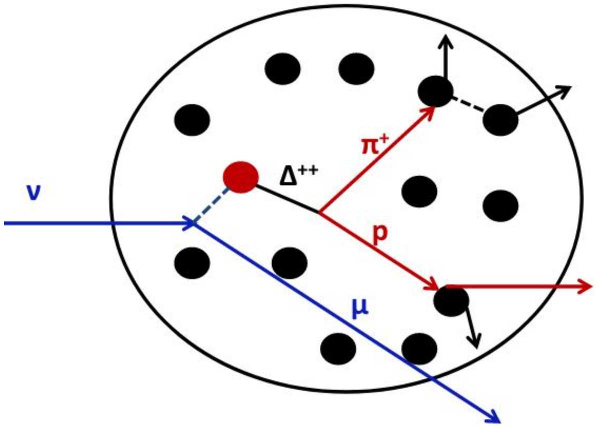
Study on FSI in electron scattering

Libo Jiang

Outline

- Motivations
- Energy dependence of neutrino oscillation
- Sources and Effect of Final State Interaction (FSI)
- Distorted - Wave Impulse Approximation (DWIA) & Spectral Function
- Remind of kinematic settings
- Comparison between theoretical predictions and experimental data
- Conclusion and Summary

Motivations



FSI: Particles can interact with nucleons before exiting the nucleus

Study on Final State Interactions(FSI)

- Understand nuclear response to electroweak interactions
- Study nuclear effect/structure through electron scattering
- All analysis result of FSI will be used in neutrino experiment
  - FSI determines the final particle states
  - Neutrino energy reconstructed from Final state particle

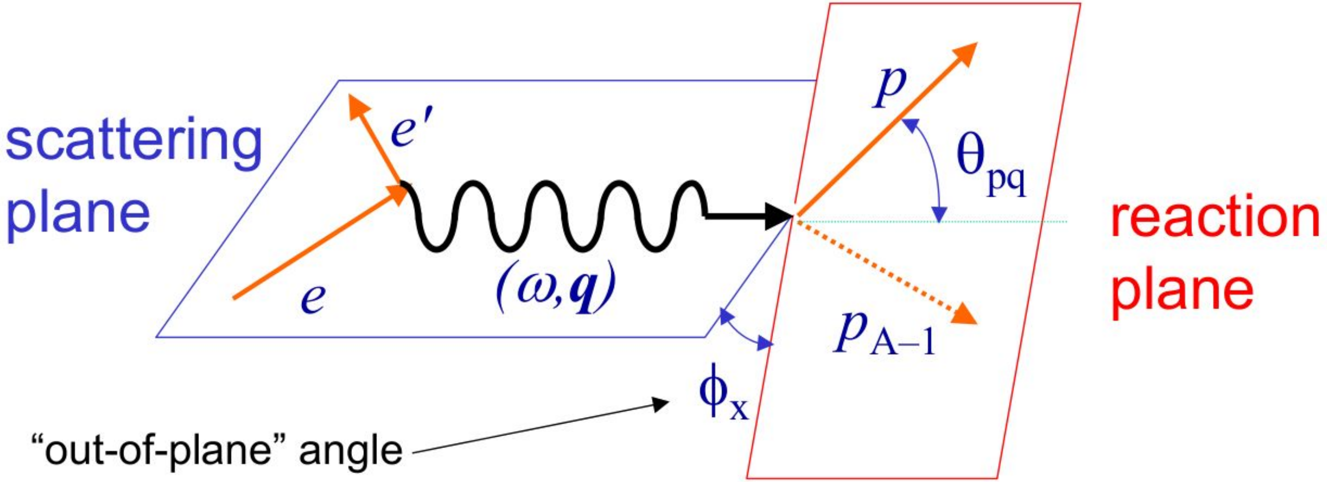
Motivations

purpose of experiment E12-14-012:

- measure the nuclear spectral function
- providing the distribution of nucleon momenta and energies
    - The knowledge of this distribution is needed to reconstruct the neutrino energy
    - In this context FSI are noise, to be removed from the measured cross section to extract the relevant information.
    - while in general FSI can give rise to all the complicated processes whose treatment would involves severe difficulties, in (e,e’p) at low missing energy they can be described in terms of interactions of the knocked out nucleon with a complex optical potential.

