Probing Nucleon Spin Structure Using Deep Inelastic Scattering

E12-06-121: Neutron $g_2$ and $d_2$

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Deep Inelastic Scattering

Unpolarized cross section:

\[
\frac{d^2 \sigma}{d \Omega dE'} = \frac{\alpha^2}{4 E^2 \sin^4 \frac{\theta}{2}} \left( \frac{2}{M} F_1(x, Q^2) \sin^2 \theta + \frac{1}{v} F_2(x, Q^2) \cos^2 \frac{\theta}{2} \right)
\]

- Unpolarized structure functions \(F_1\) and \(F_2\) contain information about the momentum structure of the target nucleon.

Polarized cross section:

\[
\frac{d^2 \sigma}{dE'd\Omega} (\downarrow \uparrow \rightarrow \uparrow \rightarrow) = \frac{4 \alpha^2 E'}{M Q^2 \sqrt{v} E} \left[ (E+E' \cos \theta) g_1(x, Q^2) - \frac{Q^2}{v} g_2(x, Q^2) \right] = \Delta \sigma_{||}
\]

\[
\frac{d^2 \sigma}{dE'd\Omega} (\downarrow \rightarrow \rightarrow \rightarrow) = \frac{4 \alpha^2 \sin \theta E'^2}{M Q^2 \sqrt{v} E} \left[ v g_1(x, Q^2) + 2 E g_2(x, Q^2) \right] = \Delta \sigma_{\perp}
\]

- Polarized structure functions \(g_1\) and \(g_2\) encode information about the spin structure of the target nucleon.

\(Q^2 = 4\text{-momentum transfer squared of the virtual photon}\)

\(v = E - E' = \text{energy transfer}\)

\(\theta = \text{scattering angle}\)

\(x = \text{Fraction of nucleon momentum carried by the struck quark}\)
\( g_2 \) and Quark-Gluon Correlations

- In naive quark parton model, nucleon is viewed as a collection of non interacting, point like constituents.

- \( g_2 \) has no interpretation in naive quark parton model, provides information on quark-gluon correlation.

- \( g_2 \) is among the cleanest higher twist observables – contributes to leading order (twist-2 is leading twist) at the transverse spin asymmetry.

\[
g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)
\]

- Twist-2 term (Wandzura & Wilczek).

\[
g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} \, dy
\]

- Twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston).

\[
\bar{g}_2(x, Q^2) = -\int \frac{\partial}{\partial y} \left( \frac{m_q}{M} h_T(y, Q^2) - \xi(y, Q^2) \right) \frac{dy}{y}
\]
**d_2: Clean Probe of Quark-Gluon Correlations**

- **d_2** is a clean probe of quark-gluon correlations / higher twist effects - third moment of the linear combination of the spin structure function.

\[
d_2(Q^2) = 3 \int_0^1 x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] dx = 3 \int_0^1 x^2 \bar{g}_2(x, Q^2) dx
\]

- Related to matrix element in OPE, which represents average color Lorentz force on the struck quark due to the remnant di-quark system and it is cleanly computable using Lattice QCD.

- Connected to “color polarizability”.

\[
\chi_E = \frac{(4d_2 + 2f_2)}{3} \quad \chi_B = \frac{(4d_2 - f_2)}{3}
\]

- \( f_2 \) is a twist-4 contribution can be extracted from the first moment of \( g_1 \).

\[
\Gamma_1 = \int_0^1 g_1 dx = \mu_2 + \frac{M^2}{9Q^2} (a_2 + 4d_2 + 4f_2) + O\left(\frac{\mu^6}{Q^4}\right)
\]

Response of the color \( \mathbf{B} \) and \( \mathbf{E} \) field to the nucleon polarization.
Hint of a negative $d_2^p$, negative twist-3 at moderate $Q^2 \sim 3 \text{ GeV}^2$.

Armstrong et al., PRL 122, 022002 (2019)

Similar hint of negative twist-3 (dips below CN elastic) in $d_2^p$ data was noted in SANE experiment.

