Measuring Polarized Gluon Distributions by Top Pair Production Spin Correlations

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Abstract

Top-antitop pairs are produced prolifically in p+p collisions at the LHC, primarily by gluon fusion. At intermediate values of momentum fraction \(x\) for each gluon in \(g+g\) to \(t+t\bar{t}\), the spin dependences of gluon distributions leave imprints on the momentum and spin correlations of the top pairs. These correlations are distinguishable from the quark-antiquark annihilation mechanism. Decays of such spin entangled top pairs produce a variety of correlations among pairs of the 3-momenta of the decay products - particles and jets. Some different angular correlations will be presented and related to measurable distributions of pairs of jets and/or leptons. Models for spin dependent gluon transverse momentum distributions and generalized transverse momentum distributions will be used to simulate top pair decay product spin correlations, illustrating how to measure the gluon or quark polarizations in the colliding protons.
Gluon Distributions

Transversity

Top quarks

Gluon Transversity → Top Pair Spin Correlations
GPD’s & TMD’s in Models—e.g. Reggeized spectator “flexible parameterization”

Electroproduction

Gluon GPDs Polarized Gluons?
Transversity NOW Seen!

$t+t$-bar production & decay to measure Gluon polarization in $p+p$ @ LHC. Inclusive $\rightarrow$ TMDs

Top spin correlations & Observable quantities
Gluon GPDs

\[
\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P', \Lambda' | G^+(\frac{1}{2}z) G^+(-\frac{1}{2}z) | P, \Lambda \rangle \bigg|_{z^+=0, \bar{z}_T=0} = \\
\frac{1}{2P^+} \tilde{U}(P', \Lambda') \left[ \hat{H}^g(x, \xi, t) \gamma^+ + E^g(x, \xi, t) \frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M} \right] U(P, \Lambda)
\]

Even t-channel parity & Gluon helicity conserving

\[
-\frac{i}{P^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P', \Lambda' | \tilde{G}^+(\frac{1}{2}z) \tilde{G}^+(-\frac{1}{2}z) | P, \Lambda \rangle \bigg|_{z^+=0, \bar{z}_T=0} = \\
\frac{1}{2P^+} \tilde{U}(P', \Lambda') \left[ \hat{H}^g(x, \xi, t) \gamma^+ \gamma_5 + E^g(x, \xi, t) \frac{\gamma_5(-\Delta^+)}{2M} \right] U(P, \Lambda)
\]

Odd t-channel parity & Gluon helicity conserving

Must have 4 more Gluon helicity NONconserving
Extension to Gluon “Transversity”

\[
- \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' \mid SF^{+i}(\frac{1}{2} z) F^{+j}(\frac{1}{2} z) \mid p, \lambda \rangle \bigg|_{z^+ = 0, z_T = 0} = S \frac{1}{2P^+} \frac{P^+\Delta^j - \Delta^+ P^j}{2mP^+} \\
\times \bar{u}(p', \lambda') \left[ H_1^g i\sigma^{+i} + \tilde{H}_1^g \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} \right. \\
\left. + E_1^g \frac{\gamma^+\Delta^i - \Delta^+ \gamma^i}{2m} + \tilde{E}_1^g \frac{\gamma^+ P^i - P^+\gamma^i}{m} \right] u(p, \lambda).
\]

4 GPDs: see M. Diehl, EPJC19, 485 (2001)

4 Gluon helicity NONconserving Double flip
Gluon “transversity”
Double helicity flip does not mix with quark distributions

Transversity for on-shell gluons or photons: no $|0\rangle$ helicity

$$|+1\rangle_{\text{trans}} = \{|+1\rangle + |-1\rangle\} / 2 = |-1\rangle_{\text{trans}}$$

$$|0\rangle_{\text{trans}} = \{|+1\rangle - |-1\rangle\} / \sqrt{2}$$

Helicity $|\pm 1\rangle = \{-/+\hat{x} - i\hat{y}\} / \sqrt{2}$

$$\hat{x} = -|0\rangle_{\text{trans}} = P_{\text{parallel}}$$  \hspace{1cm} \text{Linear polarization in the plane}

$$\hat{y} = i\sqrt{2} |+1\rangle_{\text{trans}} = P_{\text{normal}}$$  \hspace{1cm} \text{Linear polarization normal to the plane}

Using Reggeized Spectators Model
Many other models & recently


TMDs
D. Boer, Few-Body Syst. (2017); C. Pisano, et al., JHEP 10, 024 (2013);
D. Boer, et al., PRL 106, 132001 (2011); . . . .
**Helicity flip** \( A_{\Lambda', -1; \Lambda, +1} \) contributes to **DVCS** \( \sim \alpha_s \)

\[
M_{\Lambda', \Lambda' \gamma = -1; \Lambda, \Lambda \gamma = +1} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = -1; \Lambda, \Lambda g = +1}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C'(x, \xi, Q^2)
\]

Interference with Bethe-Heitler contains \( \cos 3\phi \) modulation to distinguish from (leading twist) quark contribution.