Kinematics in Electron Nucleus Scattering

- Kinematics of the **(e, e'p)** reaction
- Electron - nucleus scattering process in the final state maybe more complicated



- e and A are known
- Detect: e' and p
- Missing Momentum ***p***m = **q** - **p**
- Missing Energy *E*m = ω - Tp - TA-1

Energy Dependence of Neutrino Oscillation

- The oscillation probability after traveling a distance L

$$P_{\alpha\to\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

- Two flavor neutrino oscillation
- Mixing angle θ and mass split Δm^2 need to be determined

- Signature of neutrino oscillation
  - Distortion of neutrino energy spectrum in the far detector compared to energy spectrum in the near detector
  - Neutrino Energy reconstructed from final state particles, for example in quasi elastic scattering

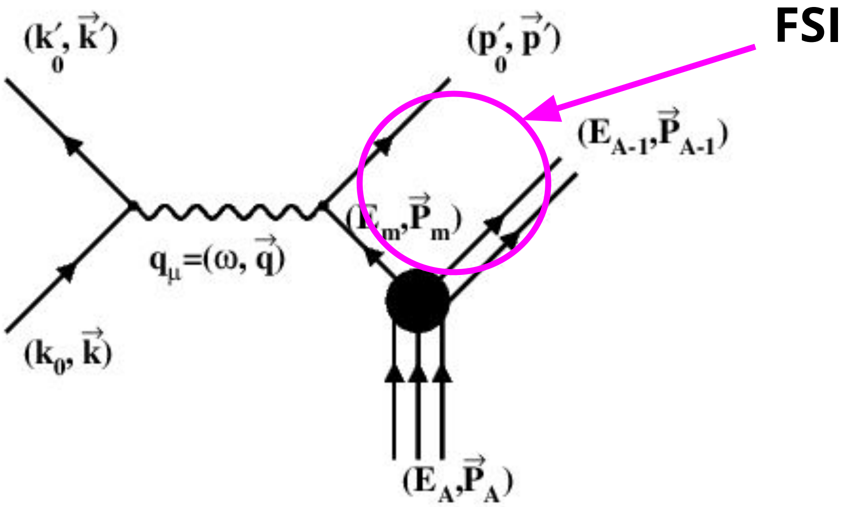
$$E_\nu = \frac{m_p^2 - m_\mu^2 - E_n{}^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |\mathbf{p}_n{}^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu|\cos\theta_\mu - |\mathbf{p}_n|\cos\theta_n)}$$

E_ν in calculated neutrino energy in quasi-elastic scattering

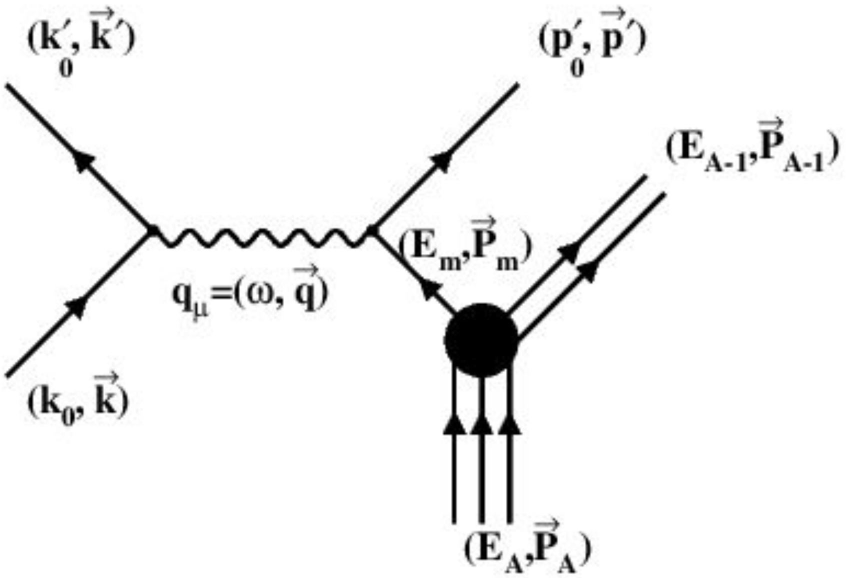
E_n and p_n are distributed according to the spectral function, which is an intrinsic property of the initial state. Understanding FSI is needed to extract the spectral function from the measured cross section.

Source of Final State Interaction (FSI)

- Re-interaction of the emitted proton with the remaining nucleus
- Others ...