Posik et al., 10.1103/PhysRevLett.113.022002  ($d_2^n$, color force extraction)
Flay et al., 10.1103/PhysRevD.94.05200 (Archival paper: $g_1^n$, $g_2^n$, $d_2^n$)
Parno, et al., 10.1016/j.physletb.2015.03.067  ($A_1^n$)
• x and $Q^2$ evolution of $g_2$ in the wide kinematic range ($0.23 < x < 0.85$) will give us knowledge about $g_2$ at higher $x$.

• Doubles number of precision data points for $g_2^n(x,Q^2)$ in DIS region.

• $d_2$ will be measured for the constant $Q^2 = 3,4,5,6 (GeV/c)^2$ for the very first time.
Precision $g_2^n$ data set over broad range of $x$ and $Q^2$.
Points are vertically offset from zero along lines that reflect different (roughly) constant $Q^2$ values from 2.5—7 GeV$^2$.

Projected results for $d_2^n$ at truly constant $Q^2 = 3, 4, 5, 6$ GeV$^2$/c$^2$.
• Hall C: Polarized $^3$He target, SHMS + HMS

• Beam energies:
  11 GeV (production), 2.2 GeV (calib.).

• Beam currents:
  30 μA (production), 40 μA (max., calib.).

• Each arm measures an absolute polarized cross section independent of the other arm ($g_1$, $g_2$).

<table>
<thead>
<tr>
<th>SHMS Production</th>
<th>HMS Production</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Setting</strong></td>
<td><strong>Setting</strong></td>
</tr>
<tr>
<td>A</td>
<td>A'</td>
</tr>
<tr>
<td>B</td>
<td>B'</td>
</tr>
<tr>
<td>C</td>
<td>C'</td>
</tr>
<tr>
<td>D</td>
<td>D'</td>
</tr>
</tbody>
</table>

• **SHMS** collects data at $\theta = 11°, 13.3°, 15.5°$ and $18.0°$ for 125 hrs each.

• **HMS** collects data at $\theta = 13.5°, 16.4°, 20.0°$ and $*25.0°$ for 125 hrs each.
E12-06-121: Run Plan

Nominal beam time allocation:

PAC 36 approved E12-06-121 for requested 700 PAC hours (29 PAC days)

- 5-pass beam (nominal 11.0 GeV/c) for ~ 676 PAC hours.
- 1-pass beam (nominal 2.2 GeV/c) for ~ 20 PAC hours + pass change → 5-pass.

1-pass running (calibration):

1-pass beam allocation: 3 calendar days

Nominal to do list:

- 8 hr Moller run
- 4 hr Optics at $p_0 = 2.2$ GeV/c
- Pressure curves for current cell
- Hydrogen elastics, delta QE meas
- $^3$He elastic data (E12-06-121A)
  (See Table)

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ [GeV]</th>
<th>$\theta$ [°]</th>
<th>$Q^2$ [fm$^{-2}$]</th>
<th>Estimated Cross Section [mb/sr]</th>
<th>Rate [Hz]</th>
<th>Time [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHMS</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2.216</td>
<td>k1</td>
<td>11</td>
<td>$4.39 \times 10^{-4}$</td>
<td>723.69</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>k2</td>
<td>13</td>
<td>$5.14 \times 10^{-5}$</td>
<td>84.89</td>
<td>1</td>
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<tr>
<td></td>
<td>k3</td>
<td>15</td>
<td>$4.37 \times 10^{-6}$</td>
<td>7.21</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>k4</td>
<td>17</td>
<td>$2.22 \times 10^{-7}$</td>
<td>0.37</td>
<td>10</td>
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<tr>
<td></td>
<td>k5</td>
<td>19</td>
<td>$5.97 \times 10^{-8}$</td>
<td>0.10</td>
<td>11</td>
</tr>
<tr>
<td>HMS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.216</td>
<td>k6</td>
<td>21</td>
<td>$3.99 \times 10^{-8}$</td>
<td>0.12</td>
<td>24</td>
</tr>
</tbody>
</table>

Projection from E12-06-121A
E12-06-121: Run Plan

5-pass running (Production):

5-pass beam allocation: 54 calendar days (162 shifts)

For each kinematic pair \((X, X')\)