- For \( \Lambda' \gamma = +1 \) and \( \Lambda' \gamma = -1 \), 
- For \( \Lambda g = +1 \) and \( \Lambda g = -1 \),

\[\Lambda^' \text{ \mu^- \mu^+ alternative method to extract}\]

Other alternatives to extract J/ψ production; jets . . . .

Polarized & unpolarized beam measurements

Evidence of gluon transversity

Fitting $\phi$ distribution requires $F_{++}$ and both $F_{+-}$ gluon transversity and $F_{0+}$ higher twist

$$\frac{d^4\sigma(h)}{dQ^2dx_Bdt\phi} = \frac{d^2\sigma_0}{dQ^2dx_B} \times \left[ |I^{BH}|^2 + |I^{DVCS}(h)|^2 - I(h) \right]$$

A glimpse of gluons through deeply virtual compton scattering on the proton, published in Nature Communications 8, 1408 (2017). doi:10.1038/s41467-017-01819-3

Analysis of 6 GeV Hall A DVCS data on the proton.

See Latifa Elouadrhiri talk at QCD Evolution 2018
LHC – many opportunities for studying gluons
p+p unpolarized $\rightarrow$ jets, hadrons, leptons
Interactions via $g+g \rightarrow Q+Q\bar{q} + X$
gluon TMDs in some kinematics
Extension to Gluon “Transversity”
c.f. $TMDs \, h_1 g (x, p_T^2)$ Mulders & Rodrigues (2001), Gluon Boer-Mulders function
see D. Boer, Frascati talk (Nov.2016) & many references for measurements at EIC, RHIC, LHC
Heuristic for spin TMDs

• Boer-Mulders for Unpolarized nucleon with “Transversly” polarized quark

<table>
<thead>
<tr>
<th>N/q</th>
<th>U</th>
<th>L</th>
<th>T</th>
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<tbody>
<tr>
<td>U</td>
<td>$f_1$</td>
<td></td>
<td>$h_1^\perp$</td>
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<tr>
<td>L</td>
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<td>$g_1$</td>
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<td>T</td>
<td>$f_{1T}^\perp$</td>
<td>$g_{1T}$</td>
<td>$h_1$ $h_{1T}^\perp$</td>
</tr>
</tbody>
</table>
The matrix representation is also convenient to find the physical meaning of the distributions. Well known is $G$ which measures the number of gluons with momentum $(x,k_T)$ in a hadron. The functions $GL$ (GT) represents the difference of the numbers of gluons with opposite circular polarizations in a longitudinally transversely polarized nucleon. The off-diagonal function $H$ also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, $H$ flips the polarization.

Mulders & Rodrigues, PRD63, 94021 (2001)
Gluon TMDs

TMD Color gauge invariance

Small x gluons, Kharzeev, Kovchegov, Tuchin, saturation,

2 gluon distributions: WW vs. DP,

Saturation issues: McLerran-Venugopalan model,
ColorGlassCondensate, . . . ?

See Mulders, et al.: small x DP is pure gauge link

\[ \Gamma^{\mu\nu}[U,U'](x, k_T) = \int \frac{d(\xi \cdot P) d^2k_T}{(P \cdot n)^2 (2\pi)^3} e^{i(xP+kt)\cdot\xi} \left[ P \mid \text{Tr}_c \left( F^{\mu\nu}(0) U_{[0,\xi]} F^{\mu\nu}(\xi) U'_{[\xi,0]} \right) \mid P \right] \bigg|_{\xi \cdot n = 0} \]

Gauge link
\[ \xi = [0^+, \xi^-, \xi_T] \]
\[ U_c[0, \xi] = \mathcal{P} \exp \left( -ig \int_{c[0,\xi]} ds_\mu A^\mu(s) \right) \]

Can be forward light front pointing link +∞ FSI. Weizsacker-Williams
Or mixed light front pointing link -∞ ISI Dipole

For \( U \) and \( U' \) have [+] or [-] (& parity opposites)

Consider TMD & GPD vs. data at intermediate x
The Model for valence quarks- Reggeized Diquarks

The Model – first for Chiral Even – then Odd Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector

Gluon GPDs, TMDs, GTMDs
From p+p to gluon TMDs to quark pairs

Form quarkonia & different possibilities for gg
Complications from f.s.i. & jets – hadronization
See Boer, Brodsky, Pisano, et al., . . .