Effect of Final State Interactions



Without FSI

$$\mathbf{p} - \mathbf{q} = -\mathbf{p}_{A-1} = \mathbf{p}_0$$

With FSI

$$\mathbf{p} - \mathbf{q} = -\mathbf{p}_{A-1} \neq \mathbf{p}_0$$

Distorted - Wave Impulse Approximation (DWIA)

Non-relativistic PWIA:

$$\frac{d^6\sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S(p_m, \varepsilon_m)$$

σ_{eN} is the half off-shell electron nucleon cross section which is related or can be expressed as a function of σ_M

nuclear spectral function

Suitable Kinematic Factor K= $E_p|\mathbf{p}_p|$

Distorted - Wave Impulse Approximation (DWIA)

FSI not negligible and destroy the simple factorization in last slide

$$\frac{d^6\sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S^D(p_m, \varepsilon_m)$$

σ_{eN} is the half off-shell electron nucleon cross section which is related or can be expressed as a function of σ_M

Distorted

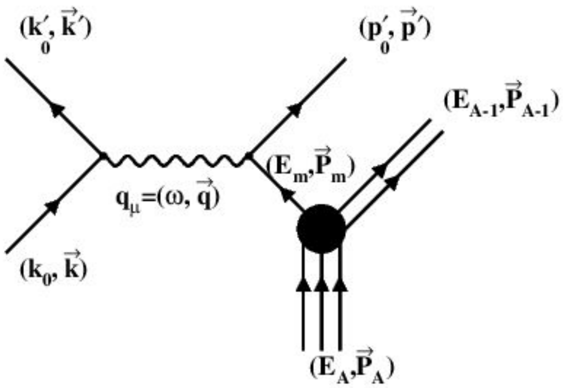
nuclear spectral function

Suitable Kinematic Factor $K = E_p|\mathbf{p}_p|$

Distorted - Wave Impulse Approximation (DWIA)

Theoretical assumption in DWIA mode

- In the final state the residual nucleus, which is composed of A-1 nucleons, is left in the bound state $|\Psi_\alpha^B(E)>$, characterized by energy E and quantum number α
- The direct knockout mechanism where the one-body nuclear electromagnetic current operator act exclusively on the space spanned by this type of many-body stage



$$J^\mu(\boldsymbol{q}) = \int \langle \Psi_f | a^\dagger(\boldsymbol{r}) | \Psi_\alpha^B(E) \rangle \, \hat{j}^\mu(\boldsymbol{r}) \times \langle \Psi_\alpha^B(E) | a(\boldsymbol{r}) | \Psi_i \rangle \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3r$$

$$= \int \chi_{E\alpha}^{(-)*}(\boldsymbol{r}) \, \hat{j}^\mu(\boldsymbol{r}) \, \phi_{E\alpha}(\boldsymbol{r}) [S_\alpha(E)]^{1/2} \times e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3r,$$

Distorted - Wave Impulse Approximation (DWIA)

Electromagnetic Current

$$J^\mu(\boldsymbol{q}) = \int \langle \Psi_f | a^\dagger(\boldsymbol{r}) | \Psi_\alpha^B(E) \rangle \hat{j}^\mu(\boldsymbol{r}) \times \langle \Psi_\alpha^B(E) | a(\boldsymbol{r}) | \Psi_i \rangle e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3r$$

$$= \int \chi_{E\alpha}^{(-)*}(\boldsymbol{r}) \, \hat{j}^\mu(\boldsymbol{r}) \, \phi_{E\alpha}(\boldsymbol{r}) [S_\alpha(E)]^{1/2} \times e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3r,$$

- a(**r**): the operator which annihilates a nucleon with coordinate **r**

$$\langle \Psi_\alpha^B(E) | a(\boldsymbol{r}) | \Psi_f \rangle = \chi_{E\alpha}^{(-)}(\boldsymbol{r}).$$

- $\chi_{E\alpha}^{(-)}$: the distorted wave function of the emitted nucleon
- $\langle \Psi_\alpha^B(E) | a(\boldsymbol{r}) | \Psi_i \rangle$: the overlap function describes the residual nucleus as a hole state in the target

$$\langle \Psi_\alpha^B(E) | a(\boldsymbol{r}) | \Psi_i \rangle = [S_\alpha(E)]^{1/2} \phi_{E\alpha}(\boldsymbol{r}).$$

Distorted - Wave Impulse Approximation (DWIA)

Electromagnetic Current

$$J^{\mu}(\boldsymbol{q}) = \int \langle \Psi_{\mathrm{f}} | a^{\dagger}(\boldsymbol{r}) | \Psi_{\alpha}^{\mathrm{B}}(E) \rangle \, \hat{j}^{\mu}(\boldsymbol{r}) \times \langle \Psi_{\alpha}^{\mathrm{B}}(E) | a(\boldsymbol{r}) | \Psi_{\mathrm{i}} \rangle \, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3 r$$

$$= \int \chi_{E\alpha}^{(-)*}(\boldsymbol{r}) \, \hat{j}^{\mu}(\boldsymbol{r}) \, \phi_{E\alpha}(\boldsymbol{r}) [S_{\alpha}(E)]^{1/2} \times \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3 r,$$

- $S_{\alpha}(E)$: spectroscopic factor, which is the norm of the overlap function and gives the probability of removing from the target a nucleon at **r**.