- Reference cell runs: \(^3\)He, \(N_2\)
- Empty cell run
- 8 hrs Optics (C-foil + Sieve)
- Positive polarity runs: 4 hrs optics, 4 hrs production
- Target NMR sweep (1–2 / shift)
- Production runs (~31 shifts)

Instrumentation / Calibration runs

- BPM calibration (2 hour)
- BCM calibration (2 hour)
- Beam energy (2 hour)

Summing Up:

Total 160 shifts (~40 shifts/setting)
- production + optics + pos. pol. Running - 35 shifts/setting
- Moller Runs (1/week) - 2 shifts/setting
- Allow ~10% overhead = ~3-4 shifts/setting

2 shifts for instrumentation and calibration runs.
E12-06-121A: Measurement of $^3$He Elastic Electromagnetic Form Factors

- Significant discrepancies between theoretical and experimental $^3$He FFs (particularly $G_M$).

- All higher $Q^2$ data are from unpolarized electron scattering results.
  - Rosenbluth separations are impossible in diffractive minima and global fits require FF parametrizations.

- **Double polarization asymmetry:**
  - Zeros of asymmetry are FF diffractive minima.
  - Constrain minima locations.
  - Hypothesis test theoretical models.

New independent tool to map FFs without the issues of unpolarized Rosenbluth measurements!
The experiment E12-06-121 (neutron $g_2$ and $d_2$) will run in 2020 right after E12-06-110 in Hall C.

High $x$ and $Q^2$ evolution of $g_2$ and $d_2$ will be explored (large precision data).

It will be the first evaluation of $d_2^n$ at truly constant $Q^2$ values.

This will give insight into quark-gluon correlations.

Several theoretical predictions (especially Lattice QCD) will be verified.
Supporting Documentations

- **Proposals**
  - [https://hallcweb.jlab.org/wiki/images/1/1a/D2n_HallC_PAC36-update_v2.pdf](https://hallcweb.jlab.org/wiki/images/1/1a/D2n_HallC_PAC36-update_v2.pdf)

- **Polarized $^3$He Target**
  - [https://hallcweb.jlab.org/wiki/index.php/Pol_He-3_Target Information](https://hallcweb.jlab.org/wiki/index.php/Pol_He-3_Target Information)
  - [https://www.jlab.org/indico/event/351/session/1/contribution/9/material/slides/0.pdf](https://www.jlab.org/indico/event/351/session/1/contribution/9/material/slides/0.pdf)

- **E06-014 (2009 $d_2^n$ experiment) wiki**
  - [https://hallaweb.jlab.org/wiki/index.php/Analysis_resources_for_d2n](https://hallaweb.jlab.org/wiki/index.php/Analysis_resources_for_d2n)
Back-up Slides
Twist Expansion

- Quark electromagnetic current in forward Compton amplitude,
  \[ T_{\mu\nu} = i \int d^4z \ e^{iz} < N \right| T \left( j_\mu(z)j_\nu(0) \right) \left| N \right> \]

- Operator product expansion (OPE) : 
  \[ j_\mu(z)j_\mu(0) = \sum C_{\mu_1...\mu_n} O_{d,n}^{\mu_1...\mu_n} \]
  \( O_{d,n}^{\mu_1...\mu_n} \) : Local quark gluon operators with mass dimension \( d \) and spin dimension \( n \)

- Dimension Analysis : 
  \[ C_{\mu_1...\mu_n} O_{d,n}^{\mu_1...\mu_n} \rightarrow \left( \frac{q_{\mu_1}}{Q} \right) ... \left( \frac{q_{\mu_n}}{Q} \right) Q^{2-d} M^{d-n-2} p^{\mu_1} ... p^{\mu_n} \]
  \[ \rightarrow \frac{p.q}{Q^n} Q^{2-d} M^{d-n-2} \]
  \[ \rightarrow \left( \frac{1}{x} \right)^n \left( \frac{Q}{M} \right)^{2+n-d} \]
  \[ \rightarrow \left( \frac{1}{x} \right)^n \left( \frac{Q}{M} \right)^{2-t} \]
  Twist, \( t = d-n \)
## Expected rates for HMS