Factorization and evolution
For Gluon fusion top production at LHC

- $g_1$ & $g_2$ carry helicity $\Lambda_1 \Lambda_2 = \pm 1$ & color 1, 8... & C=+ or -
- $t$ & $t$-bar carry helicity $\lambda_t , \lambda_{t\bar{t}} = \pm \frac{1}{2}$ & color 1 or 8
- $t$ & $t$-bar decay before hadronizing => no toponia & large scale
How is top polarization determined?
Its decay is good analyzer for transverse polarization.

\[ U_{\lambda_t, \chi_t} = \sum_{\lambda_b} B_{\lambda_b, \chi_t}^* B_{\lambda_b, \chi_t} \]
\[ \propto (I + \vec{p}_l \cdot \vec{\sigma}_t / p_l)_{\lambda_t, \chi'_t}(p_b \cdot p_\nu) \]

Calculated in top rest frame
OR

\[ U = (p_t - m_t S_t) \cdot p_l (p_b \cdot p_\nu) \]

\[ S_t = \left[ \frac{\vec{p} \cdot \vec{P}_t}{m_t}, \vec{p}_t + \frac{(\vec{p} \cdot \vec{P}_t) \vec{P}_t}{m_t (E_t + m_t)} \right] \]

Covariant form in any frame
\[ P_t = \text{strength of top polarization} \]

Dalitz & GRG, PLB287,225(1992); PRD45, 1531(1992)

lepton or u-quark moves parallel to transverse polarization
What is known about polarized top production?

Top Single Spin Asymmetry and Double Spin Correlations – Measurements

** SSA: \[ B_1 \text{ or } A_p = -0.035 \pm 0.040. \text{ (syst & stat)} \]

*** Double: \[ C_{\text{helicity}} = 0.315 \pm 0.07 \text{ vs. NLO QCD } = 0.31 \]
(Bernreuther, et al., PRL 87, 242002 (2001) QCD corrections but unpolarized gluons)

CMS PRL112, 182001 (2014): Different kinematics & selection criteria

** SSA: \[ A_p = 0.005 \pm 0.01. \]

*** Double: \[ A_{\Delta \phi} = 0.113 \pm 0.01 \text{ vs. } 0.110 \pm 0.001 \text{ (MC & QCD)} \]
\[ A_{c_1 c_2} = -0.021 \pm 0.03 \text{ vs } -0.078 \pm 0.001 \]

\[
\frac{1}{\sigma} \frac{d^2 \sigma}{d \cos \theta_1 d \cos \theta_2} = \frac{1}{4} \left( 1 + B_1 \cos \theta_1 + B_2 \cos \theta_2 - C_{\text{helicity}} \cos \theta_1 \cdot \cos \theta_2 \right)
\]
\[ \theta_1 \theta_2 \text{ decay product angles w.r.t. } t+t\text{bar CM} \]
Direct measure of hard process - top polarization
Top decays weakly before hadronizing ⇒ decay "self-analyzing"

Contributions to order $\alpha_S$ Imaginary Part (Dharmaratna & GRG 1990,1996)

Analyze $t\rightarrow W^+ b$
Dilepton events or lepton+hadron jets or all Hadron jets

Tree-level QCD

q+q (Tevatron)
or g+g (LHC)

Figure for net $p_{\text{transverse}}$

For gluon transversity

need large net $p_{\text{transverse}}$
to access transversity
For inclusive $p+p \rightarrow t+t\bar{t}+X$

$\Delta(\Lambda_N - \Lambda'_{N'} - \Lambda_{g} + \Lambda'_{g})$

Momentum transfer dependence

$G_{\Lambda N, \Lambda g, \Lambda' g}$
Gluon linear polarization with like and unlike $t$-$\bar{t}$ helicities

[work on particular gluon distributions still in progress S. Liuti, GRG, Gonzalez-Hernandez, Poage (thesis)]