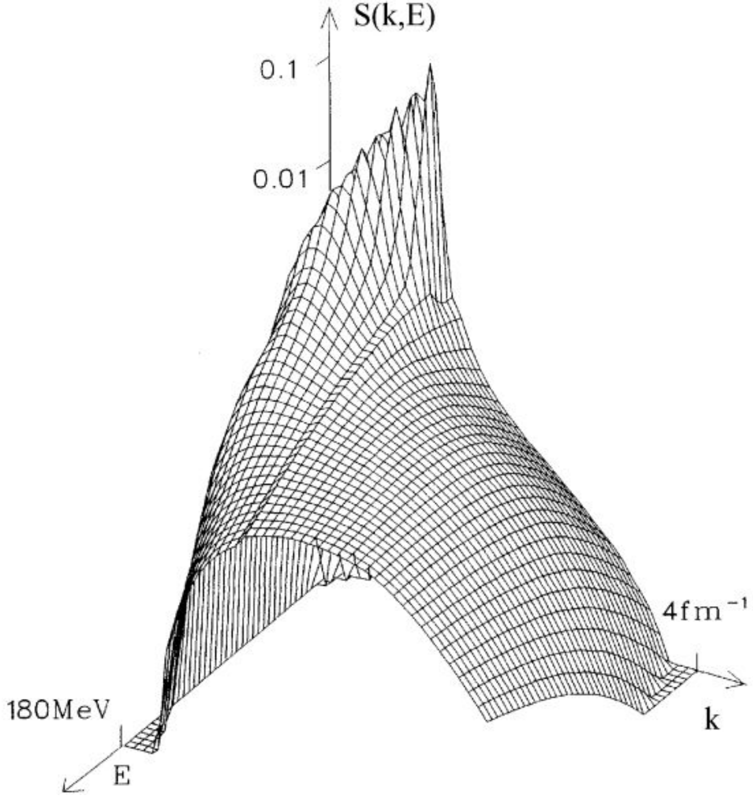
- Overlap function/spectral function contains the effect of **Mean field** and **Correlations**

- In Hartree- Fock frame: $\Phi_{E\alpha}(\mathbf{p})[S_{\alpha}(E)]^{1/2} = \langle E\alpha | a(\mathbf{p}) | \Psi_i \rangle$

Spectral Function

- Spectral function describes the probability of removing a nucleon of momentum k from the nuclear ground state, leaving the residual nucleus with excitation energy
- Plot on the right shows the distribution of the spectral function as a function of E and k
- Mean field and correlations should be considered in the calculation of spectral function(*Källén-Lehmann*)

$$P(\mathbf{k}, E) = \sum_{\alpha \in \{F\}} Z_\alpha |\phi_\alpha(\mathbf{k})|^2 F_\alpha(E - E_\alpha) + P_{\mathrm{corr}}(\mathbf{k}, E)$$



Distorted - Wave Impulse Approximation (DWIA)

For a specific value of energy E, in Hartree-Fock frame:

- the residual nucleus is in an eigenstate |Ea> of its hamiltonian characterized by quantum numbers a

$$|\Psi_f\rangle = \sum_{\nu} \chi^{(-)}_{Ea,\nu} a^{\dagger}_{\nu} |Ea\rangle$$

The index labels the natural orbitals of |Ea> corresponding to an occupation number n

- Distorted wave function

$$\chi^{(-)}_{Ea,\nu} = \frac{1}{1-n_{\nu}} \langle Ea | a_{\nu} | \Psi_f \rangle$$

: Is the eigen function of a effective hamiltonian

In **PWIA**(plane wave impulse approximation), FSI are neglected, and the wave function of knocked proton reduces to plane wave

Distorted - Wave Impulse Approximation (DWIA)

Effect of Optical Potential

- The real part
  - Shift the cross section
- The imaginary part
  - Gives a reduction by a factor between 0.5 and 0.7
- The spin-orbit component
  - Produces and asymmetry of the response for different

Choice of Optical Potential

- Wood-Saxon
- Hartree-Fock
- Relativistic

Remind of Kinematic Settings (Argon)

	E_e	$E_{e'}$	θ_e	P_p	θ_p	$\|\mathbf{q}\|$	p_m
	MeV	MeV	deg	MeV/c	deg	MeV/c	MeV/c
kin1	2222	1799	21.5	915	−50.0	857.5	57.7
kin3	2222	1799	17.5	915	−47.0	740.9	174.1
kin4	2222	1799	15.5	915	−44.5	658.5	229.7
kin5	2222	1716	15.5	1030	−39.0	730.3	299.7
kin2	2222	1716	20.0	1030	−44.0	846.1	183.9
Inc-kin5	2222	-	15.5	-	-		