| $\theta_0$ [°] | $E'_{cent}$ [GeV] | $Q^2$ [GeV$^2$] | $x$  | $W$ [GeV] | $e^-$ rate [Hz] | $\pi^-$ rate [Hz] | $t||$ [hrs] | $t\perp$ [hrs] | $\Delta A||$ [\cdot10^{-4}] | $\Delta A\perp$ [\cdot10^{-4}] |
|---------------|-------------------|-----------------|-----|----------|----------------|-----------------|------------|------------|-----------------|-----------------|
| 13.5          | 4.305             | 2.617           | 0.208 | 3.293    | 954            | 765             | 8          | 117        | 2.0             | 0.6             |
| 16.4          | 5.088             | 4.555           | 0.410 | 2.727    | 218            | 15              | 12         | 113        | 3.9             | 1.2             |
| 20.0          | 4.000             | 5.31            | 0.404 | 2.951    | 76             | 66              | 10         | 115        | 6.0             | 1.8             |
| 25.0          | 2.500             | 5.15            | 0.323 | 3.417    | 20             | 84              | 13         | 112        | 10.7            | 3.1             |

- The rate table is taken from PAC-30 proposal.
- The uncertainties for $A_\parallel$ and $A_\perp$ are statistical only.
## Kinematic bins and expected rates for SHMS

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>$\theta_0 = 11^\circ$</td>
<td>7.112</td>
<td>2.875</td>
<td>0.394</td>
<td>2.305</td>
<td>1058</td>
<td>11</td>
<td>12</td>
<td>113</td>
<td>2.0</td>
<td>0.5</td>
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<tr>
<td>$\quad E'_{cent} = 7.5$ GeV</td>
<td>7.709</td>
<td>3.116</td>
<td>0.504</td>
<td>1.988</td>
<td>708</td>
<td>3.1</td>
<td>12</td>
<td>113</td>
<td>2.3</td>
<td>0.7</td>
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<tr>
<td>$\theta_0 = 13.3^\circ$</td>
<td>6.647</td>
<td>3.922</td>
<td>0.480</td>
<td>2.267</td>
<td>268</td>
<td>3.1</td>
<td>12</td>
<td>113</td>
<td>3.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$\quad E'_{cent} = 7.0$ GeV</td>
<td>7.203</td>
<td>4.250</td>
<td>0.596</td>
<td>1.941</td>
<td>139</td>
<td>0.8</td>
<td>12</td>
<td>113</td>
<td>4.8</td>
<td>1.5</td>
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<tr>
<td>$\theta_0 = 15.5^\circ$</td>
<td>5.997</td>
<td>4.798</td>
<td>0.511</td>
<td>2.342</td>
<td>96</td>
<td>1.9</td>
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<td>113</td>
<td>5.7</td>
<td>1.8</td>
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<td>$\quad E'_{cent} = 6.3$ GeV</td>
<td>6.496</td>
<td>5.197</td>
<td>0.614</td>
<td>2.037</td>
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<td>0.47</td>
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<td>113</td>
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<td>$\theta_0 = 18.0^\circ$</td>
<td>5.348</td>
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<td>0.542</td>
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<td>113</td>
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<td>3.1</td>
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<td>$\quad E'_{cent} = 5.6$ GeV</td>
<td>5.790</td>
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## Systematic Error Table

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<tr>
<th>Item description</th>
<th>Subitem description</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target polarization</td>
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<td>1.5 %</td>
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<tr>
<td>Beam polarization</td>
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<td>3 %</td>
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<tr>
<td>Asymmetry (raw)</td>
<td></td>
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<tr>
<td></td>
<td>Target spin direction (0.1°)</td>
<td>&lt; 5 × 10⁻⁴</td>
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<td></td>
<td>Beam charge asymmetry</td>
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<tr>
<td>Cross section (raw)</td>
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<td>PID efficiency</td>
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<td></td>
<td>Background rejection</td>
<td>≈ 1 %</td>
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<td></td>
<td>Beam charge</td>
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<td></td>
<td>Beam position</td>
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<td></td>
<td>Finite Acceptance cut</td>
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<td>Radiative corrections</td>
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<td>From $^3$He to Neutron correction</td>
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<td>5 %</td>
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<tr>
<td>Total systematic uncertainty</td>
<td></td>
<td>≤ 10 %</td>
</tr>
</tbody>
</table>