$F \sim G_{XX} + G_{YY}, \ H \sim G_{XX} - G_{YY}$ or linear polarization

$$\rho_{t', \bar{t}'; t, \bar{t}} \quad \begin{array}{cccc}
F & F & \bar{H} & H \\
F & H & \bar{H} & F
\end{array}$$

$$
\begin{array}{cccc}
++;++ & \gamma^{-2} (1 + \beta^2 (1 + \sin^4 \theta)) & \gamma^{-2} (-1 + \beta^2 (1 + \sin^4 \theta)) & -2 \frac{\beta^2}{\gamma^2} \sin^2 \theta & -2 \frac{\beta^2}{\gamma^2} \sin^2 \theta \\
+-+- & \beta^2 \sin^2 \theta (2 - \sin^2 \theta) & - \beta^2 \sin^4 \theta & 0 & 0
\end{array}
$$
\[ \rho_{t',t',t,t} \propto \sum_{all-helicities-not-tops} \tilde{G}_{\Lambda_N\Lambda_N\Lambda_g'\Lambda_g} A^*_{\Lambda_g'\Lambda_g'\Lambda_g'\Lambda_g} A_{\Lambda_N\Lambda_N\Lambda_g'\Lambda_g} G_{\Lambda_N\Lambda_N\Lambda_g'\Lambda_g} \]

- The gluon spin correlations are transmitted to (determine the spin of) the decay products.

- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.

- The **gluon fusion mechanism** gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)

\( g_1, g_2 \) carry helicity \( \Lambda_1, \Lambda_2 = \pm 1 \) OR transversity 1 or 0
\( t, \bar{t} \)-bar carry helicity \( \lambda_t, \lambda_{\bar{t}} = \pm \frac{1}{2} \) OR transversity \( \pm 1/2 \)

Introduced in:
The light (polarized) quark-antiquark annihilation mechanism gives rise to the angular distribution between opposite charge lepton pairs, more information than $C_{\text{helicity}}$ or $A_{c1\ c2}$

$$W(\theta, p, p\hat{l}, p_l) = \frac{1}{4} \left\{ 1 + [\sin^2 \theta ((p^2 + m^2)(\hat{p}\hat{l})_x(\hat{p}\hat{l})_y + (p^2 - m^2)(\hat{p}\hat{l})_y(\hat{p}\hat{l})_y) \right.$$
$$- 2mp \cos \theta \sin \theta ((\hat{p}\hat{l})_x(\hat{p}\hat{l})_z + (\hat{p}\hat{l})_z(\hat{p}\hat{l})_x) + [(p^2 - m^2)$$
$$+ [p^2 + m^2] \cos^2 \theta (\hat{p}\hat{l})_x(\hat{p}\hat{l})_z] / [(p^2 + m^2) + (p^2 - m^2) \cos^2 \theta] \left\} \right.$$}

$$= \frac{1}{4} + \frac{1}{4} \left\{ (2 - \beta^2) \sin^2 \theta (\hat{p}\hat{l})_x(\hat{p}\hat{l})_x + \beta^2 (\hat{p}\hat{l})_y(\hat{p}\hat{l})_y + [\beta^2 + (2 - \beta^2) \cos^2 \theta (\hat{p}\hat{l})_z(\hat{p}\hat{l})_z$$
$$- \frac{2}{\gamma} \cos \theta \sin \theta ((\hat{p}\hat{l})_x(\hat{p}\hat{l})_z + (\hat{p}\hat{l})_z(\hat{p}\hat{l})_x) \right\} / [(2 - \beta^2) + \beta^2 \cos^2 \theta]$$

$m =$top mass, $\theta =$ t production angle in q+q-bar CM
$p =$ light quark 3-momentum in CM
Unit vectors $p$-hat are anti-lepton$^+$ and lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

\[ g_1+g_2 \rightarrow t + \bar{t}-\text{bar} \]

**Spin correlations - dilepton channel**

Correlations expressed as a weighting factor first for **unpolarized gluons**.