Theoretical Predictions of DWIA (Oxygen)

Input file of DWIA (fortran code provided by Carlotta Giusti)

```
16      8       15      100     1
1.3     0.80    0.0     1000.0
20.0    -300.0  350.0   500.0   1.000   0.85
1       1       1       1       1       1       1       -1      -1
0       1       1       0       0       2       1       1       1
1       1       1       1       0.5     12.127  1.
1.37     0.65    4.646   0.65    25.     1.3     0.85
28.356  1.215   0.703   6.877   1.434   0.545   0.00    1.41    0.570
4.808   0.997   0.66    -0.870  0.968   0.620   0.000   0.
520.6   418.1   90.     78.     -40.8
   1 # O16 (e,e'P) P1/2
```

(A,Z) = (16, 8) -> A-1 = 15

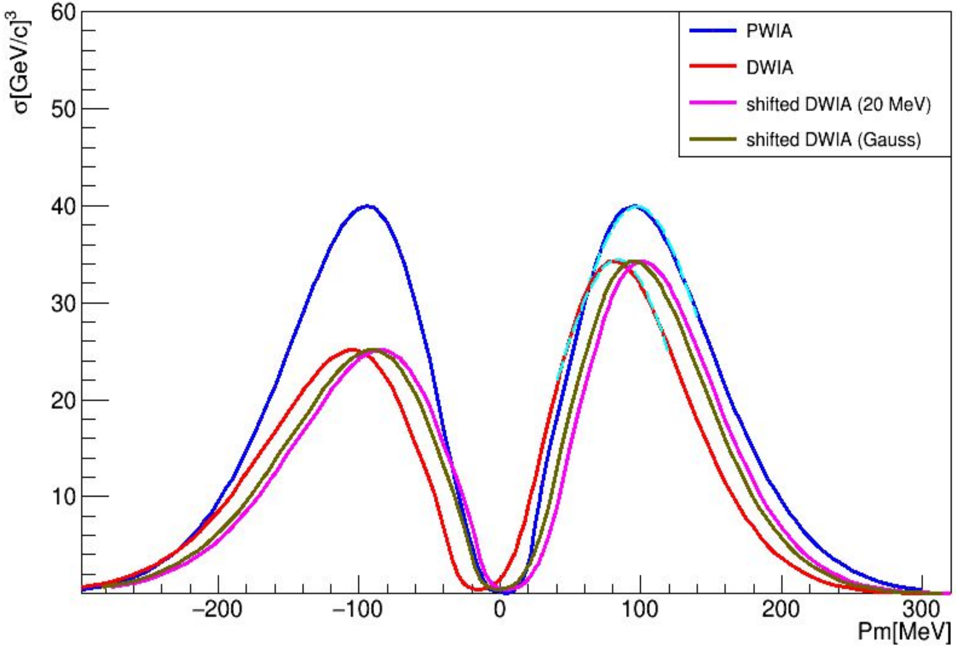
Optical Potential : Wood Saxon; Bound state wave function: Wood Saxon

Beam Energy = 520.6 MeV; Outgoing electron energy = 418.1 MeV;

Kinetic Energy of proton after knock-out from target = 90 MeV;

Electron Scattering angle = 78 degree; Proton Scattering angle = -40.8 degree;

Theoretical Predictions (DWIA vs PWIA)

