3^2)^{\frac{1}{2}} \]

- The \( q_i \)s are the distances from the centre along each of the lines A, B, C to the point \( q \)

- \( q \) and \( \omega \) are analogous to polar coordinates, and their X and Y equivalents can be defined as usual

\[ X = q \cos(\omega) \]
\[ Y = q \sin(\omega) \]
The axes divide the longitudinal phase space into six sectors, shown here for $pK^+K^-$ toy MC.

Each sector corresponds to specific directions of travel of the final state particles in the CM frame.

The arrows on the axes show the forward travel of each labelled particle.

For example, the bottommost sector has $K^+K^-$ going forward and proton going backward.
Application

- Bottommost sector has $K^+K^-$ going forward and proton going backward, like a meson decay.
- In the bottom left sector, only the $K^+$ is going forward, consistent with a Baryon decay to $K^-p$.
- By examining events in this manner, we have an additional means of separating baryon and meson decays with the same final state.
Let’s take an example from GlueX data, $\gamma p \rightarrow p\pi^+\pi^-$

Examine the Van Hove plot, with and without a sector cut to select the $\rho$
It can also be informative to examine the Van Hove angle against invariant mass of particle pairs.

Here, the $\rho$ can be clearly seen, as can possible background processes.
However, signal and background processes are not exclusively contained to individual sectors

A cut on the van Hove angle is not a silver bullet to slay unwanted mesons or baryons!!!
Returning to the toy MC example of $pK^+K^-$

Modulate the flat phase space with a 2.2 GeV baryon state and several mesons ranging in mass from 1.0 GeV to 1.8 GeV

Baryon and meson events cross sector boundaries
Toy MC, K+ K−

$t_1=1.5$ GeV$^2$

- 68.81% K+
- 20.13% K−

$t_1=1.5$ GeV$^2$ 91.44% survived

$t_1=1.5$ GeV$^2$ 74.03% survived

$t_2=3$ GeV$^2$

- 2.59% K+
- 97.41% K−

$t_2=3$ GeV$^2$ 62.78% survived

$t_2=3$ GeV$^2$ 2.05% survived
One more example from GlueX data, $\gamma p \rightarrow p\eta\pi^0 \rightarrow p\gamma\gamma\gamma\gamma$

Several mesons present (including $a_0(980)$ and $a_2(1320)$), as are some baryons
Van Hove angle against invariant mass of particle pairs

Here, the $a_0(980)$ and $a_2(1320)$ can be seen, and the baryon processes are also visible.

Instead of cutting on Van Hove angle, we can use these plots to inform baryon veto cuts, and verify them by examining the Van Hove plot.
Van Hove plot for $p\eta\pi^0$, before (left) and after (right) applying cuts on $p\pi^0$ and $p\eta$ mass.

We can see the cuts have removed much of the unwanted baryon processes, but at a cost of signal.
Generalising to more particles

- The expressions for three particles generalise to processes with greater multiplicity

\[ A + B \rightarrow C_1 C_2 \ldots C_n \]

- Total CM energy remains \( W \); \( q_i, r_i \) are still the longitudinal and transverse momentum components of particle \( C_i \), i.e. \( p_i = q_i + r_i \)

- And the assumptions

\[
\begin{align*}
\sum_{i=1}^{N} E_i &= W, \\
\sum_{i=1}^{N} q_i &= 0, \\
\sum_{i=1}^{N} r_i &= 0
\end{align*}
\]

Still hold
Four Particles

- In other words, we can describe the longitudinal phase space of a four particle final state in three dimensions.

- In this case, there are four triplets of particles, each describing a Van Hove phase space as for three particles.

- Their intersection forms a cuboctahedron, with the position of an event defined by the distances to each plane.
Extension to Four Particles

\[ q_4 = -q_1 - q_2 - q_3 \]
\[ x = \sqrt{\frac{3}{8}} (q_3 - q_2) \]
\[ y = \sqrt{\frac{1}{8}} (2q_1 + 3q_2 + 3q_3) \]
\[ z = q_1 \]
\[ r = \sqrt{x^2 + y^2 + z^2} \]
\[ \theta = \cos^{-1} \left( \frac{z}{\sqrt{x^2+y^2}} \right) \]
\[ \phi = \tan^{-1} \left( \frac{y}{x} \right) \]

Like the three particle version, different regions of the Van Hove plot correspond to specific directions of travel of the final state particles.
Example in Four Particles

- Consider the process $\gamma p \rightarrow p\eta\pi^+\pi^-$
- Different mesons can produce the same final state

$r = \sqrt{x^2 + y^2 + z^2}$
$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2}}$
$\phi = \tan^{-1}\frac{y}{x}$

- Similar to the three particle case, different regions of the Van Hove plot correspond to specific directions of travel of the final state particles and different decay processes
Great Circle Plot

- These three dimensional polar coordinates can also be visualised in terms of $\theta$ vs $\phi$
- The four hexagons forming the longitudinal phase space are circumscribed by great circles
- The different Van Hove regions can then be represented as a “Great Circle Plot”

![Great Circle Plot Diagram](image)
In $\gamma p \rightarrow p\eta\pi^+\pi^-$, the Great Circle Plot looks like this.
$\eta'$ events are dominant, but the neighbouring regions will contain events from other mesons.
Comparing $\eta\pi^+\pi^-$ invariant mass for all data (left) with just the Van Hove region for $\eta'$ (right)

![Graphs showing comparison of $\eta\pi^+\pi^-$ invariant mass for all data and just the Van Hove region for $\eta'$.]

Again, a cut is not a silver bullet, but useful as diagnostic tool nonetheless.
Van Hove plots provide another means of separating overlapping resonances.

We have an extensive set of tools in GlueX software to make and manipulate these plots for three and four particle final states.
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