- The gluon fusion mechanism gives rise to a higher order angular distribution \( \sin^4 \theta \) due to the combination of two spin 1 gluons.

\[
W(\theta, p, p_t, p_i) = \frac{1}{4} \left[ \frac{1}{4} \left\{ [p^4 \sin^4 \theta + m^4] (\hat{p}_t)_x (\hat{p}_i)_x + [p^2 (p^2 - 2m^2) \sin^4 \theta - m^4] (\hat{p}_t)_y (\hat{p}_i)_y \\
+ [p^4 \sin^4 \theta - 2p^2 (p^2 - m^2) \sin^2 \theta + m^2 (2p^2 - m^2)] (\hat{p}_t)_z (\hat{p}_i)_z \\
+ 2mp^2 \sqrt{p^2 - m^2} \cos \theta \sin^3 \theta [(\hat{p}_t)_x (\hat{p}_i)_x - (\hat{p}_t)_z (\hat{p}_i)_z] \right\} \\
/ \left[ p^2 (2m^2 - p^2) \sin^4 \theta + 2p^2 (p^2 - m^2) \sin^2 \theta + m^2 (2p^2 - m^2) \right] \right]
\]

\[
= \frac{1}{4} \left[ \frac{1}{4} \left\{ [(1 - \beta^2)^2 + \sin^4 \theta] (\hat{p}_t)_x (\hat{p}_i)_x \\
+ [-(1 - \beta^2)^2 - (1 - 2\beta^2) \sin^4 \theta] (\hat{p}_t)_y (\hat{p}_i)_y \\
+ [(1 - \beta^4) - 2\beta^2 \sin^2 \theta + \sin^4 \theta] (\hat{p}_t)_z (\hat{p}_i)_z \\
+ 2\beta^3 \sin^3 \theta \cos \theta [(\hat{p}_t)_x (\hat{p}_i)_x - (\hat{p}_t)_z (\hat{p}_i)_z] \right\} \\
/ \left[ (1 - \beta^4) + 2\beta^2 \sin^2 \theta + (1 - 2\beta^2) \sin^4 \theta \right] \right]
\]

\( m \) = top mass, \( \theta = t \) production angle in g+g CM; \( p = \) gluon 3-momentum in CM

\( p \)-hat’s are lepton 3-momenta directions in the top and anti-top rest frames.

**Use these to test SM vs. BSM – Integrated version agrees – with big errors -- GRG in process -- see also Mahlon & Parke**

**See GG& Liuti, 1710.01683; 2024742 (APS-DFP 2017)**

5/10/19

G. R. Goldstein  QCD_Evol_Argonne 2019
\[ g_1 + g_2 \to t + \bar{t}\]

**Spin correlations**

Correlations expressed as a weighting factor for polarized gluons.

- The **gluon fusion mechanism** gives rise to a higher order angular distribution \((\sin^4\theta)\) due to the combination of two spin 1 gluons.

\[
W^{(LP,LP)}(\theta, p, p_l, p_t) = -\frac{1}{4} + \frac{1}{4} \left\{ \frac{((1 - \beta^4) + \beta^2 \sin^2 \theta(-2 + (2 - \beta^2) \sin^2 \theta))(\hat{p}_l)_x(\hat{p}_l)_z}{4 + \frac{\beta^2}{\gamma} \sin^3 \theta \cos \theta[(\hat{p}_l)_x(\hat{p}_l)_z - (\hat{p}_l)_z(\hat{p}_l)_x]} \right\}
\]

**Crucial measurements**

\[
(\hat{p}_l)_x(\hat{p}_l)_z = W_{xx}, \quad (\hat{p}_l)_x(\hat{p}_l)_\bar{z} = W_{xz}, \ldots \]

**Weighting tensor**

- Use these to compare with unpolarized to extract the Gluon transversity
- or linear polarizations \(G_{xx} - G_{yy}\)
- Careful about Frames:
- Collider LAB, \(t + \bar{t}\) pair CM, separate \(t\) & \(t\)-bar rest, \(W^{+/}\) rest frames
Comparing lepton directional correlations

Weighting tensor for lepton$^+$ lepton$^-$ when $\theta=\pi/8$

or lepton$^+$ d-quark or u-quark lepton$^-$

Each event has $\mu^-$ $\mu^+$ momenta $\rightarrow p^\pm (x, y, z)$ as well as $\theta$ & $\beta$

Probability for given event configuration is given by

$G(\text{UP}) \ W(\theta, p, p^\pm l, pl)$ + $G(\text{LP}) \ W^{\text{LP}} (\theta, p, p^\pm l, pl)$

Quite distinct! $x$ & $y$ components are aligned for LP, anti-aligned for UP

Can Diagonalize (with $W_{xy}, W_{yx}$) to obtain positive ellipsoidal weighting
Comparing lepton directional correlations