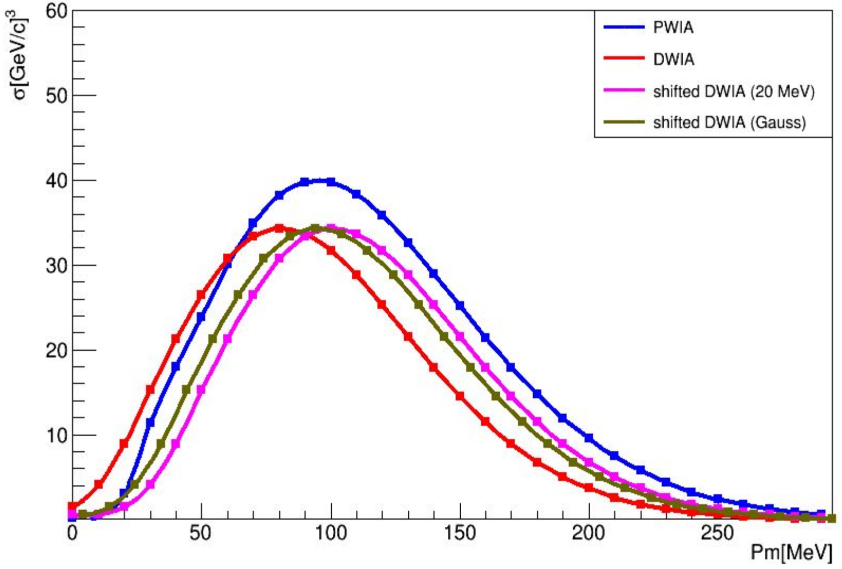


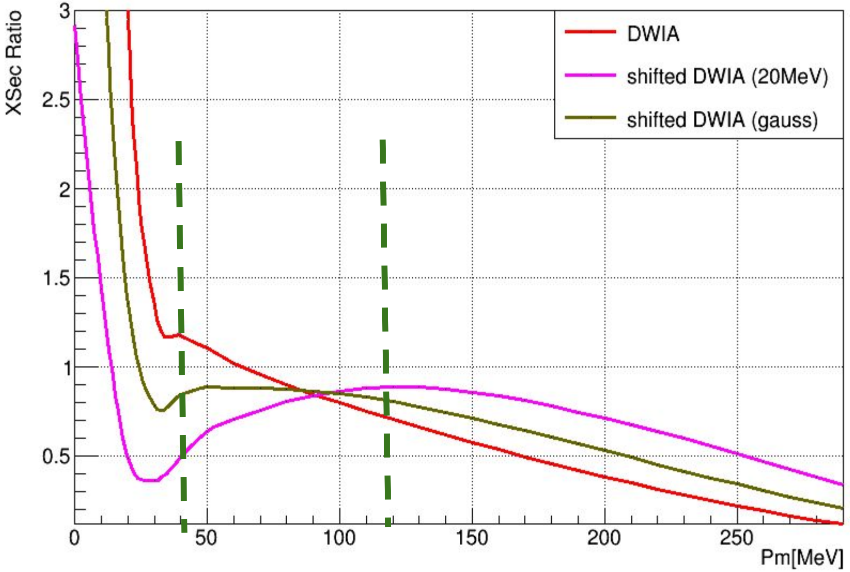
PWIA is symmetric.

Two ways for shifting

- Shifting according to the minimum value
- Shifting according to the mean value of Gaussian fit
  - Cyan curve is the result of gaussian fit