**Estimate of contributions to $d_2$ from unmeasured region**

\[
\int_{0.003}^{0.25} d_2 \, dx = 4.8 \times 10^{-4}
\]

**Projected absolute statistical uncertainty on $d_2$**

\[\Delta d_2 \approx 5 \times 10^{-4}\]

**Projected absolute systematic uncertainty on $d_2$**

(assuming $d_2 = 5 \times 10^{-3}$)

\[\Delta d_2 \approx 5 \times 10^{-4}\]
Neutron Asymmetries from $^3\text{He}$

\[ A_1^n = \frac{1}{p_n \, F_2^n \left( 1 + \frac{0.056}{p_n} \right)} \left( A_1^{3\text{He}} - 2P_p \left( 1 - \frac{0.014}{2P_p} \right) \frac{F_2^p}{F_2^{3\text{He}}} A_1^p \right) \]

\[ A_2^n = \frac{1}{p_n \, F_2^n \left( 1 + \frac{0.056}{p_n} \right)} \left( A_2^{3\text{He}} - 2P_p \left( 1 - \frac{0.014}{2P_p} \right) \frac{F_2^p}{F_2^{3\text{He}}} A_2^p \right) \]

\[ g_1^n = \frac{1}{p_n \, F_2^n \left( 1 + \frac{0.056}{p_n} \right)} \left( g_1^{3\text{He}} - 2P_p \left( 1 - \frac{0.014}{2P_p} \right) \frac{F_2^p}{F_2^{3\text{He}}} g_1^p \right) \]

\[ g_2^n = \frac{1}{p_n \, F_2^n \left( 1 + \frac{0.056}{p_n} \right)} \left( g_2^{3\text{He}} - 2P_p \left( 1 - \frac{0.014}{2P_p} \right) \frac{F_2^p}{F_2^{3\text{He}}} g_2^p \right) \]

$P_p, P_n$ : Effective proton and neutron polarizations in $^3\text{He}$
E12-06-121A: Measurement of $^3$He Elastic Electromagnetic Form Factors

- Significant discrepancies between theoretical and experimental $^3$He FFs (particularly $G_M$).

- All higher $Q^2$ data are from unpolarized electron scattering results.

$$
\left( \frac{d \sigma}{d \Omega} \right)_{\text{exp}} = \left( \frac{d \sigma}{d \Omega} \right)_{\text{Mott}} \frac{1}{1+\tau} \left[ G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right]
$$

with $\epsilon = (1+2(1+\tau)\tan^2(\frac{\theta}{2}))^{-1}$ and $\tau = \frac{Q^2}{4M^2}$

- **Double polarization asymmetry:**

$$
A_{\text{phys}} = \frac{-2\sqrt{\tau(1+\tau)}\tan(\frac{\theta}{2})}{G_E^2 + \frac{\tau}{\epsilon} G_M^2} \left[ \sin(\theta')\cos(\varphi') G_E G_M + \sqrt{\tau} \left[ 1 + (1+\tau)\tan^2(\frac{\theta}{2}) \right] \cos(\theta') G_M^2 \right]
$$

New independent tool to map FFs without the issues of unpolarized Rosenbluth measurements!
E12-06-121A: Proposed Procedure

Take data during $d_2^n$ 1-pass (~24 PAC hours)

- Polarized $^3$He target (polarization > 50 %)

- HMS:
  - Positioned at single angle centered on the anticipated FF diffractive minima for the entirety of the run.

- SHMS:
  - Start at small angles and step up in $Q^2$ passing through the $G_E$ minimum and approaching just below $G_M$'s.
  - Constrains the minima locations while mapping the asymmetry.
$^3$He Charge Form Factor

![Graph showing $|F_{ch}(Q^2)|$ vs. $Q^2$ (fm$^{-2}$)]

- Representative Fit Barcus 2019
- Uncertainty Band Barcus 2019
- Representative Fit Amroun et al. 1994
- Uncertainty Band Amroun et al. 1994
- CST Marcucci et al 2016
- $\chi$EFT 500 Marcucci et al 2016
- $\chi$EFT 600 Marcucci et al 2016
$^3$He Magnetic Form Factor