Weighting factors for lepton$^+$ lepton$^-$ when $\theta=\pi/2$
$W_{xz}=0$ for the off-diagonal

$\Theta = \pi/2$
$\beta$

Each event has $\mu^-$ $\mu^+$ momenta $p^\pm (x, y, z)$ as well as $\theta$ & $\beta$

Probability for given event configuration is given by
$G(\text{UP}) W(\theta, p, p^\perp l, pl) + G(\text{LP}) W^{\text{LP}} (\theta, p, p^\perp l, pl)$

Quite distinct! $x$ & $y$ components are aligned for LP, anti-aligned for UP
Diagonalize (with $W_{xy,yx}$) to obtain positive ellipsoidal weighting
Separating polarized gluons

* Each event has $\mu^- \mu^+$ momenta $\rightarrow p^\pm (x, y, z)$
  in t & tbar rest frame
* t+tbar CM determines $\theta$ direction as well as $\beta$ for t & tbar
* Probability for given event configuration is given by

$$G(UP) \ W(\theta,p,p^{-}l,pl) \ +G(LP) \ W^{LP} (\theta,p,p^{-}l,pl)$$
  (ignoring light quarks)

- Quite distinct! x & y components are
- aligned for LP, anti-aligned for UP
- G’s convoluted with W’s all gluon $k_T \ & \ k_T$ satisfying
- measured $p_t+p_{anti-t} \ <->$ large transverse momenta : transversity
Large transverse momentum

- t\-t\¬bar inclusive at 13 TeV
How are top pair polarizations measured at LHC?

• Purely hadronic events $\supseteq 6$ particles/jets $(b,u,d + \bar{b},\bar{u},d)$. Combinatorics!

• Dilepton events leave unknown $\nu & \bar{\nu}$ momenta $(b,e^+ \nu + \bar{b},\bar{u},d)$. Clean, lower $d\sigma$

• Single lepton events $\supseteq$ one $\nu$ missing
  – Most promising: with H. Beauchemin (ATLAS), M. Yampolskaya, T. Lachance

• What is $t$ or $\bar{t}$ momentum?

• Measuring $e$ or $\mu$ and $b$-jet fixes $t$ to an ellipse
Finding top momentum

- top leptonic decay in lab
- $\rightarrow$ momenta in lab
- $b$-jet & lepton measured $\Rightarrow$
- Ellipse of $t$-vectors determined
- Boost to rest frame
- helicity preserved
- Lepton direction correlated

**Fig. 5.** Momentum vectors $b$ and $t$ observed in the laboratory frame for bottom quark and lepton, and the construction for locating all top-quark momenta $t$ such that these three vectors can correspond to the decay sequence $t \rightarrow bW^{+}, W^{+} \rightarrow t^{+} \nu_{t}$ for a given top-quark mass $m_{t}$.

Dalitz & GG, PRD45,1531(1992)
Summary

- Gluon GPDs & TMDs (from spectator & Regge $R \times D_q$)
- *Helicity* conserving & Helicity flip $\rightarrow$ gluon *Transversity*
- Electroproduction & DVCS $\rightarrow$ gluon transversity GPDs
- $pp \rightarrow$ gluons $\rightarrow t + t\bar{t} + X \rightarrow$ gluon TMDs
- Measurements? Single Top polarization
- $t + t\bar{t}$ spin correlations $\rightarrow$ via lepton decays or hadron jets

- To Do List
  - More phenomenology to come
  - Parton showers & jets
  - Care about evolution, factorization, power counting, . . .
Collaborators: Gluons
Simonetta Liuti, Osvaldo Gonzalez Hernandez, Jon Poage

• GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
• GRG, Liuti, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
• J.Poage, Tufts U. dissertation (2016)
• GRG & Liuti, Hernandez, PoS QCDEV2017, 037 (2017)

Collaborators: Tops
Richard Dalitz,
Discussions: Krzysztof Sliwa, Hugo Beauchemin Tufts and Atlas


Collaborators: Transversity
Michael J. Moravcsik,

• GRG & M.J. Moravcsik, Ann. Phys. 98, 128 (1976); ibid. 142, 219 (1982);
• Ibid. 195, 213 (1989).