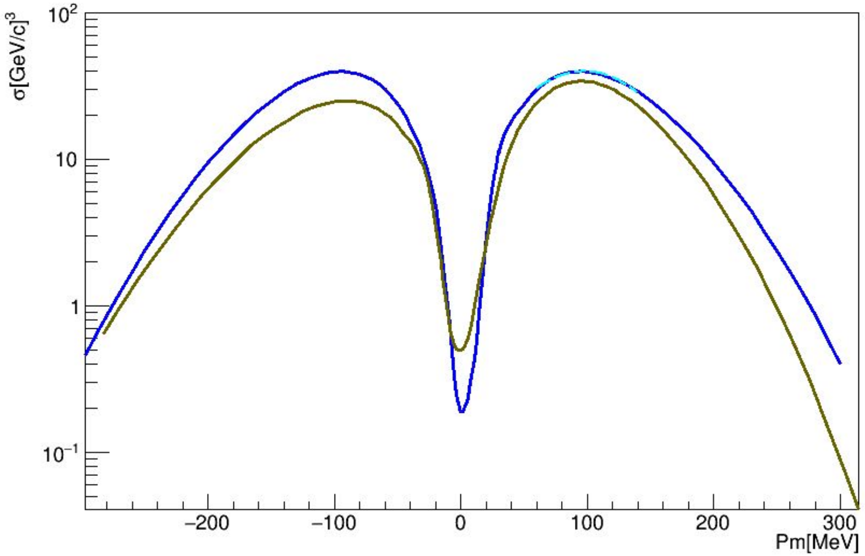
Compared to Experimental Data

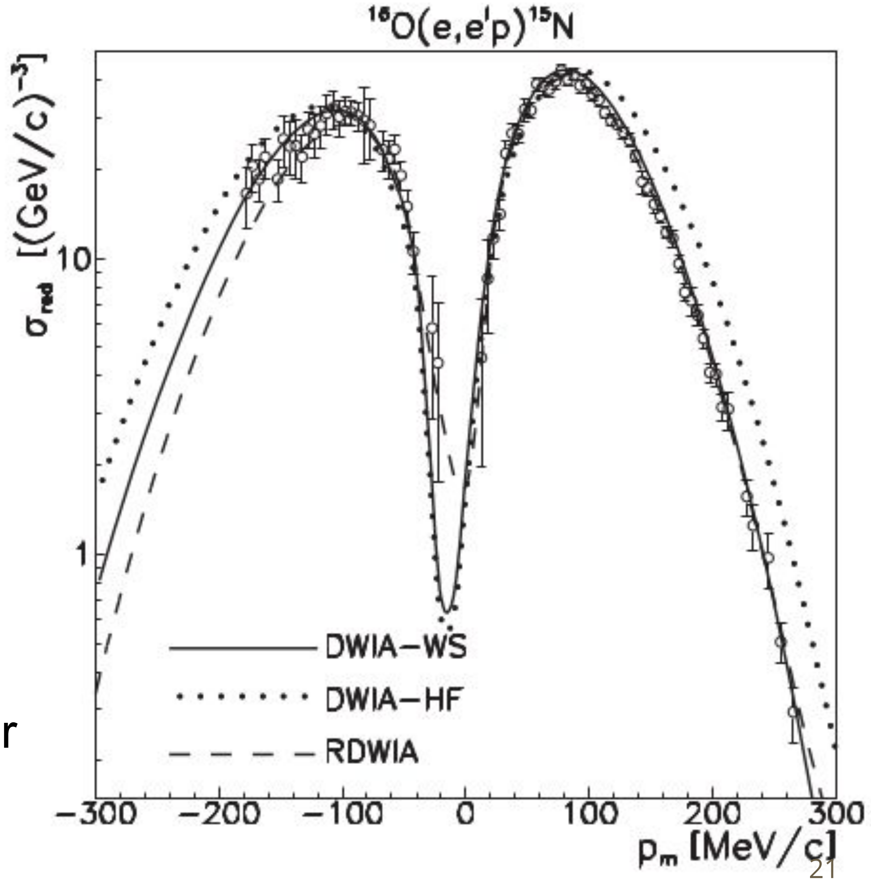




- Left: Xsec in positive missing momentum region; Right: Ratio between DWIA/shifted DWIA and PWIA
- Region between two dark green dashed line is flat