Thank you!
Construct helicity flip amps
Spectator Model, then GPDs

\[ A_{++,+-} = \sqrt{1 - \xi^2} \frac{t_0 - t}{4M^2} \left( \tilde{H}_T^g + (1 - \xi) \frac{E_T^g + \tilde{E}_T^g}{2} \right) \]
\[ A_{--,-+} = \sqrt{1 - \xi^2} \frac{t_0 - t}{4M^2} \left( \tilde{H}_T^g + (1 + \xi) \frac{E_T^g - \tilde{E}_T^g}{2} \right) \]
\[ A_{++,-+} = +e^{-i\phi}(1 - \xi^2) \frac{\sqrt{t_0 - t}}{2M} \left( H_T^g + \frac{t_0 - t}{M^2} \tilde{H}_T^g - \frac{\xi^2}{1 - \xi^2} E_T^g + \frac{\xi}{1 - \xi^2} \tilde{E}_T^g \right) \]
\[ A_{+-,+} = -e^{i\phi}(1 - \xi^2) \frac{\sqrt{t_0 - t}}{8M^3} \tilde{H}_T^g, \]

Compare to spectator model results

\[ \tilde{H}_T^g = 0 \]

\[ (1 - X)A_{+-,-+}^0 = (1 - X')A_{++,+}^0 \]

\[ \tilde{E}_T^g = 0. \]

As in Hoodbhoy & Ji, PRD58, 054006 (1998)
Measuring Gluon GPDs in Nucleons

For unpolarized $\text{e}+\text{p} \rightarrow \text{e}'+\gamma+p'$ cross section depends on azimuthal angle $\phi$. $\cos 3\phi$ modulation in interference $d\sigma$ measures gluon transversity GPDs (CFF’s).

$$\sqrt{t_0 - t^3} \over 8M^3 \left[ H^g_{T} F_2 - E^g_{T} F_1 - 2\tilde{H}^g_{T} \left( F_1 + \frac{t}{4M^2} F_2 \right) \right] \cos 3\phi$$

$$\mathcal{H}^g_T \sim \int dx \; H^g_{T} / (x-\xi)(x+\xi) \; \text{CFF’s}$$

But $\mathcal{H}^g_T \sim$ may need EIC

Evidence of gluon transversity

Fitting $\phi$ distribution requires $F_{++}$ and both $F_{+-}$ gluon transversity and $F_{0+}$ higher twist.

<table>
<thead>
<tr>
<th>Helicity states</th>
<th>LO/LT</th>
<th>Higher twist</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = -0.18$ GeV$^2$</td>
<td>250</td>
<td>204</td>
<td>206</td>
</tr>
<tr>
<td>$t = -0.24$ GeV$^2$</td>
<td>367</td>
<td>206</td>
<td>208</td>
</tr>
<tr>
<td>$t = -0.30$ GeV$^2$</td>
<td>415</td>
<td>189</td>
<td>190</td>
</tr>
</tbody>
</table>

Values of $\chi^2$ (ndf = 208) obtained in the leading-order, leading-twist (++); higher-twist (++/0+); and next-to-leading-order (++/--) scenarios. The fit is not performed at the highest value of $-t$ because of the lack of full acceptance in $\phi$, resulting in a large statistical uncertainty. The fits include statistical and point-to-point systematic uncertainties.
Constructing gluon GPDs
Gluon 'vertex functions' \( G_{\Lambda x} \); \( \Lambda g, \Lambda \)

\[ \lambda, k^+ = XP^+ \]
\[ P^+ \rightarrow P_{\chi}^+ = (1-X)P^+ \]

\[ \lambda', k'^+ = (X-\zeta)P^+ \]
\[ P'_{\chi} = (1-\zeta)P^+ \]

\[
\begin{align*}
G_{+++}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X)}} \frac{(k_x - ik_y)}{X} \\
G_{-++}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X)}} (M(1 - X) - M_x) \\
G_{++-}(x, \vec{k}_T^2) &= 0 \\
G_{-+-}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X)}} (1 - X) \frac{(k_x - ik_y)}{X} \\
G^*_{+++}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X')}} \frac{(k_x + ik_y)}{X'} \\
G^*_{-++}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X')}} (M(1 - X') - M_x) \\
G^*_{++-}(x, \vec{k}_T^2) &= 0 \\
G^*_{-+-}(x, \vec{k}_T^2) &= -\frac{2}{\sqrt{2(1-X')}} (1 - X') \frac{(k_x + ik_y)}{X'}
\end{align*}
\]

\[ X' = \frac{X - \zeta}{1 - \zeta}, \tilde{k}_i = 1,2 = k_i - \frac{1-X}{1-\zeta} \Delta_i \]

GRG, Gonzalez Hernandez, Liuti, Poage, in progress
Single Spin Asymmetry

\[ pp \rightarrow \Lambda^\uparrow (\Lambda_c^\uparrow) X \]

K. Heller, PRD1997
curves from model of
Dharmaratna & GRG PRD ‘90 & ‘97

\( \pi^- + p \rightarrow \Lambda_c + X \)
curves from GRG hep-ph/9907573
After pdf’s vs. $Q^2 \rightarrow$ fix $x$ dependence
Regge behavior determines $t$ dependence
Spectator determines $\zeta$ dependence

$t = t_{\text{min}} \text{ to } -0.7 \text{ GeV}^2$

$H_g(x,0,0)$  
$Q^2=$initial to 10 GeV$^2$

$H_g(x,\zeta,t)$  
$Q^2=$initial

from J. Poage
Top spin correlations & gluon polarizations

<table>
<thead>
<tr>
<th>( \rho_{\nu,\bar{\nu};t,\bar{t}} )</th>
<th>UP,UP</th>
<th>LP,LP</th>
<th>UP,LP + LP,UP</th>
</tr>
</thead>
<tbody>
<tr>
<td>++, ++</td>
<td>( \gamma^{-2}(1 + \beta^2(1 + \sin^4\theta)) )</td>
<td>( \gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta)) )</td>
<td>(-4\gamma^{-2}\beta^2\sin^2\theta)</td>
</tr>
<tr>
<td>+-, +−</td>
<td>( \beta^2\sin^2\theta(2 - \sin^2\theta) )</td>
<td>( -\beta^2\sin^4\theta )</td>
<td>0</td>
</tr>
<tr>
<td>++, −−</td>
<td>( \gamma^{-2}(-1 + \beta^2(1 + \sin^4\theta)) )</td>
<td>( \gamma^{-2}(+1 + \beta^2(1 + \sin^4\theta)) )</td>
<td>( +4\gamma^{-2}\beta^2\sin^2\theta )</td>
</tr>
<tr>
<td>+−, −+</td>
<td>( \beta^2\sin^4\theta )</td>
<td>( -\beta^2\sin^2\theta(2 - \sin^2\theta) )</td>
<td>0</td>
</tr>
<tr>
<td>++, +−</td>
<td>( -2\gamma^{-1}\beta^2\sin^3\theta \cos\theta )</td>
<td>( -2\gamma^{-1}\beta^2\sin^3\theta \cos\theta )</td>
<td>( -4\gamma^{-1}\beta^2\sin\theta \cos\theta )</td>
</tr>
<tr>
<td>++, −+</td>
<td>( 2\gamma^{-1}\beta^2\sin^3\theta \cos\theta )</td>
<td>( 2\gamma^{-1}\beta^2\sin^3\theta \cos\theta )</td>
<td>( 4\gamma^{-1}\beta^2\sin\theta \cos\theta )</td>
</tr>
</tbody>
</table>

\[ G^{(1)}_{\Lambda N_1, R,R} + G^{(1)}_{\Lambda N_1, L,L} = G^{(1)}_{\Lambda N_1, X,X} + G^{(1)}_{\Lambda N_1, Y,Y} = G^{(1)}_{\Lambda N_1, U,P} \]
\[ G^{(1)}_{\Lambda N_1, R,L} + G^{(1)}_{\Lambda N_1, L,R} = G^{(1)}_{\Lambda N_1, Y,Y} - G^{(1)}_{\Lambda N_1, X,X} = G^{(1)}_{\Lambda N_1, L,P} \]

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming \( g+g \rightarrow t + \bar{t} \) in single plane CM
\( \gamma \) & \( \beta \) for top & antitop in CM.
\( \theta \) = top production angle in CM relative to \( (t+\bar{t}) \) momentum direction in lab
Taking \( X-Z \) plane for \( p+p \rightarrow (t+\bar{t}) \) in X gives \( \phi \) dependence to \( t+\bar{t} \) plane for opposite helicities: \( \text{Re}(e^{\pm(1\text{or}_2)i\phi} \cdot e^{\pm(-i(1\text{or}_2)\phi)}) \)
leading to \( \cos2\phi \) for UP,LP and LP,UP and \( \cos4\phi \) modulations
for LP,LP.