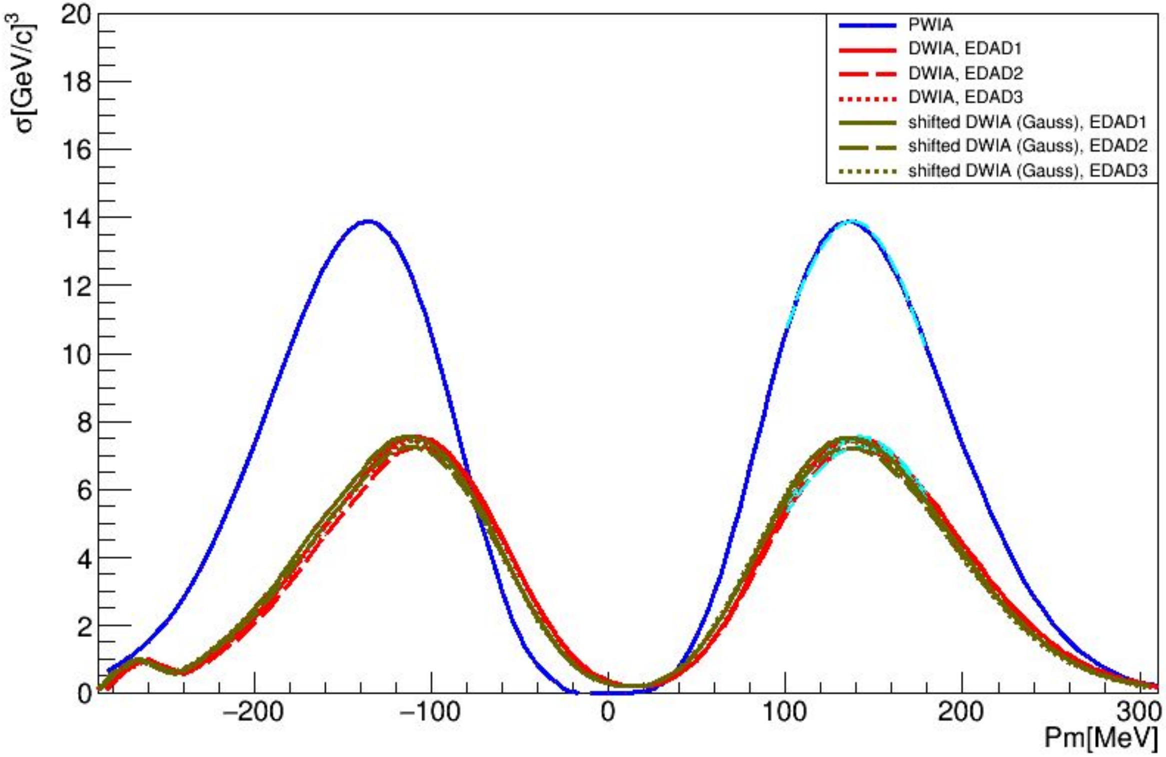
Compared to Experimental Data





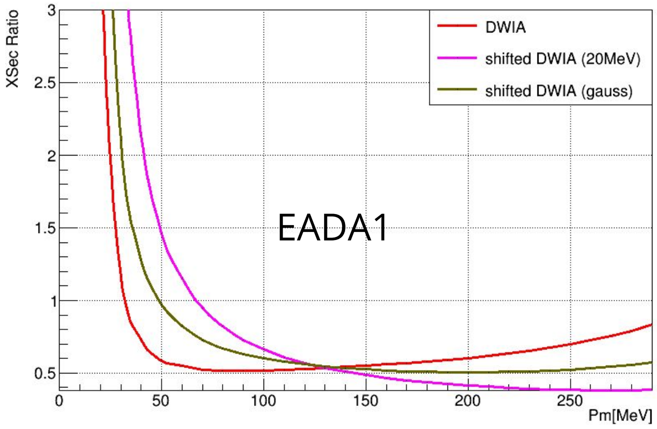
- **Blue Curve:** PWIA; **Dark green curve:** shifted DWIA
- Experimental data come from Leuschner et al PRC 49, 955 (1994)
- Plot is from **PRC 84, 024615 (2011)**

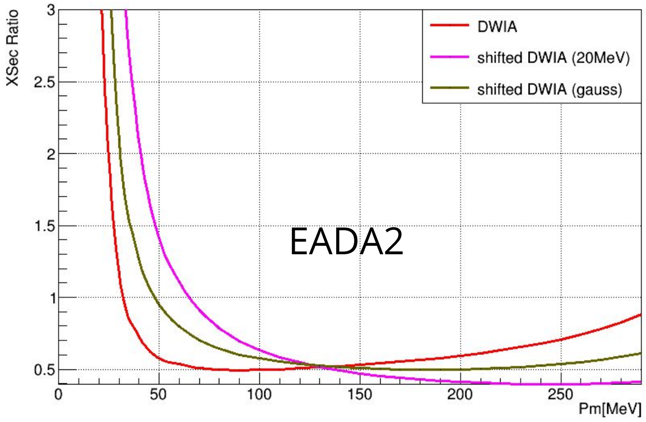
Cross Section of DWIA vs PWIA - Argon

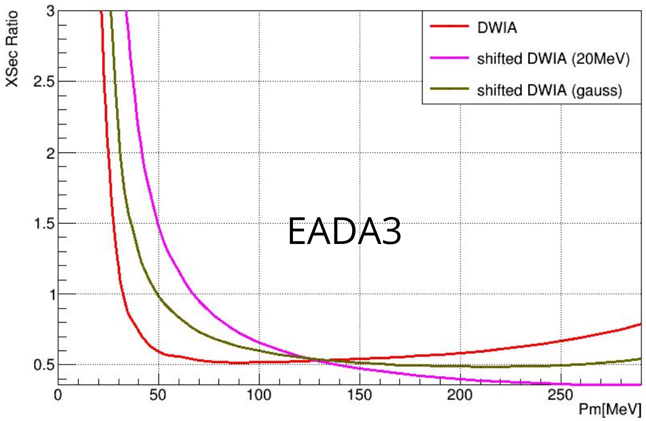


- **Proton Energy:** 210 MeV
- **EDAD**1. **EDAD**2. **EDAD**3 mean 3 different fits to optical potential
- Ref: E. D. Cooper, S. Hmma, B. C . Clark, and R. L. Mercer, PRC 47, 297 (1993)

Cross Section of DWIA vs PWIA - Argon







	EADA1	EADA2	EADA3
Ratio	0.567	0.552	0.558

Conclusion and Summary

- Final State Interaction (FSI) is not included in Current Monte Carlo(MC)
- Distorted - wave impulse approximation (DWIA) provides the cross sections with FSI considered.
- FSI can be implemented by including the ration and shift and reweighting current MC cross sections with the factor (ratio between DWIA theoretical predictions and PWIA prediction)

Backup information: another source of FSI

- The distortion of the electron wave functions produced by the nuclear coulomb field
  - Increase the momentum transfer q and increase in the electron flux in the vicinity of target nucleus